Chapter 1 The Foundations:Logic and Proofs

1.1 Propositional Logic

Question 11

Let p and q be the propositions "Swimming at the New Jerssey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

1. $\neg q$
2. $p \wedge q$
3. $\neg p \vee q$
4. $p \to \neg q$
5. $\neg q \to p$
6. $\neg p \to q$
7. $p \leftrightarrow \neg q$
8. $\neg p \wedge (p \vee \neg q)$

Solution

1 Sharks have **not** been spotted near the shore

2 Swimming at the New Jerssey shore is allowed **and** sharks have been spotted near the shore

3 Swimming at the New Jerssey shore is **not** allowed or sharks have been spotted near the shore

4 **If** swimming at the New Jerssey shore is allowed **then** sharks have **not** been spotted near the shore

5 **If** sharks have **not** been spotted near the shore, **then** swimming at the New Jerssey shore is allowed

6 **If** swimming at the New Jerssey shore is **not** allowed, sharks have been spotted near the shore

7 **If and only if** swimming at the New Jerssey shore is allowed, **then** sharks have **not** been spotted near the shore

8 Swimming at the New Jerssey shore is **not** allowed, **but** swimming at the New Jerssey shore is allowed **or** sharks have **not** been spotted near the shore

Question 17

Let p, q, and r be the propositions

p: Grizzly bears have been seen in the area.
q: Hiking is safe on the trail.
r: Berries are ripe along the trail.

Write these propositions using p, q, and r and logical connectives (including negations).

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

e) For hiking on the trail to be safe, it is necessary butnot sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area

f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

Solution

a) $r \land \neg p$

b) $(\neg p \land q) \land r$

c) $r \to (q \leftrightarrow \neg p)$

d) $\neg q \land (\neg p \land r)$

e) $q \to (\neg r \land \neg p)$

f) $\neg q \leftarrow (p \land r)$

Question 24

Write each of these statements in the form "if p, then q" in English. [*Hint*: Refer to the list of common ways to express conditional statements provided in this section.]

a) It is necessary to wash the boss's car to get promoted.

b) Winds from the south imply a spring thaw.

c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

d) Willy gets caught whenever he cheats.

e) You can access the website only if you pay a subscription fee.

f) Getting elected follows from knowing the right people

g) Carol gets seasick whenever she is on a boat.

Solution

a) **If** you get promoted, **then** the boss's car is washed.

b) **If** winds from the south, **then** a spring thaw

c) **If** you bought the computer less than a year ago, **then** the warranty is good.

d) **If** Willy cheats, **then** Willy gets caught.

e) **If** you can access the website, **then** you pay a subscription fee.

f) **If** knowing the right people, **then** getting elected.

g) **If** Carol is on a boat, **then** Carol gets seasick.

Question 30

State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.

b) I go to the beach whenever it is a sunny summer day.

c) When I stay up late, it is necessary that I sleep until noon.

Solution

a) *If it snows tonight, then I will stay at home.*

- **converse**: If I stay at home, then it will snow tonight.
- **contrapositive**: If I do not stay at home, then it will not snow tonight.
- **inverse**: If it does not snow tonight, then I will not stay at home.

b) *I go to the beach whenever it is a sunny summer day.*

- **converse**: It is a sunny summer day whenever I go to the beach.
- **contrapositive**: It is not a sunny summer day whenever I do not go to the beach.
- **inverse**: I do not go to the beach whenever it is not a sunny summer day.

c) *When I stay up late, it is necessary that I sleep until noon.*

- **converse**: When I sleep until noon, it is necessary that I stay up late.
- **contrapositive**: When I do not sleep until noon, it is necessary that I do not stay up late.
- **inverse**: When I do not stay up late, it is necessary that I would not sleep until noon.

Question 33

Construct a truth table for each of these compound propositions.

a) $p \wedge \neg p$

b) $p \vee \neg p$

c) $(p \vee \neg q) \rightarrow q$

d) $(p \vee q) \rightarrow (p \wedge q)$

e) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

f) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

Solution

a)

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

b)

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

c)

p	q	$(p \vee \neg q) \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	F

d)

p	q	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T
T	F	F
F	T	F
F	F	T

e)

p	q	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T
T	F	T
F	T	T
F	F	T

f)

p	q	$(p \to q) \to (q \to p)$
T	T	T
T	F	T
F	T	F
F	F	T

Question 34

Construct a truth table for each of these compound propositions.

