Online Appendix
Estimating Quantile Production Functions:
A Control Function Approach

Justin Doty* and Suyong Song†

1 Consistency and Asymptotic Normality

In this section we discuss consistency and asymptotic normality of our estimator. Let $y_{it}^0 = y_{it} - \omega_{it}$ and assume the following:

Assumption 1.1

(a) Sampling: $(y_{it}^0, k_{it}, l_{it}, m_{it}, \omega_{it})$ *are i.i.d. defined on the probability space* (Ω, F, P) *and take values in a compact set.*

(b) Compactness and Convexity: For all $\tau \in \mathcal{T}$, $\boldsymbol{\beta}(\tau) \in int(\mathcal{B})$, *where* $\mathcal{B}$ *is compact and convex.*

(c) Full Rank and Continuity: y^0 *has bounded conditional density a.s.* $\sup_{y\in\mathbb{R}} f_{y|k,l,\omega}(y) < K$ *and*

$$\Pi(\beta, \tau, \omega) = \mathbb{E}[g_\tau(y^0, x, \omega, \beta(\tau))] = \mathbb{E}[(\tau - \mathbb{1}\{y < \beta_k k + \beta_l l + \omega\})(k, l)].$$

The Jacobian matrix $J_\beta(\beta, \tau, \omega) = \frac{\partial}{\partial\beta}\Pi(\beta, \tau, \omega)$ *is continuous and has full rank uniformly over* $\mathcal{B} \times \mathcal{T} \times \mathcal{W}$ *and* $J_\omega(\beta, \tau, \omega) = \frac{\partial}{\partial\omega}\Pi(\beta, \tau, \omega)$ *is uniformly continuous over* $\mathcal{B} \times \mathcal{T} \times \mathcal{W}$.

*Department of Economics, University of Iowa, S321 Pappajohn Business Building, 21 E Market St, Iowa City, IA 52242. Email: justin-doty@uiowa.edu

†Department of Economics and Finance, University of Iowa, W360 Pappajohn Business Building, 21 E Market St, Iowa City, IA 52242. Email: suyong-song@uiowa.edu

(d) *First-stage Estimates: The first stage estimates $\hat{\boldsymbol{\alpha}}^{\mu} = (\hat{\theta}, \hat{\boldsymbol{\beta}}^{\mu})$ admits the following expansion:*

$$\sqrt{N}(\hat{\boldsymbol{\alpha}}^{\mu} - \boldsymbol{\alpha}^{\mu}) = \sqrt{N}\mathbb{E}_N[\psi_{it}] + o_p(1), \quad \mathbb{E}[\psi_{it}] = 0, \quad \mathbb{E}[\psi_{it}\psi_{it}^{'}] < \infty.$$

Assumption 1.1 is similar to Assumption 2 in Chernozhukov and Hansen (2006). The main differences lie in the conditions we place on the first stage estimates in our model. Assumption 1.1(d) is used to derive the influence of the variance of the first-stage estimates in the covariance matrix of our quantile regression estimates. The vector $\hat{\boldsymbol{\alpha}}^{\mu} = (\hat{\theta}, \hat{\boldsymbol{\beta}}^{\mu})$ refers to the first-stage nonparametric estimates from a linear sieve estimator and the second-stage production function coefficients using ACF. We state the main theorem below.

Theorem 1.1 *Suppose Assumption 1.1 holds, then as $N \to \infty$*

$$\sup_{\tau \in \mathcal{T}} ||\hat{\boldsymbol{\beta}}(\tau) - \boldsymbol{\beta}(\tau)|| \to_p 0,$$

and

$$\sqrt{N}(\hat{\boldsymbol{\beta}}(\cdot) - \boldsymbol{\beta}(\cdot)) = [-J_{\beta}(\cdot)]^{-1}\mathbb{E}_N[\varphi_{\tau}(u_{it}(\tau))x_{it} + J_{\omega}(\cdot)\delta_{it}] + o_p(1) \to_d \mathbb{G}(\cdot) \quad \text{in } \ell^{\infty}(\mathcal{T}),$$

where $u_{it}(\tau) \equiv y_{it}^0 - x_{it}\beta(\tau), \delta_{it} \equiv \mu_{zx}\psi_{it}, \mu_{zx} = (\mu_z, \mu_x) \equiv (E[p^{k_n}(z_{it})], E[x_{it}]), \varphi_{\tau}(u) \equiv \tau - \mathbb{1}\{u < 0\}, J_{\beta}(\tau) \equiv J_{\beta}(\beta, \tau, 0), J_{\omega}(\tau) \equiv J_{\omega}(\beta, \tau, 0), \mathbb{G}$ is a mean-zero Gaussian process with variance function $E[\mathbb{G}(\tau)\mathbb{G}(\tau^{'})] = J_{\beta}(\tau)^{-1}\Sigma(\tau, \tau^{'})[J_{\beta}(\tau)^{-1}]^{'}$ where $\Sigma(\tau, \tau^{'})$ is given by

$$\Sigma(\tau, \tau^{'}) = S(\tau, \tau^{'}) + J_{\omega}(\tau)\Gamma_{\delta g}(\tau^{'}) + \Gamma_{g\delta}(\tau)J_{\omega}(\tau^{'}) + J_{\omega}(\tau)\Gamma_{\delta\delta}J_{\omega}(\tau^{'})^{'},$$

where $S(\tau, \tau^{'}) = (\min\{\tau, \tau^{'}\} - \tau\tau^{'})E[xx^{'}], \Gamma_{g\delta}(\tau) = E[g_{\tau}(y^0, x, \beta(\tau))\delta]$, and $\Gamma_{\delta\delta} = E[\delta^2]$.

A formal proof of Theorem 1.1 can follow from using various lemmas found in Chernozhukov and Hansen (2005). Here, we discuss estimation of the covariance matrix of our estimator. The variance of the first-stage estimates appear through $\Gamma_{\delta g}(\tau^{'}), \Gamma_{g\delta}(\tau)$, and $\Gamma_{\delta\delta}$. In our application, the influence function ψ_{it} can be derived using standard expansions of M-estimators corresponding to the first and second stage moment conditions of ACF:

$$\psi_{it} = \begin{pmatrix} \psi_{it}^{\theta} \\ \psi_{it}^{\beta\mu} \end{pmatrix} = \begin{pmatrix} \Sigma_z^{-1} g_1(Z_t; \theta) \\ -(D_{\beta^{\mu}}\Sigma_x D_{\beta^{\mu}}^{'})^{-1} D_{\beta^{\mu}}\Sigma_x g_2(x; \beta^{\mu}, \theta) \end{pmatrix}.$$

In the first line, it is assumed that $\Sigma_z = \mathbb{E}[\mu_z\mu_z^{'}]$ is non-singular with finite norm and

$g_1(Z_t;\theta) = p^{k_n}(z_{it})\varepsilon_{it}$. In the second line, $g_2(x;\beta^\mu,\theta)$ is the moment equation following the second stage of ACF. Here, Σ_x is a positive-definite weighting matrix and $D_{\beta^\mu} = \frac{\partial}{\partial\beta^\mu} g_2(x;\beta^\mu,\theta)$. We do not focus on a particular choice for the weighting matrix Σ_x. The first stage estimates can be estimated jointly, as in Wooldridge (2009), then choice of Σ_x is the covariance matrix of the moment equations for g_1 and g_2. However, if the researcher prefers to estimate the first stage in two steps as we do, Σ_x should be chosen according to either Ai and Chen (2012) or Ackerberg *et al.* (2014).

