Heterogeneity in Firms: A Proxy Variable Approach to Quantile Production Functions

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Introduction

- Identification issues: optimal input choices are functions of unobserved productivity leads to transmission bias
- Popular control function approaches
  - OP (1994): Simultaneity and selection, investment policy proxy, application to telecommunications industry
  - LP (2003): Intermediate input proxy, application to Chilean manufacturing firms
  - ACF (2015): Structural value-Added production with intermediate input Proxy, identification under different DGPs
- These approaches have focused on estimates of output elasticities on location of conditional firm-size distribution
- There could be considerable heterogeneity not captured by average estimates

Levinsohn and Petrin (2003)

- Brief review of LP approach
- LP consider the following gross output production function (subscript i omitted)
$$y_t = \beta_0 + \beta_k k_t + \beta_l l_t + \beta_\iota \iota_t + \omega_t + \varepsilon_t \tag{1}$$
Where ω_t is productivity observed by the firm, but unobserved by the researcher (e.g, management quality, expected defect rates, etc.) and ε_t represents iid shocks to production after making input choices at time t With the following assumptions

  1. Information Set: $\mathcal{I}_t$ includes current and past productivity shocks, but not future productivity. $\mathbb{E}[\varepsilon_t|\mathcal{I}_t] = 0$
  2. First Order Markov: Productivity shocks evolve according to the distribution $p(\omega_t|\omega_{t-1})$
  3. Capital Accumulation: $k_t = \kappa(k_{t-1}, I_{t-1})$
  4. Scalar Unobservability: $\iota_t = \iota_t(k_t, \omega_t)$
  5. Strict Monotonicity: $\iota_t = \iota_t(k_t, \omega_t)$ is strictly increasing in ω_t

Levinsohn and Petrin (2003)

- Given these assumptions intermediate input demand can be inverted $\omega_t = \omega_t(k_t, \iota_t)$ and substituted into the production function

$$y_t = \beta_0 + \beta_k k_t + \beta_l l_t + \omega_t(k_t, \iota_t) + \varepsilon_t = \beta_l l_t + \Phi(k_t, \iota_t) + \varepsilon_t \quad (2)$$

  where $\Phi(k_t, \iota_t) = \beta_0 + \beta_k k_t + \beta_\iota \iota_t + \omega_t(k_t, \iota_t)$
- $\Phi(k_t, \iota_t)$ can be estimated nonparametrically
- Labor coefficient and other variable coefficients are identified in the first stage
- First stage estimates are $\hat{\beta}_l$ and $\hat{\Phi}(k_t, \iota_t)$
- Let $y^* = y - \hat{\beta}_l l_t$ denote the output net of labor contribution

Levinsohn and Petrin (2003)

- Two moment conditions identify β_k and β_ι
- Capital does not respond to the innovation in productivity
- Last period's input choice should not be correlated with the innovation in productivity this period

$$\mathbb{E}[(\varepsilon_t + \eta_t)Z_{t-1}] = \mathbb{E}[(y_t^* - \beta_k k_t - \beta_\iota \iota_t - \hat{\mathbb{E}}[\omega_t|\omega_{t-1}])Z_{t-1}] \quad (3)$$

- $\hat{\mathbb{E}}[\omega_t|\omega_{t-1}]$ can be estimated using the estimates of ω_t from first stage estimates evaluated at (β_k, β_ι)
- Z_{t-1} includes k_t and ι_{t-1} as well as additional instruments
- LP minimize GMM criterion function and bootstrap standard errors

Heterogeneous Coefficient Models

- The control function approaches of OP, LP, and ACF estimate homogeneous output elasticities
- Many interesting questions can be asked if we consider production functions that are heterogeneous between firms
- Kasahara, Schrimpf and Suzuki (2015) propose two methods to estimate a random coefficient production function and find evidence of unobserved heterogeneity beyond a Hicks-neutral technology term
- Balat, Brambilla, and Sasaki (2015) extend the control function approach to multi-dimensional coefficients with an application to international trade
- Li and Sasaki (2017) develop an identification strategy for heterogeneous elasticities using first order conditions for intermediate inputs

Production Functions and Quantile Regression

- Allowing firm technology to vary over the conditional output distribution is also valid for estimating a production function with unobserved heterogeneity
- Applications of quantile regression are however limited due to simultaneity bias
- Typical IV approaches such as Chernozhukov and Hansen (2005) are also limited due to the normal arguments of using input prices as IVs
- Possible to exploit the panel data structure and use fixed effects (Koenker, 2004), Lamarche (2010), or Canay (2011)
- The fixed effect shifts the location of the conditional distribution (not the quantiles)
- Incidental Parameters (Koenker, 2004)
- Independence of unobserved heterogeneity and productivity (Canay, 2011)

