Estimating Quantile Production Functions: A Control Function Approach

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Introduction

- Production function estimation is an ongoing and historical research topic
- Links inputs (capital goods, labor, intermediate inputs, etc) to output
- Estimates provide insights on how firm productivity changes over time and cross-sectional differences between firms
- Most inputs and output are observed (albeit with possible measurement error) in data such as manufacturing censuses or constructed from data from firm financial statements
- Other inputs to production, such as managerial ability, are not observed in the data.
- This type of unobserved productivity influences how much inputs a firm uses in production
- As such, if unobserved, biases OLS estimates. This is known as "simultaneity bias"

Control Function Approach

- A popular approach to address simultaneity bias and other econometric issues is the control function approach
- Introduce a policy function such as investment (Olley and Pakes, 1996) (OP), or an intermediate input (Levinsohn and Petrin, 2003) (LP) or (Ackerberg, Caves, and Frazer, 2015) (ACF)
- If the policy functions depend only on current state variables (e.g. capital and productivity) under a monotonicity restriction, they can be inverted to proxy for the unobserved productivity component
- Computationally simple and has been used in numerous applied papers to obtain consistent estimates of output elasticities, total factor productivity, markups, etc.
- Some issues still remain: identification issues related to model specification, measurement error and other unobservables

Control Function Approach

- Review of the ACF procedure for estimating a *value-added* production function (in logs):

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \varepsilon_{it}, \tag{1}$$

- y_{it} is value-added output for firm i and time t
- l_{it} denotes labor input
- k_{it} denotes capital input
- ω_{it} is unobserved productivity
- ε_{it} denotes an independent and identically distributed (i.i.d) shock to production
- The constant β_0 is omitted since it is not separately identified from the mean of productivity.

Control Function Approach

- ACF introduces an intermediate input demand function defined as

$$m_{it} = m_t(k_{it}, l_{it}, \omega_{it}) \tag{2}$$

- The function m is assumed to be strictly increasing in ω_{it} for all k_{it} and l_{it}.
- Productivity can then be expressed as

$$\omega_{it} = m_t^{-1}(k_{it}, l_{it}, m_{it}) \tag{3}$$

- Substituting this equation into the production function

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + m_t^{-1}(k_{it}, l_{it}, m_{it}) + \varepsilon_{it} = \Phi_t(k_{it}, l_{it}, m_{it}) + \varepsilon_{it}. \tag{4}$$

- The function, $\Phi_t(k_{it}, l_{it}, m_{it})$, is identified by the following first stage moment restriction

$$\mathbb{E}[\varepsilon_{it} | \mathcal{I}_{it}] = 0 \tag{5}$$

- $\mathcal{I}_{it}$ denotes the firm's information at time t.
- The first stage estimate of Φ_t can be obtained by a local linear regression or a polynomial regression in (k_{it}, l_{it}, m_{it}).

Control Function Approach

- For the second stage, assume that productivity follows an AR(1) process given by

$$\omega_{it} = \mathbb{E}[\omega_{it}|\omega_{it-1}] + \xi_{it} = \rho\omega_{it-1} + \xi_{it}, \tag{6}$$

- ξ_{it} denotes an innovation to productivity which satisfies $\mathbb{E}[\xi_{it}|\mathcal{I}_{it-1}] = 0$.
- Plugging into the production function gives

$$\begin{aligned} y_{it} &= \beta_k k_{it} + \beta_l l_{it} + \rho\omega_{it-1} + \xi_{it} + \varepsilon_{it} \\ &= \beta_k k_{it} + \beta_l l_{it} + \rho(\Phi_{t-1}(k_{it-1}, l_{it-1}, m_{it-1}) - \beta_k k_{it-1} - \beta_l l_{it-1}) + \xi_{it} \end{aligned}$$

- The production function parameters β_k, β_l and ρ are identified from the moment restrictions given by

$$\mathbb{E}[\xi_{it} + \varepsilon_{it}|\mathcal{I}_{it-1}] = 0. \tag{7}$$

- Estimation of the second stage coefficients proceeds by plugging in first stage estimates $\hat{\Phi}_{t-1}$ and forming a Generalized Method of Moments (GMM) criterion function.