a) $p \to \neg p$

b) $p \leftrightarrow \neg p$

c) $p \oplus (p \vee q)$

d) $(p \wedge q) \to (p \vee q)$

e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

Solution

a)

p	$\neg p$	$p \to \neg p$
T	F	T
F	T	T

b)

p	p	$p \leftrightarrow \neg p$
T	F	F
F	T	F

c)

p	q	$p \oplus (p \vee q)$
T	T	F
T	F	F
F	T	T
F	F	F

d)

p	q	$(p \wedge q) \to (p \vee q)$
T	T	T
T	F	T
F	T	T
F	F	T

e)

p	q	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F
T	F	T
F	T	F
F	F	T

f)

p	q	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T

Question 35

Construct a truth table for each of these compound propositions.

a) $(p \vee q) \to (p \oplus q)$

b) $(p \oplus q) \to (p \wedge q)$

c) $(p \vee q) \oplus (p \wedge q)$

d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

f) $(p \oplus q) \to (p \oplus \neg q)$

Solution

a)

p	q	$(p \vee q) \to (p \oplus q)$
T	T	F
T	F	T
F	T	T
F	F	T

b)

p	q	$(p \oplus q) \to (p \wedge q)$
T	T	T
T	F	F
F	T	F
F	F	T

c)

p	q	$(p \vee q) \oplus (p \wedge q)$
T	T	F
T	F	T
F	T	T
F	F	F

d)

p	q	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	T
T	F	T
F	T	T
F	F	T

e)

p	q	r	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

f)

p	q	$(p \oplus q) \rightarrow (p \oplus \neg q)$
T	T	T
T	F	F
F	T	F
F	F	T

Question 37

Construct a truth table for each of these compound propositions.

a) $p \to \neg q$

b) $\neg p \leftrightarrow q$

c) $(p \to q) \vee (\neg p \to q)$

d) $(p \to q) \wedge (\neg p \to q)$

e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

Solution

a)

p	q	$p \to \neg q$
T	T	F
T	F	T
F	T	T
F	F	T

b)

p	q	$\neg p \leftrightarrow q$
T	T	F
T	F	T
F	T	T
F	F	F

c)

p	q	$(p \to q) \vee (\neg p \to q)$
T	T	T
T	F	T
F	T	T
F	F	T

d)

p	q	$(p \to q) \land (\neg p \to q)$
T	T	T
T	F	F
F	T	T
F	F	F

e)

p	q	$(p \leftrightarrow q) \lor (\neg p \leftrightarrow q)$
T	T	T
T	F	T
F	T	T
F	F	T

f)

p	q	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
T	T	T
T	F	T
F	T	T
F	F	T

Question 44

If $p_1, p_2, \ldots, p_n$ are n propositions, explain why

$$\bigwedge_{i=1}^{n-1} \bigwedge_{j=i+1}^{n} (\neg p_i \vee \neg p_j)$$

is true if and only if at most one of $p_1, p_2, \ldots, p_n$ is true.

Solution

For the equation

$$\bigwedge_{i=1}^{n-1} \bigwedge_{j=i+1}^{n} (\neg p_i \vee \neg p_j)$$

is true, if and only if the term $(\neg p_i \vee \neg p_j), i \neq j$ is true.

The term $(\neg p_i \vee \neg p_j), i \neq j$ is true if and only if there is at most one of $p_1, p_2, \ldots, p_n$ is true.

Question 47

Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.

a) 101 1110, 010 0001

b) 1111 0000, 1010 1010

c) 00 0111 0001, 10 0100 1000

d) 11 1111 1111, 00 0000 0000

Solution

a) 101 1110, 010 0001

- $1011110 \vee 0100001 = 1111111$
- $1011110 \wedge 0100001 = 0000000$
- $1011110 \oplus 0100001 = 1111111$

b) 1111 0000, 1010 1010

- $11110000 \vee 10101010 = 11111010$
- $11110000 \wedge 10101010 = 10100000$
- $11110000 \oplus 10101010 = 01011010$

c) 00 0111 0001, 10 0100 1000

- $0001110001 \vee 1001001000 = 1001111001$
- $0001110001 \wedge 1001001000 = 0001000000$
- $0001110001 \oplus 1001001000 = 1000111001$

d) 11 1111 1111, 00 0000 0000

- $1111111111 \vee 0000000000 = 0000000000$
- $1111111111 \wedge 0000000000 = 0000000000$
- $1111111111 \oplus 0000000000 = 1111111111$

Question 52

Is the assertion “This statement is false” a proposition?