2 Ex-Post Shocks and Measurement Error in Output

The estimator presented in this paper is biased in the presence of measurement error in the dependent variable (output). Measurement error in output is a common occurrence in production data where output is measured as deflated sales. Measurement error in inputs has been addressed by Kim *et al.* (2016). Methods for correcting measurement error in quantile regression models have been proposed by Schennach (2008), Firpo *et al.* (2017) and Wei and Carroll (2009) for error in covariates and more recently Hausman *et al.* (2021) for error in the dependent variable. We can write the random-coefficient production function with measurement error in output as

$$y_{it} = \beta_k(\eta_{it})k_{it} + \beta_l(\eta_{it})l_{it} + \omega_{it} + u_{it},$$

where u_{it} is the measurement error in output. Hausman *et al.* (2021) assume that the measurement error is independent of covariates and the dependent variable, which implies a weaker condition $\mathbb{E}[u_{it}|\mathcal{I}_{it}] = 0$. The conditional mean of the above equation is

$$y_{it} = \beta_k^\mu k_{it} + \beta_l^\mu l_{it} + \omega_{it} + u_{it} + \varepsilon_{it},$$

where again, $\varepsilon_{it} = k_{it}[\beta_k(\eta_{it}) - \beta_k^\mu] + l_{it}[\beta_l(\eta_{it}) - \beta_l^\mu]$ with $\mathbb{E}[\varepsilon_{it}|\mathcal{I}_{it}] = 0$. Then, $\mathbb{E}[u_{it} + \varepsilon_{it}|\mathcal{I}_{it}] = 0$ so that estimating the first stage using Ackerberg *et al.* (2015) remains unchanged. Then second-stage estimates of productivity are again $\omega_{it} = \hat{\Phi}_t(k_{it}, l_{it}, m_{it}) - \hat{\beta}_k^\mu k_{it} - \hat{\beta}_l^\mu l_{it}$. Plugging into the dependent variable:

$$y_{it} - \hat{\omega}_{it} = \hat{y}_{it} = \beta_k(\eta_{it})k_{it} + \beta_l(\eta_{it})l_{it} + u_{it}.$$

Mild distributional assumptions are placed on u_{it} so that $\beta(\tau)$ can be estimated using sieve-maximum likelihood. The probability density function of u_{it} can be specified as $f_{u|\sigma}$ with parameter σ. Then, using the fact that the conditional CDFs of $y^0 = y - \omega$ are uniformly distributed, we can define a maximum likelihood estimator as

$$(\hat{\beta}(\cdot), \hat{\sigma}) = \underset{(\beta(\cdot),\sigma)\in\mathcal{B}}{\text{argmax}} \mathbb{E}_n[\log\, g(\hat{y}_{it}|k_{it}, l_{it}; \beta(\cdot), \sigma)], \quad (1)$$

where $g(y^0|k, l; \beta(\cdot), \sigma) = \int_0^1 f(y^0 - \beta_k(\eta)k - \beta_l(\eta)l|\sigma)d\eta$. Using this estimator inspired by Hausman *et al.* (2021) could correct for the measurement error in output and possibly reveal more heterogeneity in the conditional output distribution.

3 Data Appendix

3.1 U.S.

Table 1: Summary Statistics (in logs) for U.S. Manufacturing Firms

Industry (NAICS code)		1st Qu.	Median	3rd Qu.	Mean	sd
31 (Total=10000)	Output	11.61	12.8	14.26	12.96	1.92
	Capital	10.38	11.84	13.34	11.85	2.17
	Labor	-0.48	0.86	2.07	0.82	1.81
	Materials	11.44	12.6	14.02	12.72	1.92
32 (Total=20115)	Output	11.3	12.91	14.5	12.92	2.23
	Capital	10.18	11.97	13.83	12.01	2.52
	Labor	-1.09	0.34	1.76	0.36	2
	Materials	10.9	12.52	14.1	12.49	2.26
33 (Total=52804)	Output	10.24	11.74	13.25	11.78	2.12
	Capital	9.18	10.75	12.33	10.79	2.29
	Labor	-1.51	-0.15	1.19	-0.09	1.9
	Materials	9.7	11.26	12.82	11.27	2.22
All (Total=82919)	Output	10.63	12.15	13.71	12.2	2.2
	Capital	9.48	11.15	12.86	11.22	2.4
	Labor	-1.29	0.09	1.46	0.13	1.94
	Materials	10.15	11.74	13.33	11.74	2.29

Variable Construction:

- Output: Deflated Net Sales from Compustat (SALE).

- Capital: Deflated Gross Property Plant and Equipment (PPEGT).
- Labor: Number of Workers (EMPLOY).
- Labor Expense: EMPLOY times average industry wage calculated from the ratio of PAY and EMP in the NBER-CES Manufacturing Industry Database.
- Materials: Deflated Sales (SALE)-Operating Income before Depreciation (OIBDP) less labor expense.
- Value-Added: Deflated sales minus deflated materials.
- R&D: XRD in Compustat.
- Advertising Expense: XAD in Compustat.

3.2 Chile

Table 2: Summary Statistics (in logs) for Chilean Manufacturing Plants

Industry (ISIC code)		1st Qu.	Median	3rd Qu.	Mean	sd
311 (Total=18562)	Output	10.17	10.76	12.13	11.3	1.59
	Capital	6.63	7.54	8.96	7.94	1.98
	Labor	3.07	3.48	4.1	3.72	0.96
	Materials	9.81	10.4	11.7	10.9	1.57
381 (Total=5289)	Output	10.43	11.33	12.31	11.43	1.38
	Capital	7.36	8.4	9.58	8.51	1.74
	Labor	2.85	3.4	4.12	3.65	1.09
	Materials	9.81	10.7	11.71	10.79	1.42
321 (Total=5413)	Output	10.45	11.34	12.44	11.48	1.41
	Capital	7.17	8.26	9.46	8.35	1.7
	Labor	2.97	3.46	4.27	3.67	0.94
	Materials	9.68	10.56	11.68	10.72	1.47
All (Total=65886)	Output	10.31	11.18	12.44	11.48	1.61
	Capital	7.1	8.22	9.6	8.44	1.92
	Labor	2.99	3.51	4.29	3.74	1.04
	Materials	9.76	10.6	11.82	10.88	1.63

Variable Construction

- Output: Deflated sales. Sales of goods produced plus the difference of final and beginning of year inventories.
- Capital: Deflated sum of capital structures, equipment, and transportation using perpetual inventory method.
- Labor: Total number of workers.
- Materials: Total expenditure on materials plus the difference of final and beginning of year inventories. Deflated by industry level index.
- Value-added: Deflated sales minus deflated materials.

3.3 Colombia

Table 3: Summary Statistics (in logs) for Colombian Manufacturing Plants

Industry (ISIC code)		1st Qu.	Median	3rd Qu.	Mean	sd
311 (Total=12631)	Output	9.06	10.25	11.65	10.46	1.8
	Capital	6.04	7.09	8.38	7.27	1.78
	Labor	2.56	3.14	4.01	3.38	1.11
	Materials	8.43	9.77	11.31	9.91	2.01
322 (Total=11600)	Output	8.73	9.42	10.25	9.54	1.17
	Capital	5.48	6.15	6.95	6.25	1.21
	Labor	2.77	3.3	3.97	3.4	1
	Materials	7.68	8.57	9.5	8.53	1.51
381 (Total=7070)	Output	8.55	9.33	10.36	9.58	1.43
	Capital	5.94	6.8	7.86	6.99	1.54
	Labor	2.71	3.22	3.95	3.39	1
	Materials	7.81	8.69	9.77	8.84	1.58
All (Total=83684)	Output	8.75	9.67	10.93	9.97	1.67
	Capital	5.93	6.93	8.21	7.16	1.77
	Labor	2.71	3.3	4.14	3.51	1.11
	Materials	7.94	8.98	10.32	9.19	1.88

Variable Construction

- Output: Deflated sales. Sales of goods produced plus the difference of final and beginning of year inventories.