Control Function Approach

- These estimates only provide a picture of the conditional mean of firm output
- Quantile regression offers robust estimates to outliers in the data
- Not straightforward to estimate production functions with endogenous inputs and heterogeneous coefficients
- The conditional mean restrictions in ACF to do not extend naturally to conditional quantile restrictions with multiple unobservables
- Expectation operators are linear whereas quantile operators are not
- Similar issues are encountered in the quantile panel data literature
- First to address simultaneity and quantiles in the production function literature
- Propose an easy-to-implement estimator motivated by our identification argument which is consistent and asymptotically normal

Random Coefficient Production Function

- A Skorohod representation for a firm's *value-added* production function:

$$y_{it} = \beta_k(\eta_{it})k_{it} + \beta_l(\eta_{it})l_{it} + \omega_{it}, \quad \text{where} \quad \eta_{it} \sim U(0,1), \tag{8}$$

- $Q_\tau(y_{it}|\mathcal{I}_{it})$ is the conditional τ-th quantile of y_{it} given the information set $\mathcal{I}_{it}$ for $\tau \in (0,1)$.
- Skorohod representation assumes that the unobserved heterogeneity enters through the rank of a firm on the conditional output distribution
- η_{it} is a technology shock, not to be confused with ex-post shock ε_{it}
- Productivity ω_{it} is additively separable and does not depend on η_{it}
- ω_{it} is only a location-shift of the conditional output distribution
- Assume the constant in the production function does not vary over τ and subsume it into the productivity term

Random Coefficient Production Function

- A value-added specification in equation (8) is non-trivial
- Objects recovered from a value-added model such as the output elasticities and TFP can only be mapped to its gross-output counterpart under special structural production function (Leontief value-added)
- In this case, a Leontief value-added production function in this framework could be written as

$$y_{it} = \min\{\beta_k(\eta_{it})k_{it} + \beta_l(\eta_{it})l_{it} + \omega_{it}, \beta_m + m_{it}\},$$

- Thanks to the equivariance properties of quantiles we have

$$Q_\tau(y_{it}|\mathcal{I}_{it}) = \min\{\beta_k(\tau)k_{it} + \beta_l(\tau)l_{it} + \omega_{it}, \beta_m + m_{it}\},$$

- The conditional quantile of a structural value-added production function could be written as

$$Q_\tau(y_{it}|\mathcal{I}_{it}) = \beta_k(\tau)k_{it} + \beta_l(\tau)l_{it} + \omega_{it},$$

- This corresponds directly to the conditional quantile of Equation (8)

Quantiles and Production Risk

- What factors contribute to dispersion in the conditional output distribution through rank of η_{it}?
- Consider the location-scale model as a special case of (8)

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + (\mu_k k_{it} + \mu_l l_{it})\eta_{it}, \tag{9}$$

- The τ-th conditional quantile of y_{it} is given by

$$Q_\tau(y_{it}|\mathcal{I}_{it}) = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + (\mu_k k_{it} + \mu_l l_{it})F^{-1}(\tau), \tag{10}$$

- $F^{-1}(\tau)$ is the quantile function of production shocks η_{it}
- Inputs chosen by the firm have some control over the risk of production
- Just and Pope, 1978; Just and Pope, 1979 consider a specification that allowed firm's inputs to both increase or decrease the marginal variability of final output

Quantiles and Production Risk

- How to extend this idea to the entire distribution of output?
- Requires reformulating how firms form beliefs about uncertainty in profits due to production risk
- Instead of rational expectations framework, could we allow firms to have varying risk preferences and use a quantile profit maximization framework?
- Different managers may choose different optimal expenditure on inputs to maximize profits in the presence of production risk
- A short list of papers have considered quantile utility maximization such as Manski, 1988, Rostek, 2009, Chambers, 2007, and Bhattacharya, 2009
- For dynamic problems, for example, investment decisions, de Castro and Galvao, 2019 may be applicable
- Difficult to incorporate this theory into testable econometric models
- Leave as future research agenda

Quantiles and Frontier Models

- The model presented here is also related to the stochastic frontier analysis (SFA) literature
- A SFA model of production proposed by Aigner, Lovell, and Schmidt, 1977 introduces statistical error into a frontier model
- Frontier models assume that firms deviate from an optimal frontier of production
- The SFA model is typically written as

$$y_i = f(x_i, \beta) + \varepsilon_i, \tag{11}$$

- $\varepsilon_i = \eta_i - u_i$, x_i are inputs to production and β are the parameters
- The error term η_i denotes the statistical noise in the model such as measurement error
- u_{it} represents one-sided deviations from the production frontier
- Estimates of β are typically obtained using maximum likelihood which requires strong distributional assumptions on the error terms

