Chapter 9 Relations

9.1 Relations and Their Properties

Question 1

List the ordered pairs in the relation R from $A = 0, 1, 2, 3, 4$ to $B = 0, 1, 2, 3$, where $(a, b) \in R$ if and only if

a) $a = b$.

b) $a + b = 4$.

c) $a > b$.

d) $a|b$.

e) $gcd(a, b) = 1$.

f) $lcm(a, b) = 2$.

Solution

a) $(0, 0), (1, 1), (2, 2), (3, 3)$

b) $(1, 3), (2, 2), (3, 1), (4, 0)$

c) $(1, 0), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2), (4, 3)$

d) $(1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3, 0), (3, 3), (4, 0)$

e) $(1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 3)$

f) $(1, 2), (2, 1), (2, 2)$

Question 6

Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

a) $x + y = 0$.

b) $x = \pm y$.

c) $x - y$ is a rational number.

d) $x = 2y$.

e) $xy \geq 0$.

f) $xy = 0$.

g) $x = 1$.

h) $x = 1$ or $y = 1$.

Solution

a) **NOT** reflexive, symmetric, **NOT** antisymmetric

b) **NOT** reflexive, **NOT** symmetric, antisymmetric

c) reflexive, symmetric, **NOT** antisymmetric

d) **NOT** reflexive, **NOT** symmetric, antisymmetric

e) reflexive, symmetric, **NOT** antisymmetric

f) **NOT** reflexive, symmetric, **NOT** antisymmetric

g) **NOT** reflexive, **NOT** symmetric, antisymmetric

h) **NOT** reflexive, symmetric, **NOT** antisymmetric

Question 30

Let $R_1 = (1,2),(2,3),(3,4)$ and $R_2 = (1,1),(1,2),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(3,4)$ be relations from $1,2,3$ to $1,2,3,4$. Find

a) $R_1 \cup R_2$.

b) $R_1 \cap R_2$.

c) $R_1 - R_2$.

d) $R_2 - R_1$.

Solution

a) $R_1 \cup R_2 = (1,1),(1,2),(2,1),(2,3),(3,1),(3,2),(3,3),(3,4)$

b) $R_1 \cap R_2 = (1,2),(2,3),(3,4)$

c) $R_1 - R_2 = \emptyset$

d) $R_2 - R_1 = (1,1),(2,1),(3,1),(3,2),(3,3)$

Question 34

$$R_1 = \{(a, b) \in R_2 | a > b\}$$

, the greater than relation,

$$R_2 = \{(a, b) \in R_2 | a \geq b\}$$

, the greater than or equal to relation,

$$R_3 = \{(a, b) \in R_2 | a < b\}$$

, the less than relation,

$$R_4 = \{(a, b) \in R_2 | a \leq b\}$$

, the less than or equal to relation,

$$R_5 = \{(a, b) \in R_2 | a = b\}$$

, the equal to relation,

$$R_6 = \{(a, b) \in R_2 | a \neq b\}$$

, the unequal to relation.
Find

1. $R_1 \cup R_3$.
2. $R_1 \cup R_5$.
3. $R_2 \cap R_4$.
4. $R_3 \cap R_5$.
5. $R_1 - R_2$.
6. $R_2 - R_1$.
7. $R_1 \oplus R_3$.
8. $R_2 \oplus R_4$.

Solution

a) $R_1 \cup R_3 = R_6$

b) $R_1 \cup R_5 = R_2$

c) $R_2 \cap R_4 = R_5$

d) $R_3 \cap R_5 = R_4$

e) $R_1 - R_2 = \emptyset$

f) $R_2 - R_1 = R_5$

g) $R_1 \oplus R_3 = R_6$

h) $R_2 \oplus R_4 = R_6$

Question 48

Let S be a set with n elements and let a and b be distinct elements of S. How many relations R are there on S such that

a) $(a, b) \in R$?

b) $(a, b) \notin R$?

c) no ordered pair in R has a as its first element?

d) at least one ordered pair in R has a as its first element?

e) no ordered pair in R has a as its first element or b as its second element?