- Capital: Deflated sum of capital structures, equipment, and transportation using perpetual inventory method.
- Labor: Total number of workers.
- Materials: Total expenditure on materials plus the difference of final and beginning of year inventories. Deflated by industry level index.
- Value-added: Deflated sales minus deflated materials.

4 Estimates using Gross-Output Production Function

In this section, we provide the estimation results using gross-output production functions, where productivity is estimated from LP approach.

4.1 U.S.

Table 4: Coefficient Estimates and Standard Errors for U.S. Manufacturing Firms

		Capital		Labor		Materials		Returns to Scale		Capital Intensity	
NAICS	τ	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e
31	0.10	0.123	0.0181	0.132	0.0101	0.768	0.0317	1.022	0.0165	0.928	0.1503
	0.25	0.149	0.0194	0.118	0.0107	0.752	0.0328	1.019	0.0165	1.259	0.1902
	0.50	0.173	0.0199	0.095	0.0097	0.739	0.0333	1.006	0.0162	1.831	0.2536
	0.90	0.263	0.0287	0.064	0.0199	0.679	0.0391	1.006	0.0229	4.134	3.0492
32	0.10	0.138	0.0157	0.191	0.0122	0.702	0.0251	1.031	0.0120	0.724	0.0926
	0.25	0.144	0.0160	0.175	0.0121	0.704	0.0250	1.024	0.0120	0.827	0.1102
	0.50	0.142	0.0160	0.158	0.0114	0.718	0.0247	1.017	0.0122	0.898	0.1262
	0.90	0.212	0.0219	0.106	0.0123	0.676	0.0300	0.995	0.0136	1.993	0.3298
33	0.10	0.090	0.0085	0.352	0.0038	0.586	0.0118	1.027	0.0046	0.255	0.0244
	0.25	0.105	0.0086	0.328	0.0039	0.582	0.0119	1.014	0.0045	0.320	0.0265
	0.50	0.122	0.0089	0.305	0.0038	0.574	0.0121	1.001	0.0045	0.399	0.0298
	0.90	0.174	0.0103	0.273	0.0043	0.541	0.0134	0.987	0.0048	0.637	0.0401
All	0.10	0.150	0.0087	0.258	0.0057	0.624	0.0114	1.032	0.0046	0.582	0.0368
	0.25	0.153	0.0077	0.240	0.0053	0.633	0.0107	1.026	0.0043	0.639	0.0352
	0.50	0.163	0.0076	0.222	0.0052	0.634	0.0105	1.019	0.0043	0.735	0.0389
	0.90	0.222	0.0097	0.187	0.0057	0.600	0.0123	1.009	0.0050	1.185	0.0663

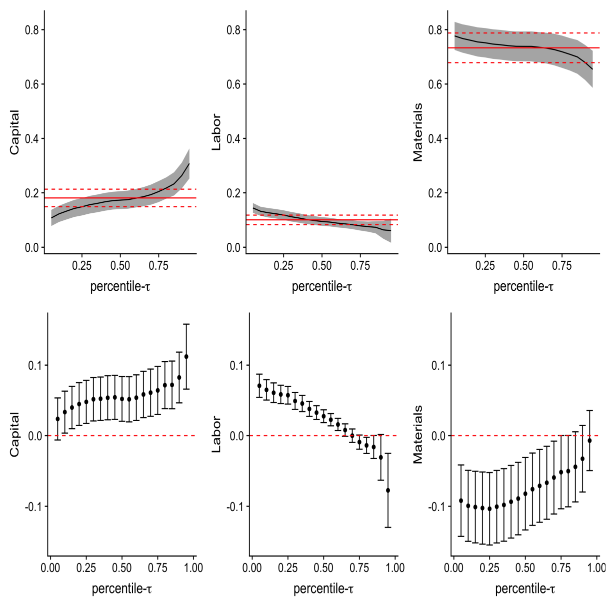
*Standard errors are obtained using bootstrap with 500 replications. The first stage uses estimates from LP.

Table 5: LP Coefficient Estimates and Standard Errors for U.S. Manufacturing Firms

	Capital		Labor		Materials		Returns to Scale		Capital Intensity	
NAICS	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e
31	0.181	0.0196	0.100	0.0107	0.733	0.0332	1.015	0.0165	1.802	0.2569
32	0.157	0.0164	0.155	0.0113	0.705	0.0258	1.016	0.0122	1.014	0.1307
33	0.123	0.0088	0.313	0.0038	0.572	0.0122	1.008	0.0045	0.394	0.0286
All	0.172	0.0078	0.224	0.0052	0.626	0.0109	1.022	0.0043	0.767	0.0393

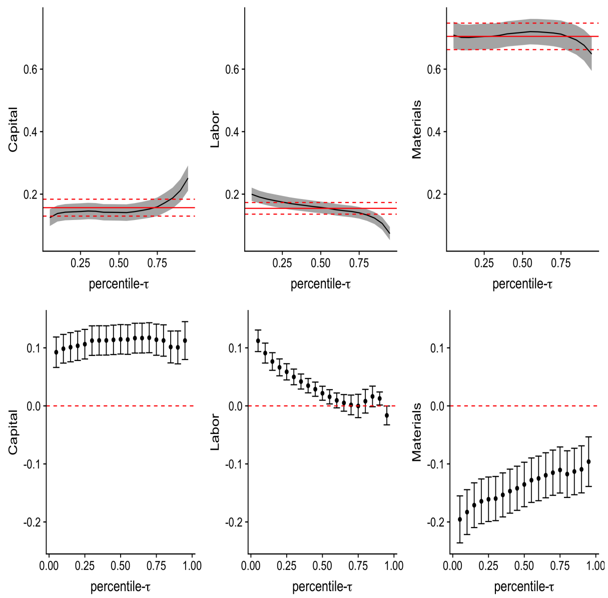
*Standard errors are obtained using bootstrap with 500 replications.

Figure 1: Estimated Coefficients of Capital and Labor for U.S.: NAICS 31



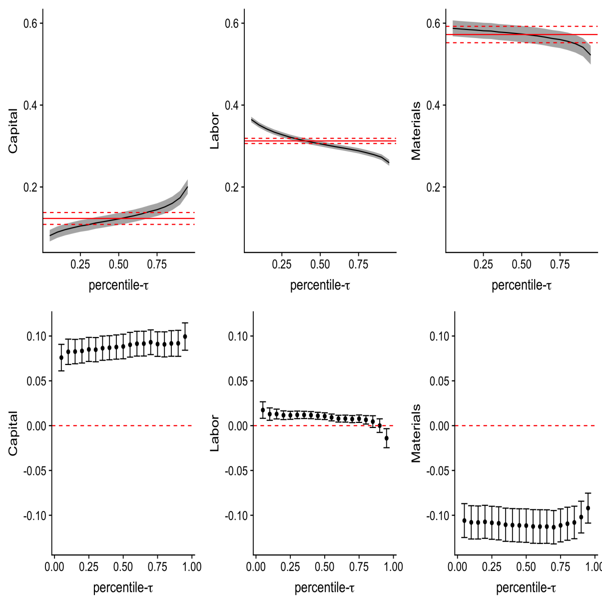
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 2: Estimated Coefficients of Capital and Labor for U.S.: NAICS 32



