Heterogeneity in Firms: A Proxy Variable Approach to Quantile Production Functions

Justin Doty and Suyong Song

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  - ACF (2015): Value-Added production with intermediate input Proxy, identification under different DGPs, timing of labor matters

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- These approaches have focused on estimates of output elasticities on location of conditional firm-size distribution
- Also, productivity heterogeneity are estimated average TFP measurements, typical dispersion measurements (e.g. 90-10 range) are average measurements.
- There could be considerable heterogeneity not captured by average estimates

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1. Information Set: $\mathcal{I}_t$ includes current and past productivity shocks, but not future productivity. $\mathbb{E}[\varepsilon_t|\mathcal{I}_t] = 0$
2. First Order Markov: Productivity shocks evolve according to the distribution $p(\omega_t|\omega_{t-1})$
3. Capital Accumulation: $k_t = \kappa(k_{t-1}, l_{t-1})$
4. Scalar Unobservability: $\iota_t = \iota_t(k_t, \omega_t)$
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- Given these assumptions intermediate input demand can be inverted $\omega_t = \omega_t(k_t, \iota_t)$ and substituted into the production function

$$y_t = \beta_0 + \beta_k k_t + \beta_l l_t + \omega_t(k_t, \iota_t) + \varepsilon_t = \beta_l l_t + \Phi(k_t, \iota_t) + \varepsilon_t \quad (2)$$

where $\Phi(k_t, \iota_t) = \beta_0 + \beta_k k_t + \beta_\iota \iota_t + \omega_t(k_t, \iota_t)$

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  where $\Phi(k_t, \iota_t) = \beta_0 + \beta_k k_t + \beta_\iota \iota_t + \omega_t(k_t, \iota_t)$
- $\Phi(k_t, \iota_t)$ can be estimated nonparametrically
- Labor coefficient and other variable coefficients are identified in the first stage
- First stage estimates are $\hat{\beta}_l$ and $\hat{\Phi}(k_t, \iota_t)$
- Let $y^* = y - \hat{\beta}_l l_t$ denote the output net of labor contribution

Levinsohn and Petrin (2003)

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- Capital does not respond to the innovation in productivity
- Last period's input choice should not be correlated with the innovation in productivity this period

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$$\mathbb{E}[(\varepsilon_t + \eta_t)Z_{t-1}] = \mathbb{E}[(y_t^* - \beta_k k_t - \beta_\iota \iota_t - \hat{\mathbb{E}}[\omega_t|\omega_{t-1}])Z_{t-1}] \tag{3}$$

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- $\hat{\mathbb{E}}[\omega_t|\omega_{t-1}]$ can be estimated using the estimates of ω_t from first stage estimates evaluated at (β_k, β_ι)
- Z_{t-1} includes k_t and ι_{t-1} as well as additional instruments
- LP minimize GMM criterion function and bootstrap standard errors

Heterogeneous Coefficient Models

- The control function approaches of OP, LP, and ACF estimate homogeneous output elasticities
- Many interesting questions can be asked if we consider production functions that are heterogeneous between firms
- Kasahara, Schrimpf and Suzuki (2015) propose two methods to estimate a random coefficient production function and find evidence of unobserved heterogeneity beyond a Hicks-neutral technology term
- Balat, Brambilla, and Sasaki (2015) extend the control function approach to infinitely supported coefficients with an application to international trade
- Li and Sasaki (2017) develop an identification strategy for heterogeneous elasticities using first order conditions for intermediate inputs

Production Functions and Quantile Regression

- Allowing firm technology to vary over the conditional output distribution is also valid for estimating a production function with unobserved heterogeneity
- Applications of quantile regression are however limited due to simultaneity bias
- Typical IV approaches such as Chernozhukov and Hansen (2005) are also limited due to availability of instruments (input prices) and arguing for the monotonicity of the unobserved heterogeneity
- Possible to exploit the panel data structure and use fixed effects (Koenker, 2004), Lamarche (2010), or Canay (2011)
- The fixed effect shifts the location of the conditional distribution (not the quantiles)
- Incidental Parameters (Koenker, 2004)
- Independence of unobserved heterogeneity and productivity (Canay, 2011)