Solution

There are 2^{n^2} relations on S.

a) 2^{n^2-1}

b) 2^{n^2-1}

c) $2^{(n-1)n}$

d) $2^{n^2-1} - 2^{(n-1)n}$

e) $2^{(n-1)^2}$

Question 52

Suppose that R and S are reflexive relations on a set A. Prove or disprove each of these statements.

a) $R \cup S$ is reflexive.

b) $R \cap S$ is reflexive.

c) $R \oplus S$ is irreflexive.

d) $R - S$ is irreflexive.

e) $S \circ R$ is reflexive.

Solution

a) Supposed a is an element of a set A, then (a, a) is in R and S. Therefore, (a, a) is also in $R \cup S$. $R \cup S$ is reflexive.

b) Supposed a is an element of a set A, then (a, a) is in R and S. Therefore, (a, a) is not in $R \oplus S$. $R \oplus S$ is irreflexive.

c) Supposed a is an element of a set A, then (a, a) is in R and S. Therefore, (a, a) is not in $R - S$. $R - S$ is irreflexive.

d) Supposed a is an element of a set A, then (a, a) is in R and S. Therefore, (a, a) is alse in $S \circ R$. $S \circ R$ is reflexive.

9.2 *n*-ary Relations and Their Applications

Question 8

The 4-tuples in a 4-ary relation represent these attributes of published books: title, ISBN, publication date, number of pages.

a) What is a likely primary key for this relation?

b) Under what conditions would (title, publication date) be a composite key?

c) Under what conditions would (title, number of pages) be a composite key?

Solution

a) ISBN. Because it is unique for each book.

b) When any two of books have same title and publication date.

c) When any two of books have same title and number of pages.

Question 9

The 5-tuples in a 5-ary relation represent these attributes of all people in the United States: name, Social Security number, street address, city, state.

a) Determine a primary key for this relation.

b) Under what conditions would (name, street address) be a composite key?

c) Under what conditions would (name, street address, city) be a composite key?

Solution

a) Social Security number. Because it is unique for each book.

b) When any two of people have same name and live at street address.

c) When any two of people have same name, live at street address and city.

9.3 Representing Relations

Question 1

Represent each of these relations on $1, 2, 3$ with a matrix (with the elements of this set listed in increasing order).

a) $(1, 1), (1, 2), (1, 3)$

b) $(1, 2), (2, 1), (2, 2), (3, 3)$

c) $(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$

d) $(1, 3), (3, 1)$

Solution

a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Question 4

List the ordered pairs in the relations on 1, 2, 3, 4 corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a) $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Solution

a) $\{(1,1),(1,2),(1,4),(2,1),(2,3),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4)\}$

b) $\{(1,1),(1,2),(1,3),(2,2),(3,3),(3,4),(4,1),(4,4)\}$

c) $\{(1,2),(1,4),(2,1),(2,3),(3,2),(3,4),(4,1),(4,3)\}$

Question 10

How many nonzero entries does the matrix representing the relation R on $A = \{1, 2, 3, \ldots, 1000\}$ consisting of the first 1000 positive integers have if R is

a) $\{(a, b)|a \leq b\}$?

b) $\{(a, b)|a = b \pm 1\}$?

c) $\{(a, b)|a + b = 1000\}$?

d) $\{(a, b)|a + b \leq 1001\}$?

e) $\{(a, b)|a \neq 0\}$?

Solution

a) $1001 * 500 = 500500$

b) $2 * 998 + 2 = 1998$

c) 999

d) $1001 * 500 = 500500$

e) $1000 * 1000 = 1000000$

Question 14

Let R_1 and R_2 be relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrices that represent

1. $R_1 \cup R_2$.
2. $R_1 \cap R_2$.
3. $R_2 \circ R_1$.
4. $R_1 \circ R_1$.
5. $R_1 \oplus R_2$.

Solution

a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

e) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Question 15

Let R be the relation represented by the matrix

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Find the matrices that represent

1. R^2
2. R^3
3. R^4

Solution

a) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Question 35

Show that if $\mathbf{M}_R$ is the matrix representing the relation R, then $\mathbf{M}_R^{[n]}$ is the matrix representing the relation R^n.