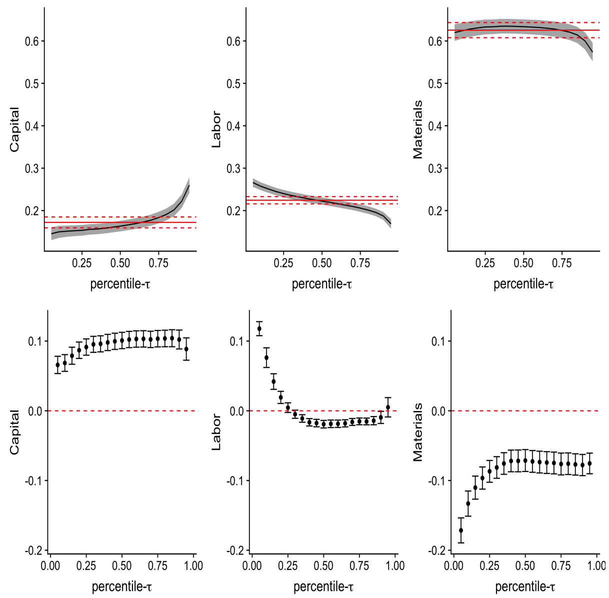
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 3: Estimated Coefficients of Capital and Labor for U.S.: NAICS 33



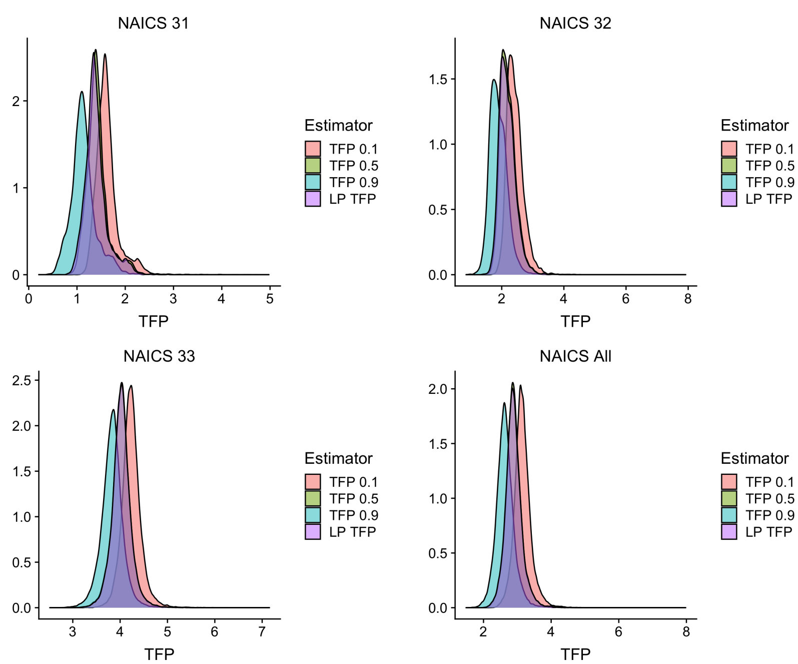
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 4: Estimated Coefficients of Capital and Labor U.S. Manufacturing Firms



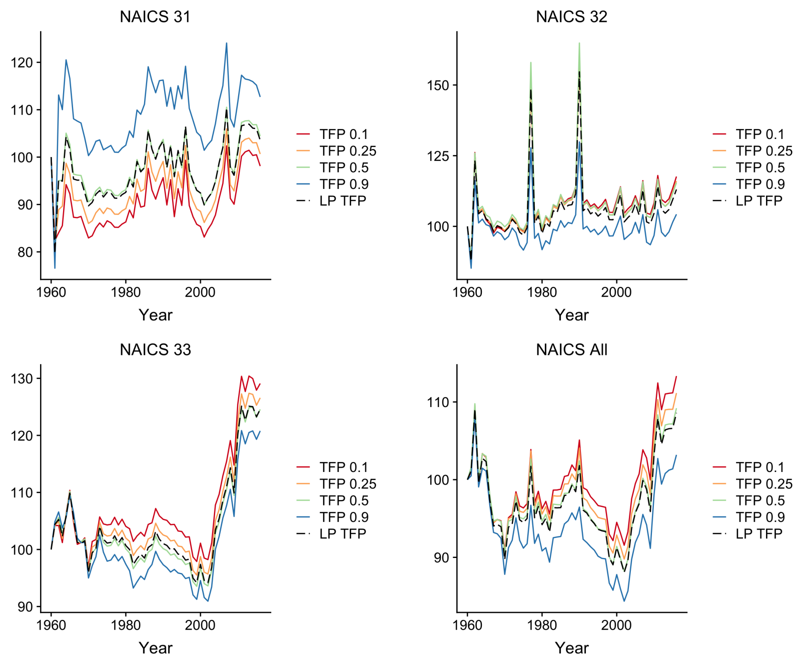
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 5: DS and LP Estimates of Log Total Factor Productivity



*Estimated distributions of TFP from the DS estimator for $\tau \in \{0.1, 0.5, 0.9\}$ and those from the LP estimator.

Figure 6: U.S. Productivity Over Time



*Estimated average productivity over time for the U.S. Base productivity in 1961 is set to 100.

Table 6: Productivity Differentials for U.S. Manufacturing Firms using DS

		R&D		Advertisements	
NAICS	τ	Coef.	s.e	Coef.	s.e
31	0.10	0.011	0.0117	0.010	0.0140
	0.25	0.011	0.0117	0.010	0.0141
	0.50	0.017	0.0115	0.016	0.0137
	0.90	0.012	0.0121	0.010	0.0140
32	0.10	-0.008	0.0088	-0.010	0.0093
	0.25	-0.004	0.0088	-0.006	0.0093
	0.50	-0.001	0.0090	-0.002	0.0095
	0.90	0.011	0.0099	0.007	0.0102
33	0.10	-0.001	0.0041	-0.001	0.0037
	0.25	0.007	0.0040	0.005	0.0037
	0.50	0.015	0.0040	0.012	0.0037
	0.90	0.023	0.0043	0.017	0.0039
All	0.10	-0.008	0.0039	-0.012	0.0037
	0.25	-0.006	0.0037	-0.010	0.0036
	0.50	-0.003	0.0037	-0.007	0.0036
	0.90	0.001	0.0040	-0.006	0.0039

*Standard errors are obtained using bootstrap with 500 replications. Log(TFP) is regressed on log(R&D) and log(Advertisements).

Table 7: Productivity Differentials for U.S. Manufacturing Firms using LP

	R&D		Advertisements	
NAICS	Coef.	s.e	Coef.	s.e
31	0.012	0.0110	0.010	0.0132
32	-0.001	0.0090	-0.003	0.0094
33	0.010	0.0040	0.008	0.0036
All	-0.005	0.0037	-0.009	0.0036

*Standard errors are obtained using bootstrap with 500 replications. Log(TFP) is regressed on log(R&D) and log(Advertisements).