Quantile Production Function

We seek to extend the control function approach to a quantile value-production function for a fixed $\tau \in (0, 1]$ defined as

$$Q_\tau(y_t | k_t, l_t) = \beta_k(\tau)k_t + \beta_l(\tau)l_t + \omega_t \tag{4}$$

- l_t is a freely variable input like labor and k_t denotes the state variable as capital input
- ω_t is an unobserved state variable to the econometrician that impacts firm's optimal input decisions
- We allow for inputs to affect quantiles of the output distribution
- A special case of this quantile production function is the location scale model

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + (\eta_k k_{it} + \eta_l l_{it} + \eta_\omega \omega_{it})\varepsilon_{it} \tag{5}$$

which implies that the τ-th conditional quantile of y_{it} is given by

$$Q_\tau(y_{it} | \mathcal{I}_{it}) = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + (\eta_k k_{it} + \eta_l l_{it} + \eta_\omega \omega_{it})F^{-1}(\tau) \tag{6}$$

Quantile Production Function: Assumptions

We maintain several timing assumptions that are standard in the control function literature

We use the set of assumptions for LP estimation, but can be easily altered for ACF as we do both in simulations

1. Information Set: $\mathcal{I}_t$ includes current and past productivity shocks, but not future productivity. $Q_\tau(\varepsilon_t|\mathcal{I}_t) = 0$
2. Firm's productivity follows an AR(1) process

$$\omega_t = \mathbb{E}[\omega_t|\omega_{t-1}] + \xi_t = g(\omega_{t-1}) + \xi_t \tag{7}$$

3. Capital Accumulation: $k_t = \kappa(k_{t-1}, i_{t-1})$
4. Scalar Unobservability: $\iota_t = \iota_t(k_t, \omega_t)$
5. Strict Monotonicity: $\iota_t = \iota_t(k_t, \omega_t)$ is strictly increasing in ω_t

Quantile Production Function: Estimation

- Given these assumptions we invert intermediate input demand $\omega_t = \omega_t(k_t, \iota_t)$ and substitute into the production function

$$Q_\tau(y_t|k_t, l_t) = \beta_l(\tau)l_t + \beta_k(\tau)k_t + \omega_t(k_t, \iota_t) = \beta_l(\tau)l_t + \Phi_\tau(k_t, \iota_t) \quad (8)$$

- First stage parameters $\beta_l(\tau)$ and $\Phi_\tau(k_t, \iota_t)$ are estimated here
- If one uses a polynomial approximation to $\Phi_\tau(k_t, \iota_t)$, $\hat{\beta}_l(\tau)$ and $\hat{\Phi}_\tau(k_t, \iota_t)$ can be estimated using a polynomial quantile regression
- Substituting into the production

$$Q_\tau(y_t|k_t, l_t) = \beta_k(\tau)k_t + \hat{\beta}_l(\tau)l_t + g(\hat{\Phi}_\tau(k_{t-1}, \iota_{t-1}) - \beta_k(\tau)k_{t-1}) + \xi_t \quad (9)$$

- This equation is complicated by the unobservable ξ_t, we will assume that its sum with the production function shock ε_t is conditional quantile zero, but this may not be valid

Quantile Production Function: Estimation

We can write a second stage conditional quantile restriction as

$$Q_\tau[y_t - \beta_k(\tau)k_t - \hat{\beta}_l(\tau)l_t - g(\hat{\Phi}_\tau(k_{t-1}, \iota_{t-1}) - \beta_k(\tau)k_{t-1})|\mathcal{I}_{it-1}] = 0 \tag{10}$$

- The model in (10) can be represented by conditional moment restrictions

$$\mathbb{E}[\mathbb{1}\{y_t - \beta_k(\tau)k_t - \hat{\beta}_l(\tau)l_t - g(\hat{\Phi}_\tau(k_{t-1}, \iota_{t-1}) - \beta_k(\tau)k_{t-1}) \leq 0\} - \tau|\mathcal{I}_{it-1}] = 0 \tag{11}$$

  where $\mathbb{1}\{\cdot\}$ is the indicator function
- To estimate the production function parameters we use the unconditional moments

$$\mathbb{E}[Z_{t-1}(\mathbb{1}\{y_t - \beta_k(\tau)k_t - \hat{\beta}_l(\tau)l_t - g(\hat{\Phi}_\tau(k_{t-1}, \iota_{t-1}) - \beta_k(\tau)k_{t-1}) \leq 0\} - \tau)] = 0 \tag{12}$$

  where Z_{t-1} includes the instruments used in the firm's information set at time $t-1$