Solution

We can proof by mathematical induction.
Basic step: Trivial for $n = 1$.
Inductive step: Assume true for k. $\mathbf{M}_R^{[k]} = \mathbf{M}_{R^k}$, Because $R_{k+1} = R^k \circ R$, and its matrix is $\mathbf{M}_{R^k} \odot \mathbf{M}_R = \mathbf{M}_R^{[k+1]}$
Therefore, it is true for positive integers n.

9.4 Closures of Relations

Question 3

Let R be the relation $\{(a, b)|\ a$ divides $b\}$ on the set of integers. What is the symmetric closure of R?

Solution

$\{(a, b)|\ a$ divides b or b divides $a\}$

Question 8

How can the directed graph representing the symmetric closure of a relation on a finite set be constructed from the directed graph for this relation?

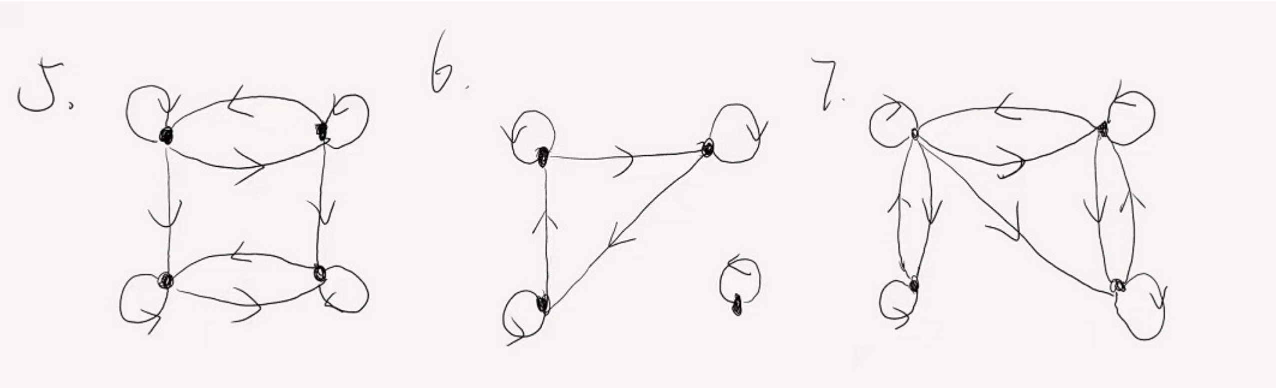
Solution

We can add an edge from x to y whenever this edge is not existed but the edge from y to x is.

Question 11

Find the directed graph of the smallest relation that is both reflexive and symmetric that contains each of the relations with directed graphs shown in Exercises 5-7.

Solution



Question 16

Determine whether these sequences of vertices are paths in this directed graph.

a) a, b, c, e

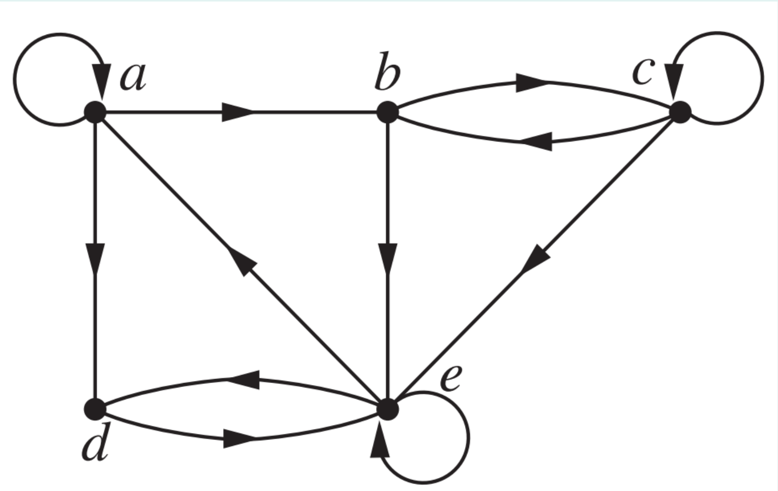
b) b, e, c, b, e

c) a, a, b, e, d, e

d) b, c, e, d, a, a, b

e) b, c, c, b, e, d, e, d

f) $a, a, b, b, c, c, b, e, d$



Solution

a) Yes.

b) No, we can not go to c from e.

c) Yes.

d) No, we can not go to a from d.

e) Yes.

f) No, we can not go to b from b.

