Debt Callability and Firm Dynamics [*]

Juste Djabakou [†]
University of Montreal and CIREQ

Job Market Paper

September 26, 2025
[**Click here for latest version**]

Abstract

Callable debt - a bond allowing issuers to redeem outstanding principal before maturity - has become dominant in U.S. corporate bond markets since the Global Financial Crisis. This paper examines the relationship between callability and firm investment, as well as its macroeconomic implications. Using an issuer–bond panel, I document a gap between contract design and effective eligibility: while 59% of outstanding debt is contractually callable, only about 1.7% is callable at the beginning of a typical quarter due to lock-out provisions; conditional on eligibility, firms call about 3.5% of eligible debt per quarter. Firms with weaker credit quality tend to rely more on callable issuance but exercise the calls less frequently. Exploiting high-frequency monetary policy surprises in a local-projection framework, I find that callable debts amplify investment responses only when firms actually call, with the strongest effects for firms holding larger eligible callable shares. To interpret these facts, I develop a heterogeneous-firm model with long-term non-callable and callable debts, endogenous default and call decisions, and investment. The model quantitatively replicates the cross-sectional incidence of contractual versus effective callability, the conditional call hazard, and the investment behavior. Callability increases the rate sensitivity of investment by about 21% relative to a baseline without callable debt, highlighting how debt callability in firms' financing structures influences aggregate investment dynamics.

Keywords: Debt Callability, Financial Frictions, Firm Investment, Firm Heterogeneity
JEL Codes: E22, E44, G31, G32, O16

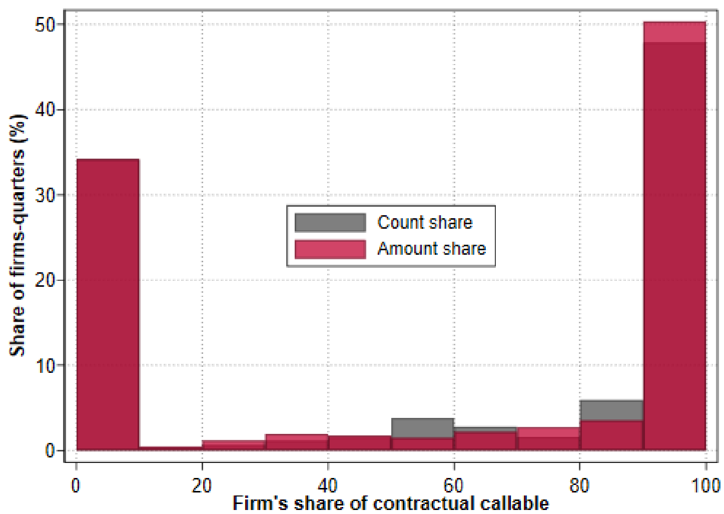
[*] I am extremely grateful to Immo Schott and Hyejin Park for their invaluable, continuous guidance and support. I thank my committee members, Réné Garcia and Guillaume Sublet, for their insightful comments and suggestions. For the helpful discussions, I also thank Firmin Ayivodji, Louphou Coulibaly, Miguel Faria-e-Castro, Bruno Feunou, Félicien Goudou, Jihyun Kim, Marlene Koffi, Julian Kozlowski, Ricardo Marto, Hannah Rubinton, Juan M. Sánchez as well as participants at various seminars and conferences at the University of Montreal, the Annual Congress of SCSE, the CIREQ Ph.D. Students' Conferences, the 57th and 58th Annual Conferences of the CEA, and the Federal Reserve Bank of St. Louis. I acknowledge financial support from the FRQSC grant and Chaire de la fondation J.W. McConnell en études américaines. All errors and shortcomings in this paper are mine.

[†] Email: juste.djabakou@umontreal.ca • Website: https://justedjabakou.github.io

1 Introduction

Firms' balance sheets are central to understanding business cycles and investment dynamics.[1] How firms manage their capital structure is therefore crucial for assessing both firm behavior and its aggregate implications.[2] While much research has focused on the composition and maturity of corporate debt, the callable nature of bonds—where issuers retain the option to redeem debt before maturity—remains largely unexplored.[3]

Figure 1: Distribution of outstanding share of callable in the U.S.



Notes: This figure overlays two distributions of firms' average share of callable bonds. The first (in gray) concerns the number of callable bonds, and the second (in red) shows the outstanding amount. The sample period is 1990Jan–2018May. Source: Mergent FISD & CRSP-Compustat.

Figure 1 shows that callable bonds are widespread but unevenly distributed across U.S. firms. The higher prevalence in terms of offering amounts indicates that callable bonds are concentrated in larger issuances, reflecting the financing strategies of firms with greater market access. Motivated by this distribution, I ask: under what conditions do firms issue callable bonds, and how does this choice vary across credit risk and firm size? How does the call option affect borrowing and investment decisions? And what factors drive the decision to redeem debt before maturity? These questions guide the analysis.

[1] An extensive macroeconomic literature starting from Bernanke and Gertler (1989); Kiyotaki and Moore (1997) has shown the importance of firms' balance sheets for macroeconomic fluctuations. Gomes (2001); Cooley and Quadrini (2001) emphasize that financial frictions account for firms' investment behavior and dynamics.

[2] Leland and Toft (1996); Covas and Haan (2011); Jermann and Quadrini (2012) stress the importance of dynamic capital structure choices.

[3] On debt maturity in macroeconomics and finance, see Demirgüç-Kunt and Maksimovic (1999); Gomes et al. (2016); Gürkaynak et al. (2022); Fabiani et al. (2022).

Using a novel issuer–bond panel covering 1990–2018, I establish three central facts. First, there is a sharp gap between contractual and effective callability: while about 59% of outstanding debt is contractually callable, only 1.7% is effectively eligible for redemption at the beginning of a typical quarter due to lockout provisions. Conditional on eligibility, firms call roughly 3.5% of the eligible amount per quarter. Second, callable issuance has become the dominant design over time—rising from about 60% before the Global Financial Crisis to more than 90% afterward—but its use and exercise are highly heterogeneous across firms. Smaller and riskier issuers rely more heavily on callable debt yet exercise calls infrequently, whereas larger and safer firms call more aggressively when conditions are favorable. Third, exploiting high-frequency monetary policy surprises in a local-projection framework, I show that investment responses are amplified only when firms actually call, with the strength of amplification increasing in the stock of eligible callable debt. Consistent patterns appear for sales and debt issuance, underscoring that callability operates as a refinancing channel that relaxes constraints only when exercised.

Bond markets have expanded markedly in recent decades, providing firms with a wider array of financing instruments (Jungherr and Schott, 2021; Darmouni et al., 2022). Among these, callability plays a distinctive role. It alters the effective maturity of debt and introduces refinancing flexibility for firms, while exposing investors to prepayment risk. For firms, callable bonds replace rollover risk with refinancing risk (Guntay et al., 2004). When interest rates fall, the call option allows refinancing at lower cost and relaxes debt overhang; when rates rise, callable bonds remain outstanding, behaving like long-term debt. For investors, the embedded option is priced as compensation for prepayment risk, making callable bonds more sensitive to the level and volatility of interest rates (Duffee, 1998; Gilchrist and Zakrajšek, 2012). These features highlight callability as a strategic margin of capital structure choice.

To interpret these findings, I first develop a three-period model with endogenous investment, default, callable share, and call decisions to build intuition. Consistent with the data, the model predicts that less productive, riskier firms issue more callable debt.[4] I then extend the framework to a quantitative heterogeneous-firm model in which firms jointly choose issuance between callable and non-callable debt, decide whether to exercise calls when eligible, invest, and default. The model introduces several innovations. First, firms can issue callable and non-callable debt simultaneously. Second, callability is an endogenous choice, so the share of callable debt is determined within the model. Third, the call decision itself is endogenous, allowing the probability of calling to vary with firm and aggregate states and to feed back into bond pricing.[5] This feature captures how shocks alter the effective callable share and how issuance choices reflect a trade-off

[4] See also Gilchrist and Zakrajšek (2012); Clymo and Rozsypal (2023); Poeschl (2023). For related evidence on firms' size and financing, see Begenau and Salomao (2019); Kochen (2023).

[5] Contrast with models where call intensity is exogenous, e.g. Guntay (2002); Jarrow et al. (2010). In Leland and Toft (1996); Goldstein et al. (2001); Chen et al. (2021), firms can restructure by fully calling existing debt.

between default risk and refinancing flexibility.[6]

A further contribution lies in the model's debt pricing. New debt is priced off the current interest rate, while outstanding debt is priced off its fixed coupon at issuance. This distinction reflects contractual rigidity and creditor heterogeneity, in contrast to models assuming a unified continuation price.[7] Finally, I distinguish callable and non-callable bonds explicitly in firms' state variables, since they follow different laws of motion and pricing rules. Callable bonds may be redeemed within the allowed window, while non-callable bonds remain until maturity. I also incorporate stochastic coupons to capture interest rate variation in new issuance. These elements provide a richer depiction of firms' capital structure dynamics, albeit at the cost of greater computational complexity.[8]

To capture these mechanisms, I build a quantitative heterogeneous-firm model in which firms jointly choose issuance between callable and non-callable debt, decide whether to exercise calls when eligible, invest, and default. The model embeds the refinancing option as an endogenous margin of capital structure, allowing me to quantify how callability affects both firm-level behavior and aggregate investment dynamics.

Related Literature. This paper contributes to two strands of research. The first is the macroeconomic literature on firm heterogeneity, financial frictions, and the aggregate implications of corporate debt structures. The second is the corporate finance literature on callable bonds and their role in firms' financing and investment decisions.

On the macroeconomic side, my work relates to models that incorporate corporate debt choices into dynamic frameworks to study the transmission of aggregate shocks. Seminal contributions include Kiyotaki and Moore (1997) and Bernanke et al. (1999), who emphasize the role of balance-sheet constraints in amplifying business cycles. More recent work develops heterogeneous-firm models with financial frictions to study monetary policy transmission and financial crises. For example, De Fiore and Uhlig (2015) highlight the substitution between bank loans and bond financing, while Crouzet (2018) and Martellini et al. (2018) study how the composition of corporate debt affects investment dynamics. Related contributions include Salomao and Varela (2022), Arellano et al. (2019), Jeenas (2024), Arellano et al. (2020), Jungherr and Schott (2021, 2022), and Ottonello and Winberry (2020), who provide frameworks to quantify how debt heterogeneity shapes the transmission of shocks and the business cycle. My paper extends this literature by introducing the endogenous choice of debt callability as a novel margin of firms' financing structure.

[6] See also Guntay et al. (2004).

[7] See Duffie and Singleton (1999) for first models that incorporate callable bond valuation, and Jarrow et al. (2010) for their extension.

[8] See Berndt (2004); Jarrow et al. (2010).

On the corporate finance side, I build on studies that examine why firms issue callable bonds and how callability affects firm behavior. Empirical work shows that callable bonds are used to manage refinancing risk (Elsaify and Roussanov, 2016), that issuers tend to call when performance and investment opportunities are strong (Chen et al., 2010), and that callable issuance is associated with under-investment problems and information asymmetries (Banko and Zhou, 2010). Theoretical contributions such as Acharya and Carpenter (2002) and Xie et al. (2009) develop option-based valuation models for callable, defaultable bonds, showing how default and call risk affect duration and pricing. More recent work emphasizes refinancing and maturity management: Xu (2018) show that early refinancing reduces interest payments, while Becker et al. (2024) document that longer-maturity, lower-quality bonds are more likely to be callable, and that call features reduce debt overhang but come at the cost of higher yields. Flor et al. (2023) compare callable and convertible bonds, showing that firms exposed to debt overhang prefer callable debt. My paper complements this literature by embedding callability into a dynamic heterogeneous-firm model with endogenous issuance, call, and default decisions, and by quantifying its implications for investment dynamics.

Finally, related work such as Gilchrist and Zakrajšek (2012) highlights how interest rate changes, term structure, and volatility affect the spreads of callable bonds. While these studies underscore the importance of callability for pricing and issuance, they typically abstract from the joint interaction of callability, default risk, and investment. This paper fills that gap by showing how the endogenous decision to issue and call debt interacts with firms' investment behavior and aggregate dynamics. Given that callable bonds have accounted for more than half of U.S. corporate debt for decades, focusing only on non-callable debt—as much of the macro-finance literature does—misses a central dimension of firms' financing structure. I therefore contribute to bridging corporate finance and macroeconomics by revealing debt callability as a first-order determinant of firm heterogeneity and macroeconomic transmission.

Layout. The rest of the current paper is organized as follows. Section 2 describes the data, the growing interest for callability in the growing importance of bond financing, and facts on callability linked to firms' characteristics. Section 3 presents a three-period model, allowing me to characterize the mechanisms under the role of callable bonds in firm decisions. Section 4 sets up a general equilibrium model with firm investment, borrowing, callability choice, default, and call decision. Section 5 presents the model parametrization, its quantitative results, and the role of debt callability in cross-sectional and aggregate investment dynamics. I conclude in section 6.

2 Empirical Results

This section describes the dataset used in my empirical analysis, the data sources, and the methodology employed to construct the sample. I also present empirical facts on the growing importance of callable bonds in corporate financing and the relationship between callability and firm characteristics.

2.1 Data

I use three different data sources to investigate the debt callability of non-financial firms in the U.S. The data sources and sample construction procedures are detailed below.

2.1.1 Bond level data

The primary source of bond characteristics is the Mergent Fixed Income Securities Database (FISD). This database provides comprehensive information on corporate bonds, including issuance and maturity dates, offering amounts, coupon rates, credit ratings, callability provisions, and call schedules. I focus on U.S. corporate bonds concerned by any event (e.g. issuance, maturity, call) between January 1990 and May 2018, excluding financial firms (SIC 6000–6999), utilities (SIC 4900–4999), foreign issuers, convertible and exchangeable securities, and preferred stock. I consider this period also to match the data availability from other sources used in this research. These filters follow standard practice in the corporate bond literature.

For each issue, I retain detailed information on the presence of call provisions. FISD distinguishes between fixed-price calls (which allow early redemption at a predetermined price) and make-whole calls (where redemption occurs at the present value of remaining cash flows discounted at a Treasury rate plus a spread). The dataset also reports call protection periods, step-down call schedules, and realized call events. This richness enables me to track both the issuance of callable bonds and their exercise over time, which is central to my analysis. Appendix A.4 provides detailed information on how I identify the call protection periods.[9]

To link bond-level information to the issuing firm, I merge FISD with Compustat and CRSP identifiers via CUSIP histories. The final output is a bond–firm–quarter panel, which records at each date the outstanding amounts of callable and non-callable bonds, their maturity profiles, and whether callable bonds are redeemed.

[9]The period between issuance of the bond and the (first) potential call date. It is also called the lockout period.

2.1.2 Firm balance sheet data

Firm-level financial and accounting information comes from Compustat Quarterly, merged with stock market data from CRSP through the link and CUSIP mapping. I construct a firm–quarter panel for all non-financial public firms that have issued bonds during the sample period.

I focus on firms for which debt accounts for at least 5% of total assets. I winsorize all firm-level variables at the 1^{st} and 99^{th} percentiles to reduce the influence of outliers. I deflate nominal variables using the quarterly CPI. Capital stocks are reconstructed using a perpetual inventory method applied to net property, plant, and equipment (ppenq), with interpolation and trimming of outliers as in Ottonello and Winberry, 2020; Jungherr et al., 2024. This construction allows me to compute both levels of capital and cumulative growth rates at multiple horizons. The final firm-level dataset includes standard balance sheet and flow variables such as size (log of assets), leverage (debt-to-assets), investment rate (capex-to-assets), and liquidity (cash-to-assets). I also construct measures of external finance flows (net debt and equity issuance) and debt composition (long-term versus short-term issuance).

My measure of investment is $\Delta^{h+1}\log k_{jt+h}$, where k_{jt+h} is the book value of the real capital stock of firm j in quarter t. To study the callability, I measure firms ' bond market exposures by classifying each bond by its callability type. A bond is deemed callable if the contract specifies an issuer option to redeem before maturity.

2.1.3 Monetary policy shocks

I measure monetary policy shocks using the high-frequency series of Miranda-Agrippino and Ricco (2021) - MIR. Their identification isolates unexpected changes in the policy stance around FOMC announcements and explicitly purges the "central-bank information" component by modeling the joint intraday response of interest-rate futures and equities. Relative to standard high-frequency surprises, the MIR shocks are cleaner (less contaminated by information effects), display more conventional transmission to macro and financial variables, and are now widely used in macro-finance. I use their publicly released series directly, aggregate meeting-day shocks to the quarterly frequency, and standardize them to unit variance for interpretation.

2.1.4 Key variables: contractual versus effective callability

I construct two firm–quarter measures that separate the *design* of debt contracts from the *effective* ability to redeem them at a given date. Let j index issuers and t quarters. Denote by $\mathcal{I}_{jt}$ the set of bonds of issuer j that are outstanding at the beginning of quarter t (i.e., alive at the end of $t-1$). For bond $i \in \mathcal{I}_{jt}$ let a_i be its outstanding amount, $\mathbb{1}\{\text{callable}_i = 1\}$ the FISD contract flag that the indenture contains an issuer call option, and q_i^{prot} the quarter when contractual call protection ends (identified as described in Appendix A.4). Importantly, q_i^{prot} is defined for *all* callable bonds, including make-whole (MW) issues.[10]

Contractual callable share (designation). This variable captures the fraction of a firm's balance sheet that is *designed* to be callable, irrespective of whether the protection period has expired. The amount–weighted measure used is

$$\text{cc}_{jt} = \frac{\sum\limits_{i \in \mathcal{I}_{jt}} a_i \, \mathbb{1}\{\text{callable}_i = 1\}}{\text{debt}_{jt}} \,, \tag{1}$$

where debt_{jt} is the debt of firm j at the beginning of the quarter t. In this baseline, I consider the average total debt of firm j over the quarters $t-1$ to $t-4$.[11] This slow-moving "design" share summarizes firms' ex ante financing choices (callable vs. non-callable) and provides a useful descriptive benchmark.

Effective (scheduled) callable share at the beginning of the quarter. For exposure to actual redemptions and refinancing, what matters is whether protection has already expired by the start of quarter t. I therefore define the scheduled/eligible share

$$\text{sc}_{jt} = \frac{\sum\limits_{i \in \mathcal{I}_{jt}} a_i \, \mathbb{1}\{q_i^{\text{prot}} \le t-1\}}{\text{debt}_{jt}} \,. \tag{2}$$

This is the paper's key state variable; it measures the stock of debt that is *legally callable at BOQ*, hence directly at risk of being redeemed in quarter t.

Conditional call rate at the beginning of the quarter. To summarize the intensity with which firms actually exercise call options once eligible, I construct a beginning-of-quarter (BOQ)

[10] For MW bonds, protection ends at the opening of the MW window; for "CC@ MW" securities, it ends at issuance.

[11] In practice, I use $\text{debt}_{jt} = \sum\limits_{i \in \mathcal{I}_{jt}} a_i$ where I proxy a_i by the FISD offering amount, with an analogous count version replacing a_i by 1. Results remain unchanged when using the count version, and when using alternative outstanding measures of debt_{jt}, such as the beginning-of-quarter outstanding debt. See Appendix B.4 for three alternative measures of the effective callable share. I do not exclude non-callable issues when forming the denominators in equations (1)–(2). First, callable bonds under protection are economically equivalent to non-callable bonds for redemption purposes; excluding non-callable would mechanically overstate the pool actually at risk of being called. Second, the firm's exposure to rollover and refinancing pressure depends on its *entire* outstanding structure, not only on the subset already eligible for call. Third, dropping non-callable conditions on a financing outcome and risks selection bias, since the choice between callable and non-callable contracts is itself endogenous to firm characteristics and macro conditions. Keeping the complete outstanding base, therefore, delivers a conservative and policy-relevant measure of callability exposure.

Table 1: DESCRIPTIVE STATISTICS

Variable	Obs	Mean	S.D.	Min	Max
$\Delta \log k_{jt+1}$	21,990	0.006	0.034	-0.124	0.310
Effective callable share sc_{jt}	13,405	0.017	0.097	0.000	1.000
Call rate (%)	8,936	3.5	18.3	0.000	100.0
Contractual callable share cc_{jt}	13,405	0.594	0.458	0.000	1.000
Maturing bond share (% of debt)	13,405	4.6	19.7	0.000	100.0
Firm size (log real assets)	22,980	8.496	1.041	5.193	12.101
Leverage	22,444	0.293	0.153	0.000	1.846
Liquidity ratio	22,943	0.073	0.080	0.000	0.802
Sales growth	22,639	0.025	0.206	-17.466	4.962
Monetary policy shock (std.)	22,980	-0.005	1.023	-5.844	2.268

Notes: All "rate" variables are shares in $[0,1]$ computed on amounts. Sample sizes differ due to availability by source/definition. $\Delta \log \mathrm{k}$ is quarterly investment growth. The standardized shock z_{mp} has mean ≈ 0 and s.d. ≈ 1 by construction.

call *hazard*:

$$\mathrm{cr}_{jt} = \frac{\sum_{i \in \mathcal{I}_{jt}} a_i \, \mathbb{1}\{\text{bond } i \text{ is called in } t\}}{\sum_{i \in \mathcal{I}_{jt}} a_i \, \mathbb{1}\{q_i^{\text{prot}} \leq t-1\} \, \mathbb{1}\{\text{alive at BOQ}\}} \,. \tag{3}$$

The numerator is the called amount during quarter t, and the denominator is the BOQ stock that was already past contractual protection (the same scheduled eligibility set used at the numerator in equation (2)). By construction, cr_{jt} is a conditional probability (hazard) among bonds that are legally callable at BOQ; it is set to missing when the denominator is zero. I report amount–weighted rates in the baseline and provide count–weighted analogs in robustness (See count-weighted variables definition in Appendix A.5). All callable bonds (including make–whole issues) are included in the sample; scheduled eligibility is defined via q_i^{prot} for all callable contracts, with MW bonds entering when their MW window opens (for "CC@ MW", at issuance).

For completeness, I also track the contemporaneous flow of bonds (newly callable) that *become* eligible in t,

$$\mathrm{nc}_{jt} = \sum_{i \in \mathcal{I}_{jt}} a_i \, \mathbb{1}\{q_i^{\text{prot}} = t\} \;, \tag{4}$$

and use this flow for robustness checks and in IV estimations.[12]

2.1.5 Descriptive statistics

The descriptive patterns reveal a pronounced gap between contractual and effective callability. On average, the contractual callable share at the beginning of the quarter is about 59%, indicating that a large fraction, but not all, of the outstanding amounts are issued under callable designs. In contrast, only about 1.7% of the amounts are effectively callable at the beginning of a typical quarter, reflecting lock-out provisions that postpone eligibility. Conditional on eligibility, issuers call roughly 3.5% of the eligible amount per quarter. The maturing share at the beginning of the quarter averages 4.6%, providing a parallel "maturity channel" exposure.[13] Firms are large (mean log assets of 8.5$), moderately leveraged, consistent with broad public issuers. I keep non-callable debt in the panel because, economically, bonds under protection behave like non-callable from the firm's perspective; including them ensures that (i) the effective exposure to potential refinancing via calls is measured relative to the full balance of outstanding debt and (ii) shifts between design-callable-but-protected tranches and true non-callable are not realistic and do not mechanically driving my results.

In these data, the contractual share is substantial on average (=59%), reflecting that many corporate bonds include a call feature in the indenture, even if it is temporarily inoperative during the lock-out period. Because callable bonds typically spend a sizeable portion of their life under protection, the effectively callable (or scheduled) share is far below the contractual share on average (=1.7% at BOQ). Eligibility can nevertheless swing sharply as large cohorts roll off protection; consistently, the effective share spans the full [0,1] range in the panel, even though its mean is small.

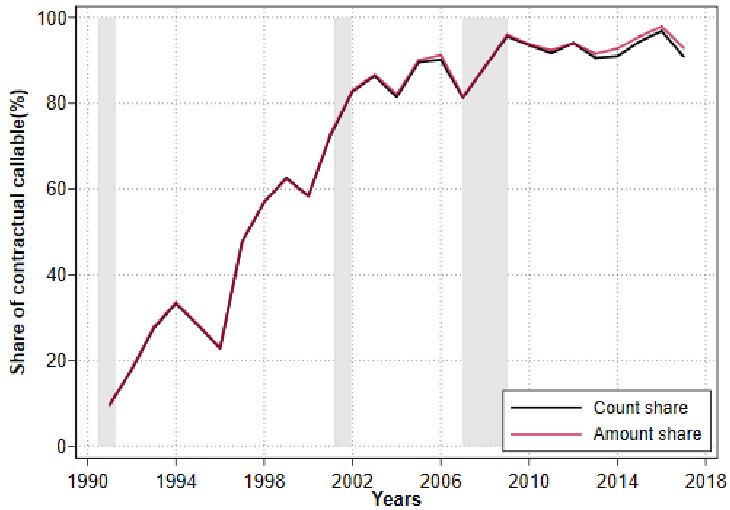
2.1.6 Callability dynamics

The prevalence of callable bonds at issuance has risen dramatically over the past three decades. Figure 2 shows that, while callable features were already common in the 1990s, their share surged after the Global Financial Crisis, reaching above 90% of issuance in recent years. This secular increase reflects a structural shift in corporate financing: firms have increasingly relied on call provisions as a built-in refinancing option, embedding flexibility into long-term debt contracts. The trend is robust across both count- and amount-weighted measures, underscoring that callability is not confined to marginal issuers but has become the dominant design in the U.S. corporate bond market. I document the cross-section of callability by credit quality, considering the callable

[12]This variable is the most important source of fluctuations in the stock of effective callable debt at the beginning of the quarter (See its variations in Appendix A1.). In Appendix B.5, I introduce it in an IV setup due to a plausible endogeneity case, arising from the timing of the call decision, the end of the protection period, and the monetary policy shock.

[13]See Almeida et al., 2012; He and Xiong, 2012; Deng and Fang, 2022; Jungherr et al., 2024 for the works on the maturity channel.

Figure 2: Callable debt issuance over time



Notes: This figure illustrates the share of callable bonds issued over the years. I consider two versions of the contractual callable share defined in equation (1), with the count version in gray and the amount version in red.

share at the issuance and the call rate for the same credit quality groups. In Figure (3a), I observe a correlation between low credit quality and a high share of callable bonds at issuance. Using the call hazard among the beginning-of-the-quarter eligible debts, reported in Figure (3b), I find that low-rated firms have, on average, a very small rate of call. These gradients quantify how callability exposure and the intensity of call exercise vary systematically with credit quality, providing a first link between contractual eligibility, realized call behavior, and refinancing margins faced by issuers with lower versus higher ratings. I provide additional links between these three dimensions within Appendix B.3, to support the first predictions.

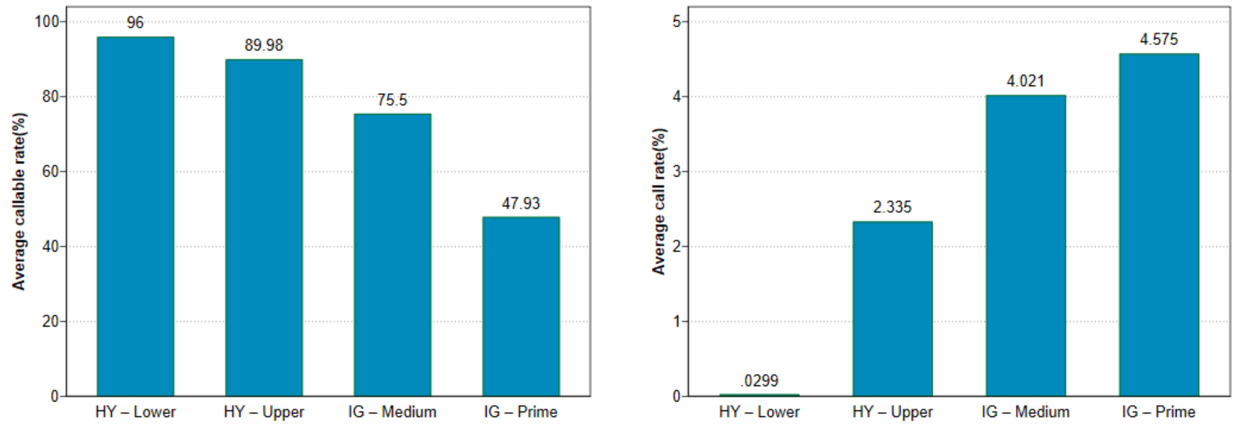
2.2 Heterogeneous responses to monetary policy shocks

To empirically document the dynamics of the investment response to monetary policy shocks associated with the presence of callable debts, I employ the panel local projections technique in the style of Jorda (2005). I start considering a simple specification without callability to show the standard (negative) response of investment to monetary policy shocks. I then augment the framework with callability features in firm-level variables.

Average firm's response. To investigate the investment response to monetary policy shock, I start by documenting the well-known average investment response. Let $\Delta^{h+1} \log k_{jt+h} \equiv \log k_{jt+h} - \log k_{jt-1}$ denote the cumulative capital growth between $t-1$ and $t+h$ of the firm j, with $h = 0...12$.

Figure 3: Callability across firms rating

(a) Issuance (b) Call exercise



Notes: These plots consider the firms group regarding their average credit rating. The left panel shows the average callable share at the issuance in every group, while the right panel reports the firm-level quarterly average of the hazard call rate.

I use the specification:

$$\Delta^{h+1} \log k_{jt+h} = \alpha_j^h + \alpha_{sq}^h + \lambda_0^h \, \varepsilon_t^{\text{mp}} + \lambda_1^h \, g_{t-1} + u_{jt}^h, \tag{5}$$

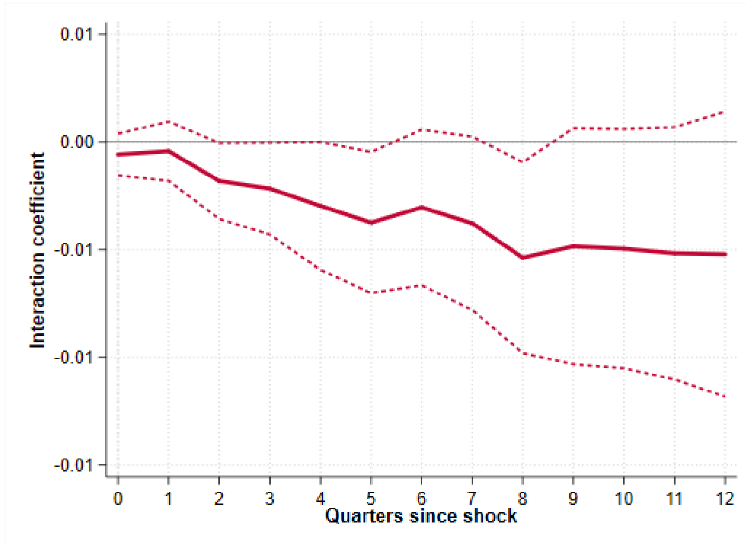
where α_j^h are issuer fixed effects; α_{sq}^h are sector$\times$fiscal-quarter fixed effects. $\varepsilon_t^{\text{mp}}$ is the quarterly monetary policy shock, and $g_{t-1} \equiv \Delta\text{GDP}_{t-1}$ is the lagged GDP growth, representing aggregate shocks. Figure (4) is consistent with prior work, saying that a positive (contractionary) monetary policy surprise depresses investment on impact and over the next few quarters ($\lambda_0^h < 0$).