Quantile Production Function

We seek to extend the control function approach to a quantile value-production function for a fixed $\tau \in (0, 1]$ defined as

$$Q_\tau(y_t | k_t, l_t) = \beta_k(\tau)k_t + \beta_l(\tau)l_t + \omega_t \tag{4}$$

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- l_t is a freely variable input like labor and k_t denotes the state variable as capital input
- ω_t is an unobserved state variable to the econometrician that impacts firm's optimal input decisions
- We allow for inputs to affect quantiles of the output distribution
- A special case of this quantile production function is the location scale model

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$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + (\eta_k k_{it} + \eta_l l_{it} + \eta_\omega \omega_{it})\varepsilon_{it} \tag{5}$$

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which implies that the τ-th conditional quantile of y_{it} is given by

$$Q_\tau(y_{it} | \mathcal{I}_{it}) = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + (\eta_k k_{it} + \eta_l l_{it} + \eta_\omega \omega_{it}) F^{-1}(\tau) \quad (6)$$

Quantile Production Function: Assumptions

We maintain several timing assumptions that are standard in the control function literature

We use the set of assumptions for LP estimation, but can be easily altered for ACF as we do both in simulations

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1. Information Set: $\mathcal{I}_t$ includes current and past productivity shocks, but not future productivity. $Q_\tau(\varepsilon_t|\mathcal{I}_t) = 0$
2. Firm's productivity follows an AR(1) process

$$\omega_t = \mathbb{E}[\omega_t|\omega_{t-1}] + \xi_t = g(\omega_{t-1}) + \xi_t \tag{7}$$

3. Capital Accumulation: $k_t = \kappa(k_{t-1}, i_{t-1})$
4. Scalar Unobservability: $\iota_t = \iota_t(k_t, \omega_t; \tau)$
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Quantile Production Function: Estimation

- Given these assumptions we invert intermediate input demand $\omega_t = \omega_t(k_t, \iota_t)$ and substitute into the production function

$$Q_\tau(y_t|k_t, l_t) = \beta_l(\tau)l_t + \beta_k(\tau)k_t + \omega_t(k_t, \iota_t) = \beta_l(\tau)l_t + \Phi(k_t, \iota_t) \quad (8)$$

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- First stage parameters $\beta_l(\tau)$ and $\Phi(k_t, \iota_t)$ are estimated here
- If one uses a polynomial approximation to $\Phi(k_t, \iota_t)$, $\hat{\beta}_l(\tau)$ and $\hat{\Phi}(k_t, \iota_t)$ can be estimated using a polynomial quantile regression

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- Substituting into the production

$$Q_\tau(y_t|k_t, l_t) = \beta_k(\tau)k_t + \hat{\beta}_l(\tau)l_t + g(\hat{\Phi}(k_{t-1}, \iota_{t-1}) - \beta_k(\tau)k_{t-1}) + \xi_t \quad (9)$$

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- This equation is complicated by the unobservable ξ_t, we will assume that its sum with the production function shock ε_t is conditional quantile zero, but this may not be valid

Quantile Production Function: Estimation

We can write a second stage conditional quantile restriction as

$$Q_\tau[y_t - \beta_k(\tau)k_t - \hat{\beta}_l(\tau)l_t - g(\hat{\Phi}_\tau(k_{t-1}, \iota_{t-1}) - \beta_k(\tau)k_{t-1})|\mathcal{I}_{it-1}] = 0 \qquad (10)$$

