Debt Callability and Firm Dynamics *

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Job Market Paper

July 15, 2025

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Abstract

Callable debt - a bond that allows the issuer to "redeem" or "call" the bond before its maturity - has become a dominant feature in the U.S. corporate bond market, particularly after the Global Financial Crisis. This paper examines the dynamic interaction between corporate bond callability and firm decisions. Using a comprehensive dataset spanning firm- and bond-level data, I find that firms with higher credit risk tend to issue more callable bonds. To capture these dynamics, I develop a heterogeneous firm model with investment decisions, non-callable and callable debt, call options, and default risk. The model shows that smaller firms and those with higher credit risk are more inclined to issue callable bonds but tend to call them less frequently. However, the share of callable bonds amplifies a firm's response to declining interest rates. Smaller firms, with a higher share of callable bonds, show a much stronger reaction when they call in response to such rate declines. The investment reaction to a 7 percentage points decline in interest rates is 27% higher compared to the baseline economy without callable debt, highlighting the amplifying effect of callable bonds on firms' sensitivity to monetary policy. Callable debt amplifies investment behavior by around 21%.

Keywords: Debt Callability, Financial Frictions, Firm Investment, Firm Heterogeneity

JEL Codes: E22, E44, G31, G32, O16

* I am extremely grateful to Immo Schott and Hyejin Park for their invaluable, continuous guidance and support. I thank my committee members, Réné Garcia and Guillaume Sublet, for their insightful comments and suggestions. For the helpful discussions, I also thank Firmin Ayivodji, Louphou Coulibaly, Miguel Faria-e-Castro, Bruno Feunou, Félicien Goudou, Jihyun Kim, Marlene Koffi, Julian Kozlowski, Ricardo Marto, Hannah Rubinton, Juan M. Sánchez as well as participants at various seminars and conferences at the University of Montreal, the Annual Congress of SCSE, the CIREQ Ph.D. Students' Conferences, the 57th and 58th Annual Conferences of the CEA, and the Federal Reserve Bank of St. Louis. I acknowledge financial support from the FRQSC grant and Chaire de la fondation J.W. McConnell en études américaines. All errors and shortcomings in this paper are mine.

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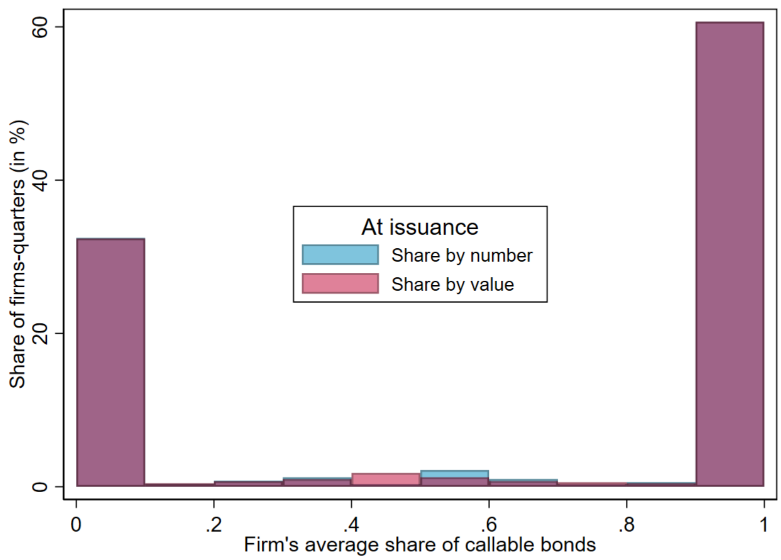
1 Introduction

Firms' balance sheets are crucial in explaining business cycles and investment dynamics.[1] Understanding how firms manage their capital structure is essential to study their behavior and its macroeconomic implications.[2] While extensive research has focused on corporate debt's composition and maturity structure, the callable nature of certain bonds—where issuers retain the option to redeem the debt before maturity—remains underexplored. [3]

Figure 1: Distribution of Callable Share in U.S.



Notes: This figure overlays two distributions of the firms' average share of callable bonds. The first (in blue) concerns the number of callable bonds, and the second (in red) shows the offering amount. The sample period is 1990Jan - 2018May.
Source: Mergent FISD & CRSP-Compustat.