Dynamics of differential response. This section of the paper presents the key message regarding the role of callability in firm behavior. I study how firm investment responds to monetary policy shocks depending on the firm's effective exposure to callability and on whether the firm actually exercises a call option in the same quarter. The baseline panel local–projection specification I use is:

$$\begin{aligned} \Delta^{h+1} \log k_{jt+h} = \alpha_j^h + \alpha_{st}^h + \beta_0^h \, \widehat{sc}_{jt} \varepsilon_t^{\text{mp}} + \beta_1^h \, x_{jt} \widehat{sc}_{jt} \varepsilon_t^{\text{mp}} \\ + \underbrace{\beta_2^h \, \widehat{sc}_{jt} g_{t-1} + \beta_3^h \, x_{jt} \widehat{sc}_{jt} g_{t-1} + \Lambda^h \, W_{jt-1}}_{\Gamma^h Z_{jt-1}} + u_{jt}^h, \end{aligned} \tag{6}$$

where α_j^h are issuer fixed effects; α_{st}^h are sector$\times$quarter fixed effects. $\varepsilon_t^{\text{mp}}$ is the monetary policy shock standardized to unit standard deviation. $\widehat{sc}_{jt} \equiv sc_{jt} - \mathbb{E}[sc_{jt}]$ is the firm-demeaned

Figure 4: AVERAGE INVESTMENT RESPONSE OVER TIME



Notes: This plot reports in the solid lines, the estimated coefficient λ_0^h over quarters h from the average response' specification $\Delta^{h+1}\log k_{jt+h} = \alpha_j^h + \alpha_{sq}^h + \lambda_0^h\,\varepsilon_t^{\text{mp}} + \lambda_1^h\,g_{t-1} + u_{jt}^h$. Dashed lines report 95% confidence bands of the two-way clustered standard errors.

exposure; $\mathbb{E}[sc_{jt}]$ denotes the firm-specific average over the estimation sample. This removes level differences in baseline callability across firms. $x_{jt} \equiv \mathbb{1}\{\text{firm } j \text{ called at } t\}$ is an indicator for a call event (any amount called) in quarter t.[14] Two set of coefficients are of interest here: (i) the coefficients β_0^h measure the differential response of investment in quarter $t+h$ for firms which have a higher share of effectively callable but do not call them at time of monetary policy shock in quarter t; the coefficients β_1^h capture the effect of the share of effectively callable on firms cumulative response of investment in quarter $t+h$, when they call their bonds at the quarter t of a monetary policy shock.

The control vector is W_{jt-1}, which includes $x_{jt}\varepsilon_t^{\text{mp}}$ and $x_{jt}g_{t-1}$ to take into account the combined effects of the call decision and the shocks. W_{jt-1} also collects lagged firm controls (logged real total assets, leverage, sales growth, liquidity, rating, and the maturing bond share), along with interactions with $\varepsilon_t^{\text{mp}}$ and g_{t-1} the previous quarter's GDP growth to flexibly soak up shock–covariate co-movement.[15] For reasons of simplicity, let Z_{jt-1} denote the vector of all control variables, $Z_{jt-1} = [\widehat{sc}_{jt}g_{t-1}\; x_{jt}\widehat{sc}_{jt}g_{t-1}\; W_{jt-1}]$. Errors are two-way clustered by firm and quarter. The coefficient β_0^h is the slope of the investment response with respect to the (demeaned) effectively callable share in quarters with no call ($x_{jt} = 0$). The incremental effect when the issuer *does* call

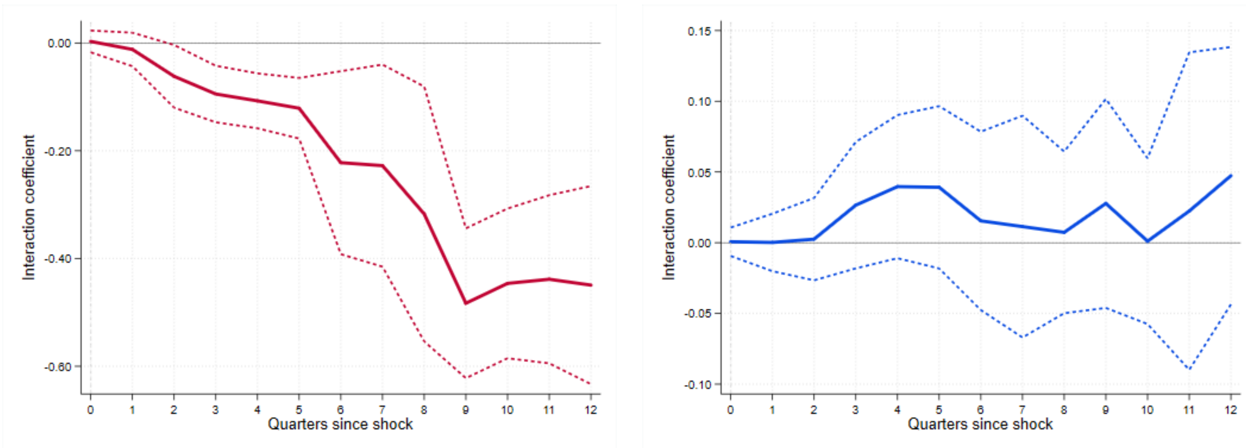
[14]Because the call decision x_{jt} is taken within quarter t, I use contract-driven variation in scheduled eligibility—and the flow of newly callable debt—as instruments in Appendix B.5; results are consistent with the baseline.

[15]The variables rating and maturing bond share are firm-level variables aggregated from FISD data; see their definitions in equations (A12) and (A13). The estimating equation includes all main effects implied by these interactions; $\varepsilon_t^{\text{mp}}$ and aggregate time variation are absorbed by the time fixed effects.

Figure 5: Differential investment response - baseline

(a) When firms call

(b) When firms do not call



Notes: These plots report in the solid lines, the estimated coefficients β_1^h (in red on the left panels) and β_0^h (in blue on the right panels) over quarters h from the baseline specification $\Delta^{h+1}\log k_{jt+h} = \alpha_j^h + \alpha_{st}^h + \beta_0^h\, \widehat{sc}_{jt}\varepsilon_t^{\text{mp}} + \beta_1^h\, x_{jt}\widehat{sc}_{jt}\varepsilon_t^{\text{mp}} + \Gamma^h Z_{jt-1} + u_{jt}^h$, where all variables are already defined above and Z_{jt-1} contains all other variables in the specification (6). Dashed lines report 95% confidence bands of the two-way clustered standard errors.

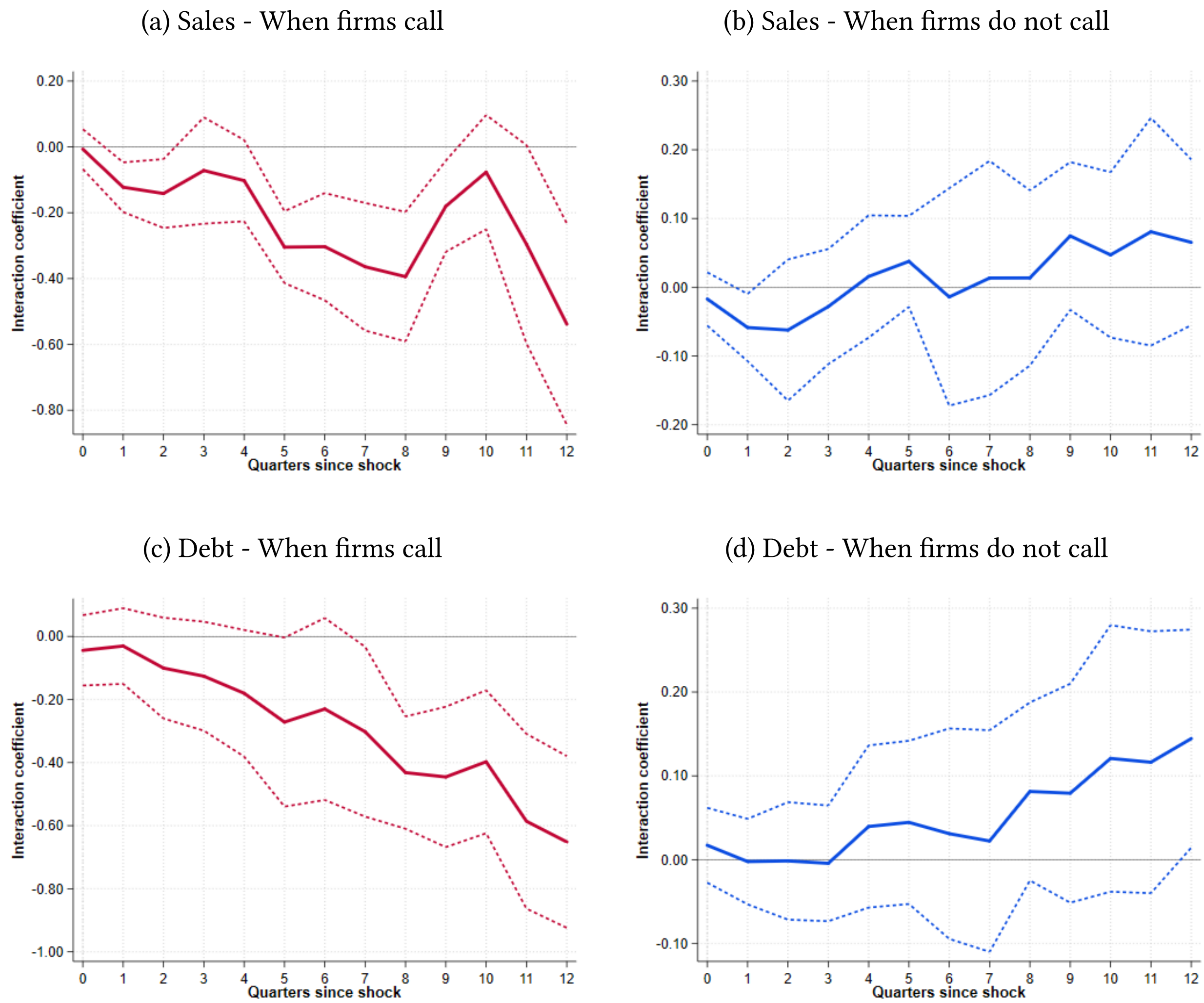
$(x_{jt} = 1)$ is β_1^h, so that the total slope in call quarters equals $\beta_0^h + \beta_1^h$.

With the shock coded so that $\varepsilon_t^{\text{mp}} > 0$ denotes a contractionary surprise, a negative β_1^h implies that, following an easing shock ($\varepsilon_t^{\text{mp}} < 0$), firms that call in t increase investment more the larger is their BOQ eligibility stock—consistent with refinancing constraints relaxing as protections expire. By contrast, β_0^h is generally statistically non-significant from zero across horizons, indicating that the presence of scheduled-callable debt does not affect the investment response when the firm does not call. Regarding Figure 5a, starting at $h = 2$, I find β_1^h with conventional significance. Because the shock is standardized, a one–standard-deviation increase in the scheduled-callable share amplifies the investment response of calling firms by 1% to 3% for the baseline specification. Firm fixed effects absorb time-invariant differences in levels and average callability, and industry×quarter effects absorb common time shocks (including the mean of $\varepsilon_t^{\text{mp}}$). Results are invariant to (i) count weights, (ii) alternative denominators; see Appendix B.4.

2.3 Sales and debt responses to monetary policy shocks

To gain a broader understanding of the role of callability in the firm's optimal decision, I now document the dynamics of the differential response of firm-level sales and debt. I use the same equation (6) from the baseline specification. I find consistent results indicating that firms' responses to monetary policy shocks are amplified by debt callability only when they actually call their bonds; see Figure 6.

Figure 6: Differential sales and debt responses - baseline



(a) Sales - When firms call

(b) Sales - When firms do not call

(c) Debt - When firms call

(d) Debt - When firms do not call

Notes: These plots report in the solid lines, the estimated coefficients β_1^h (in red on the left panels) and β_0^h (in blue on the right panels) over quarters h from the baseline specification $\Delta^{h+1} \log y_{jt+h} = \alpha_j^h + \alpha_{st}^h + \beta_0^h \, \widehat{sc}_{jt}\varepsilon_t^{\text{mp}} + \beta_1^h \, x_{jt}\widehat{sc}_{jt}\varepsilon_t^{\text{mp}} + \Gamma^h Z_{jt-1} + u_{jt}^h$, where the left-hand side variable $y \in \{\text{sales, debt}\}$, and all other variables are already defined under the equation (6). Panels (a) and (b) are about sales, while panels (c) and (d) represent debt responses. Dashed lines report 95% confidence bands of the two-way clustered standard errors.

3 Three-Period Model

This section presents the framework that depicts the interaction mechanism between debt callability and firm behavior. I consider the following five assumptions in the model. First, firms issue the two types of bonds. Second, firms only differ in productivity and face the same operations costs and aggregate shocks. Third, there is no initial outstanding debt, but it is compensated by the second period. Fourth, I allow the firms to issue new debt after the initial callable debt has been redeemed. Fifth, exits follow endogenous default on the debts. These ingredients help characterize firms' optimal decisions (capital, prices, the decision to call, and the refinancing).

Economy. The economy lasts three periods, $t = 0, 1, 2$. Risk-neutral shareholders own the firms. I consider a firm characterized by its productivity, which produces with only capital and is exposed to idiosyncratic capital shocks. Each firm is financed through debt (bond) and equity payout. Equity is initially costless issued in the first period $t = 0$, e_0 and second period $t = 1$, e_1. Equity is positive in this setup ($e > 0$), representing shareholder payout. The firm raises its debt by issuing two-period defaultable non-callable and callable bonds.

Technology. The production process occurs during the periods $t = 1, 2$. The firm produces goods y with the production function:

$$y = zk^{\alpha}, \text{ with } \alpha \in (0, 1)$$

where z is a persistent total factor productivity shock realized in periods $t = 0, 1, 2$. The initial productivity z_0 defines the type of the firm. The productivity shock evolves following a log-AR(1) process $logz_t = \rho logz_{t-1} + \epsilon_{zt}$, where $\epsilon_{zt} \sim^{i.i.d.} \mathcal{N}(0, {\sigma_z}^2)$.

Capital Quality Shocks. The firm receives, after production, an idiosyncratic capital quality shock ε i.i.d. across time and firms. It is drawn from a normal distribution $\mathcal{N}(0, \sigma_{\varepsilon}^2)$. It influences the un-depreciated capital of the firm after production. The capital quality shocks allow the model to generate the default risk and match the default rates observed in the data. It can be viewed as an unforeseen force in the efficiency of capital, such as an unmodeled technological decline that reduces the value of the firm's capital.

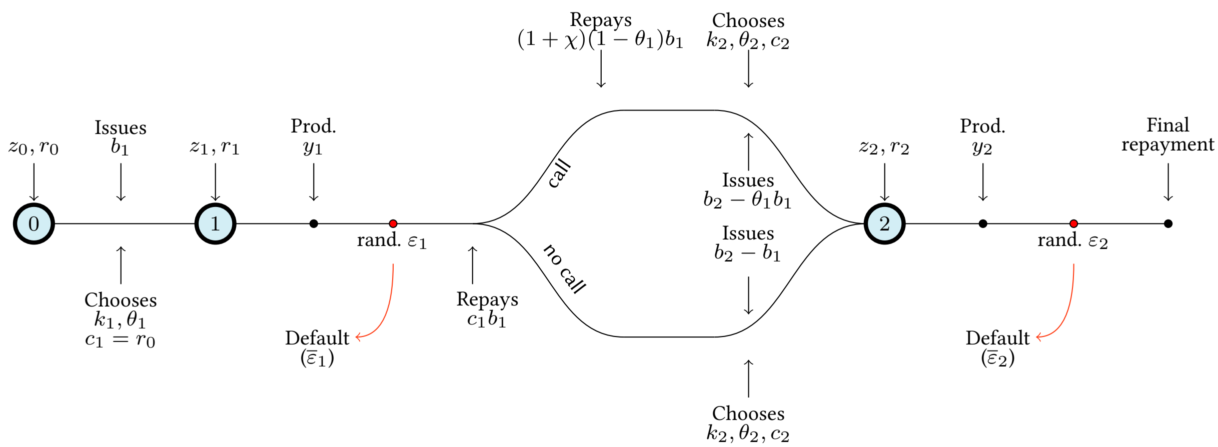
3.1 Setup

A non-callable debt issued in the period $t = 0$ is a promise to pay the fixed coupon c_1 in period $t = 1$ and to repay the principal of the debt to the bondholder together with the fixed coupon c_1, in period $t = 2$. The market price of such a non-callable bond is set at p_0^{nc} in period $t = 0$.

A callable debt issued in the period $t = 0$ with initial maturity in period $t = 2$. It has embedded in it, *a call option*, which gives the right to the issuer to call or redeem the bond in period $t = 1$. The callable bond carries two promises. If the issuer exercises the call option, it repays the debt to the bondholder with a premium χ. The other promise is to pay the fixed coupon of c_1 and continue as a non-callable bond if it is not called. The market price of the callable bond is denoted p_0^c and set in period $t = 0$.

The timing of the three-period model is depicted in Figure 7 and explained in the following

Figure 7: Timing for the three-period setup



lines. At the beginning of the period $t = 0$, the firm gets the productivity shock z_0 and the information on the interest rate r_0 prevailing. It issues equity e_0 and chooses its capital k_1 by issuing a fixed-coupon bond b_1, composed of non-callable bond in share θ_1 and callable bond in share $1 - \theta_1$. Both bonds mature in period $t = 2$. I allow the exercise of the call option to happen in period $t = 1$. I assume that issuance is costly for each type of bond, and I adopt the specification of a quadratic form.

$$\eta_b = \left(\eta_{nc}\theta_1^2 + \eta_c(1-\theta_1)^2\right) \cdot [\max(0, b)]^2$$

The capital constituted for production in the period $t = 1$ is:

$$k_1 = e_0 + p_0^{nc}\theta_1 b_1 + p_0^c(1-\theta_1)b_1 \tag{7}$$

At the beginning of period $t = 1$, the firm has a new productivity z_1 conditional on its previous one and observes the interest rate r_1. The firm produces and receives the capital quality shock ε_1. For simplicity, there is no depreciation of capital. The firm decides to default when its value is less than zero or to repay its debt obligations otherwise. It depends on the realization of capital quality, and when it is under a certain threshold $\bar{\varepsilon}_1$, the firm defaults. Conditional on surviving, the firm decides whether to call the bond $(1-\theta_1)b_1$ or not. Whether the firm calls or not, it has to pay the interest on debts. The stock of assets of the firm in the period $t = 1$, when:

- it does not call the bond is:

$$q_1^{nocall} = k_1 + z_1(k_1)^\alpha + \varepsilon_1 k_1 - c_1 b_1 \tag{8}$$

- it calls the bond is:

$$q_1^{call} = k_1 + z_1(k_1)^\alpha + \varepsilon_1 k_1 - c_1 b_1 - (1+\chi)(1-\theta_1)b_1 \tag{9}$$

The term $-c_1 b_1$ is the periodic payment of coupon on the outstanding debt b_1 when the firm chooses not to call during the period $t = 1$, while χ represents the call premium paid additionally to the principal of the callable bond $(1-\theta_1)b_1$. χ is an amount over the bond's face value paid to the bondholder if the bond is called early. This call premium compensates for the bondholder's loss of future income.

The firm considers all the state variables and new capital quality shock to decide whether to call or not $(1-\theta_1)b_1$. Following this decision, it chooses its new level of capital k_2 by choosing the amount of debt b_2 for the next period $t = 2$. The firm chooses the non-callable composition (θ_2) of the next period debt by choosing the non-callable share θ_2^{new} in the newly issued debt. I consider interest payment as set in a weighted average coupon c_2 to be paid on the next period debt b_2.[16]

- when the firm does not call the bond, the non-callable fraction, and the periodic coupon c_2 evolve as follows:

$$\begin{cases} \theta_2 &= \theta_1 \dfrac{b_1}{b_2} + \theta_2^{new} \dfrac{b_2 - b_1}{b_2} \\ c_2 &= c_1 \dfrac{b_1}{b_2} + r_1 \dfrac{b_2 - b_1}{b_2} \end{cases} \tag{10}$$

  where $\theta_2^{new} = 1$, as the new debt is a non-callable one-period bond.

- when the firm calls its callable bond, the non-callable fraction, and the periodic coupon c_2 evolve as follows:

$$\begin{cases} \theta_2 &= \theta_1 \dfrac{b_1}{b_2} + \theta_2^{new} \dfrac{b_2 - \theta_1 b_1}{b_2} \\ c_2 &= c_1 \dfrac{\theta_1 b_1}{b_2} + r_1 \dfrac{b_2 - \theta_1 b_1}{b_2} \end{cases} \tag{11}$$

In period $t = 2$, the firm gets its new productivity z_2 and observes the interest rate r_2.[17] The

[16] This weighted average coupon is essential for accurately tracking the firm's debt servicing costs over time. Ippolito et al. (2018) use a similar formulation to model the share of hedged floating-rate debt. The coupon rate determines the periodic interest payments the firm needs to make on its debt. The firm's decision to call or not call its bonds impacts future coupon payments. By incorporating the coupon rate formulas, the model captures the cost of debt accurately, influencing the firm's investment and financing decisions. The formulas also reflect changes in market interest rates over time and ensure that when the firm issues new debt, the coupon rate aligns with prevailing market conditions.

[17] The interest rate r_2 is irrelevant since the period $t = 2$ is the final period, and there is no new bond issuance.

stock of assets of the firm in the period $t = 2$, after producing, would be:

$$q_2 = k_2 + z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2)b_2 \tag{12}$$

where $(1 + c_2)b_2$ is the final debt reimbursement.

3.2 Firm Problem

In period $t = 0$, the firm chooses the capital k_1, the debt b_1, the non-callable fraction of debt θ_1, and the coupon rate c_1. In the period $t = 1$, the firm produces with k_1, and there is a threshold of capital quality shock $\overline{\varepsilon}_1$ under which the firm's value is null. I denote by $s_1 = \{z_1, r_1\}$ the state variables vector at the beginning of the period $t = 1$ and by $x_1 = \{k_1, b_1, \theta_1^{new}, c_1\}$ the policy vector.[18] The firm maximizes its present value, which is the value of the shareholders, by solving the following:

$$V_0(s_0) = \max_{x_1=\{k_1,b_1,\theta_1,c_1\}} \left\{ -k_1 + p_0 b_1 - \eta_b + \beta_0 \mathbb{E}_{s_1|s_0} \int_{\overline{\varepsilon}_1}^{\infty} \overline{q}_1 + \overline{V}_1(s_1, x_1)\varphi(\varepsilon_1)d\varepsilon_1 \right\} \tag{13}$$

$$\begin{aligned}
\text{subject to: } \overline{q}_1 &= k_1 + z_1(k_1)^\alpha + \varepsilon_1 k_1 \\
\overline{V}_1(s_1, x_1) &= \max_{\mathbb{1}_{\text{call}}(s_1,x_1)} \left\{ \overline{V}_1^{call}(s_1, x_1), \overline{V}_1^{nocall}(s_1, x_1) \right\} \\
\overline{\varepsilon}_1 : 0 &= \overline{q}_1 + \overline{V}_1(s_1, x_1) \\
p_0 &= \theta_1 p_0^{nc} + (1 - \theta_1)p_0^c \\
\eta_b &= \left(\eta_{nc}\theta_1^2 + \eta_c(1 - \theta_1)^2\right) \cdot [\max(0, b)]^2
\end{aligned}$$

The following indicator states the decision to call $(1 - \theta_1)b_1$ in $t = 1$:

$$\mathbb{1}_{\text{call}}(s_1, x_1) = \begin{cases} 1, & \text{if} \quad \overline{V}_1^{call}(s_1, x_1) > \overline{V}_1^{nocall}(s_1, x_1) \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

Where $\overline{V}_1^{call}$ ($\overline{V}_1^{nocall}$) is the firm's value when $(1 - \theta_1)b_1$ is called (not called) in $t = 1$.

When the firm does not call its callable debt in period $t = 1$, it adjusts its capital by choosing k_2 and decides the new period debt by issuing $b_2 - b_1$. In period $t = 2$, there exists a threshold $\overline{\varepsilon}_2^{nocall}$ such that under this value, $V_2^{nocall} = 0$. The firm obtains its maximal value by solving the

[18] In the period $t = 0$, the non-callable fraction $\theta_1 = \theta_1^{new}$, because there is no outstanding bond at the beginning of this period.

following:

$$\overline{V}_1^{nocall}(s_1, x_1) = \max_{k_2, b_2, \theta_2^{new}, c_2} \left\{ -e + \beta_1 \mathbb{E}_{z_2|z_1} \int_{\overline{\varepsilon}_2^{nocall}}^{\infty} V_2^{nocall}(s_2, x_2) \varphi(\varepsilon_2) d\varepsilon_2 \right\} \tag{15}$$

$$\begin{aligned}
\text{subject to: } k_2 &= e - c_1 b_1 + p_1(b_2 - b_1) \\
V_2^{nocall}(s_2, x_2) &= \max_{\overline{\varepsilon}_2^{nocall}} \{0, k_2 + z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2) b_2\} \\
\overline{\varepsilon}_2^{nocall} : 0 &= V_2^{nocall}(s_2, x_2) \\
\theta_2 &= \theta_1 \frac{b_1}{b_2} + \theta_2^{new} \frac{b_2 - b_1}{b_2} \\
c_2 &= c_1 \frac{b_1}{b_2} + r_1 \frac{b_2 - b_1}{b_2} \\
p_1 &= \theta_2^{new} p_1^{nc} + (1 - \theta_2^{new}) p_1^c
\end{aligned}$$

Where p_1 is the weighted average price of the new debt, composed by the price of the non-callable (callable) new bond $p_1^{nc}(p_1^c)$. These prices are given by:

$$p_1^{nc} = p_1^c = \frac{1}{(1 + r_1)} \mathbb{E}_{z_2|z_1} \int_{\overline{\varepsilon}_2^{nocall}}^{\infty} (1 + r_1) \varphi(\varepsilon_2) d\varepsilon_2 = \mathbb{E}_{z_2|z_1} \left(1 - \Phi(\overline{\varepsilon}_2^{nocall})\right) \tag{16}$$

$\left(1 - \Phi(\overline{\varepsilon}_2^{nocall})\right)$ is the probability of not defaulting in the period $t = 2$ after the capital quality shock when the firm does not call in period $t = 1$.

In period $t = 1$, when the firm calls its callable debt $(1 - \theta_1)b_1$, it returns the debt and issues a new one $b_2 - \theta_1 b_1$. In period $t = 2$, there exists a threshold $\overline{\varepsilon}_2^{call}$ such that under this value, $V_2^{call} = 0$. The firm then solves:

$$\overline{V}_1^{call}(s_1, x_1) = \max_{k_2, b_2, \theta_2^{new}, c_2} \left\{ -e + \beta_1 \mathbb{E}_{z_2|z_1} \int_{\overline{\varepsilon}_2^{call}}^{\infty} V_2^{call}(s_2, x_2) \varphi(\varepsilon_2) d\varepsilon_2 \right\} \tag{17}$$

$$\begin{aligned}
\text{subject to: } k_2 &= e - c_1 b_1 - (1 + \chi)(1 - \theta_1) b_1 + p_1(b_2 - \theta_1 b_1) \\
V_2^{call}(s_2, x_2) &= \max_{\overline{\varepsilon}_2^{call}} \{0, k_2 + z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2) b_2\} \\
\overline{\varepsilon}_2^{call} : 0 &= V_2^{call}(s_2, x_2) \\
\theta_2 &= \theta_1 \frac{b_1}{b_2} + \theta_2^{new} \frac{b_2 - \theta_1 b_1}{b_2} \\
c_2 &= c_1 \frac{\theta_1 b_1}{b_2} + r_1 \frac{b_2 - \theta_1 b_1}{b_2} \\
p_1 &= \theta_2^{new} p_1^{nc} + (1 - \theta_2^{new}) p_1^c
\end{aligned}$$

where χ is the call premium, the additional amount over the face value the issuer pays to redeem the callable bond $(1-\theta_1)b_1$ early. p_1 is the weighted average price of the new debt, composed by the price of the non-callable (callable) new bond $p_1^{nc}(p_1^c)$. These prices are given by:

$$p_1^{nc} = p_1^c = \frac{1}{(1+r_1)}\mathbb{E}_{z_2|z_1}\int_{\overline{\varepsilon}_2^{call}}^{\infty}(1+r_1)\varphi(\varepsilon_2)d\varepsilon_2 = \mathbb{E}_{z_2|z_1}\left(1-\Phi(\overline{\varepsilon}_2^{call})\right) \tag{18}$$

$\left(1-\Phi(\overline{\varepsilon}_2^{call})\right)$ is the probability of not defaulting in the period $t = 2$ after the capital quality shock when the firm calls in period $t = 1$.

3.3 Creditors' Problem

Creditors are risk neutral and discount the future at the same rate $\beta_0 = 1/(1+r_0)$ in period $t = 0$ and $\beta_1 = 1/(1+r_1)$ in period $t = 1$ as shareholders. This part follows the structure of creditors' problem in Jungherr and Schott (2021, 2022).

3.3.1 Non-callable bond pricing

After producing, when the firm decides to default, in $t = 1$, the creditors of the non-callable bonds recover the remaining fraction $(1-\xi)$ of the firm's liquidation value $\underline{q}_1$, with the fraction ξ being lost and:

$$\underline{q}_1 \equiv k_1 + y_1 + \varepsilon_1 k_1 \tag{19}$$

In this three-period model, for simplicity, I consider that the whole value of the firm is lost when it defaults on its debt, so $\xi = 1$. The coupon received by the debt holders in period $t = 1$ is r_0, then the price of the non-callable debt $\theta_1 b_1$ is:

$$p_0^{nc} = \frac{1}{(1+r_0)}\mathbb{E}_{s_1|s_0}\int_{\overline{\varepsilon}_1}^{\infty}\left(r_0+\overline{p}_1^{nc}\right)\varphi(\varepsilon_1)d\varepsilon_1 \tag{20}$$

with $\overline{p}_1^{nc}$ the average expected continuation price of $\theta_1 b_1$ in $t = 1$:

$$\overline{p}_1^{nc} = \mathbb{1}_{\text{call}} \times \overline{p}_1^{nc\_call}(s_1, x_1) + (1-\mathbb{1}_{\text{call}}) \times \overline{p}_1^{nc\_nocall}(s_1, x_1)$$

where $\mathbb{1}_{\text{call}}$ is the dummy that indicates the firm decision to call or not, $\overline{p}_1^{nc\_call}$ ($\overline{p}_1^{nc\_nocall}$) is the continuation price of $\theta_1 b_1$ when $(1-\theta_1)b_1$ is called (not called) in $t = 1$:

$$\overline{p}_1^{nc\_call}(s_1, x_1) = \frac{1}{(1+r_1)}\mathbb{E}_{z_2|z_1}\int_{\overline{\varepsilon}_2^{call}}^{\infty}(1+c_1)\varphi(\varepsilon_2)d\varepsilon_2 = \beta_1(1+c_1)\mathbb{E}_{z_2|z_1}\left(1-\Phi(\overline{\varepsilon}_2^{call})\right)$$

and

$$\overline{p}_1^{nc\_nocall}(s_1, x_1) = \frac{1}{(1+r_1)}\mathbb{E}_{z_2|z_1}\int_{\overline{\varepsilon}_2^{nocall}}^{\infty}(1+c_1)\varphi(\varepsilon_2)d\varepsilon_2 = \beta_1(1+c_1)\mathbb{E}_{z_2|z_1}\left(1-\Phi(\overline{\varepsilon}_2^{nocall})\right)$$

3.3.2 Callable bond pricing

The price of the callable debt $(1-\theta_1)b_1$ is generally set relative to a similar non-callable debt, such that the option is not exercised in the period $t=1$. The issuer has to consider the minimum between the non-callable bond price and the exercise price (here at par), using a callable bond premium.[19] I use the following formula to model the callable bond pricing at the issuance in the period $t=0$:

$$p_0^c = \frac{1}{(1+r_0)}\mathbb{E}_{s_1|s_0}\int_{\overline{\varepsilon}_1}^{\infty}\left[r_0 + (1-\mathbb{1}_{\text{call}})\,\overline{p}_1^{c\_nocall} + \mathbb{1}_{\text{call}}\,(1+\chi)\right]\varphi(\varepsilon_1)d\varepsilon_1 \tag{21}$$

The term $(1+\chi)$ represents the bondholder's potential payoff, ensuring that they do not receive more than the fixed call price even if the market price $\overline{p}_1^{c\_call}$ is higher at the time of the call, this aligns with the fact that the call scenario occurs when interest rates drop, increasing the continuation value of the bond. This cap protects the issuer and defines the financial outcome of the early redemption. The call premium $\chi \in (0,1)$ is typically a small percentage of the face value to compensate bondholders for the early redemption.[20]

$\overline{p}_1^{c\_call}$ ($\overline{p}_1^{c\_nocall}$) is the market price of $(1-\theta_1)b_1$ when $(1-\theta_1)b_1$ is called (not called) in $t=1$. Because of the issuance of both bonds in the same period by the same issuer facing the same conditions, and due to the fact there is only one period left at this stage of the timing, a callable bond that is not called is priced as a similar one-period non-callable one: $\overline{p}_1^{c\_call} = \overline{p}_1^{nc\_call}$ and $\overline{p}_1^{c\_nocall} = \overline{p}_1^{nc\_nocall}$.