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- The model in (10) can be represented by conditional moment restrictions

$$\mathbb{E}[\mathbb{1}\{y_t - \beta_k(\tau)k_t - \hat{\beta}_l(\tau)l_t - g(\hat{\Phi}_\tau(k_{t-1}, \iota_{t-1}) - \beta_k(\tau)k_{t-1}) \leq 0\} - \tau|\mathcal{I}_{it-1}] = 0 \quad (11)$$

where $\mathbb{1}\{\cdot\}$ is the indicator function

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  where $\mathbb{1}\{\cdot\}$ is the indicator function
- To estimate the production function parameters we use the unconditional moments

$$\mathbb{E}[Z_{t-1}(\mathbb{1}\{y_t - \beta_k(\tau)k_t - \hat{\beta}_l(\tau)l_t - g(\hat{\Phi}_\tau(k_{t-1}, \iota_{t-1}) - \beta_k(\tau)k_{t-1}) \leq 0\} - \tau)] = 0 \quad (12)$$

  where Z_{t-1} includes the instruments used in the firm's information set at time $t-1$

Smoothed GMM for Quantile Models

- The indicator function makes estimation of the production function parameters intractable
- We smooth the indicator function and rely on the theory and estimation procedure of de Castro, Galvao, Kaplan and Liu (2018)
- Develops theory for feasible estimators of parameters in general conditional quantile restrictions that include non-linear IVQR
- This approach is computationally attractive compared to approaches such as the MCMC approach proposed by Chernozhukov and Hong (2003)

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- This approach is computationally attractive compared to approaches such as the MCMC approach proposed by Chernozhukov and Hong (2003)
- To fix notation let
  - Z_{t-1} the set of instruments discussed earlier
  - x_t the set of exogenous and endogenous covariates
  - $\Lambda(\cdot)$ denote the residual function that defines the conditional quantile restriction that is known up to $\beta_k(\tau)$

Smoothed GMM for Quantile Models

- The sample analog of (12) can be written as:

$$\hat{M}_n(\beta, \tau) = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} Z_{it} \left[\tilde{I} \left(\frac{\Lambda(y_{it}, x_{it}, \beta(\tau))}{h_n} \right) - \tau \right], \qquad (13)$$

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where h_n is a bandwidth (sequence) and $\tilde{I}(\cdot)$ is a smoothed version of the indicator function $\mathbb{1}\{\cdot \leq 0\}$ used by Horowitz (1998), Whang (2006), and Kaplan and Sun (2016):

$$\tilde{I}(u) = \mathbb{1}\{-1 \leq u \leq 1\}\left[0.5 + \frac{105}{64}\left(u - \frac{5}{3}u^3 + \frac{7}{5}u^5 - \frac{3}{7}u^7\right)\right] + \mathbb{1}\{u > 1\}. \qquad (14)$$

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The smoothed GMM estimator minimizes a weighted quadratic norm of the smoothed sample moment vector

$$\hat{\beta}_{GMM} = \underset{\beta}{\operatorname{argmin}}\, \hat{M}_n(\beta,\tau)^\top \hat{W} \hat{M}_n(\beta,\tau). \tag{15}$$

Smoothed GMM for Quantile Models

- The optimal weighting matrix is the estimator of the inverse long-run variance of the sample moment $\hat{W}^* = \bar{\Omega}^{-1} \xrightarrow{p} \Omega^{-1}$
- In simulation and in application we use the long-run variance estimator as in Newey and West (1987) with a truncated kernel. We find this helps with scaling the moment conditions
- The asymptotic results for this estimator cannot be applied here due to the semi-parametric nature of the two-step procedure
- We are working to see if the asymptotic results of Ai and Chen (2007) can be applied here. This gives us extra conditions to show what rate gives an asymptotically negligible bandwidth. The smoothed sample moments are differentiable so we can take a Taylor expansion around $\beta(\tau)$, but the bandwidth problem remains
- We use bootstrap for our empirical application