4.2 Chile

Table 8: Coefficient Estimates and Standard Errors for Chilean Manufacturing Plants

		Capital		Labor		Materials		Returns to Scale		Capital Intensity	
ISIC	τ	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e
311	0.10	0.026	0.0058	0.109	0.0086	0.762	0.0106	0.898	0.0109	0.237	0.0610
	0.25	0.045	0.0056	0.102	0.0065	0.761	0.0106	0.907	0.0106	0.437	0.0618
	0.50	0.061	0.0059	0.118	0.0063	0.754	0.0105	0.934	0.0107	0.517	0.0592
	0.90	0.081	0.0069	0.153	0.0111	0.756	0.0110	0.991	0.0118	0.528	0.0665
381	0.10	0.053	0.0137	0.326	0.0266	0.589	0.0183	0.968	0.0197	0.163	0.0502
	0.25	0.084	0.0129	0.295	0.0195	0.594	0.0166	0.973	0.0161	0.285	0.0530
	0.50	0.109	0.0131	0.274	0.0167	0.599	0.0158	0.982	0.0156	0.398	0.0611
	0.90	0.132	0.0159	0.285	0.0273	0.615	0.0165	1.032	0.0213	0.462	0.0845
321	0.10	0.022	0.0121	0.272	0.0224	0.620	0.0169	0.914	0.0178	0.082	0.0475
	0.25	0.048	0.0120	0.253	0.0168	0.621	0.0152	0.923	0.0154	0.191	0.0532
	0.50	0.072	0.0121	0.232	0.0152	0.625	0.0144	0.929	0.0150	0.310	0.0646
	0.90	0.108	0.0131	0.202	0.0204	0.639	0.0150	0.949	0.0178	0.536	0.1000
All	0.10	0.038	0.0043	0.187	0.0070	0.672	0.0062	0.898	0.0063	0.205	0.0255
	0.25	0.074	0.0043	0.165	0.0062	0.667	0.0061	0.906	0.0060	0.450	0.0338
	0.50	0.104	0.0043	0.155	0.0057	0.664	0.0059	0.923	0.0057	0.670	0.0409
	0.90	0.148	0.0055	0.174	0.0100	0.663	0.0064	0.984	0.0071	0.852	0.0708

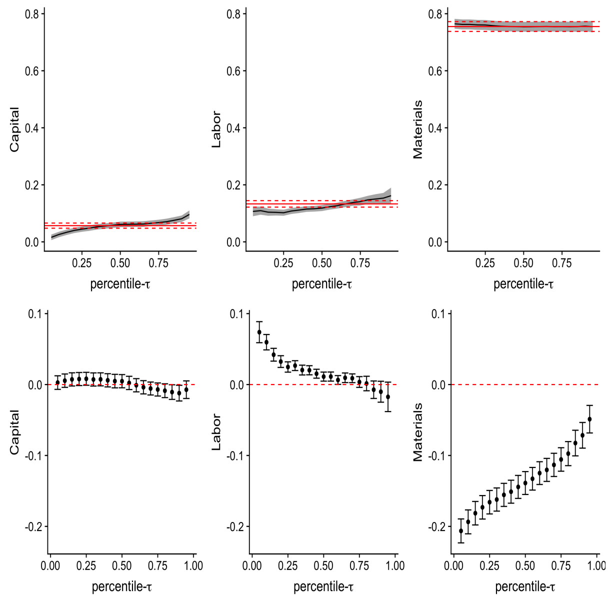
* Standard errors are obtained using bootstrap with 500 replications. The first stage uses estimates from LP.

Table 9: LP Coefficient Estimates and Standard Errors for Chilean Manufacturing Plants

	Capital		Labor		Materials		Returns to Scale		Capital Intensity	
ISIC	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e
311	0.057	0.0055	0.133	0.0069	0.755	0.0105	0.945	0.0105	0.427	0.0496
381	0.093	0.0130	0.288	0.0187	0.607	0.0154	0.988	0.0157	0.325	0.0574
321	0.067	0.0106	0.234	0.0146	0.629	0.0140	0.930	0.0150	0.286	0.0539
All	0.095	0.0039	0.171	0.0062	0.668	0.0059	0.934	0.0055	0.554	0.0343

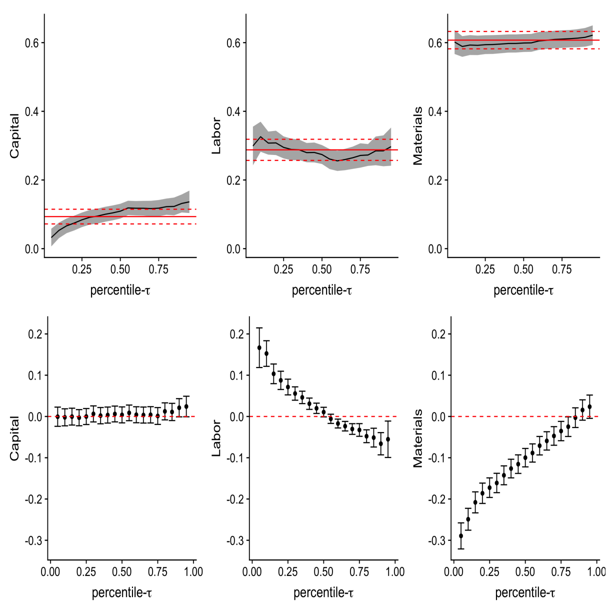
*Standard errors are obtained using bootstrap with 500 replications.

Figure 7: Estimated Coefficients of Capital and Labor for Chile: ISIC 311



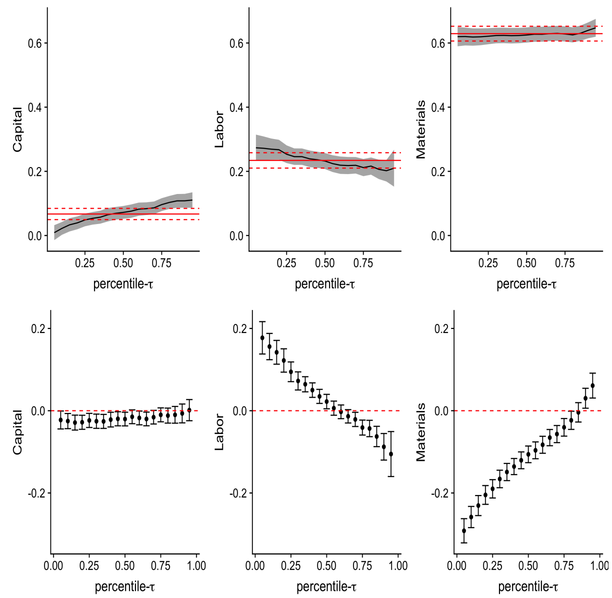
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 8: Estimated Coefficients of Capital and Labor for Chile: ISIC 381



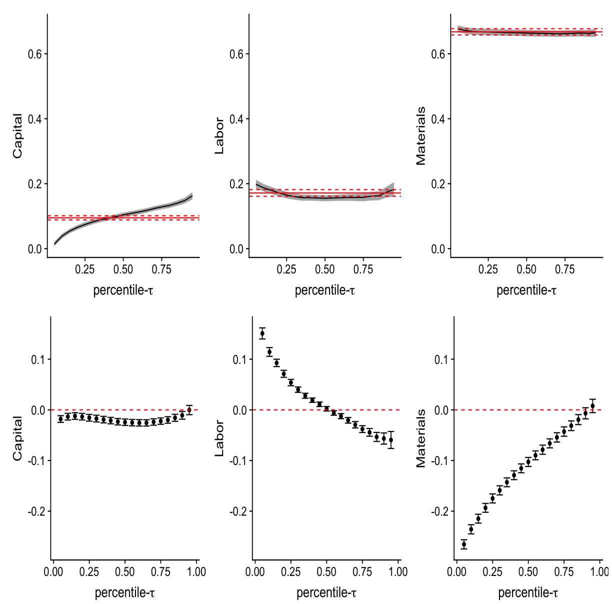
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 9: Estimated Coefficients of Capital and Labor for Chile: ISIC 321



