61 HW2 1/23/2023

1. for every $a,b,c \in \mathbb{Z}$, prove that if $a|b$ and $a|c$, then $a|(b+c)$

suppose $a|b$ and $a|c$

then $\exists\ x,y \in \mathbb{Z}$ st $a \cdot x = b$ and $a \cdot y = c$

then $b+c = (a \cdot x) + (a \cdot y) = a(x+y)$, and $a|(b+c)$ since $(x+y) \in \mathbb{Z}$

2. 2.1 ex 11

prove that $\forall\ m,n \in \mathbb{Z}$, if m odd and n even, mn is even

suppose m odd and n even

then $\exists\ x,y \in \mathbb{Z}$ st $m = 2x+1$ and $n = 2y$

$mn = (2x+1)(2y) = 4xy + 2y = 2(2xy+y)$

since $\exists\ \ell \in \mathbb{Z}$ st $mn = 2\ell$, mn is even

3. 2.2 ex 15

$x,y \in \mathbb{R}$. prove that if $x \leq y + \varepsilon\ \forall\ \varepsilon \in \mathbb{R}_{>0}$, then $x \leq y$

proof by contradiction:

assume $x > y \rightarrow x - y > 0$

for $\varepsilon = \frac{x-y}{2} > 0$, $x \leq y + \frac{x-y}{2}$

$\leq \frac{x+y}{2}$

$2x \leq x+y$

$x \leq y$ which contradicts $x > y$

$\therefore x \leq y$ if $x \leq y + \varepsilon\ \forall\ \varepsilon \in \mathbb{R}_{>0}$

4. 2.2 ex 2

converse of ex 1 ($\forall\ x \in \mathbb{R}$, if x^2 is irrational, x is irrational):

$\rightarrow \forall\ x \in \mathbb{R}$, if x is irrational, then x^2 is irrational

this is false: for $x = \sqrt{2}$, $x^2 = 2$, which is rational

5. $\forall\ x \in \mathbb{Z}$, if $x^3 - 1$ is even, then x is odd

proof by contrapositive:

suppose x is even st $\exists\ a \in \mathbb{Z}$ and $x = 2a$

then $x^3 - 1 = (2a)^3 - 1 = 8a^3 - 1$

$\exists\ \ell \in \mathbb{Z}$ st $x^3 - 1 = 2(4a^3) - 1$ eg. $\ell = 4a^3$

this means $x^3 - 1$ is odd $\rightarrow \therefore$ $x^3 - 1$ is even means x is odd