Quantiles and Frontier Models

- Quantile regression is a natural candidate for obtaining an estimate of the efficient frontier
- Aragon, Daouia, and Thomas-Agnan, 2005 interprets production functions as being from a continuous interval $\tau \in [0, 1]$ where $\tau \to 1$ converges to the efficient frontier
- Difficulty in choosing which quantile to estimate as the frontier
- Inference on extremal quantiles is difficult (Chernozhukov, 2005)
- Statistical noise is also an issue since predicted error is composite of the technical inefficiency and noise
- Other econometric issues, such as endogeneity of input choices with respect to inefficiency are difficult to incorporate in this framework
- It is possible the model proposed in this paper could be applied here

Identification

- Show that the model presented in Equation (8) is non-parametrically identified
- Results can also be applied to other production functions such as translog, provided that productivity is additively separable
- Let $\varepsilon_{it} = k_{it}[\beta_k(\eta_{it}) - \beta_k^{\mu}] + l_{it}[\beta_l(\eta_{it}) - \beta_l^{\mu}]$
- A conditional mean equation for (8) can be written as

$$y_{it} = \beta_k^{\mu} k_{it} + \beta_l^{\mu} l_{it} + \omega_{it} + \varepsilon_{it}, \tag{12}$$

- Here, $\mathbb{E}[\varepsilon_{it} | \mathcal{I}_{it}] = 0$
- The production function coefficients $\boldsymbol{\beta}(\tau) = (\beta_k(\tau), \beta_l(\tau))$ are non-parametrically identified with $T = 2$ under conditional independence assumptions and other mild regularity conditions

Identification

- For ease of notation let $x_{it} = (k_{it}, l_{it}), x_{it+1} = (k_{it+1}, l_{it+1})$, and $x_i = (x_{it}, x_{it+1})$
- Let $Z_{it} = \beta_k(\eta_{it})k_{it} + \beta_l(\eta_{ti})l_{it}$
- For any random variable X and $\rho \neq 0$, let $\tilde{X} = X/\rho$
- Two consecutive period of output can be written as $y_{it} = Z_{it} + \omega_{it}$ and $y_{it+1} = Z_{it+1} + \omega_{it+1}$
- Assume a linear AR(1) process for productivity, $\omega_{it+1} = \rho\omega_{it} + \xi_{it+1}$
- Plugging into second period observation equation gives $\tilde{y}_{it+1} = y_{it+1}/\rho = \tilde{Z}_{it+1} + \tilde{\xi}_{it+1} + \omega_{it}$
- So there are two repeated measures of productivity

$$\begin{aligned} y_{it} &= Z_{it} + \omega_{it} \\ \tilde{y}_{it+1} &= \tilde{Z}_{it+1} + \tilde{\xi}_{it+1} + \omega_{it}. \end{aligned} \tag{13}$$

Identification

- Goal is identification of the conditional quantile

$$Q_\tau(Z_{it}|x_i) = x_{it}\beta(\tau),$$

which can be identified if the conditional distribution function

$$F_{Z_{it}|x_i}(Z_{it}|x_i) = \frac{1}{2} - \lim_{\upsilon \to \infty} \int_{-\upsilon}^{\upsilon} \frac{e^{-\mathrm{i}sZ_{it}}}{2\pi \mathrm{i}s} \phi_{Z_{it}|x_i}(s|x_i)ds, \qquad (14)$$

is identified

- Since the quantile function is the inverse of the CDF, this implies identification of the conditional quantiles
- Identification relies on the conditional characteristic function $\phi_{Z_{it}|x_i}(s|x_i)$ being identified

Identification

- Utilize conditional deconvolution arguments to identify this conditional characteristic functions up to an unknown location, $\mathbb{E}[\omega_{it}|x_i]$
- Similar ideas have been used in panel data models such as Neumann, 2007 and Evdokimov, 2010
- Identification of the location relies on identification of $\boldsymbol{\beta}^{\mu} = (\beta_k^{\mu}, \beta_l^{\mu})$ from Equation (12) and the parameter ρ
- Similar to ACF, we show that these parameters are identified by the moment restriction $\mathbb{E}[\xi_{it} + \varepsilon_{it}|\mathcal{I}_{it-1}] = 0$ from Equation (7)
- Once this is established, the characteristic functions can be identified using $T = 2$ firm-year observations.