Solution

If the assertion “This statement is false” is true, it would be false. So it is both true and false, a contradiction.
If the assertion “This statement is false” is false, it would be ture. So it is both true and false, a contradiction.
Therefore, this assertion is a paradox and is not a proposition.

1.2 Applications of Propositional Logic

Question 3

You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Express your answer in terms of g: "You can graduate," m: "You owe money to the university," r: "You have completed the requirements of your major," and b: "You have an overdue library book."

Solution

$$(\neg m \land r \land \neg b) \leftarrow g$$

Question 7

Express these system specifications using the propositions p: "The message is scanned for viruses" and q: "The message was sent from an unknown system" together with logical connectives (including negations).

a) "The message is scanned for viruses whenever the message was sent from an unknown system."

b) "The message was sent from an unknown system but it was not scanned for viruses."

c) "It is necessary to scan the message for viruses whenever it was sent from an unknown system."

d) "When a message is not sent from an unknown system it is not scanned for viruses.

Solution

a) $q \to p$

b) $q \wedge \neg p$

c) $q \to p$

d) $\neg q \to \neg p$

Question 9

Are these system specifications consistent? "The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode."

Solution

Let p denote "The system is in multiuser state", q denote "The system is perating normally", r denote "The kernel is functioning" and s denote "The system is in interrupt mode".

The specifications can then be written as $p \leftrightarrow q$, $q \rightarrow r$, $\neg r \vee s$, $\neg p \rightarrow s$ and $\neg s$
An assignment of truth values that makes all three specifications true must have s false to make $\neg s$ true.

- Because we want $\neg p \rightarrow s$ to be true but s must be false, p must be true.
- Because we want $\neg r \vee s$ to be true but s must be false, r must be false.
- Because we want $q \rightarrow r$ to be true but r must be false, q must be false.
- Because we want $p \leftrightarrow q$ to be true but p must be true and q must be false, a contradiction.

We conclude that these specifications are not consistent, because they can not all be true no matter the truth value of p, q, r and s.

Question 13

What Boolean search would you use to look for Web pages about beaches in New Jersey? What if you wanted to find Web pages about beaches on the isle of Jersey (in the English Channel)?

Solution

To look for Web pages about beaches in New Jersey, we can look for pages matching NEW *AND* JERSEY *AND* BEACHES.

To look for Web pages about beaches on the isle of Jersey, we can look for pages matching (JERSEY *AND* ISLE *AND* BEACHES) *NOT* NEW.

Question 19

Each inhabitant of a remote village always tells the truth or always lies. A villager will give only a "Yes" or a "No" response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit; the other branch leads deep into the jungle. A villager is standing at the fork in the road. What one question can you ask the villager to determine which branch to take?

Solution

One of possible solutions is "If I were to ask you whether the right branch leads to the ruins, would you say 'yes'?" To explain the question, let p denote "The right branch leads to the ruins" and q denote "The villager always tells the truth." The question can be describe as

$$q \leftrightarrow (p \leftrightarrow q)$$

We can construct a truth table.

p	q	$q \leftrightarrow (p \leftrightarrow q)$
T	T	T
T	F	T
F	T	F
F	F	F

We can note that no matter the truth value of q, the result is as same as the truth value of p. Therefore, if the villager give Yes, then the right branch leads to the ruins, otherwise, the right branch leads to the jungle.

Question 22

When planning a party you want to know whom to invite. Among the people you would like to invite are three touchy friends. You know that if Jasmine attends, she will become unhappy if Samir is there, Samir will attend only if Kanti will be there, and Kanti will not attend unless Jasmine also does. Which combinations of these three friends can you invite so as not to make someone unhappy?

Solution

Let p denote "Jasmine attends the party", q denote "Samir attends the party" and r denote "Kanti attends the party", The specifications can then be written as $\neg(p \wedge q)$, $q \rightarrow r$ and $p \leftarrow r$

Assume p is false,

- Because we want $\neg(p \wedge q)$ to be true but p must be false, q can be true or false.
- Because we want $p \leftarrow r$ to be true but p must be false, r must be false.
- Because we want $q \rightarrow r$ to be true but r must be false, q must be false.

Assume p is true,

- Because we want $\neg(p \wedge q)$ to be true but p must be true, q must be false.
- Because we want $p \leftarrow r$ to be true but p must be true, r can be true or false.
- Because we want $q \leftarrow r$ to be true but p must be true, r can be true or false.