Smoothed GMM for Quantile Models

- The indicator function makes estimation of the production function parameters intractable
- We smooth the indicator function and rely on estimation procedure of Kaplan and Sun (2017)
- de Castro, Galvao, Kaplan, and Liu (2018) develop theory for feasible estimators of parameters in general conditional quantile restrictions that include non-linear IVQR
- This approach is computationally attractive compared to approaches such as the MCMC approach proposed by Chernozhukov and Hong (2003)
- To fix notation let
  - Z_{t-1} the set of instruments discussed earlier
  - x_t the set of exogenous and endogenous covariates
  - $\Lambda(\cdot)$ denote the residual function that defines the conditional quantile restriction that is known up to $\beta_k(\tau)$

Smoothed GMM for Quantile Models

- The sample analog of (12) can be written as:

$$\hat{M}_n(\beta,\tau) = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} Z_{it}\left[\tilde{I}\left(\frac{\Lambda(y_{it},x_{it},\beta(\tau))}{h_n}\right)-\tau\right], \tag{13}$$

where h_n is a bandwidth (sequence) and $\tilde{I}(\cdot)$ is a smoothed version of the indicator function $\mathbb{1}\{\cdot \leq 0\}$ used by Horowitz (1998), Whang (2006), and Kaplan and Sun (2016):

$$\tilde{I}(u) = \mathbb{1}\{-1 \leq u \leq 1\}\left[0.5 + \frac{105}{64}\left(u - \frac{5}{3}u^3 + \frac{7}{5}u^5 - \frac{3}{7}u^7\right)\right] + \mathbb{1}\{u > 1\}. \tag{14}$$

The smoothed GMM estimator minimizes a weighted quadratic norm of the smoothed sample moment vector

$$\hat{\beta}_{GMM} = \underset{\beta}{\operatorname{argmin}}\, \hat{M}_n(\beta,\tau)^\top \hat{W} \hat{M}_n(\beta,\tau). \tag{15}$$

Smoothed GMM for Quantile Models

- The optimal weighting matrix is the estimator of the inverse long-run variance of the sample moment $\hat{W}^* = \bar{\Omega}^{-1} \xrightarrow{p} \Omega^{-1}$
- In simulation and in application we use an estimate of the long-run variance using a truncated kernel with bandwidth choice of Andrews (1991). We find this helps with scaling.
- The asymptotic results for this estimator cannot be applied here due to the semi-parametric nature of the two-step procedure
- We are working to see if the asymptotic results of Ai and Chen (2007) can be applied here. This gives us extra conditions to show what rate gives an asymptotically negligible bandwidth. The smoothed sample moments are differentiable so we can take a Taylor expansion around $\beta(\tau)$, but the bandwidth problem remains
- We use bootstrap for our empirical application

Quantile Production Function: Simulation

- Simulations follow a location-scale version of LP (2003) and ACF (2015) using ACF's original set of DGPs
- Parameters are chosen to match a couple of key moments in the Chilean data used by LP
- Productivity follows a first order AR(1) process with persistence $\rho = 0.7$
- Firms make optimal choices of investment in the capital stock to maximize the expected discounted value of future profits
- Convex capital adjustment costs
- Labor input l_t is chosen either at t or $t - b$
- Possible optimization error in labor input decision

Quantile Production Function: Simulation

- Production function is assumed Leontief in materials, the location-scale specification is

$$y_t = \beta_k k_t + \beta_l l_t + \omega_t + (0.7k_t + 0.6l_t + 0.1\omega_t)\varepsilon_t \tag{16}$$

  with $\beta_k = 0.4$, $\beta_l = 0.6$
  Simulation 1: ACF Estimated Quantile Production Function
  - Firms face different wages, where the wage process following an AR(1) process
  - Assume labor is chosen at time $t - b$ where $b = 0.5$

  Simulation 2: LP Estimation Quantile Production Function
  - No wage variation across firms and labor chosen at time t
  - Added optimization error in labor
- Both simulations include 2 DGP's for the error term ε_t. In DGP 1 $\varepsilon_t \sim N(0, 0.1)$ and in DGP 2 $\varepsilon_t \sim Laplace(0, 0.1)$