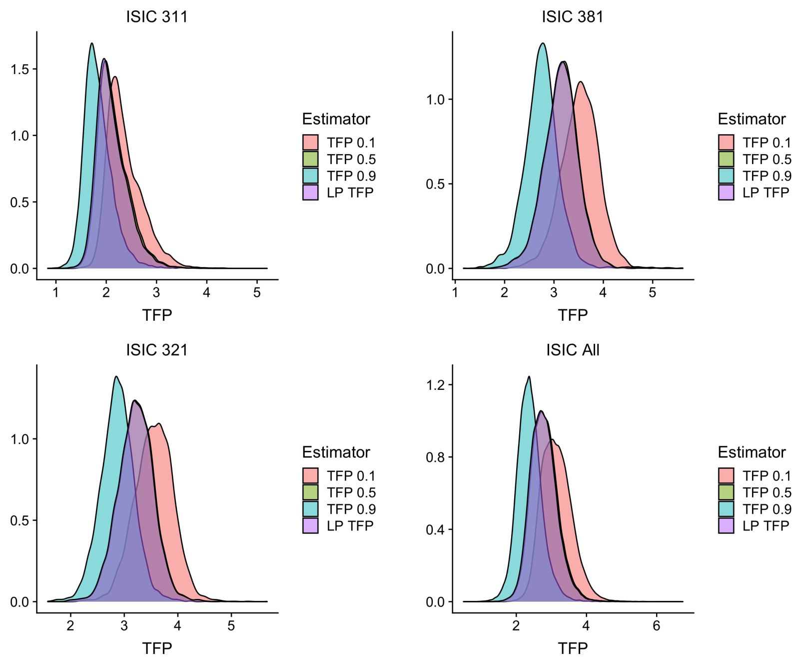
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 10: Estimated Coefficients of Capital and Labor for Chilean Manufacturing Plants



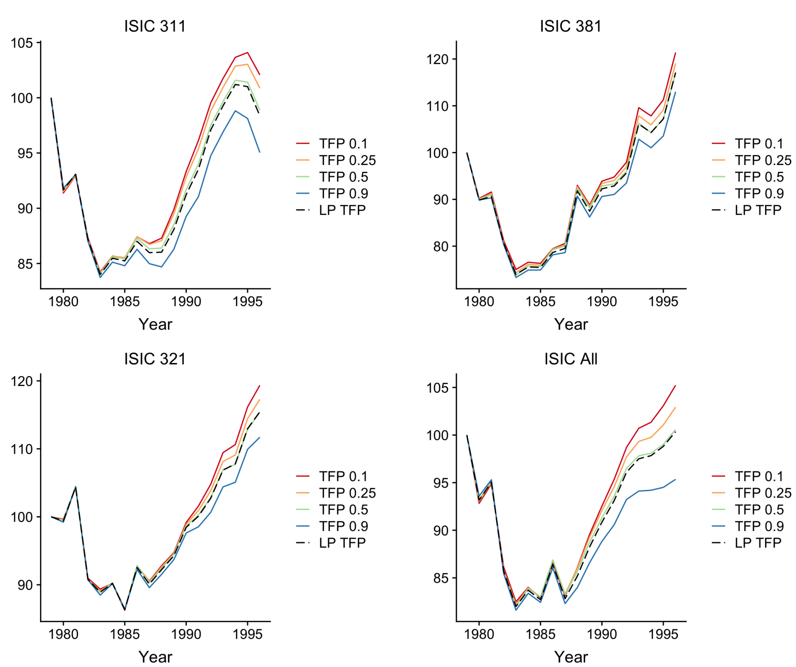
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 11: DS and LP Estimates of Log Total Factor Productivity



*Estimated distributions of TFP from the DS estimator for $\tau \in \{0.1, 0.5, 0.9\}$ and those from the LP estimator.

Figure 12: Chile Productivity Over Time



*Estimated average productivity over time for Chile. Productivity in the base year is set to 100.

Table 10: Productivity Differentials for Chilean Manufacturing Plants using DS

		Exports		Imports		Advertisements	
ISIC	τ	Coef.	s.e	Coef.	s.e	Coef.	s.e
311	0.10	0.026	0.0110	0.039	0.0122	0.019	0.0087
	0.25	0.022	0.0108	0.037	0.0120	0.017	0.0087
	0.50	0.018	0.0107	0.028	0.0116	0.012	0.0087
	0.90	0.012	0.0111	0.009	0.0120	0.002	0.0088
381	0.10	0.024	0.0125	0.049	0.0176	0.037	0.0154
	0.25	0.016	0.0123	0.037	0.0179	0.030	0.0149
	0.50	0.008	0.0125	0.026	0.0185	0.024	0.0149
	0.90	-0.011	0.0136	-0.003	0.0196	0.006	0.0165
321	0.10	0.022	0.0111	0.031	0.0145	0.043	0.0133
	0.25	0.019	0.0107	0.027	0.0140	0.040	0.0122
	0.50	0.018	0.0107	0.023	0.0142	0.038	0.0122
	0.90	0.012	0.0112	0.012	0.0152	0.031	0.0130
All	0.10	0.038	0.0047	0.061	0.0070	0.053	0.0055
	0.25	0.031	0.0046	0.054	0.0069	0.051	0.0053
	0.50	0.023	0.0046	0.045	0.0068	0.046	0.0052
	0.90	0.002	0.0046	0.017	0.0069	0.030	0.0053

*Standard errors are obtained using bootstrap with 500 replications. Log(TFP) is regressed on log(Exports), log(Imports), and log(Advertisements).

Table 11: Productivity Differentials for Chilean Manufacturing Plants using LP

	Exports		Imports		Advertisements	
ISIC	Coef.	s.e	Coef.	s.e	Coef.	s.e
311	0.018	0.0108	0.024	0.0117	0.010	0.0086
381	0.008	0.0124	0.026	0.0182	0.023	0.0149
321	0.017	0.0106	0.022	0.0139	0.037	0.0122
All	0.021	0.0046	0.042	0.0068	0.043	0.0052

*Standard errors are obtained using bootstrap with 500 replications. Log(TFP) is regressed on log(Exports), log(Imports), and log(Advertisements).