3.4 Characterization of firm optimal policies

According to this framework within a partial equilibrium setting, the model is solved by backward induction, considering the problems in (15), (17), and (13), alongside the creditors' problems

[19]The pricing of callable bonds is similar to the extension of defaultable callable bonds valuation' model in Jarrow et al. (2010) which extended the model of defaultable callable bond in Duffie and Singleton (1999). But I consider endogenous decisions to default and to call the callable bonds. In Duffie and Singleton (1999), the exercise price at the call period is set at par. Ultimately, the firm internalizes the minimal payment in the call scenario.

[20]Setting the call premium as a percentage of the face value aligns with the idea that the premium compensates for the risk relative to the prevailing interest rate environment. This ensures that the premium is proportionate to market conditions during issuance. Callable bonds usually offer a higher coupon rate than non-callable bonds to offset call risk. However, the call premium is generally lower than this higher coupon rate, as it is designed to offer a buffer rather than full compensation.

through (20), (21). The subsection discusses the first-order conditions regarding capital choices for both initial and intermediate periods ($t = 0, 1$), the central decision to call a callable bond in period $t = 1$, and the dynamics of debt issuance across these time frames. The feature of calling the callable bond $(1 - \theta_1)b_1$ at $t = 1$ serves to differentiate those optimal choices. I don't use superscripts "nocall" and "call" when there is no ambiguity in the notation.

3.4.1 Refinancing decision in the intermediate period $t = 1$

Now, I analyze the optimal decisions on issuing new debt after not calling and after calling $(1 - \theta_1)b_1$ in this period $t = 1$. The first-order conditions with respect to $b_2 - b_1$ ($b_2 - \theta_1 b_1$) when the firm does not call (calls) $(1 - \theta_1)b_1$ are the following:

nocall

$$[b_2 - b_1] : -(1 + c_2)\frac{b_2 - b_1}{k_2}\mathbb{E}_{z_2|z_1}\left(\varphi(\bar{\varepsilon}_2)\right) - \beta_1(c_1 - r_1)\frac{b_1}{b_2}\mathbb{E}_{z_2|z_1}\left[1 - \Phi\left(\bar{\varepsilon}_2\right)\right] = 0 \tag{22}$$

call

$$[b_2 - \theta_1 b_1] : -(1 + c_2)\frac{b_2 - \theta_1 b_1}{k_2}\mathbb{E}_{z_2|z_1}\left(\varphi(\bar{\varepsilon}_2)\right) - \beta_1(c_1 - r_1)\frac{\theta_1 b_1}{b_2}\mathbb{E}_{z_2|z_1}\left[1 - \Phi\left(\bar{\varepsilon}_2\right)\right] = 0 \tag{23}$$

The proof of these optimal decisions can be found in Appendix C.1. These conditions present the net cost of refinancing and indicate that the optimal refinancing after the call decision should account for the new interest rate (r_1), the level of the periodic coupon (c_1), and the expected effect on default risk ($\bar{\varepsilon}_2$).

High level b_2 (or $b_2 - b_1$) increases the default probability, as $\dfrac{\partial \bar{\varepsilon}_2}{\partial (b_2 - b_1)} = \dfrac{1 + c_2}{k_2} > 0$. It reduces the revenue $p_1(b_2 - b_1)$ from selling the new bond. In period $t = 1$, the optimal refinancing b_2 increases in the stock of existing debt b_1.[21] In the second term, the denominator is the next period stock of non-callable debt in both cases. However, for the no-call scenario, the uncalled bond in the same proportions at both the numerator and the denominator levels cancels out the effects of θ_1. The implication is that θ_1 does not influence this optimal refinancing choice when the firm does not call the fraction $(1 - \theta_1)$ of its debt b_1. When the firm calls the share $(1 - \theta_1)$ of its debt, the optimal new debt $b_2 - \theta_1 b_1$ also increases the default probability

[21]This result is consistent with the literature. See the slow debt property in Jungherr and Schott (2022).

$\frac{\partial \bar{\varepsilon}_2}{\partial(b_2 - \theta_1 b_1)} = \frac{1+c_2}{k_2} > 0$, since:

$$c_2 = \begin{cases} r_1 + (c_1 - r_1)\frac{b_1}{b_2} & \text{when no call} \\ r_1 + (c_1 - r_1)\frac{\theta_1 b_1}{b_2} & \text{when call} \end{cases} \tag{24}$$

Then, θ_1 emphasizes the role of the outstanding debt b_1 at the beginning of the period $t = 1$. It amplifies the positive impact of the stock of debt on the choice of optimal refinancing b_2 when the firm calls. Considering firms with default threshold at the right of the distribution of capital quality shock ($\bar{\varepsilon}_1 < \bar{\varepsilon}$), φ increases in $\bar{\varepsilon}_1$, so firms with high default risk face the more severe cost of refinancing (roll-over risk). I derive the following properties of debt callability in refinancing.

Proposition 1. *In period $t = 1$,*

- *θ_1 plays an (asymmetric) role in refinancing only when the callable debt is called.*
- *If the interest rate does not change, i.e., is set s.t. $r_1 = c_1$, θ_1 (or the share of callable bond) has no effect in refinancing, whether the firm decides to call or not.*

The proof of Proposition 1 can be found in Appendix C.4. Proposition 1 shows that the callable bonds are important in the amplitude of refinancing when the firm decides to call it. It also shows that the interest rate prevailing at the moment of call decision matters for this role. The asymmetry means that the share of callable bonds significantly impacts refinancing decisions when interest rates are falling, allowing firms to take advantage of lower borrowing costs. In contrast, when interest rates rise, the share of callable bonds has a less significant impact, as firms are less likely to refinance at higher rates.

3.4.2 Optimal capital choice in the intermediate period $t = 1$

I consider the optimal choices of capital under two scenarios: when the firm doesn't call the bond (k_2^{nocall}) and when it does (k_2^{call}). The first-order conditions to k_2 for these cases are respectively the following:

nocall

$$[k_2] : -1 - \frac{\partial \bar{\varepsilon}_2}{\partial k_2}\left[\beta_1 k_2 \left[1 - \Phi\left(\bar{\varepsilon}_2\right)\right] - (b_2 - b_1)\frac{\partial p_1}{\partial \bar{\varepsilon}_2}\right] + \beta_1 \mathbb{E}_{z_2|z_1}\int_{\bar{\varepsilon}_2}^{\infty}\Big(\varepsilon_2 - \bar{\varepsilon}_2\Big)\varphi(\varepsilon_2)d\varepsilon_2 = 0 \tag{25}$$

call

$$[k_2]: -1 - \frac{\partial \overline{\varepsilon}_2}{\partial k_2}\left[\beta_1 k_2\left[1-\Phi\left(\overline{\varepsilon}_2\right)\right]-(b_2-\theta_1 b_1)\frac{\partial p_1}{\partial \overline{\varepsilon}_2}\right]+\beta_1 \mathbb{E}_{z_2|z_1}\int_{\overline{\varepsilon}_2}^{\infty}\Big(\varepsilon_2-\overline{\varepsilon}_2\Big)\varphi(\varepsilon_2)d\varepsilon_2 = 0 \tag{26}$$

Where the capital effect on default risk is:

$$\frac{\partial \overline{\varepsilon}_2}{\partial k_2} = -\frac{1+\alpha z_2 k_2^{\alpha-1}+\overline{\varepsilon}_2}{k_2} = \frac{(1-\alpha)z_2 k_2^{\alpha}-(1+c_2)b_1}{(k_2)^2} \tag{27}$$

Despite the positive impact of additional capital on production, the diminishing return to capital (due to $\alpha < 1$) reduces the firm's ability to absorb shocks, contributing to a high risk of default. I then consider that capital reduces the default risk ($\partial\overline{\varepsilon}_2/\partial k_2 < 0$). In this circumstance, a high debt stock b_1 decreases the optimal choice k_2. θ_1 does not influence the optimal investment when the firm does not call $(1-\theta_1)b_1$ in $t = 1$.

As c_2 increases, the default risk induced by capital is supposed to be more important. But this effect depends on the composition of c_2, which depends in turn on θ_1 and the relative level of the interest rate r_1 to the average coupon c_1.

Corollary 3.0.1. *θ_1 (or callable share) reduces (or amplifies) the firm's optimal investment when it exercises the option to call its callable debt.*

1. *The callable fraction of bonds does not influence the optimal investment decision when the firm does not call its debt.*

2. *Following an easing of monetary policy (i.e., a decrease in r_1), firms with a high share of callable bonds (i.e., low θ_1) increase their investment more when they call these bonds compared to firms with a low share of callable bonds.*

3. *Following a tightening of monetary policy (i.e., an increase in r_1), firms with a high share of callable bonds (low θ_1) decrease their investment more when they call these bonds compared to firms with a low share of callable bonds.*

The proof of Corollary 3.0.1 can be found in Appendix C.5. Corollary 3.0.1 aligns with Becker et al. (2024) results by highlighting how callable bonds enhance a firm's flexibility in managing debt and investment, particularly under varying monetary policy conditions and for firms with high-yield ratings.

3.4.3 The decision to call (in the intermediate period $t = 1$)

The firm decides to call its callable bond $(1-\theta_1)b_1$ **iff**:

$$\overline{V}_1^{nocall}(s_1, x_1) < \overline{V}_1^{call}(s_1, x_1) \tag{28}$$

Proposition 2. *In period $t = 1$, the firm's decision to call its callable bond $(1-\theta_1)b_1$ is determined by the following inequality:*

$$\begin{aligned} &\mathbb{E}_{z_2|z_1}\left\{\left(-b_1 + \frac{(1-\alpha)y_2^{nocall}}{1+c_2}\right)\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right]\right\} \\ < -(1+\chi)(1-\theta_1)b_1 + &\mathbb{E}_{z_2|z_1}\left\{\left(-\theta_1 b_1 + \frac{(1-\alpha)y_2^{call}}{1+c_2}\right)\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\right\} \end{aligned} \tag{29}$$

The proof of Proposition 2 can be found in Appendix C.6. Proposition 2 presents, in its simplest form, the firm's decision to call the bond based on comparing the expected net returns under the no-call and call scenarios. The firm calls its callable bond if the expected net return from calling the bond (RHS of (28)) exceeds the expected net return from not calling the bond (LHS of (28)).

The term $-(1+\chi)(1-\theta_1)b_1$ represents the immediate cost of calling the bond for the firm in period $t = 1$. Conditional to not default, the term in the expectation consists of the minimal value of the capital return of the firm after refinancing through $b_2 - b_1$ ($b_2 - \theta_1 b_1$ after calling).

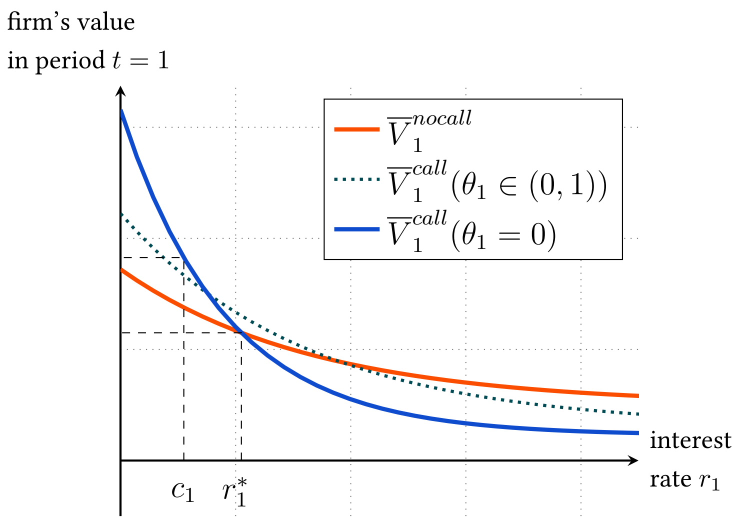
Proposition 3. *For a given state s_1, an average coupon c_1 on existing debt b_1 which has a callable bond share ($1-\theta_1 \in [0,1]$), $\exists!\, r_1^* > c_1$, such that: $\forall r_1 < r_1^*$, $\overline{V}_1^{nocall}(s_1, x_1) < \overline{V}_1^{call}(s_1, x_1)$.*

The proof of Proposition 3 can be found in Appendix C.7. The main object of interest in Proposition 3 is the level of the interest rate set for the period $t = 1$. This rate influences the value of the firm's repayment in both periods $t = 1, 2$ in the call and non-call scenarios. Therefore, depending on the level of the share of non-callable bonds θ_1, these value can change their relative position along interest rates. I derive the following Corollary from the Proposition. Both Proposition 3 and Corollary 3.0.2 are illustrated in Figure 8.

Corollary 3.0.2. *A decrease in interest rate in the period $t = 1$:*

1. *improves the credit quality of the firm;*
2. *increases the probability of calling the callable debt in this period;*
3. *these effects are stronger for firms with a smaller share of callable bonds (i.e., high θ_1) and weaker for firms with a high share of callable bonds.*

Figure 8: Illustration of call decision in period $t = 1$



Corollary 3.0.2 explores the conditions behind the decision to call for firms. The first point indicates that low interest rates improve the firm's credit quality by reducing its future default probability. To understand that, let's consider that the new low interest rate improves the future cash-flow of the firm, making it more able to repay any (new) debt. The expected future default probability of the firm is then reduced, which represents an improvement of its credit quality. Through this corollary, I am showing that the credit upgrade is common and -on average consequence- for all firms, and that does not make firms with a low credit rating become high credit rating. Therefore, the degree of callability is at the origin of the incentive to call after a decline in the interest rate. Though, a decline in interest rates can be associated with favorable opportunities that encourage firms to call.[22] This is because the default threshold is positively sensitive to changes in the periodic coupon. So, a decrease in interest rate reduces this periodic coupon, then minimizes the cutoff capital quality in the future (see Appendix C.8 for the proof.). Second, when the original coupon rate c_1 is sufficiently high (due to a high interest rate r_0 in period $t = 0$), the cost of continuing to pay these high-interest rates (reflected in the left-hand side of the (28)) may exceed the cost of facing new debt payments through a potentially lower rate r_1. Then, a decrease in the interest rate in $t = 1$ raises this benefit of calling. This demonstrates what is theoretically behind the purpose of callable bonds. Third, a higher share of callable bonds indicates a higher debt burden, which leads firms to internalize and endogenize the fact that they will face difficulties refinancing their debt once they call.

[22] Becker et al. (2024) found that calls are more likely when issuer credit quality improves, but they ignored in their analysis the level of callable debt, which exposes a firm to the need for new financing. The more the firm has callable debt, the more it will create space to fill with new debt if it calls.

3.4.4 The optimal choice of callable share $(1-\theta_1)$ in period $t=0$

The FOCs for the share of non-callable bond in the period $t=0$ is given by:

$$[\theta_1]: b_1\frac{\partial p_0}{\partial \theta_1} + p_0\frac{\partial b_1}{\partial \theta_1} - \frac{\partial \eta_b}{\partial \theta_1} \leq -\beta_0 \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \theta_1} \tag{30}$$

where I denote V_1 the firm's continuation value in the period $t=1$, given the realization of new shocks and when default is avoided:

$$V_1(s_1, x_1) = \int_{\overline{\varepsilon}_1}^{\infty} \left[\overline{q}_1 + \mathbb{1}_{\text{call}} \times \overline{V}_1^{call}(s_1, x_1) + (1-\mathbb{1}_{\text{call}}) \times \overline{V}_1^{nocall}\right] \varphi(\varepsilon_1) d\varepsilon_1 \tag{31}$$

The optimal share of non-callable bonds (θ_1) is defined by the following expression:

$$\begin{aligned} &\beta_0 \mathbb{E}_{s_1|s_0}\Bigg\{\Bigg[\underbrace{\left[r_0 + \theta_1 \overline{p}_1^{nc\_call} + (1-\theta_1)(1+\chi)\right]\frac{b_1}{k_1}}_{\text{Effective value of firm's leverage in } t=1}\varphi(\overline{\varepsilon}_1) + \underbrace{\left[1-\Phi(\overline{\varepsilon}_1)\right]}_{\text{Repayment probability in } t=1}\Bigg] \\ &\times \frac{\partial}{\partial \theta_1}\underbrace{\left[\mathbb{1}_{\text{call}} \times (\overline{V}_1^{call} - \overline{V}_1^{nocall})\right]}_{\text{net future benefit from calling in } t=1}\Bigg\} - 2\left[(\eta_{nc}+\eta_c)\theta_1 - \eta_c\right] b_1^2 = 0. \end{aligned} \tag{32}$$

The optimal share of non-callable bonds, θ_1, is defined in (32) and reflects the interplay of two forces:

1. **Call Decision (Qualitative) Effect:** Determines the likelihood of calling ($\mathbb{1}_{\text{call}}$) based on θ_1.
2. **Payoff (Quantitative) Effect:** Captures how the economic benefit of calling ($\overline{V}_1^{call} - \overline{V}_1^{nocall}$) evolves with θ_1.

Together, these forces explain the trade-offs firms face in their capital structure decisions.

Callable Bonds and Future Conditions

The presence of callable bonds allows firms to respond flexibly to changes in future firm-specific and aggregate conditions. The proposition (2) shows that:

- When the interest rate r_1 in $t=1$ is below the threshold r_1^*, the call decision ($\mathbb{1}_{\text{call}}=1$) is activated, but the net payoff ($\overline{V}_1^{call} - \overline{V}_1^{nocall}$) which is positive, decreases with θ_1.
- In contrast, when $r_1 > r_1^*$, the calling decision is inactive ($\mathbb{1}_{\text{call}}=0$), and the net payoff of the calling is negative and increases with θ_1.

The last term in (32), which incorporates issuance costs, also plays a role in determining the optimal share θ_1.[23] The following distinctions arise based on the relative issuance costs (η_c and η_{nc}):

- If $\eta_c \ll \eta_{nc}$, callable bonds dominate due to their low cost.
- If $\eta_c \gg \eta_{nc}$, callable bonds are less favorable due to their high cost.
- If $\eta_c \sim \eta_{nc}$, the issuance costs have minimal impact and θ_1 centers on 50%. θ_1 amplifies the issuance cost effect of callable bonds.

This term generates intensive margins due to the composition of the debt issued. Moreover, when the firm issues a perfect 50/50 mix of callable and non-callable bonds, the issuance cost falls by 50%, penalizing debt concentration. It also adds curvature in the θ_1 dimension.

High Initial Coupon (c_0) and Callable Bond Dynamics

When the initial coupon c_0 is high, the probability of calling in $t = 1$ increases because $r_1 < r_1^*$. In this case, we can distinguish:

- the **Call Decision (Qualitative) Effect** is dominant ($\mathbb{1}_{\text{call}} \to 1$), driving the decision to call callable bonds;
- however, the **Payoff (Quantitative) Effect** diminishes as $\partial(\overline{V}_1^{call} - \overline{V}_1^{nocall})/\partial\theta_1 < 0$, indicating reduced economic benefits from callable bonds as θ_1 increases.

Firms with lower default risk ($\bar{\varepsilon}_1 < \bar{\varepsilon}$) gain less from callable bonds because they are more likely to survive and repay, reducing the net benefit value of the call.

Low Initial Coupon (c_0) and Callable Bond Dynamics

When the initial coupon c_0 is low, the probability of calling callable bonds in $t = 1$ is minimal ($\mathbb{1}_{\text{call}} \to 0$). Here we have:

- the **Payoff (Quantitative) Effect** dominates ($\partial(\overline{V}_1^{call} - \overline{V}_1^{nocall})/\partial\theta_1 > 0$), as the economic advantage of callable bonds increases with θ_1;
- high-risk firms ($\bar{\varepsilon}_1 > \bar{\varepsilon}$) tend to issue more callable bonds to hedge against the potential for default, as calling provides a valuable net benefit.

[23]This term shows the role of considering separate issuance costs for both types of debt. It raises another wedge of a higher-up front fee on callable issues because of differences in registration fees or legal underwriting spreads. In the full model, allowing these separate issuance costs may cause an identification problem since, in practice, issuing non-contingent and contingent debt often comes with different transaction costs due to factors like credit risk and investor preferences. I then consider a unique issuance cost for both types of debt in the full model.

The three-period model highlights the core trade-offs firms face when issuing callable and non-callable debt. Firms weigh the **Call Decision (Qualitative) Effect** against the **Payoff (Quantitative) Effect**, balancing the likelihood of exercising the call option against its economic value. High-risk firms with greater default probabilities are more inclined to issue callable bonds due to the flexibility they provide. In contrast, low-risk firms prefer non-callable bonds to minimize costs. Although the three-period model provides valuable insight, it cannot capture the full dynamic interactions between bond issuance, call decisions, and investment. The following sections extend this analysis to incorporate firm heterogeneity, aggregate dynamics, and multi-period decision-making, quantifying the macroeconomic implications of callable debt and firm-level behaviors in shaping broader economic outcomes.

4 Quantitative Model

This section introduces the quantitative model, building on the three-period framework. I develop a heterogeneous firm general equilibrium model with risky debt and debt callability. The model has the following ingredients: (1) firms use labor and capital as factors of production; (2) to finance their capital investments, they combine equity and debt issuance decisions; (3) debts are sold as long-term defaultable bonds; (4) debt consists of non-callable bond and callable bond following a composition law of the share of non-callable; (5) upon non-default, firms can call their callable debt before the new optimal decisions.[24] Callable bond offers higher yields to investors in compensation for the risk of early redemption during the bond's duration. Still, it is supposed to save the costs of interest payments (when it is called) and provide greater flexibility to its issuer. The model economy has four types of agents: firms, creditors, government, and households. Firms with different persistent productivity first hire labor, produce, and then receive a capital quality shock, upon which they decide to default or not. They decide whether to call their callable bond and then invest in the capital through dividends and debt issuance. There is a continuum of risk-neutral creditors who price the bonds; a government decides the tax and interest rate levels; a representative household completes the model. Time is discrete, and as we set in a period, I use the prime symbol ($'$) to denote the future values.

[24] I assume that only public firms can access defaultable debts with a call option. They issue only long-term debts (See Karabarbounis and Macnamara (2021).)

4.1 Firms

4.1.1 Technology and Productivity

Firms are perfectly competitive and produce a single, unique, homogeneous final good. Each firm produces by combining capital k and labor l in a decreasing returns-to-scale technology and using a Cobb-Douglas production function:

$$y = z\left(k^{\psi}l^{1-\psi}\right)^{\nu}$$

where $\psi, \nu \in (0,1)$. z is the total factor productivity following a persistent shock learned from the previous period. The idiosyncratic productivity z follows an AR(1) process:

$$logz' = \rho_z logz + \epsilon_z', \quad \epsilon_z' \sim^{i.i.d.} \mathcal{N}\left(0, {\sigma_z}^2\right), \quad \rho_z \in (0,1).$$

The firm pays every period a fixed cost of operation f. The firm receives, after production, an idiosyncratic capital quality shock ε i.i.d. across time and firms. It is drawn from a normal distribution $\mathcal{N}(0, \sigma_\varepsilon^2)$. This shock defines the threshold that influences the firm's decision to default after production.

4.1.2 Financing

Now, I define the types of bonds and present how the firm issues and combines them in financing.

Non-callable debt. A long-term debt issued in the period t promises to pay a fixed coupon c each period. I assume that a fraction γ of the outstanding principal matures each period, following Hatchondo and Martinez (2009); Chatterjee and Eyigungor (2012); Gomes et al. (2016). It means that the firm pays back γ in addition to the coupon, and the debt is of finite maturity. The market price of such a non-callable bond is set at p^{nc} at the issuance period.

Callable debt. A long-term debt issued in the period t promises to pay a fixed coupon c each period. It embeds a *call option* allowing early redemption. If the issuer exercises the option, a fraction λ of the callable share is redeemed: the bondholder receives $(1+\chi)\,\lambda(1-\theta)b$, where $\chi > 0$ is the call premium over face. If the option is not exercised, the firm pays coupon c and the scheduled amortization γ on the surviving principal. The market price of the callable bond is denoted p^c and set at the issuance period.

Share of bond not to call. Considering the quantity of the outstanding debt b at the beginning of a period, I denote by θ the bond share not to be called in this period. Then, if the firm exercises its call option, it will call $(1-\theta)b$. Also, if the firm issues new debt, the share of the bond not to be called is denoted $\tilde{\theta}$. The firm can only issue new debt if $b' \geq (1-\gamma)b$ when there is no call and $b' \geq (1-\gamma)\theta b$ when there is a call. I denote by $\tilde{b}$ the new debt in each scenario when there is no ambiguity about the call decision of the firm.

Eligibility to call. I introduce $\lambda \in (0,1]$ as the fraction of the outstanding *callable* stock that is eligible to be redeemed each period. If the firm calls in t, it can redeem only $\lambda(1-\theta)b$ at the call premium, while the remaining $(1-\lambda)(1-\theta)b$ continues outstanding. This implies that after amortization the next-period outstanding principal equals:

$$(1-\gamma)\Big[\theta + (1-\theta)(1-\lambda)\Big]\, b.$$

Bond issuance cost. I assume that retiring all the outstanding debt $\left(\tilde{b} \geq 0\right)$ is costless for the firm. I adopt the specification of a quadratic form as is done in the literature (see Jungherr and Schott, 2021).[25] The cost of issuing new debt is:

$$\eta_b = \eta \cdot \left[\max(0, \tilde{b})\right]^2.$$

The firm can also issue equity, but with a lower bound $e \geq -\underline{e}$ where $\underline{e} > 0$, avoiding the possibility of financing constantly through equity Ponzi games.

Equity issuance cost. I assume that the firm incurs a cost when it issues external equity ($e \geq 0$), and no cost when it distributes dividends ($e < 0$). I adopt again the specification of a quadratic form as in Jungherr and Schott (2021). The cost of issuing new external equity is:

$$\eta_e = \eta_e \cdot [\max(0, e)]^2$$

4.1.3 Callability structure and call decision

At the call decision, the amount of debt the firm can call is $(1-\theta)b$. Whatever its decision, it pays the fraction γ on the non-callable share of its debt. Then, depending on its decision, the fraction of debt not to call in the next period in the remaining debt is different. The direct implication is that the composition of the new debt issued, denoted by $\tilde{b}$, and the periodic coupon payment

[25] I set a single debt issuance cost η to let the debt composition be driven solely by the endogenous defaultable-bond price wedge and the call premium channel. It allows me to isolate the macro role of the call option. By doing so, I argue that all cross-sectional variation in the cost of issuing callable versus non-callable debt comes exclusively from the call premium and the option value embedded in prices. The quadratic cost still dictates the quantity of new debt and helps discipline the calibration without affecting the model's mechanism and implications.

are not constant. This justifies the introduction of the firm idiosyncratic variables θ, and c, and also the index for the call decision, $j \in \{\text{nocall}, \text{call}\}$. As the debt chosen for the next period is denoted by b', the new debt issued $\tilde{b}$ is described as follows:

$$\tilde{b}_{\text{nocall}} \;=\; b' - (1-\gamma)b, \qquad \tilde{b}_{\text{call}} \;=\; b' - (1-\gamma)\Big[\theta + (1-\theta)(1-\lambda)\Big]\, b. \tag{33}$$

When deciding on the new debt, the non-callable composition (θ') of the next period debt is the weighted average of the remaining non-callable share (which is not to be called) θb in the outstanding debt $(1-\gamma)b$, and the non-callable share $\tilde{\theta}$ in the new debt issued $\tilde{b}$. It has the following law of motion:

$$\theta' \;=\; \begin{cases} \dfrac{\theta(1-\gamma)b}{b'} + \tilde{\theta}\,\dfrac{\tilde{b}_{\text{nocall}}}{b'}, & \text{(no call) and } b' > (1-\gamma)b, \\[2ex] \dfrac{\theta(1-\gamma)b}{b'} + \tilde{\theta}\,\dfrac{\tilde{b}_{\text{call}}}{b'}, & \text{(call) and } b' > (1-\gamma)[\theta + (1-\theta)(1-\lambda)]b, \\[2ex] \theta, & \text{otherwise.} \end{cases} \tag{34}$$

The periodic coupon c' of the firm is set as the weighted average of the current coupon c on the outstanding debt and the current interest rate r, which is the coupon on the new debt issued. The new interest rate r is the coupon attached to the new bond $\tilde{b}$, while the not-matured-yet and uncalled bond left in the next period, b', will have to pay the same coupon c. This formulation allows me to track the coupon associated with the not-to-call part of the bond in the next period.