Question 21

Let R be the relation on the set of all students containing the ordered pair (a, b) if a and b are in at least one common class and $a \neq b$. When is (a, b) in

a) R^2?

b) R^3?

c) R^*?

Solution

a) If for each a and b there exists a student c who is in at least on common class with a and b.

b) If for each a and b there exists students c and d such that a and c are in at least one common class, c and d are in at least one common class and d and b are in at least one common class.

c) If for each a and b there exists students $s_1,\ s_2,\ \ldots,\ s_n$ such that a and s_1 are in at least one common class, s_i and s_{i+1} when $i \geq 1$ are in at least one common class and s_n and b are in at least one common class.

Question 25

Use Algorithm 1 to find the transitive closures of these relations on $1, 2, 3, 4$

a) $(1, 2), (2, 1), (2, 3), (3, 4), (4, 1)$

b) $(2, 1), (2, 3), (3, 1), (3, 4), (4, 1), (4, 3)$

c) $(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)$

d) $(1, 1), (1, 4), (2, 1), (2, 3), (3, 1), (3, 2), (3, 4), (4, 2)$

Solution

a) $$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

b) $$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

c) $$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

d) $$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

9.5 Equivalence Relations

Question 3

Which of these relations on the set of all functions from **Z** to **Z** are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) $\{(f, g) | f(1) = g(1)\}$

b) $\{(f, g) | f(0) = g(0) \text{ or } f(1) = g(1)\}$

c) $\{(f, g) | f(x) - g(x) = 1 \text{ for all } x \in \mathbf{Z}\}$

d) $\{(f, g) | \text{ for some } C \in \mathbf{Z}, \text{ for all } x \in \mathbf{Z}, f(x) - g(x) = C\}$

e) $\{(f, g) | f(0) = g(1) \text{ and } f(1) = g(0)\}$

Solution

a) Equivalence relation.

b) **Not** an equivalence relation. Not transitive.

c) **Not** an equivalence relation. Not reflexive, Not symmetric, Not transitive.

d) Equivalence relation.

e) **Not** an equivalence relation. Not reflexive, Not transitive.

Question 11

Show that the relation R consisting of all pairs (x, y) such that x and y are bit strings of length three or more that agree in their first three bits is an equivalence relation on the set of all bit strings of length three or more.

Solution

- *Reflexive:* For (x, x) it is obviously in R.
- *Symmetric:* If (x, y) is in R, then they agree in first three bits, so (y, x) is also in R.
- *Transitive:*If (x, y) and (y, z) are in R, then they agree in first three bits, so (x, z) is also in R.

Question 15

Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.

Solution

- *Reflexive:* For $((a, b), (a, b))$ it is obviously in R.
- *Symmetric:* If $((c, d), (a, b))$ is in R, then $c + b = a + d$, so $((a, b), (c, d))$ is also in R.
- *Transitive:*If $((a, b), (c, d))$ and $((c, d), (e, f))$ are in R, then

$$a + d = c + b, c + f = d + e$$

$$c - d = a - b = e - f$$

$$a + f = b + e$$

so $((a, b), (e, f))$ is also in R.

Question 31

What are the equivalence classes of the bit strings in Exercise 30 for the equivalence relation from Exercise 12?

1. 010
2. 1011
3. 11111
4. 01010101

Solution

a) all bit strings of length 3

b) the bit strings of length 4 that end with 1

c) the bit strings of length 5 that end with 11

d) the bit strings of length 8 that end with 10101

Question 41

Which of these collections of subsets are partitions of $\{1, 2, 3, 4, 5, 6\}$?

a) $\{1, 2\}, \{2, 3, 4\}, \{4, 5, 6\}$

b) $\{1\}, \{2, 3, 6\}, \{4\}, \{5\}$

c) $\{2, 4, 6\}, \{1, 3, 5\}$

d) $\{1, 4, 5\}, \{2, 6\}$

Solution

a) not a partition. Because the number of 2 appears twice.

b) a partition.

c) a partition.

d) not a partition. Because the number of 3 does not appear.