Figure 1 illustrates the distribution of callable bonds across firms in the U.S. corporate sector. Notably, callable bonds are prevalent but unevenly distributed across firms. The higher prevalence in terms of offering amounts suggests that callable bonds are more common among more significant debt issuances, potentially reflecting the preferences or financial strategies of firms with access to capital markets. With this distribution, I examine how the callable structure of a

[1] An extensive macroeconomic literature starting from Bernanke and Gertler (1989); Kiyotaki and Moore (1997) has shown the importance of firms' balance sheets for understanding macroeconomic business cycles. Gomes (2001); Cooley and Quadrini (2001) has found that financial frictions firms face account for their investment behavior and dynamics.

[2] Leland and Toft (1996); Covas and Haan (2011); Jermann and Quadrini (2012) are among seminal papers that stress the importance of the dynamic choice of firms' capital structure.

[3] The literature in macroeconomics and finance research about firms' debt maturity includes: Demirgüç-Kunt and Maksimovic (1999); Gomes et al. (2016); Gürkaynak et al. (2022); Fabiani et al. (2022).

firm's debt interacts with its optimal choices by posing the following questions: Under what conditions do firms choose to issue callable bonds, and how does this choice vary across firms with different credit risks or sizes? How does the call option influence firms' borrowing and investment decisions? What factors influence a firm's decision to redeem debt before maturity? These questions form the foundation of this study. Empirically, I document that smaller firms and those with higher credit risk tend to issue more callable bonds but call them less frequently. Theoretically, I develop a dynamic firm financing model incorporating debt callability as an endogenous choice, linking firms' issuance and call decisions to their investment and refinancing behavior.

To address the research questions empirically, I analyze balance sheet data from publicly listed firms in the United States spanning January 1990 to May 2018, supplemented with bond-level data. The empirical analysis reveals the following key findings: (i) On average, 62% of the value of corporate bonds offered in the U.S. bond market are callable, with the prevalence of callable bonds increasing significantly—from 49.54% before the Great Financial Crisis to 95.5% in the post-recession period. (ii) When sorting issuers into five size categories based on their credit ratings, firms with low credit ratings are more likely to issue callable bonds. These firms generally have smaller assets than firms that issue less than half of their bonds as callable. (iii) Across firms, 47% of callable bonds are effectively redeemed before maturity, highlighting the call option's significant role in shaping the firm's decision and debt maturity strategies within the standard model of corporate debt. (iv) Several factors influence a firm's decision to call a bond, including interest rate movements, changes in credit risk, financial performance, firm size, and liquidity ratios.

In a theoretical framework, I first use a three-period model with an endogenous choice of investment, default, callable share, and call decision (in the intermediate period) to derive analytical solutions and illustrate intuitions. As in data, the model predictions say that less productive firms with higher credit risk issue more callable debt. These are partially in line with findings in the literature (e.g. Gilchrist and Zakrajšek, 2012; Clymo and Rozsypal, 2023; Poeschl, 2023; Kochen, 2023).[4] In a quantitative model of firm dynamics, I examine optimal decisions and implications of debt callability for firms and in aggregate.

I build the quantitative model with several innovative features. First, I allow firms to issue non-callable and callable debt simultaneously, whereas the existing literature generally treats debt as uniquely non-callable. Second, debt callability is treated as an endogenous choice. Thus, the types of debt are not issued in fixed proportions. Third, the exercise of the call option is also an endogenous decision. This means the probability of calling callable bonds each period is not exogenously fixed. By doing so, my model allows this probability to influence the pricing of both

[4] Other papers show that small firms rely less on debt than equity, but during economic booms, these firms increase both debt and equity financing, reflecting their need for external funds to fuel growth opportunities, see e.g., Begenau and Salomao (2019).

callable and non-callable bonds dynamically.[5] With this feature, I argue that the share of bonds that can be called at every period changes because of firm-level and aggregate shocks, while the choice of debt's callable structure reflects the trade-off between default risk and refinancing flexibility (e.g., Guntay et al., 2004).