For the nocall scenario, the remaining debt after repaying the fraction γ is $(1-\gamma)b$, so we have:

$$c'_{\text{nocall}} = \begin{cases} c\,\dfrac{(1-\gamma)b}{b'} + r\,\dfrac{\tilde{b}_{\text{nocall}}}{b'}, & b' > (1-\gamma)b, \\[2ex] c, & \text{otherwise.} \end{cases} \tag{35}$$

For the call scenario, the remaining debt after repaying the fraction γ and calling the fraction λ is $(1-\gamma)[\theta + (1-\theta)(1-\lambda)]b$, since the callable debt $(1-\theta)b$ is called, so we have:

$$c'_{\text{call}} = \begin{cases} c\,\dfrac{(1-\gamma)[\theta + (1-\theta)(1-\lambda)]b}{b'} + r\,\dfrac{\tilde{b}_{\text{call}}}{b'}, & b' > (1-\gamma)[\theta + (1-\theta)(1-\lambda)]b, \\[2ex] c, & \text{otherwise.} \end{cases} \tag{36}$$

Now, I define the firm's internal funds to be the net worth after production, tax, depreciation, and interest payment:

$$n_j(s,x) = \begin{cases} k - \gamma b + (1-\tau)[y + \varepsilon k - \delta k - wl - cb - f], & \text{no call,} \\ k - [\gamma + (1+\chi)\lambda(1-\theta)] + (1-\tau)[y + \varepsilon k - \delta k - wl - cb - f], & \text{call.} \end{cases}$$

The call premium $\chi \in (0,1)$, over the face value, is provided as compensation for the potential loss of future interest payments due to the early redemption of the bond. The premium might be predetermined or calculated as a percentage of the face value of the outstanding callable bond. It is a penalty during the call protection period and gradually declines as the maturity date approaches. This formulation ensures that the call premium decreases as the bond matures, aligning with the intuition that the cost of calling the bond should reduce over time.[26] Although endogenously fixed, it allows tractability in the model.[27]

4.1.4 Value Functions

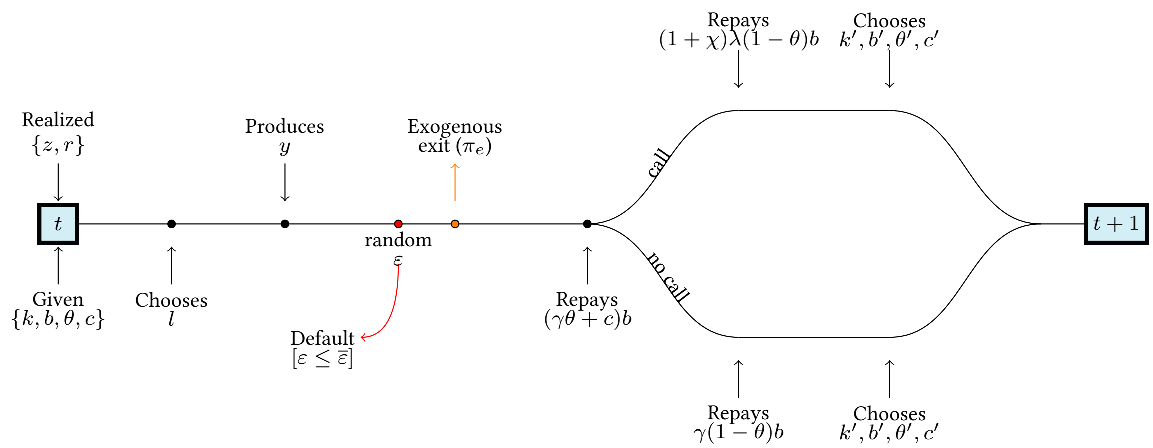
Each period, the firm has its new productivity z, it gets the information on the latest interest rate r, and based on its vector of state variables $x = \{k, b, \theta, c\}$ decided in the previous period, it chooses its labor to produce. I denote the vector of shock variables by $s = \{z, r\}$. The firm maximizes shareholder value by discounting its future cash flows at the current interest rate. Without any ambiguity, the set of state variables is (s, x) to simplify the notations. Let $V(s, x)$ be the value of the solvent firm making the finance-investment decision.

The timing within a period t is illustrated in Figure 9 and is as follows. At the beginning of a period t, the economy is characterized by the interest rate r. A firm carries capital k, debt b, the share of the not-to-call bond θ, and its weighted average coupon rate c. Given its realized productivity z, it chooses the labor l to produce y—the decision to call interacts with the firm's optimal decisions. Knowing the distribution of the capital quality shock, the firm decides on the cutoff $\bar{\varepsilon}$. Upon non-default, the firm chooses to call or not the callable share $1-\theta$ of its outstanding

[26] The call premium increases with the prevailing interest rate, reflecting the higher cost of calling the bond early when interest rates are high. It is proportional to the remaining principal of the callable bond. The call premium decreases as the bond principal decreases over time due to repayments. This relationship is consistent with the model's incorporation of the call decision indicator and default condition, which adjust the bond's value based on whether the bond is called or not. The call premium also shows an inverse relationship with the bond's original maturity. A longer original maturity typically results in a lower call premium for a given remaining principal and interest rate, as the longer duration allows more opportunity for rates to change, affecting the likelihood of calling. Note that with each period, repayment of the bond at a fraction γ leads to a geometric decrease in the outstanding principal. This geometric decay ensures that the remaining unmatured and uncalled principal of callable bonds decreases over time, which aligns with the model's treatment of the call premium and its adjustment based on the bond's callability and default conditions.

[27] The call premium adjusts dynamically with changes in the prevailing interest rate, ensuring that the model remains responsive to economic conditions. The formulation aligns with important papers in macro-finance and term structure models (refer to Joslin et al. (2014); Cochrane (2017)).

Figure 9: TIMING FOR THE FIRM'S PROBLEM



Notes: The first blue indicates the beginning of period t and the last marks the beginning of the next period $t+1$. The firm decides to continue operations or to default endogenously (red arrow) after the capital quality shock ε. The exogenous exit is illustrated in orange and happens with a probability π_e.

bond b. Thus, it can choose k', b', θ', c'. After a call, next period's outstanding principal equals $(1-\gamma)[\theta + (1-\theta)(1-\lambda)]\, b$. Then, at the beginning of the period, the value of the firm is given by:

$$V(s,x) = \max \left\{ \underbrace{V^r(s,x)}_{\text{continue}}, \underbrace{0}_{\text{default}} \right\}. \tag{37}$$

I assume that in default, the firm exists and receives 0, which implies the following definition of the default cutoff $\overline{\varepsilon}$:

$$\overline{\varepsilon} : 0 = V^r(s,x) \tag{38}$$

Upon non-default, the value received by the firm is $V^r(s,x)$, defined by:

$$V^r(s,x) = (1-\pi_e)\overline{V}(s,x) + \pi_e V^{ex}(s,x). \tag{39}$$

V^r represents the repayment value of the firm when it decides to continue. In this case, a probability π_e still exists to exit exogenously; then, with its earnings after production, the firm has to liquidate its capital and debt. It pays back its outstanding debt and pays out the dividend to shareholders. Its value in exogenous exit is:

$$V^{ex}(s,x) = k - \gamma b + (1-\tau)\left[y + \varepsilon k - \delta k - wl - cb - f\right] - (1-\gamma)bp. \tag{40}$$

Then, the firm continues operations when it survives default and exogenous exit by choosing whether to call or not the callable fraction $1-\theta$ of its outstanding bond b. Based on its choice, it chooses equity to complement the internal fund from its asset n. I now define the firm's value of continuation.

$$\overline{V}(s,x) = \max_{\mathbb{1}_{\text{call}}} \left\{ \overline{V}_{\text{nocall}}(s,x), \overline{V}_{\text{call}}(s,x) \right\} \tag{41}$$

The indicator function $\mathbb{1}_{\text{call}}(s,x)$ is one if and only the firm decides to exercise the option to call. I define the continuation price of each type of bond as their expected continuation prices:

$$\begin{cases} \overline{p}^{nc} & = (1-\mathbb{1}_{\text{call}}(s,x)) \cdot \overline{p}^{nc}_{\text{nocall}}(s,x) + \mathbb{1}_{\text{call}}(s,x) \cdot \overline{p}^{nc}_{\text{call}}(s,x) \\ \overline{p}^{c} & = (1-\mathbb{1}_{\text{call}}(s,x)) \cdot \overline{p}^{c}_{\text{nocall}}(s,x) + \mathbb{1}_{\text{call}}(s,x) \cdot \overline{p}^{c}_{\text{call}}(s,x) \end{cases} \tag{42}$$

When the firm takes the decision $j \in \{\text{nocall}, \text{call}\}$ on the callable share $1-\theta$ of its bond b, it receives:

$$\overline{V}_j(s,x) = \max_{e,k'_j,b'_j,\theta'_j,c'_j} \left\{ -e - \eta_e(e) + \frac{1}{1+r} \mathbb{E}_{s'|s} \int_{\varepsilon'} V(s',x') d\Phi(\varepsilon') \right\} \tag{43}$$

subject to:

$$\begin{aligned} k'_j &= e + n_j + p\tilde{b}_j - \eta_b(\tilde{b}_j), \\ V(s',x') &= \max \left\{ V^r(s',x'), 0 \right\}, \\ V^r(s',x') &= (1-\pi_e)\overline{V}(s',x') + \pi_e V^{ex}(s',x'), \\ \overline{V}(s',x') &= \max_{\mathbb{1}_{\text{call}}(s',x')} \left\{ \overline{V}_{\text{nocall}}(s',x'), \overline{V}_{\text{call}}(s',x') \right\}, \\ p &= \tilde{\theta}_j p^{nc} + (1-\tilde{\theta}_j) p^c. \end{aligned}$$

Where p is the weighted average price of the new debt, composed of the price of the non-callable (callable) new bond $p^{nc}(p^c)$. The vector x' is the result of the call decision: $x' \equiv \mathbb{1}'_{\text{call}} * x'_{\text{call}} + (1 - \mathbb{1}'_{\text{call}}) * x'_{\text{nocall}}$.

Firms Entry. In each period, there is free entry. I follow Hopenhayn (1992), I assume that a potential entrant draws an initial level of productivity z^e from an invariant distribution after paying the entry cost. A constant mass M of potential entrants enters the economy without any initial capital $k = 0$, any initial debt $b = 0$. The free entry condition is the following:

$$\int V(z_e, r, 0, 0, \theta, c) = c_e.$$

4.1.5 Bond Pricing

When the firm decides to default on its debts, to liquidate, and to exit, the creditors recover a fraction $1-\xi$ of the firm's liquidation $\underline{n}$ defined by:

$$\underline{n}(\varepsilon) = \max\Big\{0, k + (1-\tau)\Big[y + \varepsilon k - \delta k - wl - f\Big]\Big\} \tag{44}$$

ξ is the parameter that indicates the fraction lost when the firm is liquidated, and $(1-\xi)$ is, then, the rate of recovery of capital for the creditor. I assume that all creditors have the same seniority over the firm's liquidation value claim. I first present the pricing schedule for the non-callable and callable components of the new bond $\tilde{b}$, and second, I define the continuation price of the two types of bonds in cases of no-call and call.

Non-callable bond pricing. For the non-callable $\tilde{\theta}$ in the new debt $\tilde{b}$, the coupon rate attached to it is the current interest rate r, so next period, when the firm does not default, it has to pay the fraction γ and the interest r. After this payment, the remaining fraction of the debt will be $1-\gamma$, valued by creditors at $\overline{p}^{nc}$. But the current interest rate of r has the weight $\tilde{b}/b'$ in the composition of the new average periodic coupon for the next period c', defined in (35) and (36), and the continuation price $\overline{p}^{nc}$ contains information on this future average coupon c' instead of direct value of the current rate r.

$$\begin{aligned} p^{nc}(z, r, k', b', \tilde{\theta}, r) = \frac{1}{1+r}\mathbb{E}_{s'|s}\Bigg\{ & \int_{\overline{\varepsilon}'}^{\infty} \Big[\gamma + r + (1-\gamma)\overline{p}^{nc}(z', r', \hat{x}^k, \hat{x}^b, \tilde{\theta}, r)\Big]\, d\Phi(\varepsilon') \\ & + \int_{-\infty}^{\overline{\varepsilon}'} \frac{1-\xi}{b'}\underline{n}' d\Phi(\varepsilon')\Bigg\} \quad , \end{aligned} \tag{45}$$

where $\overline{p}^{nc}(z', r', \hat{x}^k, \hat{x}^b, \tilde{\theta}, r)$ is the continuation price of this non-callable bond ($\tilde{\theta}\tilde{b}$) in the next period.[28] I define it as the continuation price of the previous non-callable debt θb, so $\overline{p}^{nc}$ instead as shown in (42). Its formulation does not change depending on the call decision in the current period. It is important to notice that the prevailing non-callable bond θb has to pay the periodic coupon c set at its issuance. Then, it is set as:

$$\begin{aligned} \overline{p}_j^{nc}(z, r, k', b', \theta, c) = \frac{1}{1+r}\mathbb{E}_{s'|s}\Bigg\{ & \int_{\overline{\varepsilon}'}^{\infty} \left[\gamma + c + (1-\gamma)\overline{p}^{nc}(z', r', \hat{x}_j(s', x'))\right] d\Phi(\varepsilon') \\ & + \int_{-\infty}^{\overline{\varepsilon}'} \frac{1-\xi}{b'}\underline{n}' d\Phi(\varepsilon')\Bigg\} \quad , \quad \forall j \in \{\text{nocall}, \text{call}\} \end{aligned} \tag{46}$$

[28]Notice that $\overline{p}^{nc}$ is a function, and by keeping the arguments $k', b', \tilde{\theta}, r$ in the price, I am sure to keep the current interest rate r on the non-callable bond $\tilde{\theta}\tilde{b}$. See detailed explanations in Appendix (D.1.1).

where $\hat{x}_j(s,x) = \left[\hat{x}_j^k(s,x), \hat{x}_j^b(s,x), \hat{x}_j^\theta(s,x), \hat{x}_j^c(s,x)\right]$ is the vector of policy functions for k', b', θ', and c' in the scenario j.

Callable bond pricing. The price of a callable bond follows three steps: (i) a similar non-callable bond is priced; (ii) the endogenous probability of calling the callable bond in the next period; (iii) the potential loss of return for the bondholder in case of call. The callable bond is in fraction $(1-\tilde{\theta})$ in the new debt $\tilde{b}$ issued in the current period. I provide a pricing model that incorporates the expected average loss of the creditor in case of the call option exercise. Based on the pricing of the non-callable bond above, in (45), the callable bond is priced in $t=0$, at:

$$p^c(z,r,k',b',\tilde{\theta},r) = \frac{1}{1+r}\mathbb{E}_{s'|s}\Bigg\{\int_{\overline{\varepsilon}'}^{\infty}\Big[\gamma + r + (1-\gamma)\Big((1-\mathbb{1}'_{\text{call}})\overline{p}^c_{\text{nocall}} + \mathbb{1}'_{\text{call}}\left((1-\lambda)\overline{p}^c_{\text{call}} + \lambda\left(1+\chi\right)\right)\Big)\Big]d\Phi(\varepsilon') + \int_{-\infty}^{\overline{\varepsilon}'}\frac{1-\xi}{b'}\underline{n}'d\Phi(\varepsilon')\Bigg\} \tag{47}$$

where $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}\left(s', \hat{x}(s,x)\right)$ is the call decision in the next period based on $(s', \hat{x}(s,x))$, $\overline{p}^c_j(\cdot)$ is the continuation price of the callable bond $(1-\tilde{\theta})\tilde{b}$ in the next period in the scenario $j \in \{\text{nocall}, \text{call}\}$. The λ term captures that only a fraction of the callable stock is redeemed upon a call next period; the remainder continues and is valued by the corresponding continuation price. The formulation in (47) considers the potential loss to the bondholder due to the issuer's option to call the bond. For simplicity, instead of $\overline{p}^c$, I present the continuation price of the current callable bond $(1-\theta)b$.

$$\overline{p}^c(z,r,k',b',\theta,c) = (1-\mathbb{1}_{\text{call}}(s,x))\cdot\overline{p}^c_{\text{nocall}}(s,x) + \mathbb{1}_{\text{call}}(s,x)\cdot\overline{p}^c_{\text{call}}(s,x) \tag{48}$$

Contrarily to non-callable bonds, the pricing of the continuation value of callable bonds should differ depending on the call option exercise. While for the no-call scenario, an outstanding callable bond is priced similarly to a non-callable bond, in (46), in the case of calling, its pricing should stop at the final repayment, which is the principal and the call premium. This difference is then reconsidered in the continuation prices as follows.

$$\overline{p}_j^c = \frac{1}{1+r}\mathbb{E}_{s'|s}\Bigg\{\int_{\overline{\varepsilon}'}^{\infty}\Big[\gamma + c + (1-\gamma)\Big((1-\mathbb{1}'_{\text{call}})\overline{p}^{c\prime}_{\text{nocall}} + \mathbb{1}'_{\text{call}}\left((1-\lambda)\overline{p}^{c\prime}_{\text{call}} + \lambda\left(1+\chi\right)\right)\Big)\Big]d\Phi(\varepsilon') + \int_{-\infty}^{\overline{\varepsilon}'}\frac{1-\xi}{b'}\underline{n}'d\Phi(\varepsilon')\Bigg\} \tag{49}$$

where $\overline{p}^{c\prime}_j = \overline{p}^c_j(z', r', \hat{x}(s', x')), \forall j \in \{\text{nocall}, \text{call}\}$. The price above is the callable bond's market value in scenario j. The bond is priced as if it were newly issued with the current market conditions and the firm's characteristics. I provide further discussion of the pricing of callable bonds in Appendix (D.1.2).

4.2 Households

I consider a representative infinitely-lived household that owns all firms and receives the income in the economy. The household consumes, works, and invests its savings in equity and debt. Government revenue from taxation is returned to the household as a lump-sum transfer. The household has preferences over consumption C_t and labor supply L_t. The utility function is:

$$U(C_t, L_t) = ln(C_t) - \frac{L_t^{1+\vartheta}}{1+\vartheta}$$

The household discounts the future by the discount factor β (which corresponds to the average interest rate) and makes optimal choices through the marginal rate of substitution and inter-temporal substitution, i.e.

$$\begin{aligned} 1 =& \mathbb{E}\Big[\beta\frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)}\Big] \\ w =& -\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} \end{aligned} \tag{50}$$

4.3 Policy Authority

The government sets a fixed corporate tax τ on corporate periodic earnings. Concerning the monetary policy, I consider a simple framework where the authority decides on the real interest rate exogenous path. As in some works using New Keynesian Models (Jeenas, 2018; Ottonello and Winberry, 2020; Jungherr et al., 2024) or using a simple exogenous interest rate setting (e.g., Ippolito et al., 2018; Deng and Fang, 2022), I assume an AR(1) process:

$$logr' = \mu_r + \rho_r logr + \epsilon'_r, \quad \epsilon'_r \sim^{i.i.d.} \mathcal{N}\left(0, {\sigma_r}^2\right), \quad \mu_r, \rho_r \in (0,1) \tag{51}$$

4.4 Equilibrium

Now, I define the competitive recursive equilibrium of the model, considering the economy's steady state. First, I describe the law of motion of the firms' distribution and define the stationary equilibrium.

Definition 4.1 (Law of motion of the firms' distribution)**.** *Let Γ be the distribution of incumbent firms at the beginning of the period, and Ω be the distribution of entrant firms. The distribution of firms that will produce in the next period is determined as follows:*

$$\begin{aligned}\Gamma'(z',k',b') = &\int_0^\infty \int_0^\infty \Gamma(z,k,b)\ \pi_e(z'|z)\Big[1-\Phi\left(\overline{\varepsilon}'(z,r,k,b,\theta,c)\right)\Big]\cdot \mathbb{1}_{B_{inc}}\ dz\ db \\ &+ \Omega'(z',k',b')\end{aligned} \tag{52}$$

where $B_{inc} = \Big\{(z,k,b,k',b') \mid b' = (1-\gamma)\cdot b(z,r,k,b,\theta,c)\Big\}$*; and the distribution function of future entrants* $\Omega'(z',b'_{nc},b'_c)$ *is defined as:*

$$\Omega'(z',k',b') = M\cdot \mathbb{1}_{B_{entr}}$$

where $B_{entr} = \Big\{(z',k',b') \mid k'=0, b'=0\Big\}$.

Definition 4.2 (Stationary Equilibrium)**.** *A stationary equilibrium in this economy consists of a set of: (i) value functions $V(z,r,k,b,\theta,c)$, $V^r(z,r,k,b,\theta,c)$, $V^x(z,r,k,b,\theta,c)$, $\overline{V}(z,r,k,b,\theta,c)$, $\overline{V}_{nocall}(z,r,k,b,\theta,c)$, and $\overline{V}_{call}(z,r,k,b,\theta,c)$; (ii) a vector of policy functions $\hat{x}(z,r,k,b,\theta,c) = \{k',b',\theta',c'\}$; (iii) bond pricing functions $p^{nc}(z,r,k,b,\theta,c)$ and $p^c(z,r,k,b,\theta,c)$ given by (45) and (47); (iv) a mass of entrants M^* and a stationary distribution Γ^* ; (v) household consumption C^* and aggregate labor supply L^*; and (vi) a wage w^* and an interest rate r^*, such that:*

1. *Given the bond price functions $p^{nc}(z,r,k,b,\theta,c)$ and $p^c(z,r,k,b,\theta,c)$, the policy vector $\hat{x}(z,r,k,b,\theta,c)$, the value function $V(z,r,k,b,\theta,c)$, and the default decision $\overline{\varepsilon}$ solve the firm's optimization problem (43).*

2. *The free entry condition holds: $V(z_e,0,0,\theta,c)=0$.*

3. *The bond price functions $p^{nc}(z,r,k,b,\theta,c)$ and $p^c(z,r,k,b,\theta,c)$ are consistent with the zero expected profit condition for the investors and the default probabilities and expected recovery rates satisfy the repayment policy;*

4. *The representative household chooses optimally C^* and L^*, consistent with (50).*

5. *The goods market clears:*

$$
\begin{aligned}
Y \equiv & \int_0^\infty \int_0^\infty \left[y - f - H\left(\tilde{b}(z,r,k,b,\theta,c), b, \theta, c\right) \right. \\
& \qquad \left. - \xi \int_{-\infty}^{\bar{\varepsilon}(z,r,k,b,\theta,c)} \underline{n} d\Phi(\varepsilon) \right] \Gamma^* (z,b) \; dz \; db \\
= & \; C^* + I^*
\end{aligned} \tag{53}
$$

with C^*, the household optimal consumption and I^*, the aggregate investment, defined as:

$$
I^* \equiv \delta \int_0^\infty \int_0^\infty k(z, b_{nc}, b_c) \; \Gamma^*(z, b_{nc}, b_c) \; dz \; db \tag{54}
$$

5. *The labor market clears*

$$
L^* \equiv \int_0^\infty \int_0^\infty l(z, b_{nc}, b_c) \; \Gamma^*(z, b_{nc}, b_c) \; dz \; db \tag{55}
$$

I follow the solution methods in Hatchondo et al. (2016) and Jungherr and Schott (2021) to solve the model equilibrium; see details in Appendix E.2. I present the first-order conditions in Appendix D.2.

5 Quantitative Analysis

This section presents the quantitative analysis of the model, including the calibration strategy, model validation, dynamic analysis, and counterfactual experiments. After calibrating the model to match key empirical moments, I assess its ability to replicate observed firm behavior and macroeconomic dynamics, focusing on the role of callable debt in firm financing decisions and its implications for the broader economy.

5.1 Calibration

The calibration process is divided into two parts: externally fixed parameters, which are chosen based on values found in the literature or empirically observed data, and internally calibrated parameters, which are estimated by fitting the model to match key empirical moments.

5.1.1 Externally fixed parameters

The externally fixed parameters are drawn from existing studies or empirical data and represent well-established or relatively stable aspects of the economy across different models. These parameters are not estimated within the model but are held constant throughout the calibration.

These externally fixed parameters are the model's foundation, ensuring the baseline environment reflects the broader macroeconomic and firms' average-level context. By grounding these

Table 2: Externally fixed parameters

Params.	Description	Value	Source/Target
ϑ	Inverted Frisch elasticity	0.5	Arellano et al. (2019)
τ	Corporate tax rate	0.4	Gomes et al. (2016)
ψ	Capital share	0.33	Bloom et al. (2018)
ζ	Decreasing returns-to-scale	0.75	Bloom et al. (2018)
δ	Quarterly depreciation rate	0.025	Standard (BEA)
π_e	Exogenous exit rate	0.01	Ottonello and Winberry (2020)
γ_{nc}	Non-callable debt repayment rate	0.05	Jungherr and Schott (2021)
ρ_z	Productivity shock persistence	0.9	Ottonello and Winberry (2020)

parameters in empirical evidence and the literature, I ensure that the model's dynamics are consistent with well-established economic relationships. For the parameters of the production function, I take the capital share ζ and the decreasing returns-to-scale ψ from Bloom et al. (2018). The capital quarterly depreciation $\delta = 0.025$ is set to fit estimates from BEA. I use the persistence of the productivity process, $\sigma_z = 0.03$, estimated by Ottonello and Winberry (2020). Following Gomes et al. (2016), I fix the corporate tax rate τ to 0.4.

5.1.2 Internally fitted parameters

The internally fitted parameters are calibrated by matching the model to essential empirical moments. These parameters are adjusted to capture not only firm-specific financial behaviors but also aggregate dynamics. The calibration process is based on minimizing the distance between the model-implied moments and their empirical counterparts. Below, I detail the ten internally calibrated parameters, their roles within the model, their target empirical moments, and the data sources used accordingly for calibration.

Table 3: Internally fitted parameters

Params.	Description	Value	Target	Data	Model
η_{nc}	Non-callable issuance cost	0.0110	Leverage ratio	33%	29%
η_c	Callable issuance cost	0.0107	Share of callable debt	62%	64%
ξ	Default cost	0.6941	Avg credit spread (non-callable)	2.9%	2.9%
χ	Call premium	0.0082	Avg credit spread (callable)	3.2%	3.6%
f	Fixed operation cost	0.4772	Investment rate	22%	13%
σ_ε	Capital quality shock volatility	0.8874	Average call rate	47%	26%
σ_z	Productivity shock volatility	0.0180	Average exit rate	8.7%	8.6%
ρ_r	Interest rate persistence	0.8652	Average callable bond lifespan	48.6%	44.1%
σ_r	Interest rate volatility	0.0216	Average long-run interest rate	2.94%	2.91%
γ_c	Callable debt repayment rate	0.0408	Callable bond duration (years)	6.47	3.66

The issuance cost parameters (η_{nc}, η_c) are primarily identified through their impact on the leverage ratio and the share of callable debt. Higher issuance costs for callable debt (η_c) decrease its prevalence, directly affecting the share of callable debt in the capital structure. The importance of these targets is emphasized with observations and intuitions on investment dynamics and refinancing flexibility, documented by Covas and Haan (2011), Crouzet (2018), Begenau and Salomao (2019), and Becker et al. (2024). The default cost parameter (ξ) is identified through its influence on credit spreads. Higher default costs lead to wider credit spreads, aligning with the observed average credit spreads across debt types. Acharya and Carpenter (2002), who emphasize the role of default costs in shaping corporate debt dynamics.

The call premium is central to matching the average credit spread on callable debt. By adjusting χ, we ensure that the model accurately reflects the market's additional risk assessment associated with callable features. A higher call premium discourages the issuance of callable debt, influencing its relative pricing compared to non-callable debt. This approach is consistent with Duffie and Singleton (1999), who explore the pricing implications of defaultable bonds with callable features.

Fixed operating costs influence the profitability and the investment rate. By calibrating f to match the average investment rate, we ensure that firms' investment behaviors in the model reflect empirical observations. High fixed operating costs constrain firms' ability to undertake growth-enhancing investments, highlighting the role of operating expenses as an essential determinant of firm-level decisions, as discussed in the framework of Hopenhayn (1992).

Capital quality shock volatility measures the uncertainty in firms' capital effectiveness, directly influencing default risk. Higher volatility implies increased earnings uncertainty and a greater likelihood of default, leading to wider credit spreads. The calibration of σ_ε targets the average call rate, capturing the overall risk premium demanded by financial markets. It allows the model to be in the sense of essential works in the literature. (e.g., Gilchrist and Zakrajšek, 2012 on credit spreads as a reflection of firm-specific risks).

Productivity shock volatility represents the unpredictability in firms' productivity levels, affecting their operational efficiency and survival prospects. This parameter is calibrated to match the average exit rate, ensuring that the model accurately reflects the impact of productivity fluctuations on firm dynamics and macroeconomic outcomes. By capturing the variability in productivity, σ_z plays an essential role in determining firms' investment and exit decisions. On the interest rate process, its persistence ρ_r is identified through its effect on the lifespan of callable bonds. Its effect, combined with the volatility of the process σ_r, also determines the intensity of the call decision. Higher persistence leads to longer-lived callable bonds, aligning the model with empirical observations of bond lifespans.

My calibration relies on firm-level data sources, primarily from Compustat and the Financial

Information System Dataset (FISD). I use FRED data for the long-run interest rate. For instance, the leverage ratios, share of callable debt, and investment rates are sourced from Compustat. Moreover, secondary, some moments are taken coherently from key works in the literature (e.g., Arellano et al., 2019; Ottonello and Winberry, 2020).

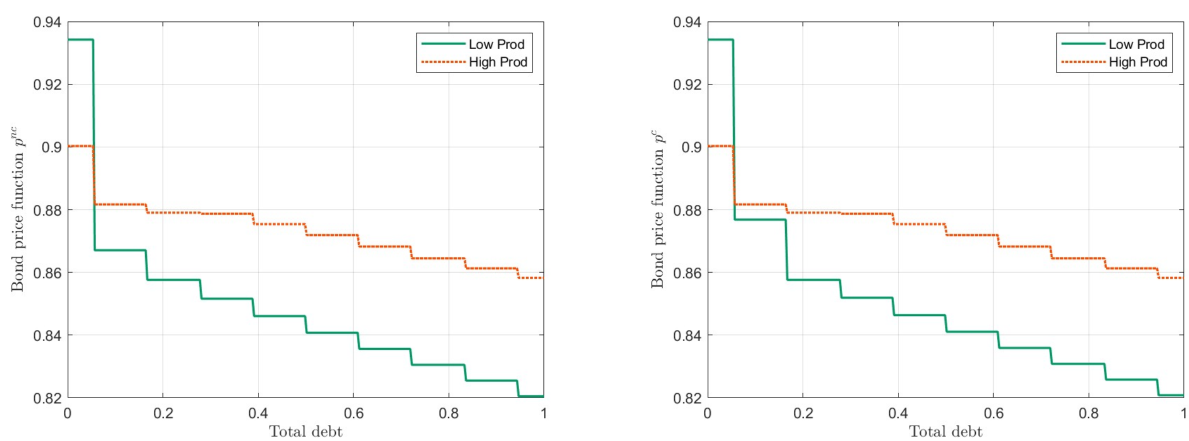
5.2 Dynamic effects of callability

In this section, we explore the dynamics of the calibrated model, focusing on the implications of callable debt on firm behavior and macroeconomic outcomes. We analyze bond prices, capital and debt decisions, and the response of key variables to shocks.

5.2.1 Bond Prices

This subsection investigates how bond prices vary with the firm's debt levels, capital stock, and productivity. Understanding the relationship between these variables and bond prices is crucial for analyzing the cost of financing and the risk premium investors require. I examine the price of non-callable and callable bonds as functions of the firm's state variables: total debt b, capital k, and productivity z. The bond price reflects the present value of future coupon payments, adjusted for the probability of default and the probability that the bond will be called in the case of callable bonds.

Figure 10: Bonds prices as function of debt



(a) Average non-callable bond prices (b) Average callable bond prices

Notes: This figure plots the prices of non-callable and callable bonds across different levels of total debt for two distinct productivity levels: low ($z = z_{low}$), and high ($z = z_{high}$).