Quantile Production Function: Simulation 1

Figure: Simulated estimators of $\beta_k(\tau)$ and $\beta_l(\tau)$. Dotted line is ACF estimator

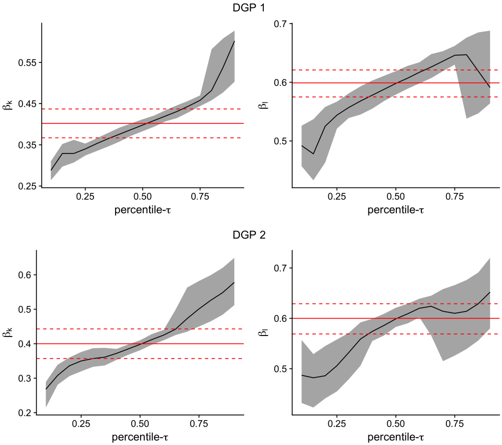


Figure: Simulated precision estimators of $\beta_k(\tau)$ and $\beta_l(\tau)$. Dotted line is ACF estimator

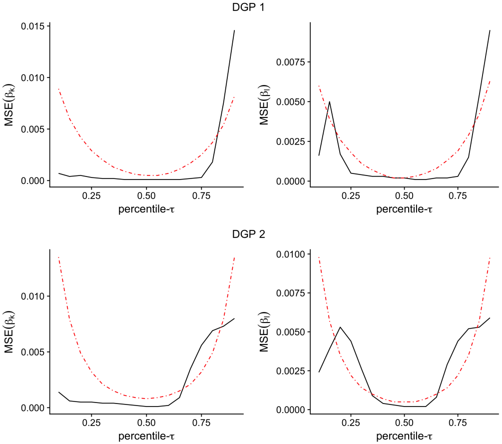
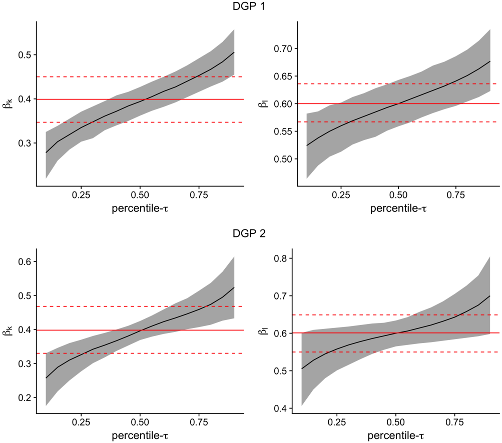
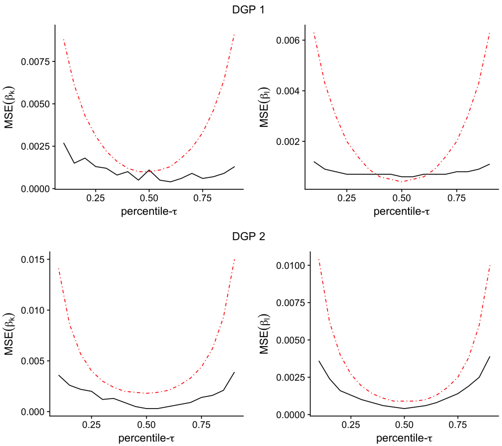


Figure: Simulated estimators of $\beta_k(\tau)$ and $\beta_l(\tau)$. Dotted line is LP estimator



Quantile Production Function: Simulation 2

Figure: Simulated precision of estimators of $\beta_k(\tau)$ and $\beta_l(\tau)$. Dotted line is LP estimator



Quantile Production Function: Application

- We apply this estimator to an 10-year panel (1987-1996) from Chile (ENIA) provided by Amil Petrin
- Focus on the four largest industries (ISIC codes in parentheses): Food Products (311), Metals (381), Textiles (321) and Wood Products (331)
- We use the QLP estimator for a value-added specification of the production function
- Standard errors are obtained from the bootstrapping method outlined by Levinsohn and Petrin (2003)