Quantile Production Function: Simulation

- Simulations follow a location-scale version of LP (2003) and ACF (2015) using ACF's original set of DGPs
- Parameters are chosen to match a couple of key moments in the Chilean data used by LP
- Productivity follows a first order AR(1) process with persistence $\rho = 0.7$
- Firms make optimal choices of investment in the capital stock to maximize the expected discounted value of future profits
- Convex capital adjustment costs
- Labor input l_t is chosen either at t or $t - b$
- Possible optimization error in labor input decision

Quantile Production Function: Simulation

- Production function is assumed Leontief in materials, the location-scale specification is

$$y_t = \beta_k k_t + \beta_l l_t + \omega_t + (0.7k_t + 0.6l_t + 0.1\omega_t)\varepsilon_t \qquad (16)$$

with $\beta_k = 0.4$, $\beta_l = 0.6$

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  Simulation 1: ACF Estimated Quantile Production Function
  - Firms face different wages, where the wage process following an AR(1) process
  - Assume labor is chosen at time $t - b$ where $b = 0.5$

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- No wage variation across firms and labor chosen at time t
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- Both simulations include 2 DGP's for the error term ε_t. In DGP 1 $\varepsilon_t \sim N(0, 0.1)$ and in DGP 2 $\varepsilon_t \sim Laplace(0, 0.1)$

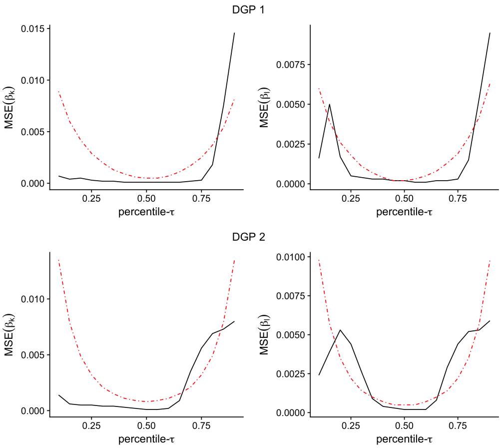
Quantile Production Function: Simulation 1

Table: Simulated precision of estimators of $\beta_k(\tau)$ and $\beta_l(\tau)$

		QACF				ACF			
		β_k		β_l		β_k		β_l	
DGP	τ	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
1	0.10	0.0223	0.0007	0.0311	0.0016	-0.0917	0.0089	-0.0759	0.0060
	0.25	0.0128	0.0003	0.0155	0.0005	-0.0492	0.0029	-0.0395	0.0018
	0.50	0.0020	0.0001	0.0020	0.0002	-0.0020	0.0005	0.0010	0.0002
	0.75	-0.0108	0.0003	-0.0055	0.0003	0.0452	0.0025	0.0415	0.0019
	0.90	-0.1123	0.0146	0.0859	0.0095	0.0877	0.0082	0.0779	0.0063
2	0.10	0.0193	0.0014	0.0164	0.0024	-0.1127	0.0135	-0.0966	0.0098
	0.25	0.0015	0.0005	0.0524	0.0044	-0.0485	0.0032	-0.0416	0.0022
	0.50	0.0020	0.0001	0.0010	0.0002	0.0000	0.0008	0.0000	0.0005
	0.75	-0.0525	0.0056	0.0316	0.0044	0.0485	0.0032	0.0416	0.0022
	0.90	-0.0653	0.0080	0.0446	0.0059	0.1127	0.0135	0.0966	0.0098

1000 replications. Under location-scale, $\beta_x(\tau) = \beta_x + \eta_x F^{-1}(\tau)$ where $x = \{k, l\}, \eta_x = \{0.7, 0.6\}$ and F is the CDF of ε_{it}

Figure: Simulated precision of estimators of $\beta_k(\tau)$ and $\beta_l(\tau)$. Dotted line is ACF estimator