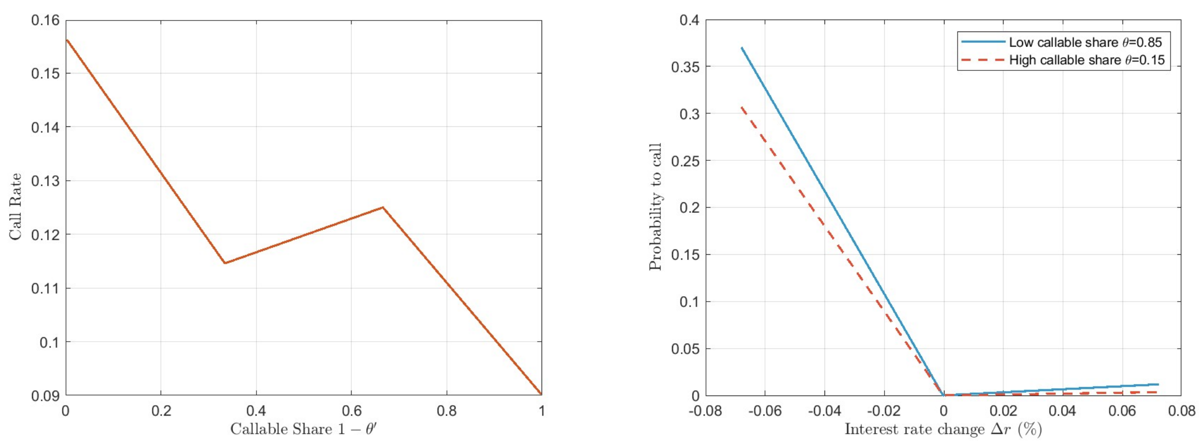
Higher debt levels: As debt increases, bond prices decline, reflecting the increased default risk associated with higher leverage. This effect is more pronounced for callable bonds, where the call option introduces additional pricing considerations.

Productivity influence: Higher productivity levels z are associated with higher bond prices, as firms are less likely to default when productivity is strong. This relationship is consistent across both callable and non-callable bonds.

Capital stock: A larger capital stock k generally leads to higher bond prices, which signals a stronger balance sheet and lower default risk. These findings highlight the link between a firm's financial position and the cost of debt financing, particularly when callable debt is involved.

5.2.2 Heterogeneous effects of callability

Figure 11: Call decision

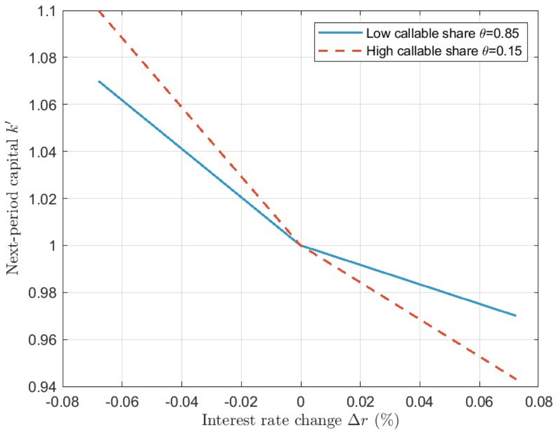


(a) Call rate (b) Decision rule to call

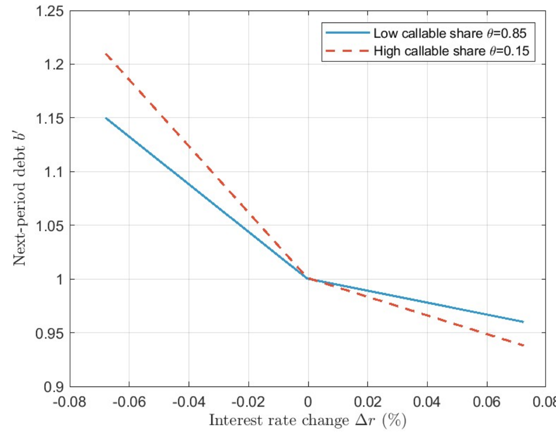
Notes: This figure plots the average call rate depending on the share of callable debt (left panel) and the response of firms' optimal decision to call (right panel). The left panel shows the average call rate of a generic firm simulated for the average level of productivity, capital, debt, and periodic coupon. The right panel displays two generic firms simulated for the average level of productivity, capital, debt, share of non-callable debt, and periodic coupon. The firms are blue for the low share of callable debt ($\theta = 0.15$) and red for the high share ($\theta = 0.85$). The right panel represents the average call rate for different values of the interest rate realization $\{0.93\%, ..., 1.09\%\}$.

The decision to call a bond is influenced by interest rate fluctuations, the firm's financial position, and the callable share of its debt portfolio. As shown in Figure 11a, firms with a higher share of callable bonds are less likely to call. Theoretically, the firm's decision to call depends on the trade-off between the savings from refinancing at lower rates and the premium paid to exercise the call option. For firms with a high callable share ($\theta = 0.15$), the average probability of calling is relatively lower across interest rate changes, reflecting limited refinancing flexibility. Conversely, firms with a low callable share ($\theta = 0.85$) exhibit a steeper response to rate declines, as refinancing significantly reduces their cost of capital and may appear more accessible. The call option allows them to benefit from declining rates. Specifically, a lower interest rate environment incentivizes firms to refinance their debt at reduced costs. In contrast, the cost of maintaining callable bonds may deter firms from exercising the option when rates rise.

Figure 12: The effect of callability on firm behavior



(a) Investment



(b) Next period debt

Notes: This figure plots the response of firms' optimal choice of investment (left panel) and next-period debt (right panel) to an interest rate change. The panels display two generic firms simulated for the average level of productivity, capital, debt, share of non-callable debt, and periodic coupon. The firms are blue for the low share of callable debt ($\theta = 0.15$) and red for the high share ($\theta = 0.85$). The panels represent the values of the variables of interest for different values of the interest rate realization $\{0.93\%, ..., 1.09\%\}$.

Figure 12 shows the firms' investment and financing response to interest rate changes in the two cases of low callable share (i.e., $\theta = 0.85$) and high share (i.e., $\theta = 0.15$) debt. The values of the variables are normalized to 1 when the rate $r_t = 1.01\%$.

Figures 12a and 12b illustrate the behavior of firms' next-period capital (k') and next-period debt (b') in response to interest rate changes (Δr), depending on their share of non-callable debt (θ). These figures reveal how callability affects firms' investment and financing strategies, with significant differences between firms with low callable shares ($\theta = 0.85$) and high callable shares ($\theta = 0.15$). Firms with a high callable share ($1 - \theta = 0.85$)—typically smaller firms—demonstrate a sharper response to declining interest rates. Callable bonds provide these firms with refinancing flexibility, enabling them to redirect savings from lower borrowing costs into investment. This heightened sensitivity mirrors traditional models of monetary policy, where smaller firms react more strongly due to their reliance on external financing. In contrast, firms with a low callable share ($1 - \theta = 0.15$)—often larger firms—exhibit more stable investment responses. While they call less frequently, these firms adjust their investment moderately, benefiting less from the refinancing flexibility offered by callable bonds. Their smoother adjustment highlights their reliance on non-callable debt, which does not allow immediate cost reductions when rates decline.

The financing behavior, reflected in next-period debt (b'), aligns with these patterns. High-callable-share firms ($1-\theta = 0.85$) increase their borrowing significantly when interest rates fall, leveraging callable debt's refinancing benefits when they call existing callable debts. However, they reduce debt more when rates rise, reflecting the higher cost burden of callable bonds in such environments.

As a quantitative exercise, I compare an economy with callable debt ($\theta = 0$) to a baseline economy without callable debt ($\theta = 1$). The results demonstrate the critical role of callable debt in amplifying firms' investment responses to interest rate changes. Specifically, for a 6.8 percentage point decline in interest rates, the optimal investment in the callable economy reaches $k' = 1.285$, compared to $k' = 1.012$ in the baseline economy. This represents a 26.98% higher investment level in the callable economy. When normalized, this implies that within the investment response to a change of interest rates, callable debt contributes to 21%. These findings confirm that callable debt introduces additional flexibility in firms' capital allocation, significantly enhancing their sensitivity to monetary policy. This amplification underscores the broader macroeconomic implications of callable bonds and their potential to reshape monetary policy transmission across firms.

6 Conclusion

This paper shows that callability is a first-order determinant of firms' financing and investment responses. Using issuer–quarter panel data, I distinguish between contractual callability and effective scheduled eligibility at the beginning of the quarter. In the data, the contractual callable share is large on average (59%), but only 1.7% of outstanding amounts are effectively callable at the beginning of quarter. I also find that conditional on eligibility, the quarterly call hazard is 3.5%. These measurement choices are central for identification and for mapping the data to the model.

Empirically, callable debt is prevalent and unevenly distributed across issuers: the share of callable design in issuance is high and has risen over time -on average 75% of bonds issued during the period 1990-2018 are callable-, with meaningful heterogeneity across credit quality groups. Smaller and riskier issuers tend to issue more callable debt yet exercise calls less often (0.03%), consistent with refinancing risk and financial constraints; larger, higher-rated firms call more aggressively (more than 4%) when conditions are favorable.

Bringing these facts to the theory side, I build a quantitative model in which firms jointly choose issuance, the share of callable debt, call exercise, and default. The model is disciplined to match spreads, the callable share, the average call rate, exit, and duration moments (Table 3). In counterfactuals, callability materially amplifies the response of investment to interest-rate movements: for a 6.8-percentage-point rate decline, optimal investment is 26.98% higher in the callable economy than in a no-call baseline, implying that callability accounts for about one-fifth (21%) of the total investment response to rates. Mechanically, callability creates a refinancing option that relaxes cash-flow and borrowing-cost constraints when rates fall, with stronger effects for firms that are able to exercise the option.

Two implications follow. First, monetary policy transmits not only through conventional balance-sheet and maturity channels but also through an issuer-call refinancing channel whose strength depends on the distribution of effective eligibility and the cross-section of credit risk. Second, the heterogeneity documented in the data implies that shocks reallocate investment toward firms with both high callable exposure and the ability to call—typically larger, safer issuers—raising questions about the aggregate incidence of policy across firm types.

The framework can be extended along several dimensions. Endogenizing call premia and issuance costs, adding secondary-market frictions around make-whole features, and letting the policy authority choose alternative rate paths would help quantify welfare and incidence. Taking the model to richer distributions of eligibility over the life cycle of bonds would also allow to study how lumpy call protection expirations shape aggregate dynamics. These steps would refine the measurement–model link introduced here and further clarify when callability stabilizes or amplifies the macroeconomy.

References

Acharya, V. V. and J. N. Carpenter (2002): "Corporate Bond Valuation and Hedging with Stochastic Interest Rates and Endogenous Bankruptcy," *The Review of Financial Studies*, 15, 1355–1383.

Almeida, H., M. Campello, B. Laranjeira, and S. Weisbenner (2012): "Corporate Debt Maturity and the Real Effects of the 2007 Credit Crisis," *Critical Finance Review*, 1, 3–58.

Arellano, C., Y. Bai, and L. Bocola (2020): "Sovereign Default Risk and Firm Heterogeneity," Working Paper 23314, National Bureau of Economic Research, series: Working Paper Series.

Arellano, C., Y. Bai, and P. J. Kehoe (2019): "Financial Frictions and Fluctuations in Volatility," *Journal of Political Economy*, 127, 2049–2103.

Banko, J. C. and L. Zhou (2010): "Callable Bonds Revisited," *Financial Management*, 39, 613–641, publisher: [Financial Management Association International, Wiley].

Becker, B., M. Campello, V. Thell, and D. Yan (2024): "Credit risk, debt overhang, and the life cycle of callable bonds," *Review of Finance*, 28, 945–985.

Begenau, J. and J. Salomao (2019): "Firm Financing over the Business Cycle," *The Review of Financial Studies*, 32, 1235–1274.

Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 79, 14–31.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework," *Handbook of macroeconomics*, 1, 1341–1393, publisher: Elsevier.

Berndt, A. (2004): "Estimating the term structure of yield spreads from callable corporate bond price data," Tech. rep., Citeseer.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2018): "Really Uncertain Business Cycles," *Econometrica*, 86, 1031–1065, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA10927.

Brennan, M. J. and E. S. Schwartz (1977): "Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion," *The Journal of Finance*, 32, 1699–1715.

Chatterjee, S. and B. Eyigungor (2012): "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 102, 2674–2699.

Chen, H., Y. Xu, and J. Yang (2021): "Systematic risk, debt maturity, and the term structure of credit spreads," *Journal of Financial Economics*, 139, 770–799.

Chen, Z., C. X. Mao, and Y. Wang (2010): "Why firms issue callable bonds: Hedging investment uncertainty," *Journal of Corporate Finance*, 16, 588–607.

Clymo, A. and F. Rozsypal (2023): "Firm cyclicality and financial frictions," Working Paper 196, Danmarks Nationalbank Working Papers.

Cochrane, J. H. (2017): "Macro-Finance*," *Review of Finance*, 21, 945–985.

Cooley, T. F. and V. Quadrini (2001): "Financial Markets and Firm Dynamics," *The American Economic Review*, 91, 1286–1310.

Covas, F. and W. J. D. Haan (2011): "The Cyclical Behavior of Debt and Equity Finance," *The American Economic Review*, 101, 877–899.

Crouzet, N. (2017): "Default, debt maturity, and investment dynamics," Working Paper, .

——— (2018): "Aggregate Implications of Corporate Debt Choices," *The Review of Economic Studies*, 85, 1635–1682.

Darmouni, O., O. Giesecke, and A. Rodnyansky (2022): "The Bond Lending Channel of Monetary Policy," *SSRN Electronic Journal*.

De Fiore, F. and H. Uhlig (2015): "Corporate Debt Structure and the Financial Crisis," *Journal of Money, Credit and Banking*, 47, 1571–1598.

Demirgüç-Kunt, A. and V. Maksimovic (1999): "Institutions, financial markets, and firm debt maturity," *Journal of Financial Economics*, 54, 295–336.

Deng, M. and M. Fang (2022): "Debt maturity heterogeneity and investment responses to monetary policy," *European Economic Review*, 144, 104095.

Duffee, G. R. (1998): "The Relation Between Treasury Yields and Corporate Bond Yield Spreads," *The Journal of Finance*, 53, 2225–2241, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/0022-1082.00089.

Duffie, D. and K. J. Singleton (1999): "Modeling Term Structures of Defaultable Bonds," *The Review of Financial Studies*, 12, 687–720, publisher: [Oxford University Press, Society for Financial Studies].

Elsaify, A. and N. Roussanov (2016): "Why Do Firms Issue Callable Bonds?" *Working Paper*, 43.

Fabiani, A., J. Heineken, and L. Falasconi (2022): "Monetary policy and corporate debt maturity," *Available at SSRN 3945615*.

Flor, C. R., K. B. Petersen, and A. Schandlbauer (2023): "Callable or convertible debt? The role of debt overhang and covenants," *Journal of Corporate Finance*, 78, 102346.

Gilchrist, S. and E. Zakrajšek (2012): "Credit spreads and business cycle fluctuations," *American Economic Review*, 102, 1692–1720.

Goldstein, R., N. Ju, and H. Leland (2001): "An EBIT-Based Model of Dynamic Capital Structure," *The Journal of Business*, 74, 483–512.

Gomes, J., U. Jermann, and L. Schmid (2016): "Sticky Leverage," *The American Economic Review*, 106, 3800–3828, publisher: American Economic Association.

Gomes, J. F. (2001): "Financing Investment," *The American Economic Review*, 91, 1263–1285.

Guntay, L. (2002): "Pricing the risks of callable defaultable coupon bonds," *Working Paper*, 28–1.

Guntay, L., N. Prabhala, and H. Unal (2004): "Callable bonds, interest-rate risk, and the supply side of hedging," *Interest-Rate Risk, and the Supply Side of Hedging (July 2004)*.

Gürkaynak, R., H. G. Karasoy-Can, and S. S. Lee (2022): "Stock Market's Assessment of Monetary Policy Transmission: The Cash Flow Effect," *The Journal of Finance*, 77, 2375–2421.

Hatchondo, J. C. and L. Martinez (2009): "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 79, 117–125.

Hatchondo, J. C., L. Martinez, and C. Sosa-Padilla (2016): "Debt Dilution and Sovereign Default Risk," *Journal of Political Economy*, 124, 1383–1422.

He, Z. and W. Xiong (2012): "Rollover Risk and Credit Risk," *The Journal of Finance*, 67, 391–430.

Hopenhayn, H. A. (1992): "Entry, Exit, and firm Dynamics in Long Run Equilibrium," *Econometrica*, 60, 1127–1150.

Ippolito, F., A. K. Ozdagli, and A. Perez-Orive (2018): "The transmission of monetary policy through bank lending: The floating rate channel," *Journal of Monetary Economics*, 95, 49–71.

Jarrow, R., H. Li, S. Liu, and C. Wu (2010): "Reduced-form valuation of callable corporate bonds: Theory and evidence," *Journal of Financial Economics*, 95, 227–248.

Jeenas, P. (2018): "Monetary Policy Shocks, Financial Structure, and Firm Activity: A Panel Approach," *SSRN Electronic Journal*.

——— (2024): "Firm Balance Sheet Liquidity, Monetary Policy Shocks, and Investment Dynamics," Working Paper, UPF.

Jermann, U. and V. Quadrini (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102, 238–271.

Jorda, O. (2005): "Estimation and inference of impulse responses by local projections," *American economic review*, 95, 161–182.

Joslin, S., M. Priebsch, and K. J. Singleton (2014): "Risk Premiums in Dynamic Term Structure Models with Unspanned Macro Risks," *The Journal of Finance*, 69, 1197–1233.

Jungherr, J., M. Meier, T. Reinelt, and I. Schott (2024): "Corporate Debt Maturity Matters For Monetary Policy," *Working paper*, 88.

Jungherr, J. and I. Schott (2021): "Optimal debt maturity and firm investment," *Review of Economic Dynamics*, 42, 110–132.

——— (2022): "Slow Debt, Deep Recessions," *American Economic Journal: Macroeconomics*, 14, 224–259.

Karabarbounis, M. and P. Macnamara (2021): "Misallocation and financial frictions: The role of long-term financing," *Review of Economic Dynamics*, 40, 44–63.

Kiyotaki, N. and J. Moore (1997): "Credit cycles," *Journal of Political Economy*, 105, 211–248, publisher: The University of Chicago Press.

Kochen, F. (2023): "Finance Over the Life Cycle of Firms," *Working Paper.*

Leland, H. E. and K. B. Toft (1996): "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *The Journal of Finance*, 51, 987–1019.

Ma, L., D. Streitz, and F. Tourre (2023): "The art of timing: Managing sudden stop risk in corporate credit markets," *Available at SSRN 4602788.*

Martellini, L., V. Milhau, and A. Tarelli (2018): "Capital structure decisions and the optimal design of corporate market debt programs," *Journal of Corporate Finance*, 49, 141–167.

Miranda-Agrippino, S. and G. Ricco (2021): "The Transmission of Monetary Policy Shocks," *American Economic Journal: Macroeconomics*, 13, 74–107.

Ottonello, P. and T. Winberry (2020): "Financial Heterogeneity and the Investment Channel of Monetary Policy," *Econometrica*, 88, 2473–2502.

Poeschl, J. (2023): “Corporate debt maturity and investment over the business cycle,” *European Economic Review*, 152, 104348.

Salomao, J. and L. Varela (2022): “Exchange Rate Exposure and Firm Dynamics,” *The Review of Economic Studies*, 89, 481–514.

Tauchen, G. (1986): “Finite state markov-chain approximations to univariate and vector autoregressions,” *Economics Letters*, 20, 177–181.

Xie, Y. A., S. Liu, C. Wu, and B. Anderson (2009): “The effects of default and call risk on bond duration,” *Journal of Banking & Finance*, 33, 1700–1708.

Xu, Q. (2018): “Kicking Maturity Down the Road: Early Refinancing and Maturity Management in the Corporate Bond Market,” *The Review of Financial Studies*, 31, 3061–3097.

Appendix to "Debt Callability and Firm Dynamics"

Juste Djabakou
University of Montreal and CIREQ

September 26, 2025

A Data Construction

This section documents the construction of the firm–quarter panel data used in this paper. The approach follows standard practices in the literature with additional emphasis on callability measurements.

A.1 Sources and linkage

I combine transaction–level bond data from Mergent FISD with quarterly firm accounts from Compustat. From FISD, I reconstruct the complete history of the outstanding par value and all principal events for each CUSIP. Using the `ACTION_TYPE` tapes, I classify changes in principal into (i) scheduled amortizations and (ii) unscheduled prepayments—calls, tenders, and open-market repurchases—and record the associated "action price." This is the standard approach to building bond life-cycle histories from FISD.

Issuer identifiers are harmonized to `gvkey` using historical CUSIPs. The resulting issue–by–quarter panel preserves, for every bond, (i) outstanding amount, (ii) call status/type, and (iii) the timing of principal changes, in the spirit of recent work on refinancing and prepayment (see Ma et al., 2023). I then aggregate to the issuer–quarter level when constructing exposure, eligibility, and hazard measures used in the analysis.

A.2 Sample selection

The baseline sample consists of non-financial, publicly listed U.S. firms from January 1990 through May 2018.

Mergent FISD. I retain U.S. corporate bonds and exclude issues by utilities (SIC 4900–4999) and financial firms (SIC 6000–6999). FISD provides issuance date, original amount, callability, convertibility, covenants, and other bond-specific attributes used to build bond-level histories and eligibility measures.

Compustat. I extract quarterly balance-sheet and income-statement items to form standard firm-level controls. Firms are required to have total debt of at least 5% of assets at some point during the sample to ensure meaningful exposure to corporate credit. All continuous variables are winsorized at the 1st and 99th percentiles.

CRSP. I obtain stock prices and related equity variables from CRSP for ancillary controls (e.g., market capitalization).

Linkage. Datasets are merged using historical CUSIPs mapped to `gvkey`, following the standard procedure in Jungherr and Schott, 2021; Ma et al., 2023. The final dataset is an issue–quarter panel linked to issuer fundamentals, which I use to construct issuer–quarter measures of scheduled callability and call hazards for the empirical analysis.

A.3 Issuer - quarter panel

I expand each bond's timeline at a quarterly frequency from its offering quarter to the earlier of maturity (termination) or the first "end event" (call, exchange, default, etc.). For each issue-quarter (i, t) I define:

- Outstanding: if the bond is alive in quarter t.
- End events: indicators for contractual maturity and for called status (a call that retires principal before maturity).
- Beginning-of-quarter (BOQ) status: I compute BOQ "liveness" as lagged outstanding, which pins down life-cycle indicators and hazards defined at the start of t.

I build on the accounting technic in the construction of bond history in Ma et al. (2023). I use the bond's contractual call protection end date to determine whether the bond is callable by BOQ. For quarter t, a bond is "scheduled callable at BOQ" if (i) it was alive at $t-1$ and (ii) its protection expired on or before $t-1$. This object is the key building block for issuer-level callable shares below and is distinct from (and more policy-relevant than) "ever callable during t." I also flag the flow of amount that becomes callable in quarter t (combining protection expiration in t and outstanding at that time). This variable is predetermined at BOQ and underpins additional variables on the measurements that can be used as excluded instruments.

A.4 Identifying the end of call protection (scheduled vs. make-whole)

This paper requires a bond–quarter measure of when an issue becomes *effectively* callable at prespecified (par/premium) prices. I construct that timing from FISD's redemption data, combining the rich textual field `initial_call_data` with the dated make-whole fields (`make_whole_start_date`, `make_whole_end_date`, `make_whole_spread`) and the offering and maturity dates. I keep all callable bonds, including those with make-whole (MW) provisions. The primary identification defines eligibility using the *scheduled* (par/premium) window and then extends it to MW.

Parsing the schedule from text. I first convert `initial_call_data` to lowercase and parse all explicit dates with regular expressions that capture common patterns (e.g., *M/D/YY*, *MM/DD/YYYY*, compact *MMDDYYYY*). Two refinements are important in practice: (i) two–digit years are mapped using a pivot (YY$< 30 \Rightarrow$20YY, else 19YY), and (ii) strings of the form "NC k YEARS" are translated into an implied first scheduled call date equal to the offering date plus k years.[29]

The result of this pass is a daily variable `first_call_date`: the *first date of the scheduled call window at Par or predetermined premiums.* When multiple milestones appear (e.g., quarterly or annual schedules), I retain the earliest scheduled date.

How Make-Whole provisions are handled. Make-whole creates an additional, earlier window in which the issuer can call at a Treasury–based make-whole price. I explicitly identify MW using (i) the make-whole dated fields, (ii) the spread field, and (iii) textual hits such as "`cc @ make whole`" or "`make-whole`". From these, I build `mw_start` $= \max\{$offering, `make_whole_start_date`$\}$ and `mw_end` $= \min\{$maturity, `make_whole_end_date`$\}$, defaulting to `mw_start` = offering when the text indicates "currently callable (make-whole)" but no dates are provided, and `mw_end` = maturity if an end date is missing. I tag bonds as `mw_flag=1` when any MW signal is present; `mw_only=1` marks those with an MW window but *no* scheduled window.

Critically, `first_call_date` remains a *scheduled* concept, even for issues that also have a make-whole component. When the text explicitly transitions from MW to par (e.g., "`CC @ MAKE WHOLE; 07/15/2011 DISCRETE ANNUAL @ PRICES`"), `first_call_date` is the first scheduled date (here, 07/15/2011). When the transition is only implicit through `make_whole_end_date` followed by "to maturity @ par", I set `first_call_date` $= \min\{\, \max($`make_whole_end_date`$+ 1,$ offering$),$ maturity$\}$, which captures the first day of the *post–MW* par window and corrects occasional date typos.

Quarterly eligibility variables. The baseline exposure is the end of scheduled protection:

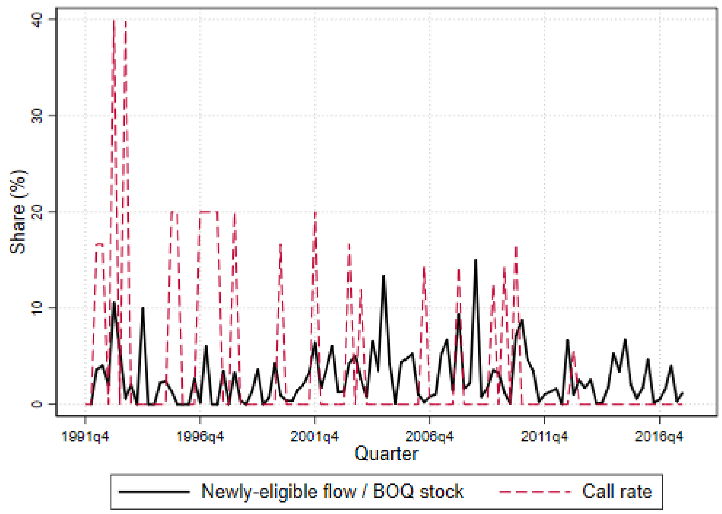
$$\texttt{q\_prot\_end}_i \equiv \text{quarter}(\texttt{first\_call\_date}), \quad \texttt{prot\_expired\_boq}_{it} = \mathbf{1}\{\, \texttt{q\_prot\_end}_i \leq t{-}1\,\}.$$

Intuitively, a bond is in the scheduled call risk set at the beginning of quarter t if its scheduled protection has expired by $t - 1$. These variables underpin the "scheduled callable share" used in the baseline rates and hazards. This callable share accounts for the dynamics of the newly callable bonds, as defined in equation (4), presented in Figure (A1). The time series illustrates the lumpy nature of the supply of newly eligible debt and the strength and frequency of actual calls, conditional on eligibility. Spikes in the "Newly-eligible flow / BOQ stock" reflect cohorts exiting call protection, whereas the "Call rate" —the hazard of being called among bonds already eligible at the beginning of the quarter—stays low and sporadic, especially after the late 1990s. This gap is consistent with substantial contractual callability in design but limited effective exercising of

[29]This step fills in cases where the text gives only a lockout length rather than a calendar date.

calls at any point in the period post-GFC.

Figure A1: New eligibility vs. conditional call rate



Notes: Both series are quarterly cross-sectional means across issuers. "Newly-eligible flow / BOQ stock" (black, solid) is the issuer-level ratio defined in equation (4), where the numerator is the amount whose scheduled call protection ends in quarter t (based on `q_prot_end`) and the denominator is the issuer's outstanding amount at the beginning of t. "Call rate" (red, dashed) is the issuer-level call hazard defined in equation (3), i.e., called amount over the beginning of quarter stock that was already past call protection (scheduled clauses). Make-whole bonds are included in the sample, but eligibility is defined by scheduled protection only.

Why the scheduled definition is my baseline. Economic callability relevant for refinancing at par emerges when the *scheduled* lockout expires, not merely when a costly make-whole option exists. Using `q_prot_end` therefore (i) yields a meaningful and low eligible stock at the beginning of the quarter, (ii) produces accurate conditional call hazards, and (iii) generates clear dynamic responses in investment.

A.5 Analog key variables construction

I present here the construction of the analogs to the key variables used in the empirical analysis. Throughout, let j index issuers and t quarters, and let $\mathcal{I}_{jt}$ denote the set of bonds that are outstanding at the *beginning* of quarter t (alive at the end of $t-1$). For bond $i \in \mathcal{I}_{jt}$, let a_i be its outstanding amount proxy, $\mathbb{1}\{\text{callable}_i = 1\}$ the contract flag for an issuer call option, and q_i^{prot} the quarter in which contractual call protection ends (identified as in Appendix A.4).

Count (number-weighted) analogs. The count versions replace amounts by indicators in both numerator and denominator.

Contractual (designation) share, count:

$$\mathrm{cc}_{jt}^{(N)} = \frac{\sum_{i\in\mathcal{I}_{jt}} \mathbb{1}\{\text{callable}_i = 1\}}{\sum_{i\in\mathcal{I}_{jt}} 1}. \tag{A1}$$

Effective (scheduled) share at BOQ, count:

$$\mathrm{sc}_{jt}^{(N)} = \frac{\sum_{i\in\mathcal{I}_{jt}} \mathbb{1}\{q_i^{\text{prot}} \le t-1\}}{\sum_{i\in\mathcal{I}_{jt}} 1}. \tag{A2}$$

Conditional call rate at BOQ, count:

$$\mathrm{cr}_{jt}^{(N)} = \frac{\sum_{i\in\mathcal{I}_{jt}} \mathbb{1}\{\text{bond } i \text{ is called in } t\}}{\sum_{i\in\mathcal{I}_{jt}} \mathbb{1}\{q_i^{\text{prot}} \le t-1\}\,\mathbb{1}\{\text{alive at BOQ}\}}. \tag{A3}$$

By construction, $\mathrm{cr}_{jt}^{(N)}$ is a BOQ call *hazard*; it is set to missing when the denominator is zero.

Amount at BOQ analogs. The BOQ amount versions use the BOQ amounts in both numerator and denominator.