Identification

Assumption 1

- *Random Sample: The random variables* $(y_i, Z_i, \omega_i)_{i=1}^{N}$ *are independently and identically distributed and* $T = 2$.
- *Conditional Independence: (i)* $f_{Z_{it}|\omega_{it},x_i}(Z_{it}|\omega_{it}, x_i) = f_{Z_{it}|x_i}(Z_{it}|x_i)$, *(ii)* $f_{Z_{it+1}|Z_{it},\xi_{it+1},\omega_{it},x_i}(Z_{it+1}|Z_{it}, \xi_{it+1}, \omega_{it}, x_i) = f_{Z_{it+1}|x_i}(Z_{it+1}|x_i)$, *and (iii)* $f_{\xi_{it+1}|Z_{it},\omega_{it},x_i}(\xi_{it+1}|Z_{it}, \omega_{it}, x_i) = f_{\xi_{it+1}|x_i}(\xi_{it+1}|x_i)$.
- *Characteristic Functions: The conditional characteristic functions* $\phi_{Z_{it}|x_i}(s|x_i)$, $\phi_{\omega_{it}|x_i}(s|x_i)$, *and* $\phi_{\xi_{it+1}|x_i}(s|x_i)$ *do not vanish.*
- *Quantiles:* $\eta_{it} \perp\!\!\!\perp (x_{it}, \omega_{it})$ *where* $\eta_{it} \sim U(0, 1)$ *and* Z_{it} *is strictly increasing in* η_{it}.

Identification

Assumption 2

- *Information Set:* $\mathcal{I}_{it}$ *only includes current and past productivity shocks.*
- *Productivity: Productivity follows* $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$, *where* ξ_{it} *is a shock that satisfies* $\mathbb{E}[\xi_{it}|\mathcal{I}_{it-1}] = 0$
- *Timing of Input Choices: Firms accumulate capital according to*

$$K_{it} = \kappa(K_{it-1}, I_{it-1}).$$

- *Scalar Unobservable: Firm's intermediate input demand is given by*

$$m_{it} = m_t(k_{it}, l_{it}, \omega_{it}).$$

- *Strict Monotonicity:* $m_t(k_{it}, l_{it}, \omega_{it})$ *is strictly increasing in* ω_{it}.
- *Identification: There exists a neighborhood of* (β^μ, ρ) *such that* (β^μ, ρ) *is the unique solution to Equation* (7).

Identification

- Assumption 1(a) places restrictions on the data generating process
- Assumption η_{it} is independent of ω_{it} conditional on two-period of inputs x_i
- Assumption 1(b)(ii) implies that η_{it+1} is independent of η_{it}, ξ_{it+1}, and ω_{it} conditional on x_i
- Assumption 1(b)(iii) implies that ξ_{it+1} is independent of η_{it} and ω_{it} conditional on x_i
- Assumption 1(c) are mild technical conditions on characteristic functions
- Assumption 2 is a modification of the ACF assumptions used to prove identification of $\mathbb{E}[\omega_{it}|x_i]$

Identification

Theorem 1

Under Assumptions 1 and 2, the location parameters β^{μ} *and* ρ*, the function* $\beta(\tau)$ *for each* $\tau \in (0,1)$ *and the distribution of productivity are identified.*

Econometric Procedure

- Estimation can proceed by constructing sample moments based on the conditional characteristic functions
- Then these estimates can be plugged into (14) and the conditional quantiles can constructed from the inverse relationship between CDF and quantiles
- This approach would be computationally burdensome. Provide a simple estimator that is consistent and asymptotically normal
- Recall $\varepsilon_{it} = k_{it}[\beta_k(\eta_{it}) - \beta_k^\mu] + l_{it}[\beta_l(\eta_{it}) - \beta_l^\mu]$
- Here $(\beta_k^\mu, \beta_l^\mu) = \boldsymbol{\beta}^\mu = \mathbb{E}[\boldsymbol{\beta}(\eta_{it})]$ is the mean of the random coefficients
- The conditional mean version of the random coefficient production function is

$$y_{it} = \beta_k^\mu k_{it} + \beta_l^\mu l_{it} + \omega_{it} + \varepsilon_{it}, \tag{15}$$