US Manufacturing: Compustat

Industry (NAICS code)		1st Qu.	Median	3rd Qu.	Mean	sd
31 (N=3271)	Output	19.05	20.24	21.57	20.3	1.77
	Capital	18.66	20.37	21.76	20.19	2.12
	Labor	17.42	19.08	20.61	19.02	2.21
	Materials	17.96	19.59	21.15	19.54	2.21
32 (N=7207)	Output	15.67	17.04	18.51	17.01	2.05
	Capital	15.65	17.51	19.13	17.31	2.41
	Labor	14.44	16.01	17.57	16.01	2.29
	Materials	14.89	16.53	18.25	16.52	2.37
33 (N=13978)	Output	7.38	8.58	9.8	8.5	1.67
	Capital	6.67	8.29	9.74	8.15	1.95
	Labor	6.01	7.42	8.91	7.48	1.93
	Materials	6.33	7.82	9.29	7.82	1.95
All (N=24456)	Output	18.58	19.78	21.23	19.85	1.79
	Capital	18.14	19.86	21.26	19.67	2.16
	Labor	16.98	18.59	20.13	18.56	2.17
	Materials	17.49	19.12	20.66	19.06	2.2

US Manufacturing: Compustat

		Capital		Labor		Returns to Scale	
Industry (NAICS code)	τ	Coef.	s.e.	Coef.	s.e.	Coef.	s.e
31	0.10	0.059	0.0308	0.612	0.0328	0.671	0.0369
	0.25	0.099	0.0213	0.558	0.0358	0.657	0.0374
	0.50	0.119	0.0275	0.474	0.0352	0.594	0.0404
	0.75	0.082	0.0247	0.441	0.0356	0.523	0.0401
32	0.10	0.039	0.0274	0.687	0.0422	0.726	0.0466
	0.25	0.079	0.0244	0.627	0.0311	0.706	0.0359
	0.50	0.154	0.0326	0.589	0.0252	0.742	0.0399
	0.75	0.146	0.0307	0.548	0.0230	0.695	0.0356
33	0.10	0.136	0.0366	0.210	0.0579	0.346	0.0624
	0.25	-0.031	0.0343	0.357	0.0401	0.327	0.0539
	0.50	0.174	0.0416	0.446	0.0289	0.620	0.0485
	0.75	0.100	0.0531	0.470	0.0203	0.571	0.0570
All	0.10	-0.091	0.0959	0.365	0.0350	0.274	0.0997
	0.25	0.072	0.0370	0.412	0.0226	0.485	0.0422
	0.50	0.192	0.0498	0.475	0.0181	0.667	0.0547
	0.75	0.420	0.0457	0.485	0.0149	0.905	0.0474

US Manufacturing: Compustat

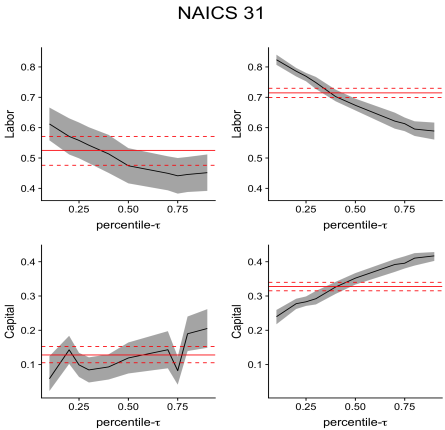
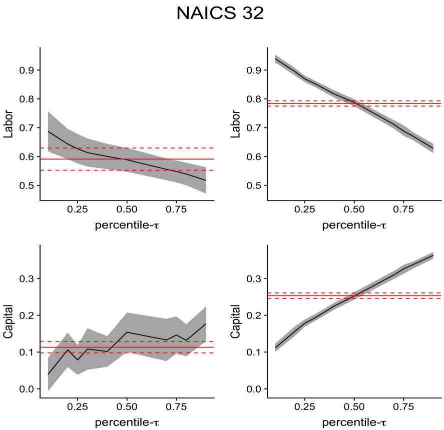


Figure: Estimated values of production function coefficients and their 90% confidence interval

US Manufacturing: Compustat



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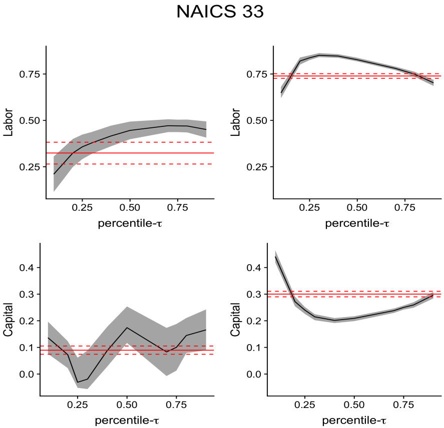