Contractual (designation) share, amount at BOQ:

$$\mathrm{cc}_{jt}^{(A,\text{BOQ})} = \frac{\sum_{i\in\mathcal{I}_{jt}} a_i\,\mathbb{1}\{\text{callable}_i = 1\}}{\sum_{i\in\mathcal{I}_{jt}} a_i}. \tag{A4}$$

Effective (scheduled) share at BOQ, amount:

$$\mathrm{sc}_{jt}^{(A,\text{BOQ})} = \frac{\sum_{i\in\mathcal{I}_{jt}} a_i\,\mathbb{1}\{q_i^{\text{prot}} \le t-1\}}{\sum_{i\in\mathcal{I}_{jt}} a_i}. \tag{A5}$$

Conditional call rate at BOQ, amount:

$$\mathrm{cr}_{jt}^{(A,\text{BOQ})} = \frac{\sum_{i\in\mathcal{I}_{jt}} a_i\,\mathbb{1}\{\text{bond } i \text{ is called in } t\}}{\sum_{i\in\mathcal{I}_{jt}} a_i\,\mathbb{1}\{q_i^{\text{prot}} \le t-1\}\,\mathbb{1}\{\text{alive at BOQ}\}}. \tag{A6}$$

Newly eligible (flow) analogs. For completeness, I also use BOQ normalizations of the *flow* of issues that become eligible in t:

$$\mathrm{nc}_{jt}^{(A,\mathrm{BOQ})} = \frac{\sum_{i\in\mathcal{I}_{jt}} a_i \, \mathbb{1}\{q_i^{\mathrm{prot}} = t\}}{\sum_{i\in\mathcal{I}_{jt}} a_i}, \tag{A7}$$

with a count analog obtained by replacing a_i with 1.

Notes. (i) All denominators are BOQ stocks and include non-callable issues: callable bonds under protection are de facto equivalent to non-callable for redemption at t, and excluding non-callable would overstate exposure. (ii) By design, the BOQ variants (A4)–(A6) are the "unsmoothed" counterparts of the main-text amount measures, which use rolling averages of BOQ stocks to reduce noise; results are robust across definitions. (iii) MW issues contribute to eligibility when q_i^{prot} indicates the opening of the MW window (or issuance for CC@ MW).

A.6 Other variables and transformations

This appendix complements the callability objects defined in Appendix A.5 with the firm-, bond-, and macro-level variables used in the empirical analysis. Throughout, j indexes issuers and t calendar quarters; BOQ denotes "beginning of quarter." Unless stated otherwise, firm-level controls enter regressions lagged one quarter as $Z_{j,t-1}$.

Firm-level controls (Compustat, quarterly). Let Assets_{jt} denote real total assets (constant dollars), Debt_{jt} total debt, Cash_{jt} cash and short-term investments, Sales_{jt} net sales, and MVE_{jt} market capitalization. I define

$$\mathrm{size}_{jt} \equiv \log(\mathrm{Assets}_{jt}), \tag{A8}$$

$$\mathrm{levq}_{jt} \equiv \frac{\mathrm{Debt}_{jt}}{\mathrm{Assets}_{jt}}, \qquad \mathrm{cashq}_{jt} \equiv \frac{\mathrm{Cash}_{jt}}{\mathrm{Assets}_{jt}}, \tag{A9}$$

$$\mathrm{sale\_growth}_{jt} \equiv \frac{\mathrm{Sales}_{jt} - \mathrm{Sales}_{j,t-1}}{\mathrm{Sales}_{j,t-1}}, \tag{A10}$$

$$q_{jt} \equiv \frac{\mathrm{MVE}_{jt} + \mathrm{Debt}_{jt}}{\mathrm{Assets}_{jt}} \quad \text{(Tobin's } q\text{)}. \tag{A11}$$

I use the lagged vector $Z_{j,t-1} = \{\mathrm{size}_{j,t-1}, \mathrm{levq}_{j,t-1}, \mathrm{cashq}_{j,t-1}, q_{j,t-1}, \mathrm{sale\_growth}_{j,t-1}\}$, in all baseline specifications. To limit the influence of outliers, I winsorize each continuous control at the 1st/99th percentiles within the estimation sample.

Bond-level offering controls (FISD, issue quarter). For the offering-level regressions, I collect contemporaneous bond covariates in the vector

$$\mathbf{B}_{ijt} = \{\,\log(\text{offer}_{ijt}),\ \text{maturity}_{ijt},\ \text{coupon}_{ijt}\},$$

where $\log(\text{offer}_{ijt})$ is the log of the initial offering amount of issue i by issuer j in quarter t, and maturity_{ijt} and coupon_{ijt} are the original maturity (in years) and the coupon rate at issuance, respectively. The dependent variable in the issuance logit is the indicator $C_{ijt} \in \{0, 1\}$ equal to one if the issue is contractually callable at origination.

Macro controls. Quarterly real activity is summarized by $g_t \equiv \Delta \log(\text{RGDP}_t)$, and I include g_{t-1} as a lagged macro control in all regressions involving firm outcomes.

Other firm-level controls (FISD-based). I complement Compustat controls with two issuer-level measures constructed from FISD bonds:

- *Issuer rating (bond-based average).* For each issue, I map the letter-grade rating into a numeric scale where lower values denote weaker credit quality (e.g., $1 = \text{D}$, $6 = \text{CCC+}$ or worse, $\ldots$, $19 = \text{AA-}$ or better), following the harmonization used in bond markets. At the issuer-quarter level, I define

$$\text{rating}_{jt} \;\equiv\; \frac{\sum_{i\in\mathcal{I}_j(t)} w_{ijt}\,\text{rating}_{ijt}}{\sum_{i\in\mathcal{I}_j(t)} w_{ijt}}, \tag{A12}$$

  where $\mathcal{I}_j(t)$ is the set of the firm's rated bonds in quarter t and w_{ijt} are amount weights (issue amounts) unless otherwise noted. Thus, *smaller* rating_{jt} indicates *lower* credit quality. In the regressions I use $\text{rating}_{j,t-1}$.

- *Maturing debt share.* Let amt_{ij} denote the face amount of bond i of issuer j. I define the BOQ outstanding amount as $\text{Out}^{\text{BOQ}}_{jt} \equiv \sum_{i\in\mathcal{I}_j} \mathbf{1}\{i \text{ outstanding at BOQ}(t)\}\,\text{amt}_{ij}$. The share of debt maturing in quarter t is

$$\text{mat}_{jt} \;\equiv\; \frac{\sum_{i\in\mathcal{I}_j} \mathbf{1}\{i \text{ matures in } t\}\,\text{amt}_{ij}}{\text{Out}^{\text{BOQ}}_{jt}}, \tag{A13}$$

  with an analogous count-based version replacing amounts by issue counts. This variable is used as a predetermined control capturing rollover pressure at t. This variable is the central variable in Jungherr et al. (2024).

B Additional Empirical Results

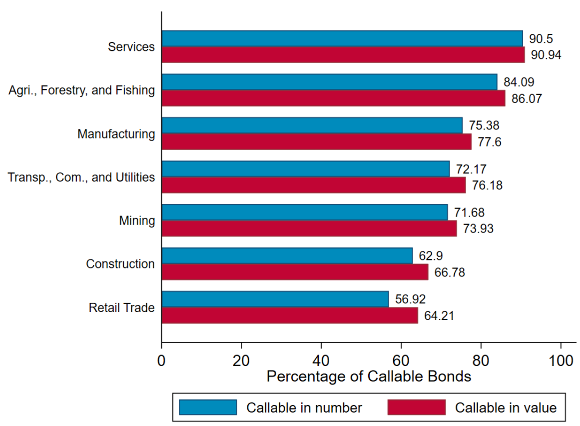
This subsection includes additional empirical exercises that support differences brought by callable bond issuance.

B.1 Callable Bond prevalence across Industry

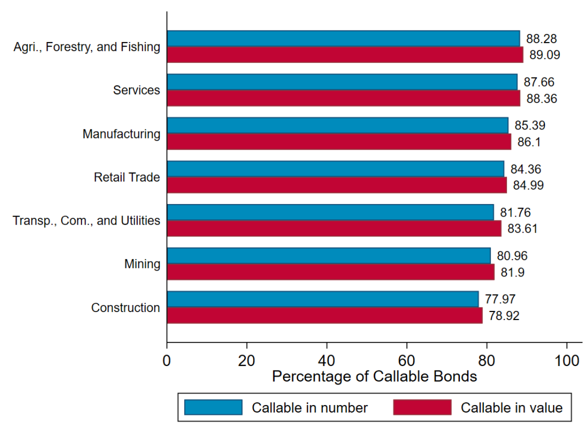
Using SIC codes, I categorized bonds into different industries and calculated the number of callable and total bonds issued by each industry. I also computed the percentage of callable bonds in terms of both the number of bonds issued and the total value of bonds issued for each industry. Industries like services, agriculture, forestry, fishing, and manufacturing tend to have higher shares of callable debts. These industries often experience significant fluctuations in cash flows due to seasonal demand, market conditions, debt overhangs, and other external factors. These are relevant reasons for the high callable share for small -high risky- firms.

Figure A1: Callable bonds by industry

(a) At the issuance



(b) Outstanding bonds



Notes: I calculate the share of callable bonds for both the issuance case and the outstanding bonds case. Data are from Mergent FISD & CRSP-Compustat.

B.2 Bonds Characteristics

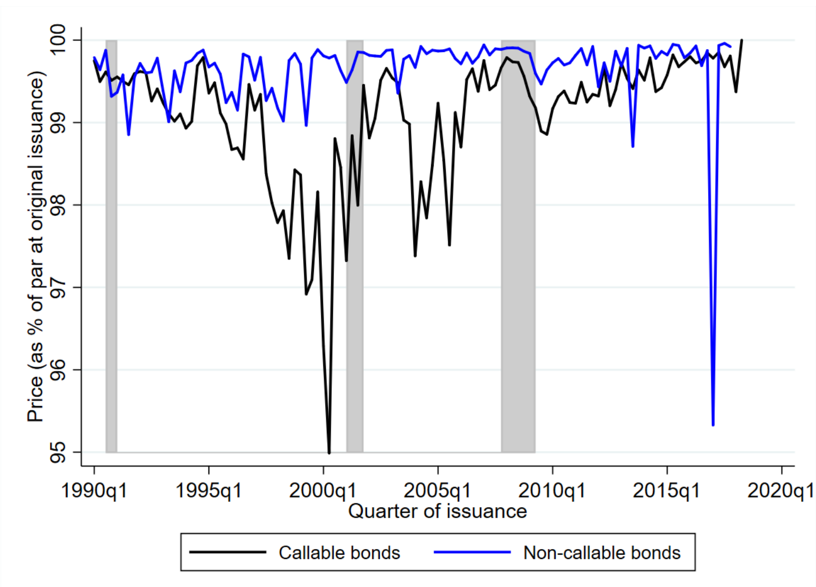
Callable bonds are generally issued at lower prices compared to non-callable bonds. Their coupon rate is, on average, higher because it makes them more attractive and justifies the risk of reinvesting for the bondholders. I observe the following trends in the data, indicating that callable bond issuance is associated with specific bond characteristics.

Maturity: Callable bonds exhibit a longer (statistically) average maturity (13 years) compared to non-callable bonds (12 years). This could be because issuing firms prefer to have the option to call back bonds when interest rates change or their credit quality improves, which is more likely to happen over a longer period.

Issue Size: Callable bonds have larger average issue sizes (49.11 $ mn) than non-callable bonds (24.77 $ mn). This could suggest that firms issuing callable bonds require more significant financing, and having the option to call back bonds provides them greater financial flexibility.

Coupon Rate: Callable bonds have higher average coupon rates compared to non-callable bonds. This trend can be explained by the additional risk investors take when investing in callable bonds. Since investors face the risk of early redemption and the potential loss of future interest payments, they require higher coupon rates as compensation for this risk.

Figure A2: Bonds prices over time



Notes: This figure plots the average bond price by firms for each type of bond (callable bonds in black). The sample period is 1990 Jan - 2018 May.
Source: Mergent FISD.

Offering Price: On average, the offering price of callable bonds is 98.20, while for non-callable bonds, it is 99.56. This difference in offering prices can be attributed to the fact that callable bonds come with a call option, which gives the issuer the right to redeem the bond before its maturity. Recognizing this additional risk, investors may require a lower offering price for callable bonds as compensation for the potential loss of future interest payments if the bond is called. This results in a lower average offering price for callable bonds than non-callable bonds, as shown in Figure A2. Becker et al. (2024) found that the difference between the callable bonds' average yield and the non-callable bonds' one, at issuance, is 2.67%.

B.3 Additional results on callability

B.3.1 Issuance of callable vs non-callable

I estimate five pooled logit models at the offering level where the dependent variable equals one if the initial issue in quarter t is contractually callable. Across specifications, worse credit quality (higher numerical rating) significantly raises the likelihood of issuing a callable bond, whereas firm size is imprecisely estimated; the monetary-policy shock and lagged GDP growth are not significant at conventional levels. All models include bond controls (log offering amount plus the standard issuance-side covariates), lagged firm controls, and quarter fixed effects; standard errors are clustered by issuer (gvkey). These results mirror the descriptive patterns in Section 2.1.5 and serve as supporting evidence rather than as a core contribution.

Table A1: DETERMINANTS OF CALLABLE/NON-CALLABLE ISSUANCE

	(1)	(2)	(3)	(4)	(5)
size_{t-1}	-0.596**		-0.382	-0.382	-0.382
	(0.273)		(0.263)	(0.263)	(0.263)
rating_{t-1}		-0.279***	-0.194**	-0.194**	-0.194**
		(0.102)	(0.092)	(0.092)	(0.092)
m.p. shock (ε_{t-1})	1.065	0.729		-6.437	0.764
	(1.267)	(1.337)		(13.828)	(1.319)
ΔGDP_{t-1}	1.190	1.821	1.855		2.075
	(3.423)	(3.562)	(3.985)		(3.670)
Observations	1,621	1,621	1,621	1,621	1,621
Pseudo R^2	0.139	0.140	0.149	0.149	0.149
Firm controls	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes
Firms clustering	Yes	Yes	Yes	Yes	Yes

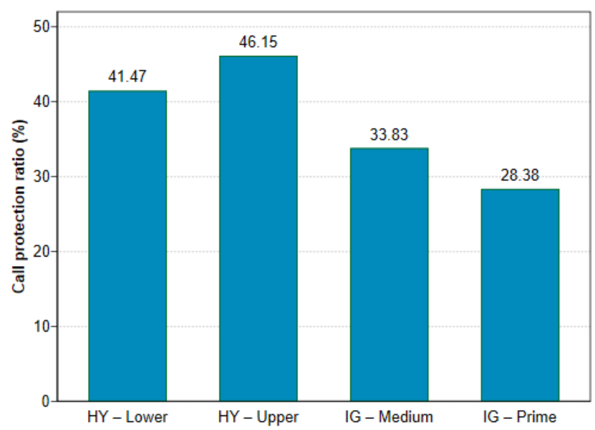
Notes: Columns (1)–(5) report pooled logit estimates of the probability that the initial offering of the bond i in quarter t by the firm j is contractually callable. The estimated specification is:
$\Pr(C_{ijt} = 1) = \Lambda\Big(\alpha_j + \delta_t + \beta_1\,\text{Size}_{j,t-1} + \beta_2\,\text{Rating}_{j,t-1} + \beta_3\,\varepsilon^{\text{MP}}_{t-1} + \beta_4\,g_{t-1} + \gamma'\mathbf{B}_{ijt} + \theta'\mathbf{F}_{j,t-1}\Big)$, where $\Lambda(\cdot)$ is the logistic link, α_j is an issuer fixed effect absorbed by clustering and quarter FE, δ_t are calendar-quarter fixed effects, $\varepsilon^{\text{MP}}_{t-1}$ is the standardized monetary-policy shock (signed so that positive denotes an easing), g_{t-1} is lagged GDP growth, $\mathbf{B}_{ijt}$ collects bond-level offering controls (including log offering amount), and $\mathbf{F}_{j,t-1}$ collects lagged firm controls containing leverage, sales growth, liquidity. Robust standard errors (in parentheses) are clustered by issuer (gvkey). Asterisks denote significance at the 10% (*), 5% (**), and 1% (***) levels.

B.3.2 Call protection and call decision

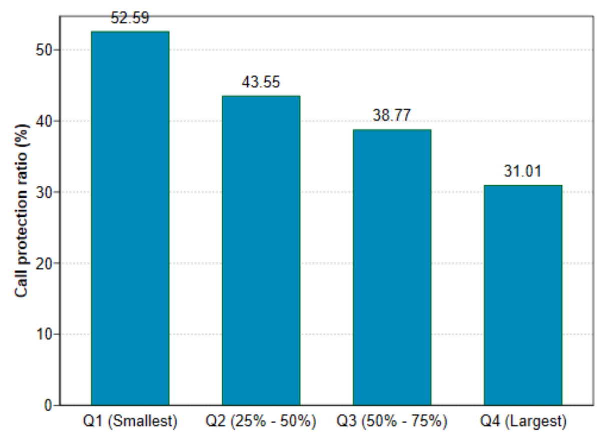
This section adds empirical evidence about the average timing of the call option exercise. The figure shows that callable bonds are typically called on average at the middle of their initial maturity.

Figure A3: Call protection period across firms groups

(a) Rating groups

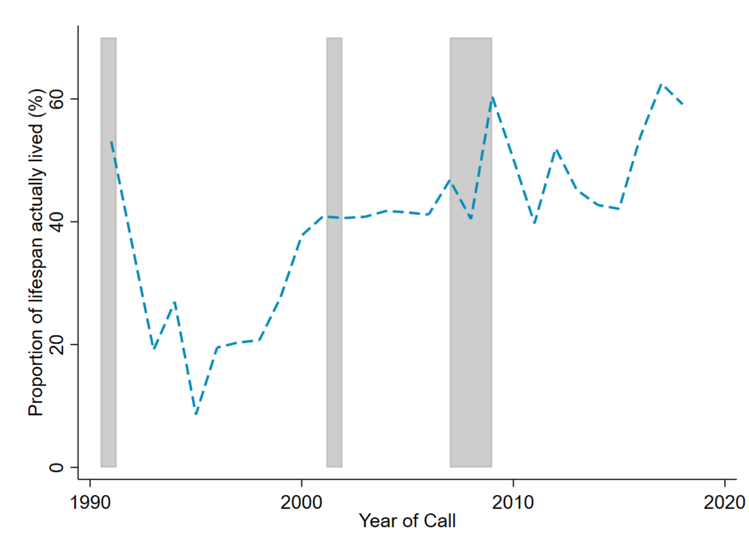


(b) Size groups



Notes: These plots show the average (callable) bonds protection period in every class of firms. The left panel reports the call protection period across rating classes, and the right panel shows those periods for the size (log of real total assets) categories.

Figure A4: After which percentage of their lives are bonds called?



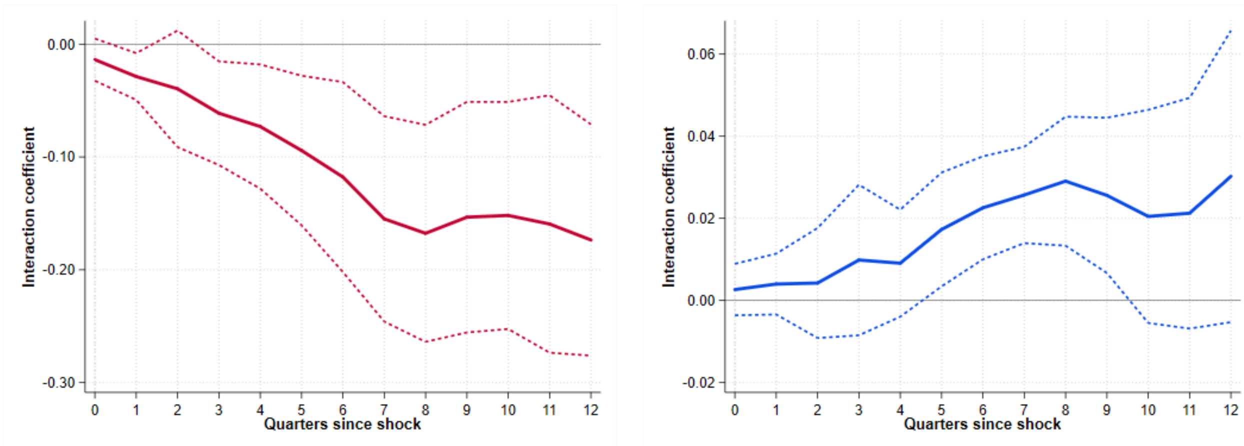
Notes: I identify what percentage of the initial maturity of the debt has elapsed between the issuance and the redemption of the debt. This percentage is computed as the annual average over our sample, with the call option exercise happening from 1991 to 2018, in our data. Data are from Mergent FISD.

B.4 Alternative regressors

This subsection presents robust results on the differential responses to monetary policy shocks, using three regressors in the baseline specification (6).

Figure A5: Differential investment response - alternative callable share denominators

(a) When firms call (b) When firms do not call



Notes: These plots report in the solid lines, the estimated coefficients β_1^h (in red on the left panels) and β_0^h (in blue on the right panels) over quarters h from the baseline specification $\Delta^{h+1} \log k_{jt+h} = \alpha_j^h + \alpha_{st}^h + \beta_0^h \, \widehat{sc}_{jt}\varepsilon_t^{\text{mp}} + \beta_1^h \, x_{jt}\widehat{sc}_{jt}\varepsilon_t^{\text{mp}} + \Gamma^h Z_{jt-1} + u_{jt}^h$, where all variables are already defined above and Z_{jt-1} contains all other variables in the specification (6). Here, the denominator of the variable $\widehat{sc}_{jt}$ is the beginning-of-quarter outstanding debt, as defined in equation (A5). Dashed lines report 95% confidence bands of the two-way clustered standard errors.

B.5 Instrumenting callability exposure and call exercise

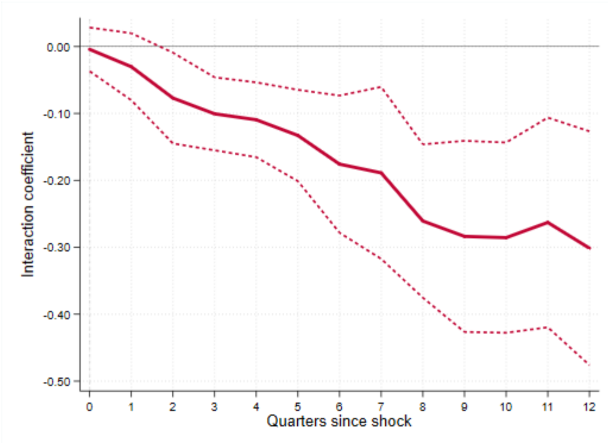
The regressors $\widehat{sc}_{jt}\varepsilon_t^{\text{mp}}$ and $x_{jt}\widehat{sc}_{jt}\varepsilon_t^{\text{mp}}$ capture how (i) the stock of debt that is legally callable at the beginning of the quarter and (ii) the firm's contemporaneous exercise of calls shape the transmission of a monetary policy shock. While $\widehat{sc}_{jt}$ is predetermined by indenture schedules set at issuance (see eqs. (2)–(3)), the call decision x_{jt} is taken within quarter t and can comove with unobserved investment shocks. To mitigate simultaneity and measurement concerns, I instrument the interaction terms using contract–driven variation in scheduled eligibility.

The eligibility flow (contractual "newly callable"). $\text{IV}_{jt-1}^{(flow)} \equiv nc_{jt-1}/\text{debt}_{jt-1}$, where nc_{jt-1} is defined in equation (4). This instrument is predetermined by indenture calendars and independent of the quarter-t investment shock; strong relevance for both $\widehat{sc}_{jt}$ and $x_{jt}\widehat{sc}_{jt}$. I also consider a peer instrument.

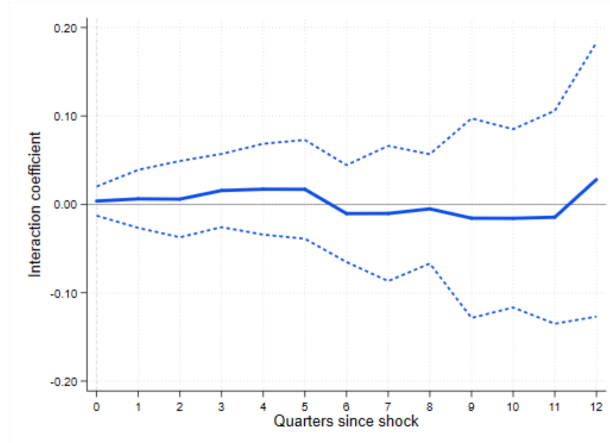
Leave-one-out peer eligibility. $\text{IV}_{jt-1}^{(peer)}$, the industry×quarter average of scheduled eligibility excluding firm j. Captures exogenous clustering in protection expires induced by vintage waves and contract norms, strengthening first-stage variation for $x_{jt}\widehat{sc}_{jt}$. The heterogeneous responses are consistent with our main results.

Figure A6: DIFFERENTIAL INVESTMENT RESPONSE - ALTERNATIVE AGGREGATION OF THE CALLABLE SHARE

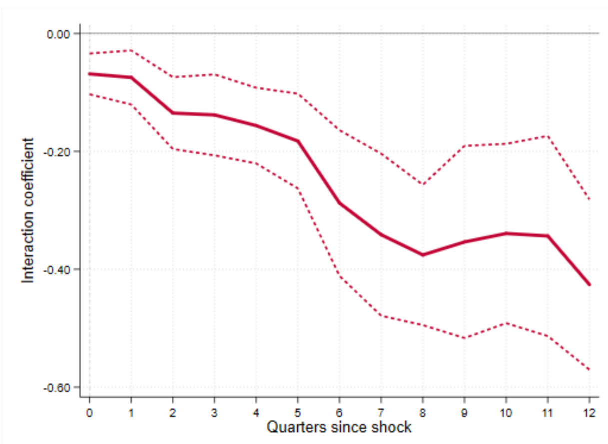
(a) When firms call



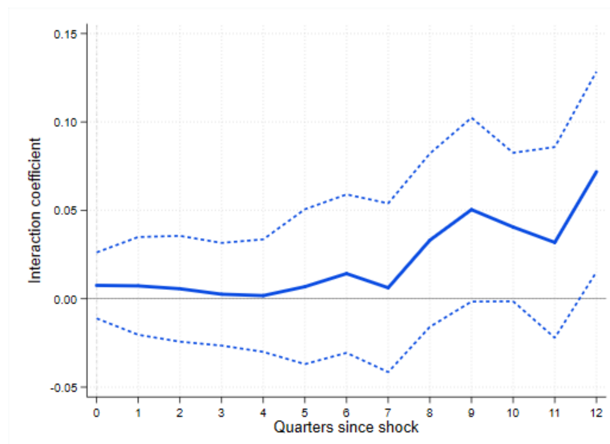
(b) When firms do not call



(c) When firms call



(d) When firms do not call



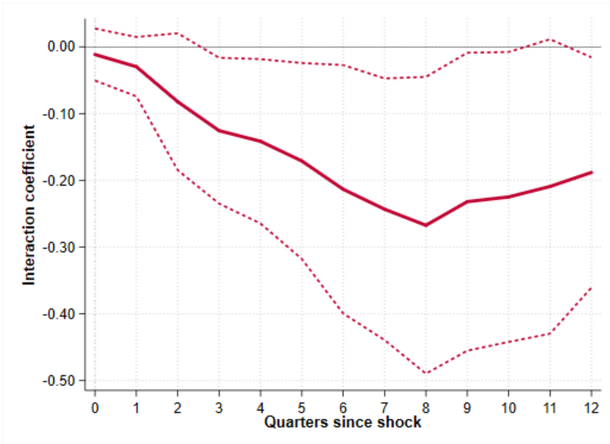
Notes: These plots report in the solid lines, the estimated coefficients β_1^h (in red on the left panels) and β_0^h (in blue on the right panels) over quarters h from the baseline specification $\Delta^{h+1} \log k_{jt+h} = \alpha_j^h + \alpha_{st}^h + \beta_0^h \, \widehat{sc}_{jt}\varepsilon_t^{\text{mp}} + \beta_1^h \, x_{jt}\widehat{sc}_{jt}\varepsilon_t^{\text{mp}} + \Gamma^h Z_{jt-1} + u_{jt}^h$, where all variables are already defined above and Z_{jt-1} contains all other variables in the specification (6). The aggregation method to build the key variable $\widehat{sc}_{jt}$ (and also the maturing share variable in the control variables) is based on the use of count-weighted indicators; see definitions in equations (A2) - (A3). Dashed lines report 95% confidence bands of the two-way clustered standard errors.

C Three-period Model Appendix

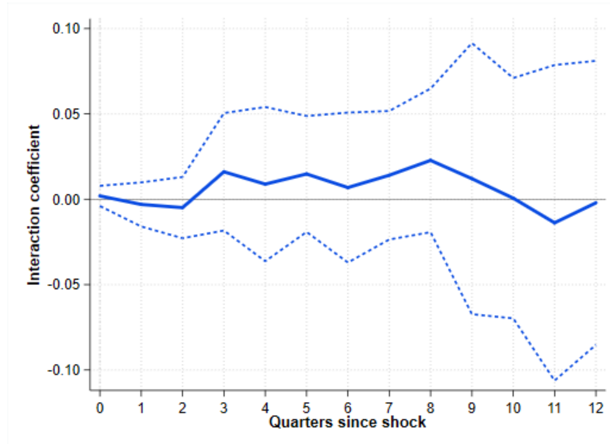
This subsection provides details on theoretical derivations and proofs of propositions, corollaries, and other optimal characteristics of the three-period model presented in the section (3).

Figure A7: DIFFERENTIAL INVESTMENT RESPONSE - CALLABILITY INSTRUMENTED

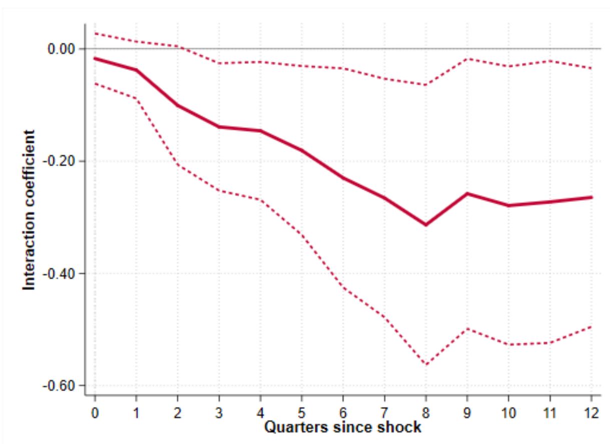
(a) When firms call



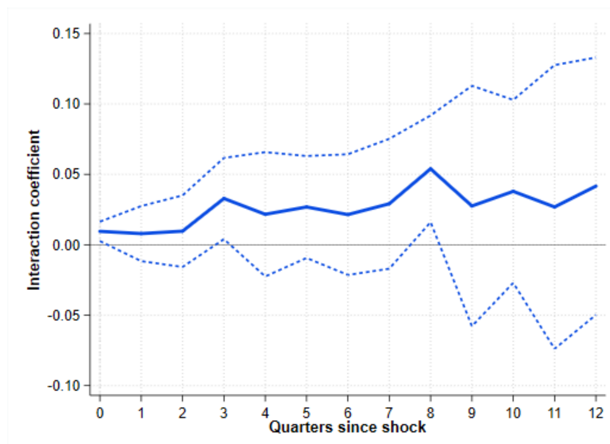
(b) When firms do not call



(c) When firms call



(d) When firms do not call



Notes: These plots report in the solid lines, the 2SLS estimated coefficients β_1^h (red, left panels) and β_0^h (blue, right panels) over quarters h from the specification $\Delta^{h+1} \log k_{jt+h} = \alpha_j^h + \alpha_{st}^h + \beta_0^h \, \widehat{sc}_{jt}\varepsilon_t^{\text{mp}} + \beta_1^h \, x_{jt}\widehat{sc}_{jt}\varepsilon_t^{\text{mp}} + \Gamma^h Z_{jt-1} + u_{jt}^h$, where all variables are already defined above and Z_{jt-1} contains all other variables in the specification (6). The call decision in the presence of monetary policy shocks, $x_{jt}\widehat{sc}_{jt}\varepsilon_t^{\text{mp}}$ is instrumented by the flow instrument defined in equation (4) and the share of leave-one-out peer schedule bond. The panels (a) and (b) report the results using amount-weighted variables, and the panels (c) and (d) show the results using count-weighted variables. Dashed lines report 95% confidence bands of the two-way clustered standard errors.