where $\mathbb{E}[\varepsilon_{it} | \mathcal{I}_{it}] = 0$

Econometric Procedure

Estimation Procedure

1. Let $\hat{\beta}_k^\mu$ and $\hat{\beta}_l^\mu$ be consistent estimators of β_k^μ and β_l^μ from a value-added production function. Construct the estimator, $\hat{\omega}_{it} = \hat{\Phi}_t(k_{it}, l_{it}, m_{it}) - \hat{\beta}_k^\mu k_{it} - \hat{\beta}_l^\mu l_{it}$, using these estimates.
2. Let $\boldsymbol{\beta}(\tau) = (\beta_k(\tau), \beta_l(\tau))$ and $\hat{y}_{it} = y_{it} - \hat{\omega}_{it}$. For $\tau \in (0,1)$, define the two-step estimator of $\boldsymbol{\beta}(\tau)$ as:

$$\hat{\boldsymbol{\beta}}(\tau) = \underset{\boldsymbol{\beta} \in \mathcal{B}}{\operatorname{argmin}} \, \mathbb{E}\left[\rho_\tau(\hat{y}_{it} - \beta_k k_{it} - \beta_l l_{it})\right],$$

   where $\mathcal{B}$ is a compact and convex parameter space, $\rho_\tau(u) = u[\tau - \mathbb{I}\{u < 0\}]$, and $\mathbb{I}\{\cdot\}$ denotes the indicator function.

Large Sample Properties

- The two-step estimator relies on an initial consistent estimator of productivity
- Standard errors from the estimator of the asymptotic covariance matrix include the variance from these estimate
- The model falls under a class of generated dependent variables in quantile regression
- The main challenge of our approach is two-fold
  1. the first stage is semi-parametric due to the non-parametric function, $\Phi_t(k_{it}, l_{it}, m_{it})$
  2. The finite parameters β_k^{μ} and β_l^{μ} and the asymptotic covariance matrix for β_k^{μ} and β_l^{μ} include the variance from estimating $\Phi_t(k_{it}, l_{it}, m_{it})$
- In the main text, it is shown that following Chernozhukov and Hansen, 2005 the quantile regression estimates are consistent and asymptotically normal

Large Sample Properties

- Estimation of the asymptotic covariance matrix is complicated in the two-step semi-parametric approach
- Need to estimate an influence function that is derived from ACF estimator

$$\psi_{it} = \begin{pmatrix} \psi_{it}^{\theta} \\ \psi_{it}^{\beta\mu} \end{pmatrix} = \begin{pmatrix} \Sigma_z^{-1} g_1(Z_t;\theta) \\ -(D_{\beta^\mu}\Sigma_x D_{\beta^\mu}^{'})^{-1} D_{\beta^\mu}\Sigma_x g_2(x;\beta^\mu,\theta) \end{pmatrix}.$$

- Assumed that $\Sigma_z = \mathbb{E}[\mu_z \mu_z^{'}]$ is non-singular with finite norm
- $g_1(Z_t;\theta) = p^{k_n}(z_{it})\varepsilon_{it}$ from first stage
- $g_2(x;\beta^\mu,\theta)$ from second stage
- Σ_x is a positive-definite weighting matrix
- $D_{\beta^\mu} = \frac{\partial}{\partial\beta^\mu} g_2(x;\beta^\mu,\theta)$
- Instead, nonparametric bootstrap is used for inference

Monte Carlo Experiments

- A location-scale model for the production function is specified as

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + (\gamma_k k_{it} + \gamma_l l_{it})\eta_{it} \tag{16}$$

  where $\beta_k = 0.4$ and $\beta_l = 0.6$
- Replicate ACF simulations by sampling 1000 datasets consisting of 1000 firms
- Simulate optimal input choices for 100 time periods, using the last 10 periods for estimation
- Consider three different data generating processes (DGPs) for the scale parameters and the distribution of η_{it}
  1. DGP 1: $\gamma_k = 0.6$, $\gamma_l = -0.6$ and $\eta_{it} \sim N(0, \sigma_\eta^2)$ where $\sigma_\eta^2 = 0.01$
  2. DGP2: $\gamma_k = 0.4$, $\gamma_l = -0.4$ and $\eta_{it} \sim (\sqrt{3\sigma_\eta^2/5})t_5$
  3. DGP3: $\gamma_k = 0.5$, $\gamma_l = -0.5$, and $\eta_{it} \sim \textit{Lognormal}(0.15, \sigma_\eta^2)$
- Productivity follows $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$ where $\rho = 0.7$

