School of Mathematics and Statistics
*Te Kura Mātai Tatauranga*

**DATA304/COMP312/DATA474** **Assignment 2** **Due 5 April, 18:00**

Use LaTeX, RMarkdown, Quarto or Jupiter Notebook for typing your answers. Upload a single PDF file.

The book by Gradshteyn & Ryzhik (1980, already in Talis) is one of the most complete references with expressions about series and products. Please cite it if you used it, indicating the equation numbers. Alternatively, you may use Wolfram Mathematica (Wolfram 2003, also in Talis). As a Te Herenga Waka-Victoria University of Wellington, you have a free license for this awesome software. Cite it, if you use it. It is worth noticing that the experience of using an actual book is irreplaceable and highly encouraged.

The icons have the following meaning:

Students are not allowed to use AI tools (ChatGPT, Bing Chat, Github Copilot, Google Bard, Moonbeam, etc.) to generate submitted material, or complete coursework in this problem.

## Problem #1 (7 marks)

A system with a single server has infinite capacity to serve requests, but customers are aware of the system state and they are discouraged to enter the system when they see people. Assume that the servings are Markovian with rate $\mu > 0$. If there is nobody in the system, a customer will enter with rate $\lambda > 0$. If there is one person in the system, the next customer will enter with rate $\lambda/2$. If there are two persons in the system, the next customer will enter with rate $\lambda/4\ldots$ and if there are $n$ persons in the system, the next customer will enter with rate $\lambda/2^n$.

(a) Describe the system graphically.

(b) Compute the steady-state distribution $\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2, \ldots)$.

(c) What is the expected queue length?

(d) Find a relationship between $\lambda$ and $\mu$ that grants system stability.

(e) Obtain and analyze the expected number of customers in the system. Use the ratio $\eta = \mu/\lambda$ as your parameter. Interpret.

## Problem #1 (3 marks)

Customers arrive, following a Markovian process with rate $\lambda > 0$, at a store with two counters and no waiting room. The counters serve the customers according to a Markovian process with rate $\mu > 0$. Model the system as a birth-and-death process. When is this system stable? Compute its steady-state distribution, and plot the probability function $\boldsymbol{\pi}$ that characterizes it as a function of the ratio $\lambda/\mu$. Show your code. Interpret.

## References

Gradshteyn, I. S. & Ryzhik, I. M. (1980), *Tables of Integrals, Series and Products*, Academic Press, New York.

Wolfram, S. (2003), *The Mathematica Book*, 5 edn, Wolfram Media.