Therefore, the combinations of friends should be one of below:

- Jasmine and Kanti
- Jasmine
- no one

Question 37

Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.

Solution

Let *f* denote "Fred is the highest paid of the three", *j* denote "Janice is the highest paid of the three" *r* denote "Janice is the lowest paid of the three" and *m* denote "Maggie is the highest paid of the three", The facts can then be written as $\neg(j \wedge r)$, $\neg f \rightarrow j$, $\neg r \rightarrow m$ and $(f \wedge \neg j \wedge \neg m) \vee (\neg f \wedge j \wedge \neg m) \vee (\neg f \wedge \neg j \wedge m)$

Assume *j* is true,

- Because we want $(f \wedge \neg j \wedge \neg m) \vee (\neg f \wedge j \wedge \neg m) \vee (\neg f \wedge \neg j \wedge m)$ to be true but *j* must be true, *f* and *m* must be false.
- Because we want $\neg(j \wedge r)$ to be true but *j* must be true, *r* must be false.
- Because we want $\neg r \rightarrow m$ to be true but *p* must be true and *m* must be false , a contradiction

Assume *j* is false,

- Because we want $\neg f \rightarrow j$ to be true but *j* must be false, *f* must be true.
- Because we want $p \leftarrow r$ to be true but *p* must be false, *r* must be false.
- Because we want $(f \wedge \neg j \wedge \neg m) \vee (\neg f \wedge j \wedge \neg m) \vee (\neg f \wedge \neg j \wedge m)$ to be true but *f* must be true, *m* must be false.
- Because we want $\neg r \rightarrow m$ to be true but *m* must be false, *r* must be true.

Therefore, Fred is the highest paid of the three and Janice is the lowest paid of the three.

Question 40

Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. Alice said, “Carlos did it.” John said, “I did not do it.” Carlos said, “Diana did it.” Diana said, “Carlos lied when he said that I did it.”

a) If the authorities also know that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning.

b) If the authorities also know that exactly one is lying, who did it? Explain your reasoning.

Solution

a) John did it. There are four posible cases to consider.

   i) If Alice is the sole truth-teller, then Carlos did it; but this means that John is telling the truth, a contradiction.

   ii) If John is the sole truth-teller, then Diana must be lying, so she did it, but then Carlos is telling the truth, a contradiction.

   iii) If Carlos is the sole truth-teller, then Diana did it, but that makes John truthful, a contradiction.

   iv) If Diana is the sole truth-teller,then John did it.

b) Since Carlos and Diana are making contradictory statements, the liar must be one of them. Therefore Alice is telling the truth, so Carlos did it.

Question 42

Solve this famous logic puzzle, attributed to Albert Einstein, and known as the zebra puzzle. Five men with different nationalities and with different jobs live in consecutive houses on a street. These houses are painted different colors. The men have different pets and have different favorite drinks. Determine who owns a zebra and whose favorite drink is mineral water (which is one of the favorite drinks) given these clues: The Englishman lives in the red house. The Spaniard owns a dog. The Japanese man is a painter. The Italian drinks tea. The Norwegian lives in the first house on the left. The green house is immediately to the right of the white one. The photographer breeds snails. The diplomat lives in the yellow house. Milk is drunk in the middle house. The owner of the green house drinks coffee. The Norwegian's house is next to the blue one. The violinist drinks orange juice. The fox is in a house next to that of the physician. The horse is in a house next to that of the diplomat. [*Hint*: Make a table where the rows represent the men and columns represent the color of their houses, their jobs, their pets, and their favorite drinks and use logical reasoning to determine the correct entries in the table.]

Solution

The following table shows a solution consistent with all the clues.

HOUSE ORDER	1	2	3	4	5
COLOR	YELLOW	BLUE	RED	IVORY	GREEN
NATIONALITY	NORWEGIAN	UKRANIAN	ENGLISHMAN	SPANIARD	JAPANESE
PET	FOX	HORSE	SNAILS	DOG	ZEBRA
DRINK	WATER	TEA	MILK	OJ	COFFEE
CIGARETTES	KOOLS	CHESTERFIELDS	OLD GOLD	LUCKY STRIKE	PARLIAMANETS

Figure 1

In this solution the Japanese man owns the zebra, and the Norwegian drinks water.

Question 45

Find the output of each of these combinatorial circuits.