Monte Carlo Results

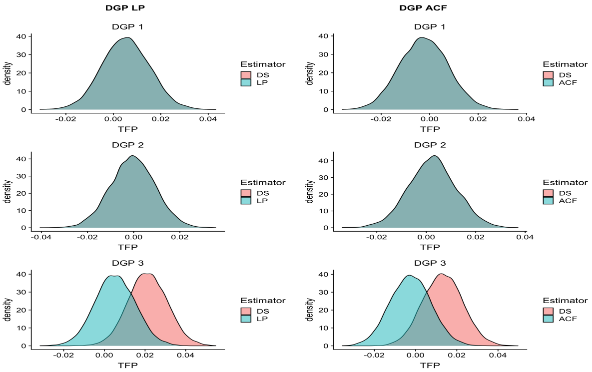
		DS				QR			
		Capital		Labor		Capital		Labor	
DGP	τ	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
ACF 1	0.10	-0.0099	0.0003	0.0129	0.0003	0.2881	0.0831	-0.2941	0.0865
	0.25	-0.0035	0.0003	0.0045	0.0001	0.3115	0.0971	-0.3175	0.1008
	0.50	-0.0010	0.0002	0.0000	0.0001	0.3380	0.1143	-0.3440	0.1184
	0.75	0.0025	0.0003	-0.0035	0.0001	0.3645	0.1329	-0.3715	0.1380
	0.90	0.0089	0.0003	-0.0119	0.0002	0.3889	0.1513	-0.3949	0.1560
ACF 2	0.10	-0.0057	0.0002	0.0087	0.0002	0.3053	0.0611	-0.3103	0.0635
	0.25	-0.0015	0.0002	0.0025	0.0001	0.3225	0.0870	-0.3285	0.0906
	0.50	0.0000	0.0002	0.0000	0.0001	0.3390	0.1150	-0.3450	0.1190
	0.75	0.0025	0.0002	-0.0025	0.0001	0.3555	0.1461	-0.3615	0.1506
	0.90	0.0067	0.0003	-0.0077	0.0001	0.3727	0.1850	-0.3787	0.1910
ACF 3	0.10	-0.0043	0.0003	0.0043	0.0001	0.3067	0.0975	-0.3127	0.1013
	0.25	-0.0015	0.0003	0.0015	0.0001	0.3145	0.0998	-0.3205	0.1036
	0.50	-0.0006	0.0003	0.0006	0.0001	0.3294	0.1057	-0.3354	0.1096
	0.75	0.0019	0.0003	-0.0029	0.0001	0.3519	0.1196	-0.3579	0.1245
	0.90	0.0097	0.0004	-0.0137	0.0003	0.3817	0.1459	-0.3877	0.1513

Monte Carlo Results

		DS				QR			
		Capital		Labor		Capital		Labor	
DGP	τ	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
LP 1	0.10	-0.0149	0.0004	0.0199	0.0004	0.2691	0.0830	-0.2771	0.0865
	0.25	-0.0045	0.0002	0.0065	0.0000	0.2905	0.0971	-0.2975	0.1008
	0.50	0.0010	0.0002	0.0000	0.0000	0.3140	0.1143	-0.3210	0.1183
	0.75	0.0065	0.0002	-0.0065	0.0000	0.3375	0.1329	-0.3445	0.1380
	0.90	0.0169	0.0005	-0.0199	0.0004	0.3589	0.1513	-0.3659	0.1560
LP 2	0.10	-0.0097	0.0003	0.0127	0.0002	0.2833	0.0610	-0.2903	0.0635
	0.25	-0.0035	0.0002	0.0035	0.0000	0.2985	0.0870	-0.3065	0.0905
	0.50	0.0000	0.0002	0.0000	0.0000	0.3140	0.1149	-0.3210	0.1190
	0.75	0.0035	0.0002	-0.0035	0.0000	0.3285	0.1460	-0.3365	0.1506
	0.90	0.0097	0.0003	-0.0127	0.0002	0.3447	0.1849	-0.3517	0.1910
LP 3	0.10	-0.0053	0.0002	0.0083	0.0001	0.2857	0.0975	-0.2927	0.1013
	0.25	-0.0015	0.0002	0.0035	0.0000	0.2935	0.0998	-0.3005	0.1036
	0.50	0.0004	0.0002	-0.0004	0.0000	0.3064	0.1056	-0.3144	0.1096
	0.75	0.0049	0.0002	-0.0059	0.0000	0.3269	0.1196	-0.3349	0.1245
	0.90	0.0167	0.0005	-0.0197	0.0004	0.3537	0.1459	-0.3617	0.1513

Monte Carlo Results

Figure 1: Monte Carlo Results for Total Factor Productivity Estimates

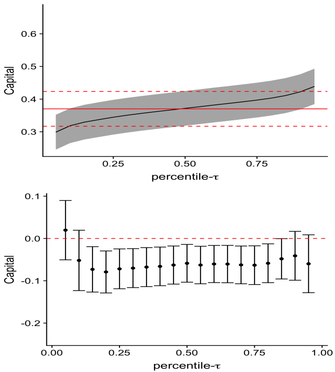


*Estimated TFP from LP, ACF, and the median DS estimator for three DGPs: The left panel compares the LP estimator and the DS estimator when productivity is estimated using LP; The right panel compares the ACF estimator and the DS estimator when productivity is estimated using ACF.