Question 47

List the ordered pairs in the equivalence relations produced by these partitions of $\{0, 1, 2, 3, 4, 5\}$.

a) $\{0\}, \{1, 2\}, \{3, 4, 5\}$

b) $\{0, 1\}, \{2, 3\}, \{4, 5\}$

c) $\{0, 1, 2\}, \{3, 4, 5\}$

d) $\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}$

Solution

a) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (3, 5), (4, 3), (4, 4), (4, 5), (5, 3), (5, 4), (5, 5)\}$

b) $\{(0, 0), (0, 1), (1, 0), (1, 1), (2, 2), (2, 3), (3, 2), (3, 3), (4, 4), (4, 5)\}$

c) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 3), (3, 4), (3, 5), (4, 3), (4, 4), (4, 5), (5, 3), (5, 4), (5, 5)\}$

d) $\{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$

Question 55

Find the smallest equivalence relation on the set $\{a, b, c, d, e\}$ containing the relation $\{(a, b), (a, c), (d, e)\}$.

Solution

The partition is $\{\{a, b, c\}, \{d, e\}\}$
And the equivalence relation is $\{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c),$
$(c, a), (c, b), (c, c), (d, d), (d, e), (e, d), (e, d)\}$

9.6 Partial Orderings

Question 1

Which of these relations on $0, 1, 2, 3$ are partial orderings? Determine the properties of a partial ordering that the others lack.

a) $(0, 0), (1, 1), (2, 2), (3, 3)$

b) $(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)$

c) $(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)$

d) $(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$

e) $(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)$

Solution

a) Is a partial ordering.

b) Not a partial ordering. Because it is not antisymmetric, not transitive.

c) Is a partial ordering.

d) Is a partial ordering.

e) Not a partial ordering. Because it is not antisymmetric, not transitive.

Question 4

Is (S, R) a poset if S is the set of all people in the world and $(a, b) \in R$, where a and b are people, if

a) a is no shorter than b?

b) a weighs more than b?

c) $a = b$ or a is a descendant of b?

d) a and b do not have a common friend?

Solution

a) Is a poset.

b) Not a poset. Because it is not reflexive.

c) Is a poset.

d) Not a poset. Because it is not reflexive.

Question 7

Determine whether the relations represented by these zero-one matrices are partial orders.

a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

Solution

a) Not a partial ordering. Because it is not antisymmetric.

b) Is a partial ordering.

c) Not a partial ordering. Because it is not transitive.

Question 17

Find the lexicographic ordering of these n-tuples:

a) $(1, 1, 2), (1, 2, 1)$

b) $(0, 1, 2, 3), (0, 1, 3, 2)$

c) $(1, 0, 1, 0, 1), (0, 1, 1, 1, 0)$

Solution

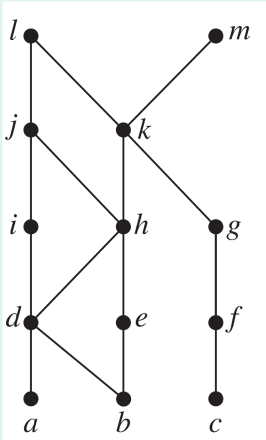
a) $(1, 1, 2) < (1, 2, 1)$

b) $(0, 1, 2, 3) < (0, 1, 3, 2)$

c) $(0, 1, 1, 1, 0) < (1, 0, 1, 0, 1)$

Question 32

Answer these questions for the partial order represented by this Hasse diagram.



a) Find the maximal elements.

b) Find the minimal elements.

c) Is there a greatest element?

d) Is there a least element?

e) Find all upper bounds of $\{a, b, c\}$.

f) Find the least upper bound of $\{a, b, c\}$, if it exists.

g) Find all lower bounds of $\{f, g, h\}$.

h) Find the greatest lower bound of $\{f, g, h\}$, if it exists.