4.3 Colombia

Table 12: Coefficient Estimates and Standard Errors for Colombian Manufacturing Plants

		Capital		Labor		Materials		Returns to Scale		Capital Intensity	
ISIC	τ	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e
311	0.10	0.049	0.0139	0.199	0.0174	0.648	0.0170	0.896	0.0167	0.244	0.0799
	0.25	0.083	0.0137	0.179	0.0103	0.642	0.0169	0.904	0.0142	0.466	0.0856
	0.50	0.110	0.0143	0.155	0.0078	0.642	0.0173	0.906	0.0139	0.707	0.1035
	0.90	0.198	0.0166	0.144	0.0151	0.612	0.0188	0.954	0.0156	1.373	0.1989
322	0.10	0.112	0.0157	0.394	0.0198	0.410	0.0165	0.916	0.0233	0.285	0.0451
	0.25	0.157	0.0155	0.353	0.0162	0.408	0.0161	0.917	0.0218	0.444	0.0522
	0.50	0.198	0.0167	0.302	0.0155	0.412	0.0163	0.912	0.0216	0.655	0.0705
	0.90	0.270	0.0231	0.256	0.0243	0.415	0.0185	0.942	0.0228	1.053	0.1670
381	0.10	0.070	0.0251	0.339	0.0244	0.378	0.0221	0.787	0.0320	0.206	0.0811
	0.25	0.113	0.0249	0.291	0.0176	0.380	0.0208	0.784	0.0308	0.388	0.0922
	0.50	0.153	0.0249	0.254	0.0170	0.377	0.0203	0.785	0.0312	0.602	0.1137
	0.90	0.261	0.0287	0.218	0.0294	0.348	0.0239	0.827	0.0333	1.197	0.2541
All	0.10	0.072	0.0060	0.309	0.0070	0.468	0.0070	0.849	0.0078	0.234	0.0210
	0.25	0.110	0.0060	0.280	0.0058	0.468	0.0066	0.859	0.0075	0.393	0.0247
	0.50	0.148	0.0061	0.256	0.0048	0.466	0.0066	0.870	0.0071	0.578	0.0285
	0.90	0.238	0.0072	0.257	0.0090	0.442	0.0073	0.937	0.0082	0.927	0.0492

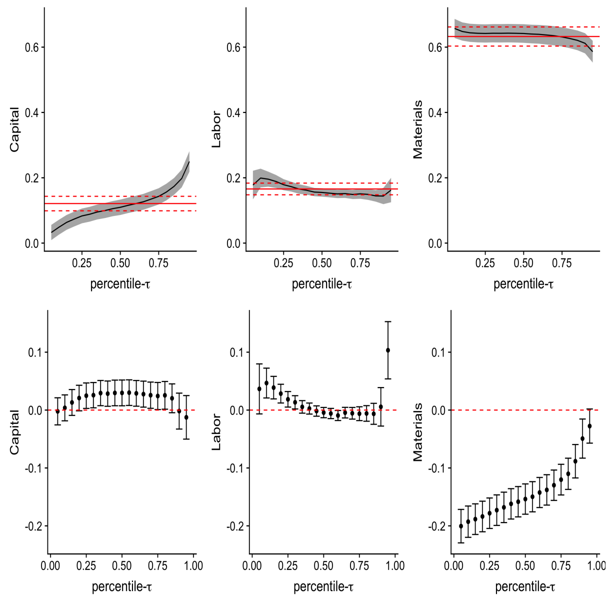
* Standard errors are obtained using bootstrap with 500 replications. The first stage uses estimates from LP.

Table 13: LP Coefficient Estimates and Standard Errors for Colombian Manufacturing Plants

	Capital		Labor		Materials		Returns to Scale		Capital Intensity	
ISIC	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e
311	0.121	0.0135	0.166	0.0109	0.633	0.0178	0.919	0.0142	0.730	0.0960
322	0.189	0.0166	0.305	0.0175	0.420	0.0162	0.914	0.0219	0.621	0.0717
381	0.162	0.0256	0.244	0.0212	0.378	0.0205	0.784	0.0308	0.663	0.1385
All	0.153	0.0059	0.266	0.0069	0.461	0.0068	0.881	0.0077	0.576	0.0294

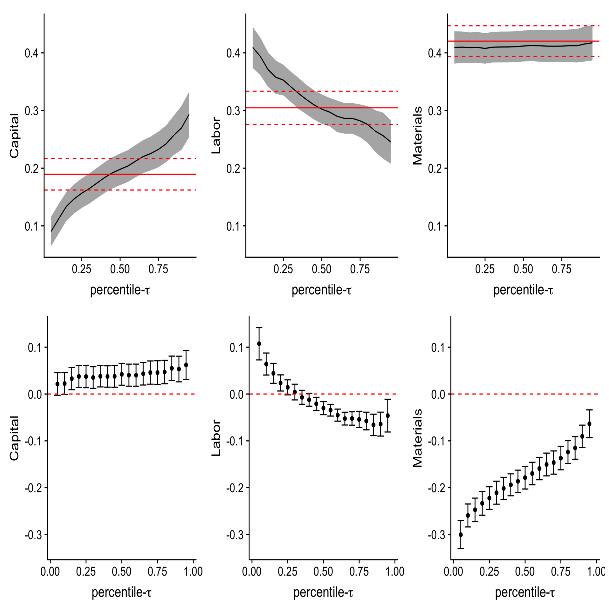
*Standard errors are obtained using bootstrap with 500 replications.

Figure 13: Estimated Coefficients of Capital and Labor for Colombia: ISIC 311



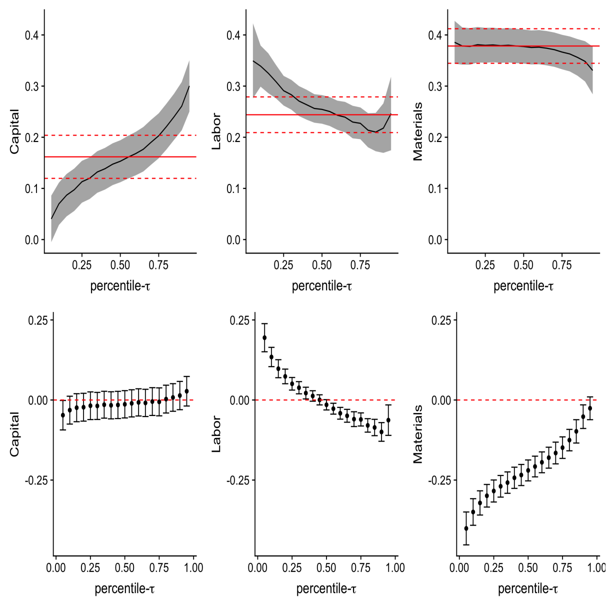
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 14: Estimated Coefficients of Capital and Labor for Colombia: ISIC 321



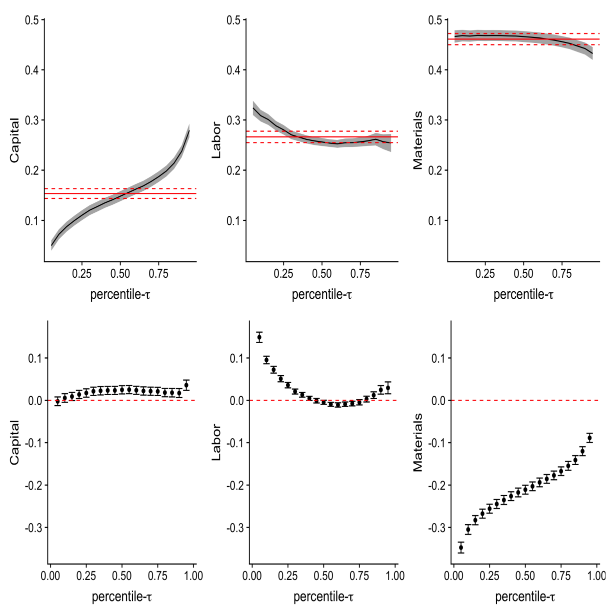
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 15: Estimated Coefficients of Capital and Labor for Colombia: ISIC 381