C.1 Proof of the refinancing decision

In this proof of (22) and (23), I provide the details of the optimal decision on refinancing in period $t = 1$ after respectively, not calling and calling the callable debt $(1 - \theta_1)b_1$. The new debt chosen is denoted by b_2. The first order condition with respect to the debt issued, respectively $b_2 - b_1$ and $b_2 - \theta_1 b_1$, is described in the following lines.

For the no-call scenario, we have:

$$[b_2 - b_1] : p_1 + \frac{\partial p_1}{\partial(b_2 - b_1)}(b_2 - b_1) + \beta_1 \frac{\partial}{\partial(b_2 - b_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{nocall}}^{\infty} V_2^{nocall}(s_2, x_2)\varphi(\varepsilon_2) d\varepsilon_2 \quad \leq 0 \tag{A1}$$

Using the definition of the price of the new debt in (15) and its components in (16), I have the derivatives of the price and the expected future value of the firm with respect to the default risk:

$$\frac{\partial p_1}{\partial \bar{\varepsilon}_2} = \theta_2^{new} \frac{\partial p_1^{nc}}{\partial \bar{\varepsilon}_2} + (1 - \theta_2^{new}) \frac{\partial p_1^c}{\partial \bar{\varepsilon}_2} = -\mathbb{E}_{z_2|z_1} \left[\varphi(\bar{\varepsilon}_2)\right] \tag{A2}$$

$$\frac{\partial}{\partial \bar{\varepsilon}_2} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} V_2(s_2, x_2)\varphi(\varepsilon_2) d\varepsilon_2 = -k_2 \mathbb{E}_{z_2|z_1} \left[1 - \Phi(\bar{\varepsilon}_2)\right] \tag{A3}$$

A high level of of next period debt b_2 increases the default risk, shown by the following:

$$\frac{\partial \bar{\varepsilon}_2}{\partial(b_2 - b_1)} = \frac{1 + c_2}{k_2} \tag{A4}$$

When combining (A2), (A3), and (A4), I have the variations in the new debt, of the price of the new debt and the expected future value of the firm:

$$\frac{\partial p_1}{\partial(b_2 - b_1)} = -\frac{1 + c_2}{k_2} \mathbb{E}_{z_2|z_1} \left[\varphi(\bar{\varepsilon}_2)\right] \tag{A5}$$

$$\frac{\partial}{\partial(b_2 - b_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} V_2(s_2, x_2)\varphi(\varepsilon_2) d\varepsilon_2 = -(1 + c_2) \mathbb{E}_{z_2|z_1} \left[1 - \Phi(\bar{\varepsilon}_2)\right] \tag{A6}$$

I rewrite the FOC of the new debt $b_2 - b_1$:

$$[b_2 - b_1] : -(1 + c_2) \frac{b_2 - b_1}{k_2} \mathbb{E}_{z_2|z_1} \left(\varphi(\bar{\varepsilon}_2)\right) + \left(1 - \beta_1(1 + c_2)\right) \mathbb{E}_{z_2|z_1} \left[1 - \Phi\left(\bar{\varepsilon}_2\right)\right] = 0 \tag{A7}$$

I can deduce the form shown in (22) by using the composition of the average coupon defined in (24).

Concerning the call scenario, the method is the same, except for the debt b_1 which is replaced by $\theta_1 b_1$ in the outstanding debt. I write then the FOC of the new debt $b_2 - \theta_1 b_1$:

$$[b_2 - \theta_1 b_1] : -(1 + c_2) \frac{b_2 - \theta_1 b_1}{k_2} \mathbb{E}_{z_2|z_1} \left(\varphi(\bar{\varepsilon}_2)\right) + \left(1 - \beta_1(1 + c_2)\right) \mathbb{E}_{z_2|z_1} \left[1 - \Phi\left(\bar{\varepsilon}_2\right)\right] = 0 \tag{A8}$$

In turn, I consider the composition of the average coupon when the firm exercises its call option, to obtain finally the optimal refinancing in the call scenario in (23).

■

C.2 Optimal values when $(1-\theta_1)b_1$ is not called in $t=1$

The default threshold $\overline{\varepsilon}_2^{nocall}$ in $t=2$, when the callable bond is not called in $t=1$, is defined as follows:

$$\begin{aligned}\overline{\varepsilon}_2^{nocall} &: 0 = V_2^{nocall}(s_2, x_2)\\ &: 0 = z_2(k_2)^{\alpha} + \varepsilon_2 k_2 - (1+c_2)b_2\end{aligned}$$

I derive the threshold $\overline{\varepsilon}_2^{nocall}$:

$$\overline{\varepsilon}_2^{nocall} = -\frac{k_2 + y_2 - (1+c_2)b_2}{k_2} \tag{A9}$$

I reformulate the optimal value of the firm (in (15)) when it doesn't call its callable bond:

$$V_1^{nocall}(s_1, x_1) = -c_1 b_1 - k_2 + p_1(b_2 - b_1) + \beta_1 \cdot k_2 \cdot \mathbb{E}_{z_2|z_1} \int_{\overline{\varepsilon}_2}^{\infty} (\varepsilon_2 - \overline{\varepsilon}_2)\varphi(\varepsilon_2) d\varepsilon_2 \tag{A10}$$

The variation in the value function above in (A10), induced by changes in the default threshold $\overline{\varepsilon}_2^{nocall}$ is defined as follows:

$$\Delta\overline{\varepsilon}_2^{nocall} = -(b_2 - b_1)(1+c_2)\mathbb{E}_{z_2|z_1}\left[\varphi\left(\overline{\varepsilon}_2\right)\right] - \beta_1 \cdot k_2 \cdot \mathbb{E}_{z_2|z_1}\Big[1 - \Phi(\overline{\varepsilon}_2)\Big] \tag{A11}$$

The first-order condition with respect to k_2 in the period $t=1$ is explained in:

$$-1 + \frac{\partial \overline{\varepsilon}_2}{\partial k_2} \cdot \Delta\overline{\varepsilon}_2^{nocall} + \beta_1 \cdot \mathbb{E}_{z_2|z_1} \int_{\overline{\varepsilon}_2}^{\infty} \Big(\varepsilon_2 - \overline{\varepsilon}_2\Big)\varphi(\varepsilon_2) d\varepsilon_2 = 0 \tag{A12}$$

with

$$\frac{\partial \overline{\varepsilon}_2}{\partial k_2} = -\frac{1 + mpk_2 \ + \ \overline{\varepsilon}_2}{k_2} = \frac{(1-\alpha)y_2 - (1+c_2)b_2}{k_2^2} \tag{A13}$$

Where $mpk_2 = \alpha z_2(k_2)^{\alpha-1}$ is the marginal productivity of the capital k_2 in the period $t=2$. I use the optimal condition in (A12) multiplied by k_2 to write the optimal value of the function in (A10):

$$V_1^{nocall}(s_1, x_1) = -c_1 b_1 + p_1(b_2 - b_1) - \frac{\partial \overline{\varepsilon}}{\partial k_2} \cdot \Delta\overline{\varepsilon}_2^{nocall} \cdot k_2 \tag{A14}$$

Now, I replace $\dfrac{\partial \overline{\varepsilon}_2}{\partial k_2}$ and $\Delta\overline{\varepsilon}_2^{nocall}$ by their respective expressions in (A13) and (A11), and I use the optimal refinancing formula in (22). I obtain the expression of the optimal value of the firm used in the proposition (2):

$$V_1^{nocall}(s_1, x_1) \;=\; -\,c_1 b_1 + \mathbb{E}_{z_2|z_1}\left\{\left[-b_1 + \frac{(1-\alpha)y_2}{1+c_2}\right] \times \left[1 - \Phi(\bar{\varepsilon}_2^{nocall})\right]\right\} \tag{A15}$$

■

C.3 Optimal values when $(1-\theta_1)b_1$ is called in $t = 1$

When the firm calls its the callable bond in $t = 1$, the default threshold $\bar{\varepsilon}_2^{call}$ in $t = 2$, is obtained as follows:

$$\begin{aligned}\bar{\varepsilon}_2^{call} &: 0 = V_2^{call}(s_2, x_2)\\ &: 0 = z_2(k_2)^{\alpha} + \varepsilon_2 k_2 - (1+c_2)b_2\end{aligned}$$

I derive the threshold $\bar{\varepsilon}_2^{call}$:

$$\bar{\varepsilon}_2^{call} \;=\; -\,\frac{k_2 + y_2 - (1+c_2)b_2}{k_2} \tag{A16}$$

I reformulate the optimal value of the firm (in (17)) when it doesn't call its callable bond:

$$\begin{aligned}V_1^{call}(s_1, x_1) = -(1+\chi)(1-\theta_1)b_1 - c_1 b_1 - k_2^{call} + p_1(b_2 - \theta_1 b_1)\\ + \beta_1 \cdot k_2^{call} \cdot \mathbb{E}_{z_2|z_1}\int_{\bar{\varepsilon}_2^{call}}^{\infty}(\varepsilon_2 - \bar{\varepsilon}_2^{call})\varphi(\varepsilon_2)d\varepsilon_2\end{aligned} \tag{A17}$$

I derive here the variation in the value function above in (A17), induced by changes in the default threshold $\bar{\varepsilon}_2^{call}$ defined as:

$$\Delta\bar{\varepsilon}_2^{call} = -(b_2 - \theta_1 b_1)(1+c_2)\mathbb{E}_{z_2|z_1}\left[\varphi\left(\bar{\varepsilon}_2^{call}\right)\right] - \beta_1 \cdot k_2^{call} \cdot \mathbb{E}_{z_2|z_1}\left[1 - \Phi(\bar{\varepsilon}_2^{call})\right] \tag{A18}$$

The first-order condition with respect to k_2 in the period $t = 1$ is shown in:

$$-1 + \frac{\partial\bar{\varepsilon}_2}{\partial k_2} \cdot \Delta\bar{\varepsilon}_2^{call} + \beta_1 \cdot \mathbb{E}_{z_2|z_1}\int_{\bar{\varepsilon}_2}^{\infty}\Big(\varepsilon_2 - \bar{\varepsilon}_2\Big)\varphi(\varepsilon_2)d\varepsilon_2 = 0 \tag{A19}$$

with

$$\frac{\partial\bar{\varepsilon}_2}{\partial k_2} \;=\; -\,\frac{1 + mpk_2 \;+\; \bar{\varepsilon}_2}{k_2} \;=\; \frac{(1-\alpha)y_2 - (1+c_2)b_2}{k_2^2} \tag{A20}$$

I multiply by k_2 the optimal condition in (A19) to write the optimal value of the function in (A17):

$$V_1^{call}(s_1, x_1) = -(1+\chi)(1-\theta_1)b_1 - c_1 b_1 + p_1(b_2 - \theta_1 b_1) - \frac{\partial\bar{\varepsilon}_2}{\partial k_2} \cdot \Delta\bar{\varepsilon}_2^{call} \cdot k_2 \tag{A21}$$

Now, I replace $\dfrac{\partial \bar{\varepsilon}_2}{\partial k_2}$ and $\Delta \bar{\varepsilon}_2^{call}$ by their respective expressions in (A20) and (A18), and I consider the final expression of the optimal refinancing in (23). I get the expression of the optimal value of the firm used in the proposition 2:

$$V_1^{call}(s_1, x_1) = -(1+\chi)(1-\theta_1)b_1 - c_1 b_1 + \mathbb{E}_{z_2|z_1}\left\{\left[-\theta_1 b_1 + \frac{(1-\alpha)y_2}{1+c_2}\right] \times \left[1 - \Phi(\bar{\varepsilon}_2^{call})\right]\right\} \tag{A22}$$

■

C.4 Proof of Proposition 1

Proof. First, let's recall the expression of the optimal refinancing in the no-call scenario.

$$[b_2 - b_1] : -(1+c_2)\frac{b_2 - b_1}{k_2}\mathbb{E}_{z_2|z_1}\left(\varphi(\bar{\varepsilon}_2)\right) - \beta_1(c_1 - r_1)\frac{b_1}{b_2}\mathbb{E}_{z_2|z_1}\left[1 - \Phi\left(\bar{\varepsilon}_2\right)\right] = 0 \tag{A23}$$

θ_1 does not affect the optimal refinancing when the callable part is not called. This is due to the same impact of θ_1 in the second fraction in front of the expectation term.

To complete the proof, I discuss the relative levels of the coupon rate c_1 on existing debt and the new interest rate r_1.

If $c_1 < r_1$, refinancing is not so beneficial for the firm, whatever its level of debt (same for calling). A high interest rate of r_1 is not ideal for refinancing in the no-call scenario. But if the firm calls its debt despite the interest rate level, the share θ_1 of non-callable will reduce the debt cost since it carries smaller interest expenses. So firms with high θ_1 are more likely to refinance after calling when $c_1 < r_1$ through the reduction in the cost of their new total debt. θ_1 is important for cheaper refinancing when a bond is called in a $c_1 < r_1$ period.

If $c_1 > r_1$, it is the right time to refinance (eventually to call), and the refinancing cost can be reduced. Moreover, firms with small θ_1 will be more encouraged to refinance because more weight is put on the interest rate (which is smaller) for future debt costs. Even if refinancing is more supportable in the future when the new interest rate is smaller, firms with high θ_1 are stuck with the relatively high-interest payment, discouraging the refinancing. θ_1 is then detrimental for lower-cost refinancing in the call scenario in a $c_1 > r_1$ period. ■

C.5 Proof of Corollary 3.0.1

Proof. I recall the definition of the firm-specific periodic coupon in (24) after the call decision. The change in this coupon concerning the change in the interest rate is given by:

$$\frac{\partial c_2}{\partial r_1} = \begin{cases} 1 - \dfrac{\theta_1 b_1}{b_2} & \text{when no call} \\ 1 - \dfrac{\theta_1 b_1}{b_2} & \text{when call} \end{cases} \tag{A24}$$

If $c_1 < r_1$, when the firm does not call, having existing debt b_1 will attenuate the rise in the cost of capital, whatever the stock of (non)callable bond. When the firm calls, having high θ_1 will serve to drag down the rising cost of capital c_2. So firms with a high θ_1 face a relatively low new cost of capital in c_2 than firms with low θ_1. Then, their investment decreases less when they call. This is because they have a high share of existing callable bonds, so they will face more fully the new rate r_1 after calling it.

Suppose $c_1 > r_1$, when the firm decides not to call its callable debt, having a small stock of debt b_1 contributes to the reduction of the new cost of capital c_2. This is independent of the share of callable in this existing debt. In the case of calling, the part that is not called $\theta_1 b_1$ has to pay the differential interest $c_1 - r_1$. So firms with a high share of callable bonds (which means low θ_1) increase the optimal capital more when they call than firms with lower callable shares.

This completes the proof of the corollary. ■

C.6 Proof of Proposition 2

Proof. To prove Proposition 2, I first recall the price formulas of new bonds in period $t = 1$:

$$p_1 = \begin{cases} \mathbb{E}_{z_2|z_1}\left[1 - \Phi(\bar{\varepsilon}_2^{nocall})\right] & \text{when no call} \\ \mathbb{E}_{z_2|z_1}\left[1 - \Phi(\bar{\varepsilon}_2^{call})\right] & \text{when call} \end{cases} \tag{A25}$$

I consider the inequality with the transformed formulations of the firm's optimal value in the cases of not calling ((A15)) and calling((A22)), to establish the inequality:

$$\cancel{-c_1 b_1} + \mathbb{E}_{z_2|z_1}\left\{\left(-b_1 + \frac{(1-\alpha)y_2^{nocall}}{1+c_2}\right)\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right]\right\}$$
$$< -(1+\chi)(1-\theta_1)b_1 \cancel{-c_1 b_1} + \mathbb{E}_{z_2|z_1}\left\{\left(-\theta_1 b_1 + \frac{(1-\alpha)y_2^{call}}{1+c_2}\right)\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\right\} \tag{A26}$$

To complete the proof, we make the term $-c_1 b_1$ disappear at both sides of the inequality. ■

C.7 Proof of Proposition 3

Proof. The proposition concerns the possibility of crossing for the firm's value in both cases of not calling and calling. I prove the proposition through four points. It is important to note that the signs of both values do not matter because they are monotone and continue functions.

1. Based on the form of the firm's value in the no-call scenario, I have:

$$\frac{\partial \overline{V}_1^{nocall}(s_1, x_1)}{\partial \theta_1} = 0. \tag{A27}$$

2. Now, I derive the change in the firm's value in both scenarios, following a change in interest rate r_1.

$$\begin{aligned}
\frac{\partial \overline{V}_1^{nocall}(s_1, x_1)}{\partial r_1} &= \frac{\partial}{\partial r_1}\mathbb{E}_{z_2|z_1}\left\{\left(-b_1 + \frac{(1-\alpha)y_2^{nocall}}{1+c_2}\right)\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right]\right\} \\
&= \mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right](1-\alpha)y_2^{nocall}\frac{\partial}{\partial r_1}\left(\frac{1}{1+c_2}\right)\right\} \\
&= -\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right]\frac{(1-\alpha)y_2^{nocall}}{(1+c_2)^2}\frac{b_2-b_1}{b_2}\right\} < 0
\end{aligned} \tag{A28}$$

$$\begin{aligned}
\frac{\partial \overline{V}_1^{call}(s_1, x_1)}{\partial r_1} &= \frac{\partial}{\partial r_1}\mathbb{E}_{z_2|z_1}\left\{\left(-\theta_1 b_1 + \frac{(1-\alpha)y_2^{call}}{1+c_2}\right)\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\right\} \\
&= \mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right](1-\alpha)y_2^{call}\frac{\partial}{\partial r_1}\left(\frac{1}{1+c_2}\right)\right\} \\
&= -\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\frac{(1-\alpha)y_2^{call}}{(1+c_2)^2}\frac{b_2-\theta_1 b_1}{b_2}\right\} < 0
\end{aligned}$$

3. At this step, I need to study the (the sign of) second derivative of $\overline{V}_1^{nocall}$ and $\overline{V}_1^{call}$ with respect to the interest rate r_1:

$$\begin{aligned}
\frac{\partial^2 \overline{V}_1^{nocall}(s_1,x_1)}{\partial r_1^2} &= -\frac{\partial}{\partial r_1}\left(\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right]\frac{(1-\alpha)y_2^{nocall}}{(1+c_2)^2}\frac{b_2-b_1}{b_2}\right\}\right)\\
&= -\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right](1-\alpha)y_2^{nocall}\frac{b_2-b_1}{b_2}\frac{\partial}{\partial r_1}\left(\frac{1}{(1+c_2)^2}\right)\right\}\\
&= 2\cdot\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right]\frac{(1-\alpha)y_2^{nocall}}{(1+c_2)^3}\left(\frac{b_2-b_1}{b_2}\right)^2\right\} > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \overline{V}_1^{call}(s_1,x_1)}{\partial r_1^2} &= -\frac{\partial}{\partial r_1}\left(\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\frac{(1-\alpha)y_2^{call}}{(1+c_2)^2}\frac{b_2-\theta_1 b_1}{b_2}\right\}\right)\\
&= -\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right](1-\alpha)y_2^{call}\frac{b_2-\theta_1 b_1}{b_2}\frac{\partial}{\partial r_1}\left(\frac{1}{(1+c_2)^2}\right)\right\}\\
&= 2\cdot\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\frac{(1-\alpha)y_2^{call}}{(1+c_2)^3}\left(\frac{b_2-\theta_1 b_1}{b_2}\right)^2\right\} > 0
\end{aligned}$$

Therefore, considering the two above inequalities, I arrive at both functions $\overline{V}_1^{nocall}$ and $\overline{V}_1^{call}$ are convex in r_1, with other four parameters fix.

4. Lastly, to know the relative positions of the slope of both curves, I derive their asymptotic behavior with respect to θ_1, as follows:

$$\begin{aligned}
\frac{\partial^2 \overline{V}_1^{call}(s_1,x_1)}{\partial \theta_1 \partial r_1} &= -\frac{\partial}{\partial \theta_1}\left(\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\frac{(1-\alpha)y_2^{call}}{(1+c_2)^2}\frac{b_2-\theta_1 b_1}{b_2}\right\}\right)\\
&= -\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right](1-\alpha)y_2^{call}\frac{\partial}{\partial \theta_1}\left(\frac{b_2-\theta_1 b_1}{b_2}\frac{1}{(1+c_2)^2}\right)\right\}\\
&= 2\cdot\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\frac{(1-\alpha)y_2^{call}}{(1+c_2)^2}\frac{b_1}{b_2}\left[1+2\cdot\frac{b_2-\theta_1 b_1}{b_2}\frac{c_1-r_1}{1+c_2}\right]\right\}
\end{aligned}$$

The sign of the derivative in the previous line depends on the sign and the amplitude of $c_1 - r_1$. Considering the case of $r_1 \to 0$, the y-intercept of $\overline{V}_1^{call}(s_1,x_1)|_{r_1=0} > \overline{V}_1^{nocall}(s_1,x_1)|_{r_1=0}, \forall \theta_1 < 1$.

$$\left.\frac{\partial^2 \overline{V}_1^{call}(s_1,x_1)}{\partial \theta_1 \partial r_1}\right|_{r_1\to 0} > 0 \tag{A29}$$

$$\left.\frac{\partial^2 \overline{V}_1^{call}(s_1,x_1)}{\partial \theta_1 \partial r_1}\right|_{r_1\to +\infty} < 0 \tag{A30}$$

5. $\overline{V}_1^{nocall}$ coincides with $\overline{V}_1^{call}$ when $\theta_1 = 1$: $\overline{V}_1^{call}(s_1,x_1)|_{\theta_1=1} = \overline{V}_1^{nocall}(s_1,x_1), \forall c_1$ and $\forall r_1$.

When I combine the five points above, I can now deduce the proof of the existence of the interest rate r_1^*, such that $\overline{V}_1^{call}$ and $\overline{V}_1^{nocall}$ cross at r_1^*, based on the intermediate value theorem.

The existence of r_1^* is then proved, therefore through the signs described in (A29) and (A30), I state that when $r_1 < r_1^*$, the exercise of the call option is the unique optimal decision: $\overline{V}_1^{nocall}(s_1, x_1) < \overline{V}_1^{call}(s_1, x_1)$. This completes the proof of Proposition 3.

■

C.8 Proof of Corollary 3.0.2

Proof. To prove Corollary 3.0.2, I need two ingredients:

1. The last terms at both sides of the inequality in (29) concern the probability of not defaulting. This term decreases in the capital quality cutoff $\overline{\varepsilon}_2$. This threshold value, in turn, increases in the periodic coupon c_2:

$$\frac{\partial \overline{\varepsilon}_2}{\partial c_2} = \frac{b_2}{k_2} > 0 \tag{A31}$$

Within the decomposition of c_2, we showed through (A24) that θ_1 amplifies the sensitivity of the default threshold.

2. I recall the sign of $\dfrac{\partial^2 \overline{V}_1^{call}(s_1, x_1)}{\partial \theta_1 \partial r_1}$ in the the last line (4) and use what the signs in (A29) and (A30) say. This sign shows how strong the effect of the share of non-callable θ_1 (so that the share of callable) is in amplifying the decision to call regarding the variation in market interest rates.

$$\frac{\partial^2 \overline{V}_1^{call}(s_1, x_1)}{\partial \theta_1 \partial r_1} \begin{cases} > 0 & \text{if } r_1 < r_1^* \text{ (when call)} \\ < 0 & \text{if } r_1 > r_1^* \text{ (when no call)} \end{cases} \tag{A32}$$

With these inequalities, I have all elements illustrated in Figure 8.

■

C.9 Proof of the optimal callability in the condition (32)

Proof. To establish the expression of the optimal condition for the choice of θ_1 in (32), let's rewrite the F.O.C. expressed in (30) as an equality.

$$\begin{aligned} [\theta_1] : b_1 \frac{\partial p_0}{\partial \theta_1} + p_0 \frac{\partial b_1}{\partial \theta_1} - \frac{\partial \eta_b}{\partial \theta_1} + \beta_0 \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \theta_1} &= 0 \\ : \frac{\partial \overline{\varepsilon}_1}{\partial \theta_1} \left[b_1 \frac{\partial p_0}{\partial \overline{\varepsilon}_1} + p_0 \frac{\partial b_1}{\partial \overline{\varepsilon}_1} + \beta_0 \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \overline{\varepsilon}_1} \right] - \frac{\partial \eta_b}{\partial \theta_1} &= 0 \end{aligned} \tag{A33}$$

First, I derive the default cutoff $\overline{\varepsilon}_1$:

$$\overline{\varepsilon}_1 = -\frac{k_1 + y_1 + \mathbb{1}_{\text{call}} \times \overline{V}_1^{call}(s_1, x_1) + (1 - \mathbb{1}_{\text{call}}) \times \overline{V}_1^{nocall}(s_1, x_1)}{k_1} \tag{A34}$$

Now, I define the components of the above equation. I get the default risk derivative relative to the non-callable bond's share.

$$\frac{\partial \overline{\varepsilon}_1}{\partial \theta_1} = -\frac{1}{k_1} \times \frac{\partial}{\partial \theta_1}\left[\mathbb{1}_{\text{call}} \times (\overline{V}_1^{call} - \overline{V}_1^{nocall})\right] \tag{A35}$$

I then consider the effects of the default risk on the price of the initial bond p_0 within both types of bond, and the expected firm's value in the intermediate period $\mathbb{E}_{s_1|s_0} V_1$.

$$\begin{aligned}
\frac{\partial p_0^{nc}}{\partial \overline{\varepsilon}_1} &= -\beta_0 \mathbb{E}_{s_1|s_0}\left\{ (r_0 + \overline{p}_1^{nc})\, \varphi(\overline{\varepsilon}_1) \right\} \\
\frac{\partial p_0^{c}}{\partial \overline{\varepsilon}_1} &= -\beta_0 \mathbb{E}_{s_1|s_0}\left\{ (r_0 + 1 + \chi)\, \varphi(\overline{\varepsilon}_1) \right\} \\
\frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \overline{\varepsilon}_1} &= -k_1 \mathbb{E}_{s_1|s_0}\left[1 - \Phi(\overline{\varepsilon}_1)\right]
\end{aligned}$$

Concerning the issuance cost, its variation on the share of non-callable is described by:

$$\frac{\partial \eta_b}{\partial \theta_1} = 2\left[(\eta_{nc} + \eta_c)\theta_1 - \eta_c\right] b_1^2 \tag{A36}$$

By replacing the five above derivatives in the expression (A33), I obtain the four following components of the optimal choice of non-callable share:

1. for the non-callable share of bond, I obtain:

$$\beta_0 \mathbb{E}_{s_1|s_0}\left\{ \left[(r_0 + \overline{p}_1^{nc}) \frac{b_1}{k_1} \varphi(\overline{\varepsilon}_1) \right] \times \frac{\partial}{\partial \theta_1}\left[\mathbb{1}_{\text{call}} \times (\overline{V}_1^{call} - \overline{V}_1^{nocall})\right] \right\}$$

2. for the callable share of bond, I obtain:

$$\beta_0 \mathbb{E}_{s_1|s_0}\left\{ \left[\mathbb{1}_{\text{call}} (r_0 + 1 + \chi) \frac{b_1}{k_1} \varphi(\overline{\varepsilon}_1) \right] \times \frac{\partial}{\partial \theta_1}\left[\mathbb{1}_{\text{call}} \times (\overline{V}_1^{call} - \overline{V}_1^{nocall})\right] \right\}$$

3. for the continuation value in next period $t = 1$, the component is:

$$\beta_0 \mathbb{E}_{s_1|s_0}\left\{ [1 - \Phi(\overline{\varepsilon}_1)] \times \frac{\partial}{\partial \theta_1}\left[\mathbb{1}_{\text{call}} \times (\overline{V}_1^{call} - \overline{V}_1^{nocall})\right] \right\}$$

4. for the issuance costs, the derivation is: $-2\left[(\eta_{nc} + \eta_c)\theta_1 - \eta_c\right] b_1^2$

I recall the key property of any indicator variable which is idempotent, then I use the following for the rest of the proof:

$$\mathbb{1}_{\text{call}} \times \mathbb{1}_{\text{call}} = \mathbb{1}_{\text{call}} \tag{A37}$$

I use also the following simplification:

$$\mathbb{1}_{\text{call}} \times \overline{p}_1^{nc} = \overline{p}_1^{nc\_call} \tag{A38}$$

Back to the optimal equation on θ_1, we have:

$$\begin{aligned} \beta_0 \mathbb{E}_{s_1|s_0} \Bigg\{ \Bigg[\theta_1 \left(r_0 + \overline{p}_1^{nc\_call} \right) + (1-\theta_1)\,(r_0 + 1 + \chi)\, \frac{b_1}{k_1} \varphi(\overline{\varepsilon}_1) + [1 - \Phi(\overline{\varepsilon}_1)] \Bigg] \\ \times \frac{\partial}{\partial \theta_1} \left[\mathbb{1}_{\text{call}} \times (\overline{V}_1^{call} - \overline{V}_1^{nocall}) \right] \Bigg\} - 2\left[(\eta_{nc} + \eta_c)\theta_1 - \eta_c \right] b_1^2 = 0. \end{aligned} \tag{A39}$$

Last, I bring the interest rate r_0 out of the continuation returns of the bond to derive the final expression:

$$\begin{aligned} \beta_0 \mathbb{E}_{s_1|s_0} \Bigg\{ \Bigg[\left[r_0 + \theta_1 \overline{p}_1^{nc\_call} + (1-\theta_1)\,(1+\chi) \right] \frac{b_1}{k_1} \varphi(\overline{\varepsilon}_1) + [1 - \Phi(\overline{\varepsilon}_1)] \Bigg] \\ \times \frac{\partial}{\partial \theta_1} \left[\mathbb{1}_{\text{call}} \times (\overline{V}_1^{call} - \overline{V}_1^{nocall}) \right] \Bigg\} - 2\left[(\eta_{nc} + \eta_c)\theta_1 - \eta_c \right] b_1^2 = 0. \end{aligned} \tag{A40}$$

■

D Quantitative Derivations

D.1 Discussion of bonds pricing

D.1.1 Importance of distinct continuation prices for callable and non-callable bonds

The continuation price, denoted as $\overline{p}^{nc}$ (or $\overline{p}^{c}$), reflects the valuation of *outstanding debt* in the next period based on the initial terms established at issuance. Distinguishing between continuation prices for non-callable and callable bonds is essential due to differences in contractual terms and payment expectations between these bonds. Specifically, the continuation price for a non-callable bond, $\overline{p}^{nc}$, and for a callable bond, $\overline{p}^{c}$, serve distinct functions by reflecting either fixed or adjustable payment obligations.

- **Non-callable bond and fixed coupon rate:** The continuation price $\overline{p}^{nc}$ for a non-callable bond incorporates the original coupon rate c determined at issuance. This fixed coupon ensures that creditors holding non-callable debt are paid at a stable rate, unaffected by shifts in the current market interest rate r. Thus, $\overline{p}^{nc}$ accurately represents the creditor's expectation of receiving consistent payments as per the bond's initial terms, upholding the contractual integrity of non-callable debt.