Solution

a) $\{l, m\}$

b) $\{a, b, c\}$

c) There is not a greatest element.

d) There is not a least element.

e) $\{k, l, m\}$

f) $\{k\}$

g) $\{k, l, m\}$

h) There is not a greatest lower bound of $\{f, g, h\}$.

Question 34

Answer these questions for the poset $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$.

a) Find the maximal elements.

b) Find the minimal elements.

c) Is there a greatest element?

d) Is there a least element?

e) Find all upper bounds of $\{2, 9\}$.

f) Find the least upper bound of $\{2, 9\}$, if it exists.

g) Find all lower bounds of $\{60, 72\}$.

h) Find the greatest lower bound of $\{60, 72\}$, if it exists.

Solution

a) $\{27, 48, 60, 72\}$

b) $\{2, 9\}$

c) There is not a greatest element.

d) There is not a least element.

e) $\{18, 36, 72\}$

f) $\{18\}$

g) $\{2, 4, 6, 12\}$

h) $\{12\}$.

Question 35

Answer these questions for the poset $(\{\{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}, \subseteq)$.

a) Find the maximal elements.

b) Find the minimal elements.

c) Is there a greatest element?

d) Is there a least element?

e) Find all upper bounds of $\{\{2\}, \{4\}\}$.

f) Find the least upper bound of $\{\{2\}, \{4\}\}$, if it exists.

g) Find all lower bounds of $\{\{1,3,4\}, \{2,3,4\}\}$.

h) Find the greatest lower bound of $\{\{1,3,4\}, \{2,3,4\}\}$, if it exists.

Solution

a) $\{1\}, \{2\}, \{4\}$

b) $\{1,3,4\}, \{2,3,4\}$

c) There is not a greatest element.

d) There is not a least element.

e) $\{2,4\}, \{2,3,4\}$

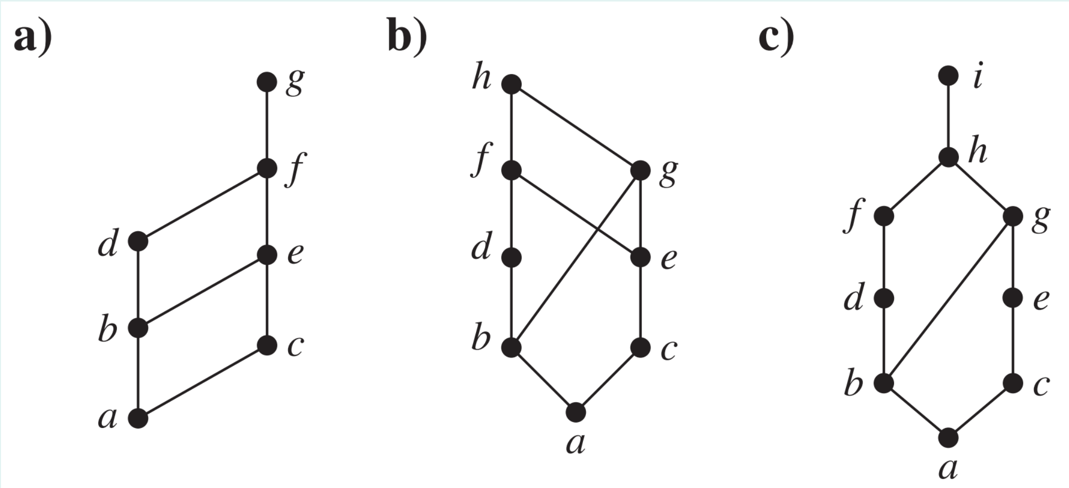
f) $\{2,3,4\}$

g) $\{4\}, \{3,4\}$

h) $\{3,4\}$

Question 43

Determine whether the posets with these Hasse diagrams are lattices.



Solution

In each case, we need to decide whether every pair of elements has a least upper bound and a greatest lower bound.

a) This is a lattice.

b) This is not a lattice.

c) This is a lattice.