Lab_5_sol

March 27, 2023

1 Tutorial 5 (SimPy)

1.1 Question 1

a. The idea is to monitor the *number of customers in the system* (call this the *state*). At any *event* (customer arrival, begin service, end service), we need to know if this changes the number of customers in the system, and update the state accordingly.

1. Use a class variable **n** in the **Arrival** class to keep track of exactly how many customers there are in the system.
2. Set up a new monitor called **numbermon** in a similar way to **delaymon**.
3. Modify the **run** method (PEM) within the **Arrival** class so that whenever an event occurs (customer arrival, begin service, end service) we update the state (class variable **Arrival.n**) and then the **numbermon** monitor will **observe** the new value of the state.
4. Modify the **model** function to return both **W** and **L** in a tuple where `L = G.numbermon.timeAverage()`
5. Add a new list **allL** to the simulation experiment in a similar way to **allW**.

Test your simulation model with $c = 1$, $\lambda = 3$ and $\mu = 5$. - For each performance measure (W and L) produce both a point estimate and a 95% confidence interval from 50 replications. - *Compare* your results to the expected values of W and L for an M/M/1 queueing system. - *Determine* whether *Little's Law* appears to hold in your simulation results by forming a confidence interval for λ_{eff}, i.e., each replication will give one estimate of $\lambda_{eff} = \frac{L}{W}$.

b. Adding to part (a), we wish monitor the *proportion of time the server is busy.*

- Set up a new monitor called **busymon** in a similar way to **delaymon** and **numbermon**.
- Modify the **run** method (PEM) within the **Arrival** class so that whenever the state **Arrival.n** is updated, the **busymon** monitor will **observe* a $\boxed{1}$ if there are any customers in the system and a $\boxed{0}$ if there are no customers in the system.
- Modify the **model** function to return a tuple (W, L, B) where `B = G.busymon.timeAverage()`
- Add a new list **allB** to the simulation experiment in a similar way to **allL** and **allW**.
- Test your simulation model as in part (a).

[1]:
```
"""(q4.py) M/M/c queueing system with several monitors and multiple␣
↪replications"""

from SimPy.Simulation import *
import random
```

```
import numpy
import math
```

[2]:
```
def conf(L):
    """confidence interval"""
    lower = numpy.mean(L) - 1.96*numpy.std(L)/math.sqrt(len(L))
    upper = numpy.mean(L) + 1.96*numpy.std(L)/math.sqrt(len(L))
    return lower, upper
```

[3]:
```
class Source(Process):
    """generate random arrivals"""
    def run(self, N, lamb, mu):
        for i in range(N):
            a = Arrival(str(i))
            activate(a, a.run(mu))
            t = random.expovariate(lamb)
            yield hold, self, t
```

[4]:
```
class Arrival(Process):
    """an arrival"""
    n = 0  # class variable (number in system)

    def run(self, mu):
        # Event: arrival
        Arrival.n += 1   # number in system
        arrivetime = now()
        G.numbermon.observe(Arrival.n)
        if (Arrival.n>0):
            G.busymon.observe(1)
        else:
            G.busymon.observe(0)

        yield request, self, G.server
        # ... waiting in queue for server to be empty (delay) ...

        # Event: service begins
        t = random.expovariate(mu)

        yield hold, self, t
        # ... now being served (activity) ...

        # Event: service ends
        yield release, self, G.server # let go of server (takes no simulation␣
↪time)
        Arrival.n -= 1
        G.numbermon.observe(Arrival.n)
        if (Arrival.n>0):
```

```
            G.busymon.observe(1)
        else:
            G.busymon.observe(0)
        delay = now()-arrivetime
        G.delaymon.observe(delay)
```

[5]:
```
class G:
    server = 'dummy'
    delaymon = 'Monitor'
    numbermon = 'Monitor'
    busymon = 'Monitor'
```

[6]:
```
def model(c, N, lamb, mu, maxtime, rvseed):
    # setup
    initialize()
    random.seed(rvseed)
    G.server = Resource(c)
    G.delaymon = Monitor()
    G.numbermon = Monitor()
    G.busymon = Monitor()

    Arrival.n = 0

    # simulate
    s = Source('Source')
    activate(s, s.run(N, lamb, mu))
    simulate(until=maxtime)

    # gather performance measures
    W = G.delaymon.mean()
    L = G.numbermon.timeAverage()
    B = G.busymon.timeAverage()
    return W, L, B
```

[7]:
```
## Experiment ----------------
allW = []
allL = []
allB = []
allLambdaEffective = []
for k in range(50):
    seed = 123*k
    result = model(c=1, N=10000, lamb=3, mu=5, maxtime=2000000, rvseed=seed)
    allW.append(result[0])
    allL.append(result[1])
    allB.append(result[2])
    allLambdaEffective.append(result[1]/result[0])
```

[8]:
```
print("Estimate of W:", numpy.mean(allW))
print("Conf int of W:", conf(allW))
print("Estimate of L:", numpy.mean(allL))
print("Conf int of L:", conf(allL))
print("Estimate of B:", numpy.mean(allB))
print("Conf int of B:", conf(allB))
print("Estimate of LambdaEffective:", numpy.mean(allLambdaEffective))
print("Conf int of LambdaEffective:", conf(allLambdaEffective))
```