Application

- Estimator is applied to firm and plant-level manufacturing datasets from US, Chile, and Colombia to examine heterogeneity in production
- US data comes from Compustat and covers public firms. Sample manufacturing industries from 1961 to 2016
- Chile data comes from the census of Chilean manufacturing plants conducted by the INE
- Colombia data comes from the census of Colombian manufacturing firms conducted by the Departamenta Administrativo Nacional de Estadistica
- Estimates are examined across different manufacturing and industries
- Bootstrap to estimate standard errors of $\beta_k(\tau)$ and $\beta_l(\tau)$ with the number of iterations set to 500.

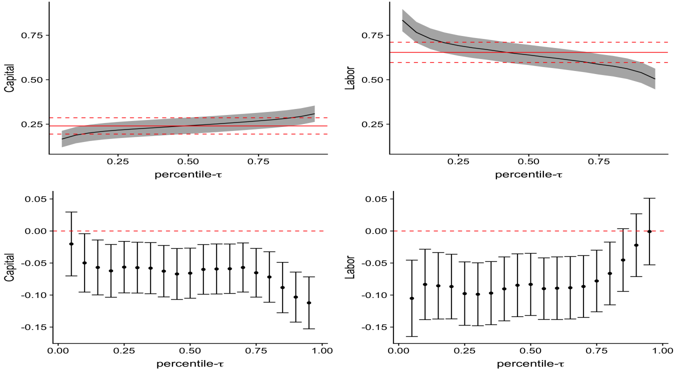
Figure 2: Estimated Coefficients of Capital and Labor for U.S.: NAICS 31



*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

US Compustat

Figure 3: Estimated Coefficients of Capital and Labor for U.S.: NAICS 32



*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

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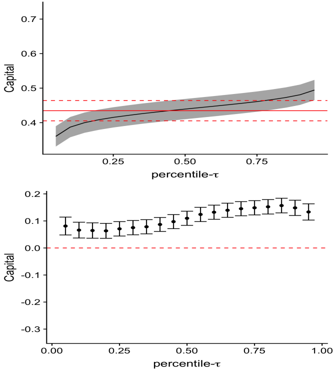
Figure 4: Estimated Coefficients of Capital and Labor for U.S.: NAICS 33



*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

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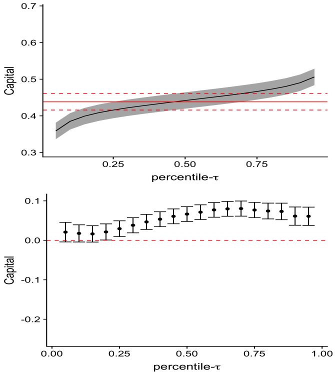
Figure 5: Estimated Coefficients of Capital and Labor for U.S. Manufacturing Firms



*Top row: Estimated values of production function coefficients and their point-wise 90% confidence interval. Bottom row: Difference between DS and QR estimates that does not control for endogeneity and their 95% confidence intervals.