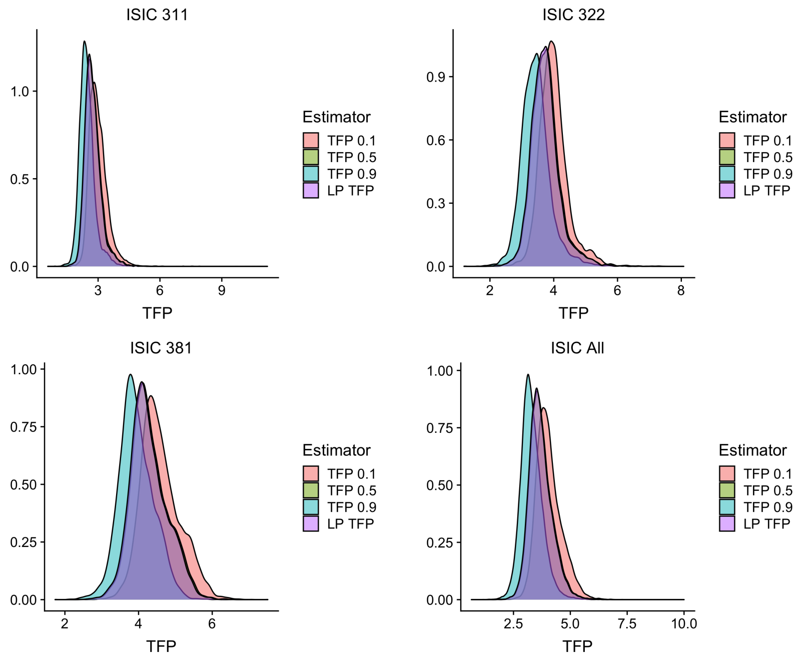
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 16: Estimated Coefficients of Capital and Labor for Colombian Manufacturing Plants



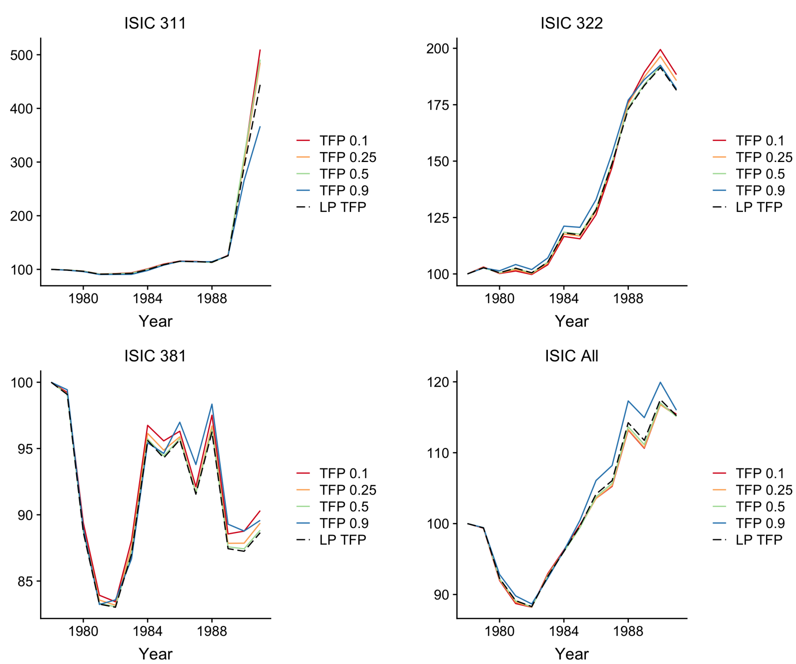
*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

Figure 17: DS and LP Estimates of Log Total Factor Productivity



*Estimated distributions of TFP from the DS estimator for $\tau \in \{0.1, 0.5, 0.9\}$ and those from the LP estimator.

Figure 18: Colombian Productivity Over Time



*Estimated average productivity (in levels) over time for Colombia. Base year productivity is set to 100.

Table 14: Productivity Differentials for Colombian Manufacturing Plants using DS

		Exports		Imports		Advertisements	
ISIC	τ	Coef.	s.e	Coef.	s.e	Coef.	s.e
311	0.10	0.041	0.0144	0.087	0.0167	0.064	0.0144
	0.25	0.036	0.0139	0.084	0.0152	0.063	0.0134
	0.50	0.032	0.0137	0.082	0.0141	0.063	0.0126
	0.90	0.015	0.0135	0.069	0.0124	0.056	0.0113
322	0.10	0.049	0.0126	0.092	0.0147	0.073	0.0117
	0.25	0.046	0.0118	0.086	0.0141	0.066	0.0115
	0.50	0.044	0.0114	0.079	0.0140	0.060	0.0116
	0.90	0.034	0.0106	0.060	0.0142	0.043	0.0120
381	0.10	0.118	0.0226	0.155	0.0246	0.142	0.0227
	0.25	0.114	0.0221	0.151	0.0238	0.137	0.0220
	0.50	0.109	0.0219	0.146	0.0233	0.131	0.0216
	0.90	0.089	0.0210	0.126	0.0216	0.108	0.0202
All	0.10	0.074	0.0059	0.129	0.0058	0.101	0.0049
	0.25	0.068	0.0056	0.120	0.0056	0.094	0.0047
	0.50	0.061	0.0053	0.111	0.0055	0.088	0.0046
	0.90	0.042	0.0048	0.086	0.0055	0.068	0.0044

*Standard errors are obtained using bootstrap with 500 replications. Log(TFP) is regressed on log(Exports), log(Imports), and log(Advertisements).

Table 15: Productivity Differentials for Colombian Manufacturing Plants using LP

	Exports		Imports		Advertisements	
ISIC	Coef.	s.e	Coef.	s.e	Coef.	s.e
311	0.029	0.0135	0.079	0.0139	0.061	0.0124
322	0.043	0.0114	0.078	0.0139	0.060	0.0115
381	0.108	0.0216	0.145	0.0230	0.130	0.0212
All	0.060	0.0053	0.109	0.0055	0.086	0.0046

*Standard errors are obtained using bootstrap with 500 replications. Log(TFP) is regressed on log(Exports), log(Imports), and log(Advertisements).

References

Ackerberg, D., Caves, K. and Frazer, G. (2015). Identification properties of recent production function estimators. *Econometrica*, **83** (6), 2411–2451.

—, Chen, X., Hahn, J. and Liao, Z. (2014). Asymptotic efficiency of semiparametric two-step GMM. *The Review of Economic Studies*, **81** (3), 919–943.

Ai, C. and Chen, X. (2012). The semiparametric efficiency bound for models of sequential moment restrictions containing unknown functions. *Journal of Econometrics*, **170** (2), 442–457.

Chernozhukov, V. and Hansen, C. (2005). An IV model of quantile treatment effects. *Econometrica*, **73** (1), 245–261.

— and — (2006). Instrumental quantile regression inference for structural and treatment effect models. *Journal of Econometrics*, **132** (2), 491–525.

Firpo, S., Galvao, A. F. and Song, S. (2017). Measurement errors in quantile regression models. *Journal of Econometrics*, **198** (1), 146–164.

Hausman, J., Liu, H., Luo, Y. and Palmer, C. (2021). Errors in the dependent variable of quantile regression models. *Econometrica*, **89** (2), 849–873.

Kim, K. I., Petrin, A. and Song, S. (2016). Estimating production functions with control functions when capital is measured with error. *Journal of Econometrics*, **190** (2), 267–279.

Schennach, S. M. (2008). Quantile regression with mismeasured covariates. *Econometric Theory*, **24** (4), 1010–1043.

Wei, Y. and Carroll, R. J. (2009). Quantile regression with measurement error. *Journal of the American Statistical Association*, **104** (487), 1129–1143.

Wooldridge, J. M. (2009). On estimating firm-level production functions using proxy variables to control for unobservables. *Economics Letters*, **104** (3), 112–114.