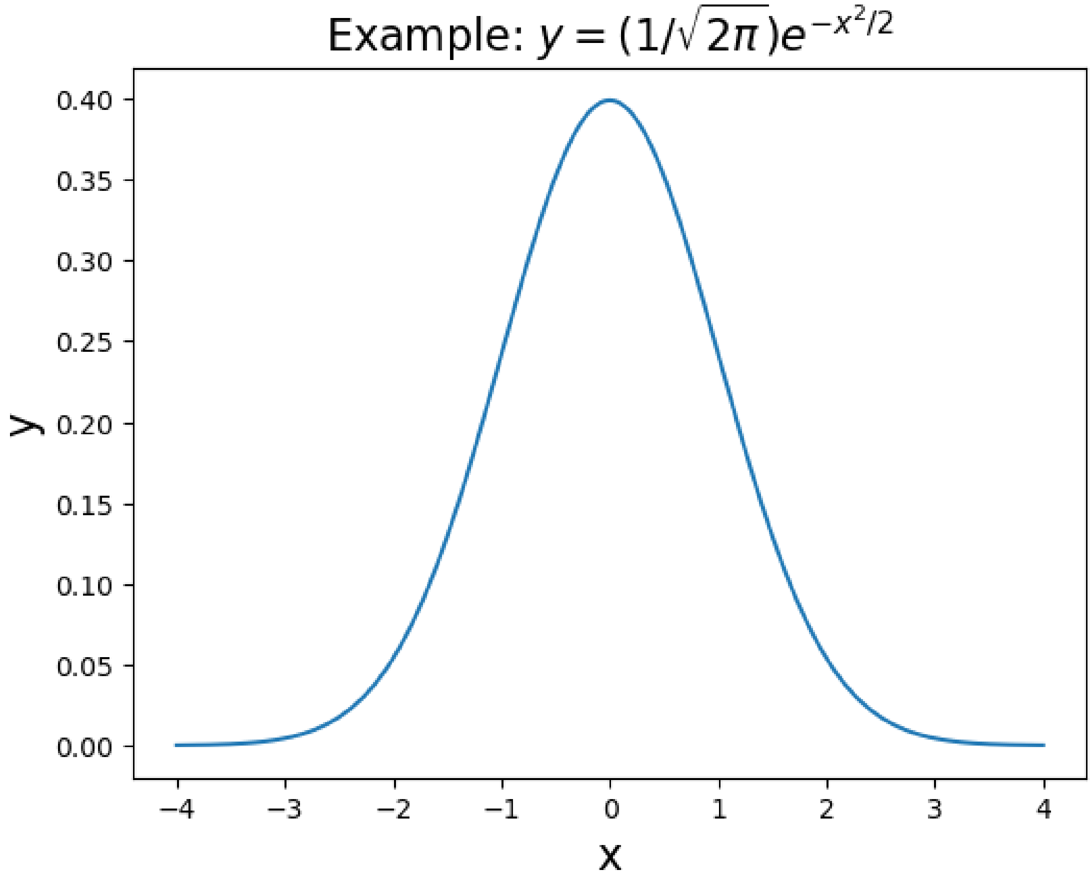
```
Estimate of W: 0.5002552383402216
Conf int of W: (0.49503594147899854, 0.5054745352014446)
Estimate of L: 1.5019964292934942
Conf int of L: (1.4841970173280463, 1.519795841258942)
Estimate of B: 0.5998344143844396
Conf int of B: (0.5977099946713437, 0.6019588340975356)
Estimate of LambdaEffective: 3.002010391868243
Conf int of LambdaEffective: (2.993370764645074, 3.010650019091412)
```

Comments: - For an M/M/1 queueing system with $\lambda = 3$ and $\mu = 5$ we expect $\rho = \frac{\lambda}{\mu} = 0.6$, $L = \frac{\rho}{1-\rho} = 1.5$ and $W = \frac{L}{\lambda} = 0.5$. Clearly all three of these values are inside their respective confidence intervals (the expected value of **B** is ρ). - Little's Law appears to hold as $\lambda = 3$ is inside the confidence interval for λ_{eff}.

1.2 Question 2

The module **matplotlib** is a Python 2D plotting library which produces publication quality figures. Run the follow fragment of Python code and explain roughly what each of the commands appear to do.

[9]:
```
import numpy
import matplotlib.pyplot as pl
x = numpy.linspace(-4,4,1000)
y = (1/numpy.sqrt(2*numpy.pi))*numpy.exp(-0.5*x**2)
pl.clf()
pl.plot(x,y)
pl.title("Example: $y=(1/\sqrt{2\pi})e^{-x^2/2}$", fontsize=16)
pl.xlabel("x", fontsize=16)
pl.ylabel("y", fontsize=16)
pl.axis("tight")
pl.savefig("myfig.png")
```


Example: $y = (1/\sqrt{2\pi})e^{-x^2/2}$
y
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−4
−3
−2
−1
0
1
2
3
4
x