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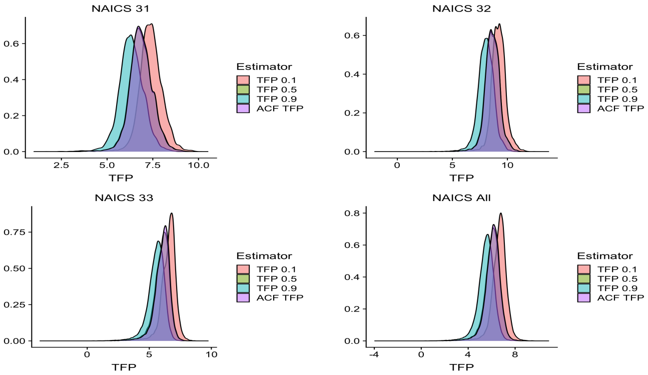
		Capital		Labor		Returns to Scale		Capital Intensity	
NAICS	τ	Coef.	s.e	Coef.	s.e	Coef.	s.e	Coef.	s.e
31	0.10	0.319	0.0323	0.554	0.0383	0.873	0.0161	0.575	0.0787
	0.25	0.345	0.0324	0.500	0.0375	0.845	0.0143	0.689	0.0915
	0.50	0.372	0.0323	0.450	0.0369	0.821	0.0133	0.827	0.1073
	0.90	0.422	0.0327	0.374	0.0390	0.797	0.0204	1.127	0.1420
32	0.10	0.189	0.0280	0.766	0.0359	0.955	0.0132	0.246	0.0478
	0.25	0.217	0.0279	0.692	0.0349	0.909	0.0119	0.313	0.0555
	0.50	0.242	0.0279	0.639	0.0346	0.881	0.0114	0.378	0.0630
	0.90	0.293	0.0278	0.540	0.0354	0.833	0.0127	0.543	0.0835
33	0.10	0.387	0.0179	0.605	0.0226	0.992	0.0070	0.640	0.0283
	0.25	0.415	0.0178	0.522	0.0220	0.937	0.0061	0.796	0.0330
	0.50	0.439	0.0178	0.468	0.0217	0.907	0.0058	0.939	0.0371
	0.90	0.481	0.0179	0.385	0.0216	0.866	0.0057	1.250	0.0459
All	0.10	0.385	0.0136	0.576	0.0172	0.962	0.0055	0.668	0.0262
	0.25	0.416	0.0136	0.495	0.0168	0.910	0.0049	0.841	0.0311
	0.50	0.442	0.0136	0.445	0.0168	0.887	0.0052	0.992	0.0354
	0.90	0.490	0.0137	0.363	0.0171	0.853	0.0064	1.352	0.0458

Figure 6: DS and ACF Estimates of Log Total Factor Productivity



*Estimated distributions of TFP from the DS estimator for $\tau \in \{0.1, 0.5, 0.9\}$ and those from the ACF estimator.

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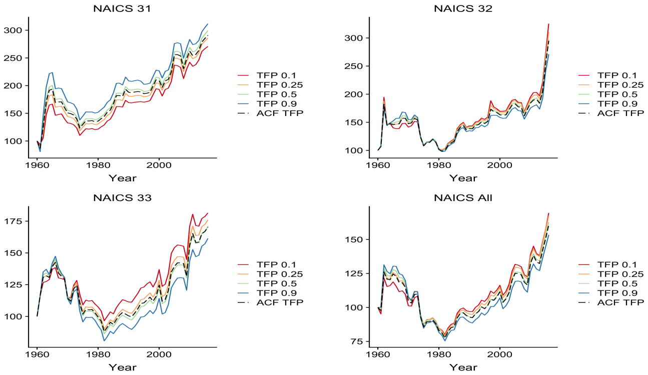
Figure 7: U.S. Productivity Over Time



*Estimated average productivity (in levels) over time for the U.S. Base year productivity is set to 100.

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NAICS	τ	R&D		Advertisements	
		Coef.	s.e	Coef.	s.e
31	0.10	0.157	0.0160	0.187	0.0197
	0.25	0.170	0.0143	0.200	0.0178
	0.50	0.181	0.0133	0.211	0.0162
	0.90	0.190	0.0139	0.219	0.0159
32	0.10	0.105	0.0092	0.112	0.0105
	0.25	0.133	0.0093	0.139	0.0103
	0.50	0.148	0.0088	0.154	0.0098
	0.90	0.175	0.0088	0.180	0.0099
33	0.10	0.064	0.0054	0.048	0.0054
	0.25	0.098	0.0047	0.076	0.0047
	0.50	0.115	0.0046	0.091	0.0045
	0.90	0.138	0.0050	0.109	0.0047
All	0.10	0.097	0.0047	0.082	0.0051
	0.25	0.126	0.0042	0.109	0.0045
	0.50	0.138	0.0040	0.120	0.0043
	0.90	0.154	0.0042	0.133	0.0042

Conclusion

- Proposed a method that extends the control function approach to quantiles of firm output
- Computationally attractive, easy to implement
- Estimator works well in finite samples, consistent and asymptotically normal
- Limitations of control function approach apply to this model as well
- Future work could find the extension to gross-output production function in the framework of Gandhi, Navarro, and Rivers, 2020
- Possible extension to unconditional quantile estimates
- Allowing for richer distributional effects: in productivity and inputs is desirable