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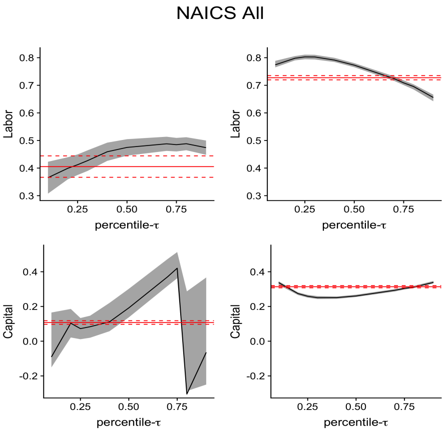

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US Manufacturing: Compustat

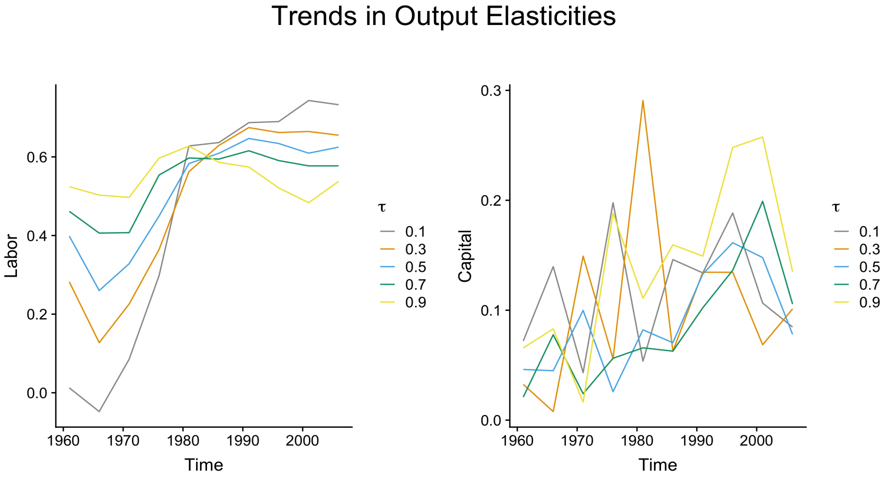


Figure: Estimated values of production function coefficients over time

Chilean Manufacturing

Industry (ISIC code)		1st Qu.	Median	3rd Qu.	Mean	sd
311 (N=13838)	Output	10.21	10.84	12.22	11.36	1.58
	Capital	10.56	11.4	12.4	11.52	1.37
	Labor	10.49	11.4	12.54	11.53	1.43
	Materials	10.38	11.28	12.53	11.56	1.6
381 (N=4311)	Output	6.69	7.66	9.06	8.02	1.98
	Capital	7.52	8.51	9.7	8.65	1.68
	Labor	7.21	8.34	9.56	8.4	1.72
	Materials	7.22	8.35	9.72	8.54	1.92
321 (N=4302)	Output	2.77	3.22	3.91	3.49	0.99
	Capital	2.89	3.47	4.22	3.71	1.08
	Labor	2.94	3.48	4.37	3.69	0.95
	Materials	2.89	3.43	4.28	3.67	1.02
All (N=22451)	Output	9.84	10.46	11.81	10.94	1.56
	Capital	9.91	10.75	11.79	10.86	1.41
	Labor	9.68	10.62	11.75	10.73	1.48
	Materials	9.81	10.68	11.89	10.93	1.62

Chilean Manufacturing

Industry (ISIC code)	τ	Capital Coef.	Capital s.e.	Labor Coef.	Labor s.e.	Returns to Scale Coef.	Returns to Scale s.e
311	0.10	0.367	0.1054	0.390	0.0438	0.757	0.1137
	0.25	0.214	0.0442	0.411	0.0325	0.565	0.0520
	0.50	0.243	0.0366	0.413	0.0249	1.066	0.0409
	0.75	0.167	0.0428	0.415	0.0220	0.721	0.0480
381	0.10	0.246	0.1005	0.733	0.0687	0.728	0.1215
	0.25	0.249	0.0593	0.624	0.0449	0.616	0.0715
	0.50	0.266	0.0404	0.564	0.0369	0.802	0.0495
	0.75	0.093	0.0599	0.520	0.0390	0.715	0.0709
321	0.10	0.356	0.1264	0.710	0.0761	0.625	0.1419
	0.25	0.320	0.0778	0.628	0.0523	0.979	0.0840
	0.50	0.201	0.0686	0.582	0.0457	0.948	0.0685
	0.75	0.193	0.0586	0.554	0.0418	0.827	0.0681
All	0.10	0.301	0.0469	0.526	0.0205	0.647	0.0482
	0.25	0.315	0.0360	0.502	0.0174	0.912	0.0387
	0.50	0.292	0.0370	0.466	0.0140	0.839	0.0387
	0.75	0.194	0.0404	0.436	0.0150	0.880	0.0434