Quantile Production Function: Simulation 2

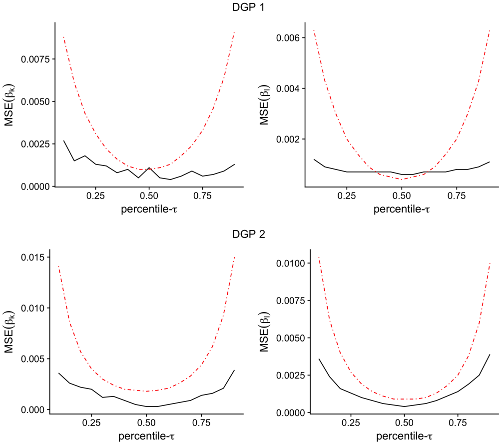
Table: Simulated precision of estimators of $\beta_k(\tau)$ and $\beta_l(\tau)$

		QLP				ACF			
		β_k		β_l		β_k		β_l	
DGP	τ	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
1	0.10	0.0323	0.0027	-0.0009	0.0012	-0.0887	0.0088	-0.0769	0.0063
	0.25	0.0178	0.0013	-0.0005	0.0007	-0.0462	0.0031	-0.0405	0.0020
	0.50	0.0060	0.0011	0.0000	0.0006	0.0010	0.0010	0.0000	0.0004
	0.75	-0.0058	0.0006	-0.0005	0.0008	0.0482	0.0033	0.0405	0.0020
	0.90	-0.0163	0.0013	-0.0001	0.0011	0.0907	0.0092	0.0769	0.0063
2	0.10	0.0303	0.0036	-0.0016	0.0036	-0.1107	0.0141	-0.0976	0.0104
	0.25	0.0255	0.0020	0.0004	0.0013	-0.0465	0.0040	-0.0426	0.0027
	0.50	0.0040	0.0003	0.0000	0.0004	0.0020	0.0018	-0.0010	0.0009
	0.75	-0.0105	0.0014	-0.0014	0.0014	0.0505	0.0044	0.0406	0.0025
	0.90	-0.0113	0.0039	-0.0034	0.0039	0.1147	0.0150	0.0956	0.0100

1000 replications. Under location-scale, $\beta_x(\tau) = \beta_x + \eta_x F^{-1}(\tau)$ where $x = \{k, l\}, \eta_x = \{0.7, 0.6\}$ and F is the CDF of ε_{it}

Quantile Production Function: Simulation 2

Figure: Simulated precision of estimators of $\beta_k(\tau)$ and $\beta_l(\tau)$. Dotted line is LP estimator



Quantile Production Function: Application

- We apply this estimator to an 10-year panel (1987-1996) from Chile (ENIA) provided by Amil Petrin
- Focus on the four largest industries (ISIC codes in parentheses): Food Products (311), Metals (381), Textiles (321) and Wood Products (331)
- We use the QLP estimator for a value-added specification of the production function
- Standard errors are obtained from the bootstrapping method outlined by Levinsohn and Petrin (2003)
- Comparison to LP approach is provided and implemented by prodest.R package (Gabrielle Rovigatti)

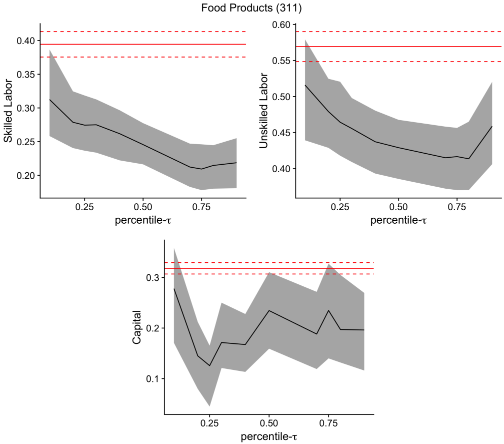
Quantile Production Function: Simulation 2

Table: Parameter estimates for four industries (bootstrapped standard errors)