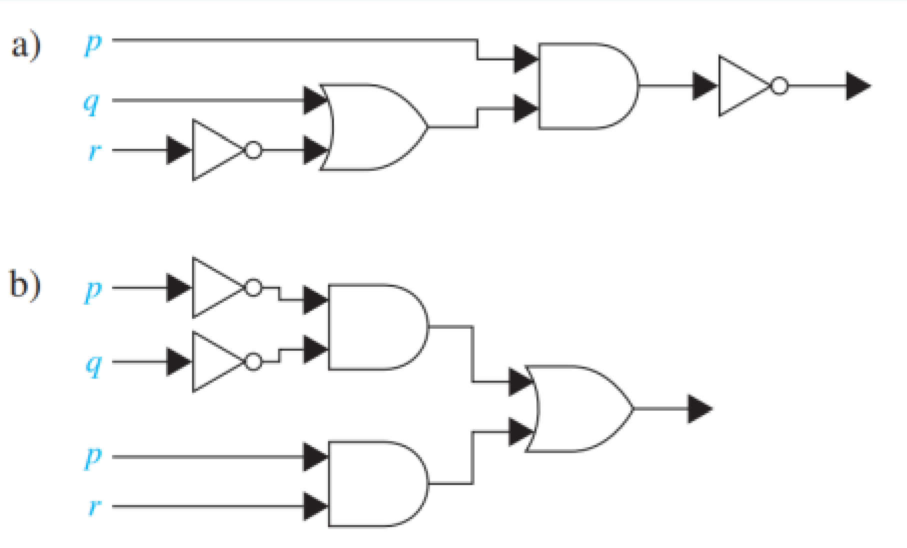

Figure 2

Solution

a) $\neg(p \wedge (q \vee \neg r))$

b) $((\neg p) \wedge (\neg q)) \vee (p \wedge r)$

1.3 Propositional Equivalences

Question 7

Use De Morgan's laws to find the negation of each of the following statements.

a) Jan is rich and happy.

b) Carlos will bicycle or run tomorrow.

c) Mei walks or takes the bus to class.

d) Ibrahim is smart and hard working.

Solution

a) Jan is not rich or not happy.

b) Carlos will not bicycle and not run tomorrow.

c) Mei does not walk and does not take the bus to class.

d) Ibrahim is not smart or not hard working.

Question 9

For each of these compound propositions, use the conditional-disjunction equivalence (Example 3) to find an equivalent compound proposition that does not involve conditionals.

a) $p \to \neg q$

b) $(p \to q) \to r$

c) $(\neg q \to p) \to (p \to \neg q)$

Solution

a) $\neg p \lor q$

b) $\neg(p \lor \neg q) \lor r$

c) $\neg(q \lor \neg p) \lor (\neg p \lor q)$

Question 19

Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology

Solution

$$
\begin{aligned}
(\neg q \wedge (p \to q)) \to \neg p &\equiv \neg(\neg q \wedge (\neg p \vee q)) \vee \neg p && \text{by the example 3} \\
&\equiv (q \vee (p \wedge \neg q)) \vee \neg p && \text{by De Morgan's laws} \\
&\equiv ((q \vee p) \wedge (q \vee \neg q)) \vee \neg p && \text{by Distributive laws} \\
&\equiv ((q \vee p) \wedge T) \vee \neg p && \text{by Negation laws} \\
&\equiv (q \vee p) \vee \neg p && \text{by Identity laws} \\
&\equiv q \vee (p \vee \neg p) && \text{by Associative laws} \\
&\equiv q \vee T && \text{by Negation laws} \\
&\equiv T && \text{by Identity laws}
\end{aligned}
$$

Question 20

Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

Solution

The first proposition is true if and only if p and q have the same truth value.
The second proposition is true if and only if either p and q are both true, or p and q are both false
Therefore, the two propositionare are logically equivalent.

Question 21

Show that $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.

Solution

The first proposition is true if and only if p and q have different truth values.
The second proposition is true if and only if either p is true and q is false, or p is false and q is true.
Therefore, the two propositionare are logically equivalent.

Question 22

Show that $p \to q)$ and $\neg q \to \neg p$ are logically equivalent.

Solution

The first proposition is false if and only if p is true and q is false.
The second proposition is false if and only if q is false and p is true
Therefore, the two propositionare are logically equivalent.

Question 23

Show that $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent.