- **Callable bond and call decision flexibility:** The continuation price $\overline{p}^{c}$ for a callable bond varies according to the firm's call decision. If the firm refrains from calling, $\overline{p}^{c}$ aligns with $\overline{p}^{nc}$, continuing under the original coupon payments. However, if the call option is exercised, the continuation price reflects only the principal repayment plus the call premium χ, omitting future coupon payments. This adjustment is crucial for callable bondholders, who face the potential for early repayment, including a compensatory premium for their investment. Therefore, $\overline{p}^{c}$ accounts for both no-call scenarios, where future coupons are expected, and call scenarios, where payment is limited to principal plus premium.

Can a single continuation price be used? Although it might seem appealing to simplify the model by using a single continuation price based on a weighted average of periodic coupon payments (as defined in (35) and (36)), this approach would introduce inaccuracies:

1. *Creditor composition differences:* Newly issued debt and outstanding debt are held by different creditor groups with distinct expectations. New creditors enter under current market conditions, while existing creditors rely on previously set coupon terms. A unified continuation price could obscure these differences, misrepresenting the expectations of each creditor class.[30]

[30] See e.g. Ippolito et al. (2018) for the weighted average periodic coupon, in the context of fixed vs floating rate debts.

2. *Contractual integrity of fixed coupons:* Outstanding debt is typically structured with fixed coupons that do not vary with interest rate changes. Averaging coupons across different debt types would imply contractual adjustments to these fixed payments, contradicting the standard fixed-rate structure in financial contracts.

Thus, maintaining distinct continuation prices—one based on the fixed coupon for outstanding debt and another on the current interest rate for newly issued debt—provides an accurate depiction of the firm's obligations and aligns with the varying expectations of each creditor group. For non-callable bonds, this distinction ensures stability through a fixed coupon rate, while for callable bonds, it allows for flexible adjustment based on the firm's optimal call decision. This dual pricing approach is particularly valuable for accurately modeling scenarios with mixed debt instruments, as it reflects the financial strategies of firms in fluctuating market conditions and their impact on bondholder returns.

D.1.2 Callable bond pricing

The callable bond pricing model must incorporate both the firm's endogenous call decision and the potential loss for bondholders when the call option is exercised. Equation (47) reflects this structure, providing a dynamic framework that captures how the firm optimizes its value under the possibility of calling the bond.

1. **Endogenous call decision.** In this framework, the firm decides whether to call the bond by comparing the **value of calling** with the value of holding the bond until maturity. The decision to call is endogenous, depending on the firm's state variables s, and policy functions $\hat{x}(s, x)$. This decision is modeled through the call indicator $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}(s', \hat{x}(s, x))$, which determines whether the callable bond will be called or not. This allows for a more accurate representation of the firm's incentives and behavior, as opposed to assuming a fixed probability or exogenously determined call timing. The literature, such as Duffie and Singleton (1999); Jarrow et al. (2010), often relies on exogenous factors to model the call decision, which may not fully capture the issuer's strategic behavior in response to market conditions (See also Chen et al. (2010).).

2. **Min function and redundancy.** In traditional models, the **min function** is used to ensure that the bondholder's payoff does not exceed the call price, even if the market price of the bond exceeds it. However, in our extended model, this comparison is **internalized** in the firm's value-maximizing decision to call or not. Therefore, the **min function** becomes redundant since the firm already optimizes between calling the bond or not based on the comparison of its continuation value. The decision rule internalizes the market price dynamics and the call price, making the explicit use of **min** unnecessary (See Brennan and

Schwartz (1977) for classical discussions on callable bond pricing.).

3. The model incorporates a dynamic continuation price for both callable and non-callable bonds, taking into account future state variables and policy functions, as is done in the dynamic debt model literature in macroeconomics (e.g. Crouzet, 2017; Karabarbounis and Macnamara, 2021; Jungherr and Schott, 2021). This contrasts with simpler models where the continuation value is often static or based on less granular assumptions. By considering the future coupon payments and the possibility of a call, the model provides a more comprehensive valuation that aligns with real market dynamics.

4. The call premium is intrinsically set as a function of the prevailing interest rate r, making the model sensitive to market conditions at issuance. This allows for the call premium to decrease as the bond matures without being called, reflecting the decreasing call risk. This feature provides a more accurate representation of the bond's pricing over its life, something not always addressed in the literature, where call premiums may be assumed constant or independent of market rates.

D.2 First Order Conditions

In this subsection, I present the first-order conditions for the decision variables: the capital k', the debt b', and the share of non-callable θ'. Without any specificity, I consider the rewritten value function defined in (43), so for a given state variables $s = \{z, r\}, x = \{k, b, \theta, c\}$, for the case $j \in \{\text{nocall}, \text{call}\}$ we have:

$$\overline{V}_j(s, x) = \max_{e, k'_j, b'_j, \theta'_j, c'_j} \left\{ -k'_j + n_j + p\tilde{b}_j - \eta_b(\tilde{b}_j) - \eta_e(e) + \frac{1}{1+r} \mathbb{E}_{s'|s} \int_{\overline{\varepsilon}'}^{\infty} V^r(s', x') d\Phi(\varepsilon') \right\} \tag{A1}$$

The weighted average price of the new debt is: $p = \tilde{\theta}_j p^{nc} + (1 - \tilde{\theta}_j) p^c$. I define a new pseudo-value function of the firm that considers the part of its value function influenced by its optimal choices.

$$W_j(s', x') = -k'_j + p\tilde{b}_j - \eta_b(\tilde{b}_j) - \eta_e(e) + \frac{1}{1+r} \mathbb{E}_{s'|s} \int_{\overline{\varepsilon}'}^{\infty} V^r(s', x') d\Phi(\varepsilon') \tag{A2}$$

D.2.1 Optimal capital k'

The firm's first-order condition for capital k'_j follows:

$$\left(1+\frac{\partial\eta_e(e)}{\partial e}\right)\left\{-1+b'\left[\tilde{\theta}_j\frac{\partial p^{nc}}{\partial k'_j}+(1-\tilde{\theta}_j)\frac{\partial p^{c}}{\partial k'_j}\right]\right\}+\frac{1}{1+r}\mathbb{E}_{s'|s}\int_{\overline{\varepsilon}'}^{\infty}\frac{\partial V^r(s',x')}{\partial k'_j}d\Phi(\varepsilon')=0 \tag{A3}$$

Where the repayment value evolves depending on the capital as follows:

$$\begin{aligned}\frac{\partial V^r(s',x')}{\partial k'_j}=&\left[1-\mathbb{1}'_{\text{call}}\right]\left\{\frac{\partial n'_{\text{nocall}}}{\partial k'_j}\cdot\left[(1-\pi_e)\mathbb{E}_{s''|s'}\left(1+\frac{\partial\eta_e(e')}{\partial e'}\right)+\pi_e\left(1-(1-\gamma)b'\mathbb{E}_{s''|s'}\frac{\partial p'_{\text{nocall}}}{\partial n'_{\text{nocall}}}\right)\right]\right\}\\&+\mathbb{1}'_{\text{call}}\left\{\frac{\partial n'_{\text{call}}}{\partial k'_j}\cdot\left[(1-\pi_e)\mathbb{E}_{s''|s'}\left(1+\frac{\partial\eta_e(e')}{\partial e'}\right)+\pi_e\left(1-(1-\gamma)\theta' b'\mathbb{E}_{s''|s'}\frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}}\right)\right]\right\}\end{aligned}$$

where $\mathbb{1}'_{\text{call}}=\mathbb{1}_{\text{call}}(s',x')$ indicates whether the firm will choose to call the callable debt $(1-\theta')$ in the next period; $n'_{\text{nocall}}=n_{\text{nocall}}(s',x')$, and $n'_{\text{call}}=n_{\text{call}}(s',x')$ are respectively the internal fund of the firm after producing for both future choices. Then, we have:

$$\forall j\in\{\text{nocall},\text{call}\},\frac{\partial n'_j}{\partial k'_j}=\frac{\partial n_j(s',x')}{\partial k'_j}=1+(1-\tau)\left[z'\alpha k'^{\alpha-1}_j+\epsilon'-\delta\right] \tag{A4}$$

To obtain the derivatives of the prices relative to the next period capital, I need the effects on the default threshold, $\overline{\varepsilon}'$, which is defined implicitly in (38).

$$\frac{\partial\overline{\varepsilon}}{\partial k'_j}=-\frac{1+(1-\tau)\left(z'\alpha k'^{\alpha-1}_j+\epsilon'-\delta\right)}{(1-\tau)k'_j} \tag{A5}$$

I can then compute the non-callable bond price derivative with respect to k'_j:

$$\begin{aligned}\frac{\partial p^{nc}}{\partial k'_j}=&\beta\mathbb{E}_{s'|s}\left\{\int_{\overline{\varepsilon}'}^{\infty}(1-\gamma)\mathbb{E}_{s''|s'}\left[\left(1-\mathbb{1}'_{\text{call}}\right)\frac{\partial p'^{nc}_{\text{nocall}}}{\partial n'_{\text{nocall}}}\frac{\partial n'_{\text{nocall}}}{\partial k'_j}+\mathbb{1}'_{\text{call}}\frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}}\frac{\partial n'_{\text{call}}}{\partial k'_j}\right]d\Phi(\varepsilon')\right.\\&+\int_{-\infty}^{\overline{\varepsilon}'}\frac{1-\xi}{b'}\left[1+(1-\tau)\left(z'\alpha k'^{\alpha-1}_j+\epsilon'-\delta\right)\right]d\Phi(\varepsilon')\\&\left.+\varphi(\overline{\varepsilon}')\left[-\left(\gamma+r+(1-\gamma)\,\mathbb{E}_{s''|s'}p'^{nc}\right)+\frac{1-\xi}{b'}\underline{n}(\overline{\varepsilon}')\right]\frac{\partial\overline{\varepsilon}'}{\partial k'_j}\right\}\end{aligned}$$

I derive the variation of the callable bond price with respect to k'_j:

$$\begin{aligned}\frac{\partial p^c}{\partial k'_j} =& \beta \mathbb{E}_{s'|s}\Bigg\{\int_{\overline{\varepsilon}'}^{\infty}(1-\gamma)\mathbb{E}_{s''|s'}\left[(1-\mathbb{1}'_{\text{call}})\frac{\partial p'^c}{\partial n'_{\text{nocall}}}\frac{\partial n'_{\text{nocall}}}{\partial k'_j}\right]d\Phi(\varepsilon')\\ &+\int_{-\infty}^{\overline{\varepsilon}'}\frac{1-\xi}{b'}\left[1+(1-\tau)\left(z'\alpha k_j'^{\alpha-1}+\epsilon'-\delta\right)\right]d\Phi(\varepsilon')\\ &+\varphi(\overline{\varepsilon}')\left[-\left[r+(1-\mathbb{1}'_{\text{call}})\left[\gamma+(1-\gamma)\,\mathbb{E}_{s''|s'}p'^c_{\text{call}}\right]+\mathbb{1}'_{\text{call}}(1+\chi)\right]+\frac{1-\xi}{b'}\underline{n}(\overline{\varepsilon}')\right]\frac{\partial \overline{\varepsilon}'}{\partial k'_j}\Bigg\}\end{aligned}$$

D.2.2 Optimal debt b'

Now, I derive the optimal condition for the choice of the next period level of debt b'.

$$\left(1+\frac{\partial \eta_e(e)}{\partial e}\right)\left\{p+b'\left[\tilde{\theta}_j\frac{\partial p^{nc}}{\partial b'}+(1-\tilde{\theta}_j)\frac{\partial p^c}{\partial b'}\right]-\frac{\partial \eta_b(b')}{\partial b'}\right\}+\frac{1}{1+r}\mathbb{E}_{s'|s}\int_{\overline{\varepsilon}'}^{\infty}\frac{\partial V^r(s',x')}{\partial b'}d\Phi(\varepsilon')=0 \tag{A6}$$

Where the derivative of the repayment value with respect to b' is:

$$\begin{aligned}\frac{\partial V^r(s',x')}{\partial b'} = \left[1-\mathbb{1}'_{\text{call}}\right]\mathbb{E}_{s''|s'}\Bigg\{ & \frac{\partial n'_{\text{nocall}}}{\partial b'}\cdot\left[(1-\pi_e)\left(1+\frac{\partial \eta_e(e')}{\partial e'}\right)+\pi_e\left(1-(1-\gamma)b'\frac{\partial p'_{\text{nocall}}}{\partial n'_{\text{nocall}}}\right)\right]\\ &+\frac{\partial \tilde{b}'_{\text{nocall}}}{\partial b'}\cdot\left[(1-\pi_e)\frac{\partial W(s'',x''_{\text{nocall}})}{\partial \tilde{b}'_{\text{nocall}}}-\pi_e\left(p'_{\text{nocall}}+b'\frac{\partial p'_{\text{nocall}}}{\partial n'_{\text{nocall}}}\right)\right]\Bigg\}\\ +\mathbb{1}'_{\text{call}}\mathbb{E}_{s''|s'}\Bigg\{ & \frac{\partial n'_{\text{call}}}{\partial b'}\cdot\left[(1-\pi_e)\left(1+\frac{\partial \eta_e(e')}{\partial e'}\right)+\pi_e\left(1-(1-\gamma)\theta' b'\frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}}\right)\right]\\ &+\frac{\partial \tilde{b}'_{\text{call}}}{\partial b'}\cdot\left[(1-\pi_e)\frac{\partial W(s'',x''_{\text{call}})}{\partial \tilde{b}'_{\text{call}}}-\pi_e\left(p'^{nc}_{\text{call}}+b'\frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}}\right)\right]\Bigg\}\end{aligned}$$

where the function $W(s'',x''_j)$ is the function defined in (A2) when the decision j will be taken the next period. $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}(s',x')$ indicates whether the firm will choose to call the callable debt $(1-\theta')$ in the next period; $n'_{\text{nocall}} = n_{\text{nocall}}(s',x')$, and $n'_{\text{call}} = n_{\text{call}}(s',x')$ are respectively the internal fund of the firm after producing for both future choices. Additionally, I define the following components of the above derivatives.

When the firm does not call in the next period, $\mathbb{1}'_{\text{call}} = 0$, $\frac{\partial n'_{\text{nocall}}}{\partial b'} = -(\gamma+(1-\tau)c')$, and $\frac{\partial \tilde{b}'_{\text{nocall}}}{\partial b'} = 1-\gamma$, while when the firm decides to call in the next period, $\mathbb{1}'_{\text{call}} = 1$, we have $\frac{\partial n'_{\text{call}}}{\partial b'} = -\left[\gamma\theta'+(1+\chi)(1-\theta')+(1-\tau)c'\right]$, and $\frac{\partial \tilde{b}'_{\text{call}}}{\partial b'} = \theta'(1-\gamma)$.

I derive the variation of the pseudo-value functions. In the no-call scenario in the next period, we have:

$$\frac{\partial W(s'', x''_{\text{nocall}})}{\partial \tilde{b}'_{\text{nocall}}} = -p'_{\text{nocall}} \left(1 + \frac{\partial \eta_e(e')}{\partial e'}\right) - \frac{\partial \eta_b(\tilde{b}'_{\text{nocall}})}{\partial \tilde{b}'_{\text{nocall}}} \tag{A7}$$

and for the call scenario, we have:

$$\frac{\partial W(s'', x''_{\text{call}})}{\partial \tilde{b}'_{\text{call}}} = -p'^{nc}_{\text{call}} \left(1 + \frac{\partial \eta_e(e')}{\partial e'}\right) - \frac{\partial \eta_b(\tilde{b}'_{\text{call}})}{\partial \tilde{b}'_{\text{call}}} \tag{A8}$$

Lastly, to complete the optimal conditions components, I derive the prices sensitivities to the next period debt level.

I compute the non-callable bond price derivative with respect to b':

$$\begin{aligned}
\frac{\partial p^{nc}}{\partial b'} = \beta \mathbb{E}_{s'|s} \Bigg\{ \int_{\overline{\varepsilon}'}^{\infty} (1-\gamma) \mathbb{E}_{s''|s'} \Bigg[(1 - \mathbb{1}'_{\text{call}}) \left(\frac{\partial p'^{nc}_{\text{nocall}}}{\partial n'_{\text{nocall}}} \frac{\partial n'_{\text{nocall}}}{\partial b'} + \frac{\partial p'^{nc}_{\text{nocall}}}{\partial \tilde{b}'_{\text{nocall}}} \frac{\partial \tilde{b}'_{\text{nocall}}}{\partial b'} \right) \\
+ \mathbb{1}'_{\text{call}} \left(\frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}} \frac{\partial n'_{\text{call}}}{\partial b'} + \frac{\partial p'^{nc}_{\text{call}}}{\partial \tilde{b}'_{\text{call}}} \frac{\partial \tilde{b}'_{\text{call}}}{\partial b'} \right) \Bigg] d\Phi(\varepsilon') \\
- \int_{-\infty}^{\overline{\varepsilon}'} \frac{1-\xi}{(b')^2} \left[k' + (1-\tau) \left(z' k'^{\alpha} + \epsilon' k' - \delta k' - f \right) \right] d\Phi(\varepsilon') \\
+ \varphi(\overline{\varepsilon}') \left[- \left(\gamma + r + (1-\gamma) \mathbb{E}_{s''|s'} p'^{nc} \right) + \frac{1-\xi}{b'} \underline{n}(\overline{\varepsilon}') \right] \frac{\partial \overline{\varepsilon}'}{\partial b'} \Bigg\}
\end{aligned} \tag{A9}$$

For the callable bond, the difference stands from the call price in the call scenario. I compute the non-callable bond price derivative with respect to b':

$$\begin{aligned}
\frac{\partial p^{c}}{\partial b'} = \beta \mathbb{E}_{s'|s} \Bigg\{ \int_{\overline{\varepsilon}'}^{\infty} (1-\gamma) \mathbb{E}_{s''|s'} \Bigg[(1 - \mathbb{1}'_{\text{call}}) \left(\frac{\partial p'^{c}_{\text{nocall}}}{\partial n'_{\text{nocall}}} \frac{\partial n'_{\text{nocall}}}{\partial b'} + \frac{\partial p'^{c}_{\text{nocall}}}{\partial \tilde{b}'_{\text{nocall}}} \frac{\partial \tilde{b}'_{\text{nocall}}}{\partial b'} \right) \\
- \int_{-\infty}^{\overline{\varepsilon}'} \frac{1-\xi}{(b')^2} \left[k' + (1-\tau) \left(z' k'^{\alpha} + \epsilon' k' - \delta k' - f \right) \right] d\Phi(\varepsilon') \\
+ \varphi(\overline{\varepsilon}') \left[- \left[r + (1 - \mathbb{1}'_{\text{call}}) \left[\gamma + (1-\gamma) \mathbb{E}_{s''|s'} p'^{c}_{\text{call}} \right] + \mathbb{1}'_{\text{call}} (1+\chi) \right] + \frac{1-\xi}{b'} \underline{n}(\overline{\varepsilon}') \right] \frac{\partial \overline{\varepsilon}'}{\partial b'} \Bigg\}
\end{aligned} \tag{A10}$$

E Quantitative Appendix

In this section, I describe the model counterparts of key empirical moments for the full model, present the solution method, and discuss further quantitative results.

E.1 Model Moments

This subsection guides the connection between the theoretical framework and the empirical evidence presented earlier. The empirical moments we focus on include bond pricing, firm default, debt issuance, and investment dynamics, which are all essential for understanding the impact of debt callability on firm behavior and aggregate outcomes.

At the stationary equilibrium, I compute model moments as cross-sectional expectations under the invariant distribution $\pi(s)$ of firm states $s = (z, r, k, b, \theta, c)$. I do *not* impose $c^* = r^*$ nor normalize prices to 1. Instead, all prices and decisions are those implied by the model's policy functions and continuation prices, aggregated with weights $\pi(s)$. Throughout, expectations are taken with respect to the stationary distribution $\pi(s)$ over firm states $s = (z, r, k, b, \theta, c)$. Let $\mathbb{E}[\,\cdot\,] \equiv \sum_s \pi(s)\,(\cdot)$.

E.1.1 Bond Pricing and Credit Spreads

The pricing of callable and non-callable debt within the model is fundamental for understanding corporate financing strategies. The bond price is determined by evaluating the firm's decision to either call or not call the debt. This decision hinges on comparing the present value of callable debt, which includes the call premium, against the value of non-callable debt.

The price of callable debt in the model is determined by comparing the call and no-call decisions. The bond price depends on whether the firm exercises the call option and the associated future expected value of the callable and non-callable debt. The price computation is done for each state, accounting for firm productivity, interest rates, debt levels, and capital.

average non-callable price : $\hat{p}^{nc}$

$$= \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} (1 - \mathbb{1}_{\text{call}}(s)) \cdot \overline{p}^{nc}_{\text{nocall}} + \mathbb{1}_{\text{call}}(s) \cdot \overline{p}^{nc}_{\text{call}}, \tag{A1}$$

where $\overline{p}^{nc}_j = \overline{p}^{nc}_j(z, r, k', b', \theta, c)$ for $j \in \{\text{nocall}, \text{call}\}$ are defined in Equation (46). K, B, Θ, and $\bar{C}$ are the state spaces for respectively the state variables k, b, θ, and c, they are defined in the subsection below. The indicator $\mathbb{1}_{\text{call}}$ captures the endogenous call decision in the current period.

Similarly, the price of callable debt is computed as:

$$\textbf{average callable price} : \hat{p}^c$$
$$= \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} (1 - \mathbb{1}_{\text{call}}(s)) \cdot \overline{p}^c_{\text{nocall}} + \mathbb{1}_{\text{call}}(s) \cdot \overline{p}^c_{\text{call}}, \tag{A2}$$

where $\overline{p}^c_j = \overline{p}^c_j(z, r, k', b', \theta, c)$ for $j \in \{\text{nocall}, \text{call}\}$ are defined in Equation (49). These prices are essential for calculating the average credit spreads, which reflect the compensation investors require for bearing credit risk.

Now, I derive the credit spreads. The credit spread measures the additional yield that investors demand to hold corporate debt over risk-free government bonds. It is the key indicator of the perceived riskiness of the firm's debt. At the steady state equilibrium, the prices have characteristics with fixed constant average call rate, and constant rate, and the price of riskless non-callable and callable debts is 1.

The credit spread on non-callable debt is calculated as:

$$\textbf{spread}^{nc} := \left[\frac{\gamma + c^*}{\hat{p}^{nc}} + (1 - \gamma) \right]^4 - (1 + r^*)^4, \tag{A3}$$

Here, γ represents the quarterly rate of debt repayment, and c^* is the steady-state coupon rate on the firm's debts, equalized to the risk-free rate r^*. The exponent 4 annualizes the quarterly returns, aligning with the typical frequency of debt repayments.

For callable debt, the spread is given by:

$$\textbf{spread}^c := \left\{ \frac{\gamma + c^* + (1 - \gamma) \left[(1 - q^*) \, \widehat{p}^c_{\text{nocall}} + q^* \left(\widehat{p}^c_{\text{call}} + \chi \right) \right]}{\widehat{p}^c} \right\}^4 - (1 + r^*)^4, \tag{A4}$$

where q^* denotes the average call rate, and χ is the call premium. The spread on callable debt accounts for the additional costs associated with the option to call, reflecting the increased risk for investors.

The overall average credit spread is a weighted average of the spreads on non-callable and callable debt:

$$\textbf{spread} := \theta^* \cdot \text{spread}^{nc} + (1 - \theta^*) \cdot \text{spread}^c \tag{A5}$$

where θ^* is the steady-state share of non-callable debt. This aggregation ensures that the model captures the combined effect of both debt types on the overall credit risk premium.

E.1.2 Rates of exit

First, the default rate reveals the probability that a firm will fail to meet its debt obligations, influenced by productivity shocks, debt levels, and most importantly capital quality shock. It is determined by the threshold $\bar{\varepsilon}$, below which a firm defaults.

$$\textbf{default rate } := \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} \mathbb{P}[\varepsilon \leq \bar{\varepsilon}] = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} \Phi[\bar{\varepsilon}] = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} \int_{\bar{\varepsilon}}^{\infty} d\Phi(\varepsilon) \tag{A6}$$

where Φ is the cumulative distribution function of capital quality shocks, and $\bar{\varepsilon}$ is the default threshold in the current period. This expectation integrates over all possible states, providing an average default probability across the economy.

The exit rate accounts for the probability that a firm exits the market, either through default or exogenously. It is a combination of the default rate and the probability of an exogenous exit, π_e. As described on the timing of the model (9), the firm decides to default or not before the exogenous exit shock:

$$\textbf{exit rate } := \text{default rate} + (1 - \text{default rate}) \cdot \pi_e \tag{A7}$$

E.1.3 Debt duration

I approximate durations from scheduled hazards implied by the model. Let recall that $(1-\theta)b$ denote the callable stock at the beginning of the quarter and let $\lambda \in (0,1]$ be the exogenous eligibility share per quarter. Define the scheduled callable share and conditional call rate:

$$s^{\text{sched}} := \lambda \cdot \frac{\mathbb{E}_{s\sim\pi}[(1-\theta)b]}{\mathbb{E}_{s\sim\pi}[b]}, \tag{A8}$$

$$c^{\text{sched}} := \frac{\mathbb{E}_{s\sim\pi}[\mathbb{1}_{\text{call}}(s') \cdot (1-\theta)b]}{\mathbb{E}_{s\sim\pi}[(1-\theta)b]}. \tag{A9}$$

The implied unconditional call hazard is $h_{\text{call}} = s^{\text{sched}} \cdot c^{\text{sched}}$. Quarterly durations are then

$$\textbf{duration}^{nc} = \frac{1+r^*}{\gamma + r^*}, \qquad \textbf{duration}^{c} = \frac{1+r^*}{\gamma + (1-\gamma)\, h_{\text{call}} + r^*}, \tag{A10}$$

and I report the stock-weighted average

$$\textbf{duration} = (1 - \widehat{cc}_{\text{boq}}) \cdot \text{duration}^{nc} + \widehat{cc}_{\text{boq}} \cdot \text{duration}^{c}, \tag{A11}$$

where $\widehat{cc}_{\text{boq}} = \mathbb{E}_{s\sim\pi}[(1-\theta)b]/\mathbb{E}_{s\sim\pi}[b]$ is the callable share by *amounts* at the beginning of the quarter.

E.1.4 Callability

The callable share rate measures the proportion of a firm's debt that is callable, reflecting the firm's reliance on callable debt instruments for financing flexibility. I consider the average value of the optimal share of callable within the solution of the model.

$$\textbf{callable share rate} \;:\; (1-\theta^*) = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} \left(1-\theta'\left(z,r,k,b,\theta,c\right)\right) \tag{A12}$$

where K, B, Θ, and $\bar{C}$ are the state spaces for respectively the state variables k, b, θ, and c.

I also track the issuance-based callable share by amounts,

$$\widehat{\mathbf{cc}}^{\text{iss}} := \frac{\mathbb{E}_{s\sim\pi}[(1-\tilde{b}(s)) \cdot \text{GI}(s)]}{\mathbb{E}_{s\sim\pi}[\text{GI}(s)]}, \tag{A13}$$

where $\tilde{b}(s)$ is newly issued debt and $\text{GI}(s) = \max\{b'(s) - b_{\text{old}}(s), 0\}$ is gross issuance implied by policies.

The decision to call the bond is based on comparing the value of calling vs. not calling, considering the cost of paying the call premium and the potential gains from refinancing. Therefore, the average call rate quantifies the frequency with which firms exercise their option to call debt before maturity, influenced by factors such as interest rate movements, debt levels, and firm productivity.

$$\textbf{average call rate} \;:\; q^* = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} \mathbb{1}'_{\text{call}}\left(s', \hat{x}(s,x)\right) \tag{A14}$$

where $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}\left(s', \hat{x}(s,x)\right)$ is the indicator of the call decision the next period, and $\hat{x}(s,x)$ is the vector of policy functions. This formula ensures that the call rate reflects only those bonds eligible for calling. I separately report the scheduled metrics $(s^{\text{sched}}, c^{\text{sched}})$ defined above, which map into the duration calculation.

E.1.5 Investment

The investment rate reflects the proportion of a firm's earnings allocated to capital investments, influenced by factors such as profitability, fixed operating costs, and available financing.

$$\textbf{Investment Rate} = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} I(z,r,k,b,\theta,c) \tag{A15}$$

Where $I(z, r, k, b, \theta, c)$ denotes the firm's investment decision function. This moment serves to control the aggregate investment behavior of firms, and the capital accumulation.

E.2 Solving the Model

The components of the model solution are the value functions, the bond price schedules, and the firm-level policies on k', b', θ', c'. To solve it, I use dynamic programming and numerical algorithms. The procedure iterates on the value functions and the price functions until the convergence. I start by discretizing the state space. Using the method in Tauchen (1986), I transform the AR(1) process of the productivity z into a 25-point grid, and the interest rate turns into a 10-point grid. For other state variables, I set a 25-point grid for bonds b, a 10-point grid for non-callable share θ, and a 10-point grid for the periodic coupon c.

At every step of the algorithm, I use interpolation on b, θ, c grids to approximate the policy functions. The main algorithm to solve the model works through the following steps.

1. I start with the finite-horizon (T periods) version of the model. I set the final value and price to be null, $V_{T+1} = 0, p = 0$.

2. I set the initial guesses for value and price functions, by solving the period $T - 1$. In this case, I set the policy on new debt issued such as only non-callable bonds are issued $\tilde{\theta}_T = 1$), and the parameter of debt repayment at $\gamma = 1$, i.e. full repayment at the final period of non-callable bonds and unmatured and uncalled callable bonds. The following lines detail this step:

   (a) Guess of capital k_T, deduction of the labor supply, and using labor market clearing, get the wage. Guess future non-callable bonds share $\tilde{\theta}$ (for the period T, $\tilde{\theta}_T = 1$), and compute the periodic coupon for the next period, given by (35) and (36).

   (b) Using these values, for each point on the space state grid, solve the two optimization problems in (43), subsequently the call decision problem in (41), then in (39), and the optimal value in (37). To do it, first, given the guessed k_T and continuation value of decision variables, I compute the default $\overline{\varepsilon}$ given by (38) and the default probability. Second, I compute the prices p^{nc}, $\overline{p}_j^{nc}$, p^c, and $\overline{p}_j^c, j \in \{\text{nocall}, \text{call}\}$, using (45)-(49) through inner loop-outer loop procedure.

   (c) Based on these objects, I optimize the firm objective function to obtain the optimal firm's decision k, b, θ, c and the prices' schedules.

   (d) The continuation values and prices obtained here serve as the new guesses for the rest of the algorithm.

3. For each period $t < T-1$, I use these guesses as the continuation values $\overline{V}_j$ and prices p^{nc}, $\overline{p}_j^{nc}$, p^c, and $\overline{p}_j^c$ for the rest of the algorithm.

4. The solution is found if the distance between the new and previous continuation values $\overline{V}_j$, and the distance between the new and previous prices p^{nc}, p^c are below the tolerance level (set at 10^{-6}). If this criterion is not satisfied, I get back to step (2) using these new $\overline{V}_j$ and prices p^{nc}, $\overline{p}_j^{nc}$, p^c, and $\overline{p}_j^c$ as the new continuation values and prices.