		Capital		Skilled Labor		Unskilled Labor		Returns to Scale	
Industry (ISIC code)	τ	Coef.	s.e.	Coef.	s.e.	Coef.	s.e.	Coef.	s.e
311	0.10	0.278	0.0570	0.313	0.0402	0.516	0.0443	1.107	0.0793
	0.25	0.126	0.0375	0.274	0.0249	0.464	0.0298	0.865	0.0490
	0.50	0.235	0.0444	0.246	0.0187	0.429	0.0241	0.909	0.0522
	0.75	0.235	0.0561	0.209	0.0206	0.417	0.0263	0.861	0.0652
381	0.10	0.166	0.1123	0.465	0.0846	0.596	0.0772	1.228	0.1188
	0.25	0.371	0.0981	0.393	0.0534	0.524	0.0675	1.288	0.1063
	0.50	0.244	0.0783	0.399	0.0401	0.417	0.0469	1.061	0.0838
	0.75	0.218	0.0622	0.367	0.0347	0.396	0.0305	0.981	0.0707
321	0.10	0.262	0.1278	0.545	0.0585	0.476	0.0692	1.283	0.1351
	0.25	0.160	0.1144	0.511	0.0521	0.385	0.0558	1.056	0.1126
	0.50	0.171	0.0665	0.492	0.0495	0.386	0.0545	1.049	0.0778
	0.75	0.227	0.0613	0.442	0.0524	0.392	0.0627	1.061	0.0744
331	0.10	0.208	0.1020	0.435	0.0522	0.583	0.0728	1.225	0.1271
	0.25	0.144	0.0662	0.378	0.0610	0.481	0.0676	1.003	0.0942
	0.50	0.176	0.0580	0.351	0.0396	0.431	0.0498	0.959	0.0742
	0.75	0.127	0.0600	0.378	0.0365	0.314	0.0410	0.819	0.0683

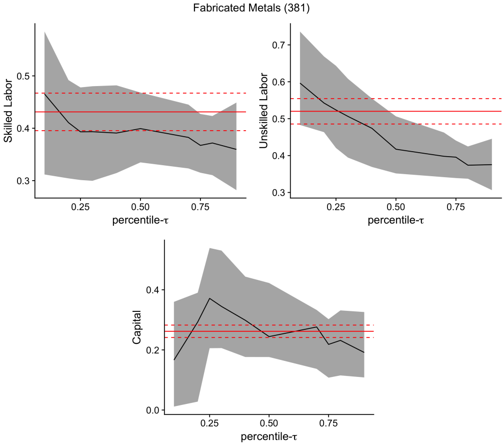
Quantile Production Function: Application

Figure: Estimated values of production function coefficients and their 90% confidence interval



Quantile Production Function: Application

Figure: Estimated values of production function coefficients and their 90% confidence interval



Quantile Production Function: Application

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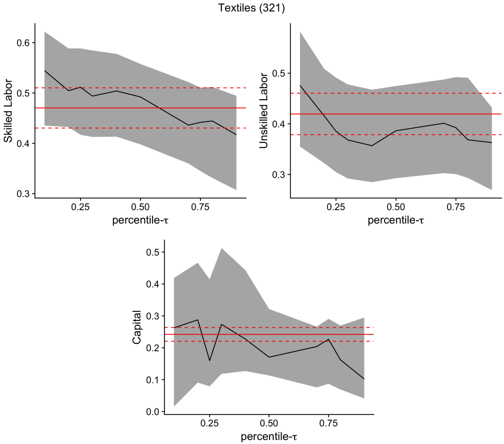
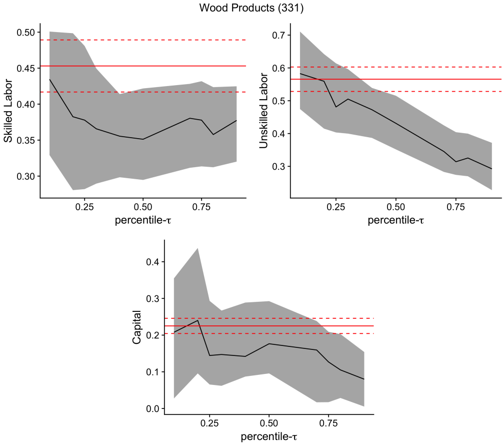