Solution

The first proposition is true if and only if either p is true and q is false, or p is false and q is true.
The second proposition is true if and only if either p is true and q is false, or p is false and q is true.
Therefore, the two propositionare are logically equivalent.

Question 24

Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

Solution

The first proposition is true if and only if p and q have the same truth value.
The second proposition is true if and only if p and q have the same truth value.
Therefore, the two propositionare are logically equivalent.

Question 25

Show that $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$ are logically equivalent.

Solution

The first proposition is true if and only if p and q have different truth values.
The second proposition is true if and only if either p is true and q is false, or p is false and q is true.
Therefore, the two propositionare are logically equivalent.

Question 26

Show that $(p \to q) \land (p \to r)$ and $p \to (q \land r)$ are logically equivalent.

Solution

The first proposition is false if and only if either p is true and q is false, or p is true and r is false.
The second proposition is false if and only if p is true and either q is false or r is false.
Therefore, the two propositionare are logically equivalent.

Question 27

Show that $(p \to r) \land (q \to r)$ and $(p \lor q) \to r$ are logically equivalent.

Solution

The first proposition is false if and only if either p is true and r is false, or q is true and r is false.
The second proposition is false if and only if either p or q is true and r is false.
Therefore, the two propositionare are logically equivalent.

Question 28

Show that $(p \to q) \lor (p \to r)$ and $p \to (q \lor r)$ are logically equivalent.

Solution

The first proposition is false if and only if p is true and q is false, and p is true and r is false.
The second proposition is false if and only if p is true and both q and r are false.
Therefore, the two propositionare are logically equivalent.

Question 29

Show that $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

Solution

The first proposition is false if and only if *p* is true and *r* is false, and *q* is true and *r* is false.
The second proposition is false if and only if either *p* or *q* is true and *r* is false.
Therefore, the two propositionare are logically equivalent.

Question 30

Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)a$ are logically equivalent.

Solution

The first proposition is false if and only if *p* is false, *q* is true and *r* is false.
The second proposition is false if and only if *q* is true and both *p* and *r* are false.
Therefore, the two propositionare are logically equivalent.

Question 31

Show that $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are logically equivalent.

Solution

The first proposition is true if and only if *p* and *q* have same truth value.
The second proposition is true if and only if *p* is not true when *q* is false, and if *q* is not true when *p* is false.
Therefore, the two propositionare are logically equivalent.

Question 32

Show that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are logically equivalent.

Solution

The first proposition is true if and only if p and q have same truth value.
The second proposition is true if and only if $\neg p$ and $\neg q$ have same truth value, equivalent to p and q have same truth value.
Therefore, the two propositionare are logically equivalent.

Question 33

Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

Solution

We can construct a truth table.

p	q	r	$(p \to q) \wedge (q \to r) \to (p \to r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

The last column is all Ts.

Question 46

Suppose that a truth table in n propositional variables is specified. Show that a compound proposition with this truth table can be formed by taking the disjunction of conjunctions of the variables or their negations, with one conjunction included for each combination of values for which the compound proposition is true. The resulting compound proposition is said to be in disjunctive normal form.
A collection of logical operators is called functionally complete if every compound proposition is logically equivalent to a compound proposition involving only these logical operators.

Solution

For each line of truth table, we can write down a conjunction of n propositional variables corresponds to this case. Only when these atomic propositions takes the corresponding truth value of this line that the conjunction can be true.
If we pick the lines of truth table that make the compound proposition true, and write down their conjunctions, and take the disjunction of the resulting propositions, then we have the proposition in its disjunctive normal form.

Question 50

Construct a truth table for the logical operator *NAND*.

Solution

p	q	$p \mid q$
T	T	T
T	F	F
F	T	F
F	F	F

Question 51

Show that $p \mid q$ is logically equivalent to $\neg(p \wedge q)$.

Solution

$\neg(p \wedge q)$ is false if and only if either p or q, or both, are false ,this was the definition of $p \mid q$, the two are logically equivalent.

Question 52

Construct a truth table for the logical operator *NOR*.

Solution

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

Question 53

Show that $p \downarrow q$ is logically equivalent to $\neg(p \vee q)$.

Solution

$\neg(p \vee q)$ is true when both p and q are false, and is false otherwise. Because this was the definition of $p \downarrow q$, the two are logically equivalent.

Question 57

Show that $p \mid q$ and $q \mid p$ are equivalent.

Solution

From the truth table of Question 50, we can proof that $p \mid q$ and $q \mid p$ are equivalent.