Chilean Manufacturing

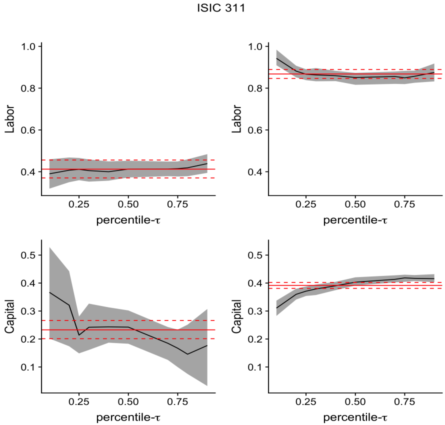
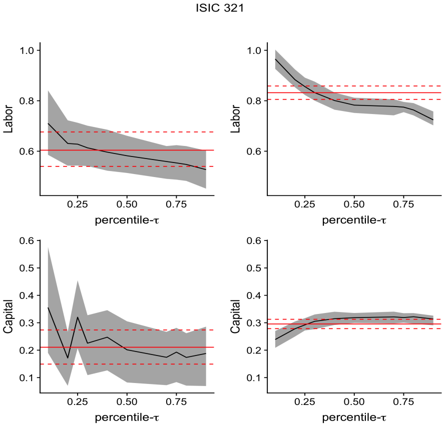


Figure: Estimated values of production function coefficients and their 90% confidence interval

Chilean Manufacturing



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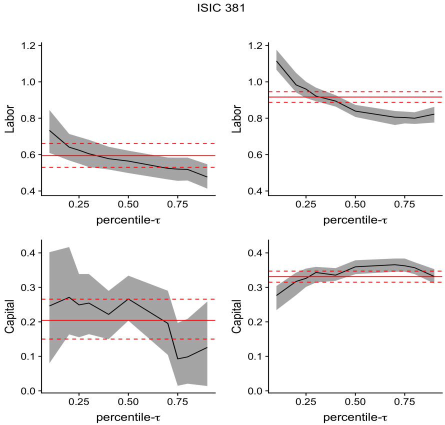
Chilean Manufacturing



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Chilean Manufacturing

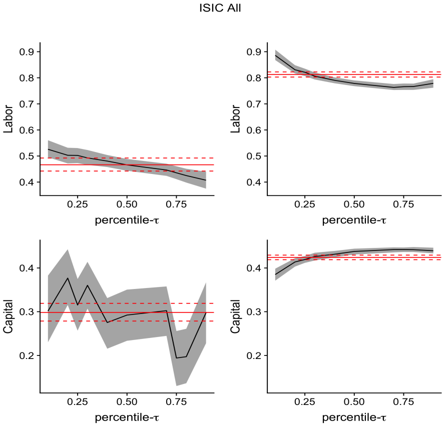

Figure: Estimated values of production function coefficients and their 90% confidence interval

Colombian Manufacturing

Industry (ISIC code)		1st Qu.	Median	3rd Qu.	Mean	sd
311 (N=13215)	Output	9.03	10.21	11.59	10.42	1.8
	Capital	8.69	9.37	10.22	9.49	1.18
	Labor	8.52	9.3	10.33	9.54	1.43
	Materials	8.7	9.62	10.88	9.92	1.67
322 (N=12182)	Output	6.02	7.07	8.35	7.24	1.78
	Capital	5.47	6.14	6.93	6.23	1.21
	Labor	5.89	6.75	7.81	6.93	1.55
	Materials	5.9	6.89	8.16	7.12	1.77
381 (N=7411)	Output	2.56	3.09	3.97	3.36	1.1
	Capital	2.77	3.3	3.95	3.42	0.92
	Labor	2.64	3.18	3.91	3.37	0.98
	Materials	2.71	3.3	4.11	3.5	1.09
All (N=32808)	Output	8.39	9.73	11.26	9.87	2
	Capital	7.62	8.53	9.46	8.48	1.51
	Labor	7.77	8.65	9.72	8.8	1.58
	Materials	7.89	8.93	10.26	9.15	1.88