Quantile Production Function: Application

Figure: Estimated values of production function coefficients and their 90% confidence interval



Quantile Production Function: Conclusion

- While these quantile estimates exhibit interesting trends, they are not statistically different from the mean estimates
- Perhaps a different data application would be better
- Still working on asymptotic results and a numerically feasible implementation such as Ackerberg, Chen, Hahn, and Liao (2014)
- Implications for TFP measurements, a new way to look at TFP dispersion
- Working on implementing additional tests: Wald test for constant effects across quantiles, over-identification tests and an alternative model specification

Alternative Estimator: Integrated Moment Condition

We have the following moment condition from earlier (now including ξ (i, t omitted))

$$\mathbb{E}[Z(\mathbb{1}\{y-\beta_k(\tau)k-\hat{\beta}_l(\tau)l-g(\hat{\Phi}_\tau(k,\iota)-\beta_k(\tau)k)-\xi \leq 0\}-\tau)] = 0 \quad (17)$$

or

$$\mathbb{E}[Z(\mathbb{1}\{y-\beta_k(\tau)k-\hat{\beta}_l(\tau)l-g(\hat{\Phi}_\tau(k,\iota)-\beta_k(\tau)k) \leq \xi\}-\tau)] = 0 \quad (18)$$

Assume $\xi \sim N(\beta_0(\tau), \sigma^2)$. Applying law of iterated expectations we obtain the following integrated moment condition

$$\mathbb{E}[\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} Z(\mathbb{1}\{y-\beta_k(\tau)k-\hat{\beta}_l(\tau)-g(\hat{\Phi}_\tau(k,\iota)-\beta_k(\tau)k) \leq \xi\}-\tau)e^{\frac{-\xi}{2\sigma^2}}] = 0 \quad (19)$$

Need to consider the case when $\xi \geq 0$ and $\xi < 0$ inside the indicator function. For example, consider when $\xi \geq 0$

Alternative Estimator: Integrated Moment Condition

The integral of the indicator function when $\xi \geq 0$

$$\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} \mathbb{1}\{y - \beta_k(\tau)k - \hat{\beta}_l(\tau)l - g(\hat{\Phi}_\tau(k,\iota) - \beta_k(\tau)k) \leq \xi\} \quad (20)$$

Is equal to (with simplification)

$$1 - \Psi\left(\frac{\Lambda(y,x,\beta(\tau)) - \beta_0(\tau)}{\sigma}\right) \quad (21)$$

Similarly when $\xi < 0$ we obtain

$$\Psi\left(\frac{\Lambda(y,x,\beta(\tau)) - \beta_0(\tau)}{\sigma}\right) \quad (22)$$

where $\Lambda(y,x,\beta(\tau))$ is the residual function defined earlier (without ξ) and $\Psi(\cdot)$ is the CDF of standard normal

Alternative Estimator: Integrated Moment Condition

The integrated moment function becomes

$$\mathbb{E}\Bigg[Z\Bigg(\mathbb{1}\{\Lambda(y,x,\beta(\tau))\geq 0\}\Bigg(1-\Psi\Big(\frac{\Lambda(y,x,\beta(\tau))-\beta_0(\tau)}{\sigma}\Big)\Bigg)+ \\ \mathbb{1}\{\Lambda(y,x,\beta(\tau))< 0\}\Bigg(\Psi\Big(\frac{\Lambda(y,x,\beta(\tau))-\beta_0(\tau)}{\sigma}\Big)\Bigg)-\tau\Bigg)\Bigg]=0 \tag{23}$$

- This removes the needs for smoothing since this function is continuous and differentiable
- With standard assumptions we can use GMM and iteratively solve for the production function parameters for optimal values of the nuisance parameters