Colombian Manufacturing

		Capital		Labor		Returns to Scale	
Industry (ISIC code)	τ	Coef.	s.e.	Coef.	s.e.	Coef.	s.e
311	0.10	0.155	0.0601	0.757	0.0284	0.912	0.0629
	0.25	0.347	0.0561	0.627	0.0181	0.709	0.0572
	0.50	0.314	0.0352	0.517	0.0185	1.555	0.0377
	0.75	0.297	0.0322	0.426	0.0204	1.028	0.0358
322	0.10	0.237	0.0632	0.829	0.0264	0.929	0.0659
	0.25	0.377	0.0418	0.754	0.0223	0.560	0.0423
	0.50	0.302	0.0304	0.643	0.0241	1.347	0.0338
	0.75	0.312	0.0370	0.522	0.0291	0.884	0.0417
381	0.10	0.374	0.1131	1.181	0.0563	0.974	0.1139
	0.25	0.294	0.0406	0.948	0.0304	1.067	0.0451
	0.50	0.269	0.0525	0.759	0.0315	1.243	0.0547
	0.75	0.464	0.0621	0.641	0.0292	1.027	0.0661
All	0.10	0.156	0.1969	0.872	0.0152	0.929	0.1983
	0.25	0.150	0.0252	0.744	0.0100	1.145	0.0264
	0.50	0.239	0.0236	0.642	0.0083	1.227	0.0243
	0.75	0.323	0.0244	0.567	0.0092	0.893	0.0258

Colombian Manufacturing

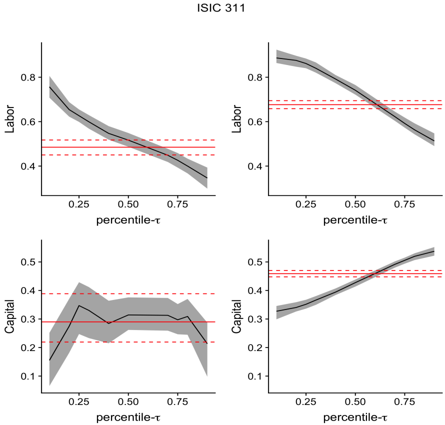

Figure: Estimated values of production function coefficients and their 90% confidence interval

Colombian Manufacturing

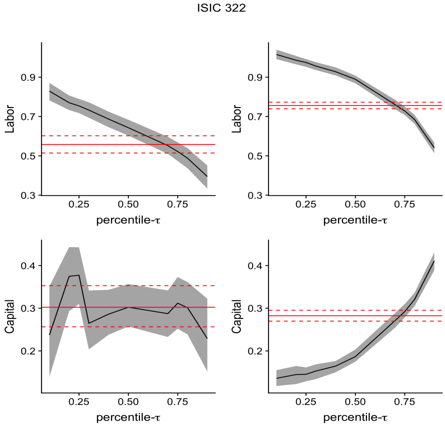

Figure: Estimated values of production function coefficients and their 90% confidence interval

Colombian Manufacturing

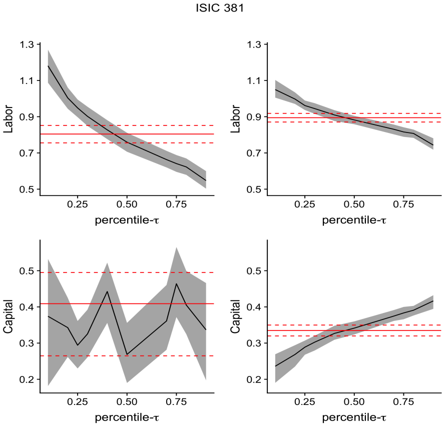

Figure: Estimated values of production function coefficients and their 90% confidence interval

Colombian Manufacturing

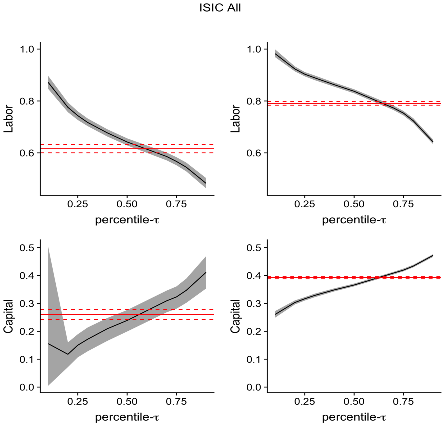

Figure: Estimated values of production function coefficients and their 90% confidence interval