An additional contribution lies in my model's distinct approach to debt pricing. Specifically, the continuation price of new debt considers the current interest rate, directly affecting future coupon payments. Conversely, the continuation price of outstanding debt is anchored to the fixed coupon rate set at issuance. This distinction accurately reflects the contractual obligations for each debt class, capturing creditors' expectations under different interest rate environments. By distinguishing these pricing characteristics, my model provides a more realistic depiction of firm debt dynamics, accommodating creditor heterogeneity that arises from separate pricing rules for new and existing debt.[6] Lastly, instead of treating net worth as a firm-specific variable alongside productivity, I distinguish both types of bonds separately in the structure of firms' capital as they have neither the same law of motion nor the same pricing characteristics.[7] While non-callable bonds continue their path until maturity, callable bonds face the potential of being called within the allowed period. Then, I incorporate periodic stochastic coupons into firms' state variables to account for changing interest rates carried by new debt issued. These elements capture the complexity and dynamics of firms' capital structure decisions, but the downside of these innovations is the increase in computational complexity (e.g., Berndt, 2004; Jarrow et al., 2010).

Over the years, advancements in the financial industry have led to a proliferation of financial instruments designed to facilitate financing for firms and households. Typically, bond funding has grown significantly in the last decade (Jungherr and Schott, 2021; Darmouni et al., 2022). As such, it is of particular interest to examine the potential impacts and implications of this growth on our understanding of the transmission of policies, especially monetary policy. One notable feature of bonds is the call option, which allows for the issuer's early redemption of a bond. To enrich our knowledge of the implications of firms' debt heterogeneity, we need to study key characteristics of firms' debt. Then, my paper shows why it may be essential to consider debt callability, which significantly impacts the behavior of firms and their optimal decisions. Callable bonds differ from short-term bonds because they typically have a more extended maturity period. Additionally, callable bonds offer a higher yield to compensate for the added risk of early redemption by the

[5] The firm's call policy has been previously modeled from the market perspective through a stochastic intensity process (see Guntay, 2002; Jarrow et al., 2010, where the call intensity does not result from an endogenous firm's decision. In Leland and Toft (1996); Goldstein et al. (2001); Chen et al. (2021), firms can restructure their capital by calling entirely the existing debt.

[6] This approach contrasts with standard models that assume a unified continuation price, which may overlook contractual rigidity in outstanding debt.

[7] In the model of callable bond valuation from seminal paper Duffie and Singleton (1999), the firm's balance sheet is not considered in the decision to call. See Jarrow et al. (2010) for an extension of their approach.

issuer. In contrast, short-term bonds typically provide lower yields due to their shorter maturity and lower risk. On the other hand, callable bonds differ from long-term bonds in that the issuer can redeem them before maturity, whereas long-term bonds are held until their maturity date. This early redemption feature of callable bonds adds an element of uncertainty, making them distinct from traditional long-term bonds. Callable bonds are a middle ground between short-term and long-term bonds, offering a particular combination of yield and risk that sets them apart from both.

The callability of bonds presents distinct trade-offs for firms and investors. For investors, callable bonds introduce prepayment risk, which is compensated through adjustments in bond pricing to reflect the call option's expected value. For firms, callable bonds offer greater flexibility by allowing them to refinance or deleverage when interest rates decline, potentially reducing their overall cost of debt. This refinancing flexibility makes callable bonds an attractive alternative to non-callable bonds, replacing rollover risk with refinancing risk (Guntay et al., 2004). Moreover, the uncertainty surrounding future interest rates significantly affects bond prices, with callable bonds being more sensitive to changes in interest rate term structures and volatility compared to non-callable bonds (Duffee, 1998; Gilchrist and Zakrajšek, 2012). This responsiveness underscores the strategic importance of callability as a risk management tool and a determinant of firms' capital structure decisions.

Firms' behavior may not be the same depending on the callability of their debts. Callable bonds serve as a hedging tool against rising debt costs, allowing firms to refinance when conditions are favorable. Firms holding callable bonds in their debt portfolios typically exercise the call option during episodes of declining interest rates or improved credit risk, as highlighted by Becker et al., 2024. Conversely, when interest rates rise, firms are less likely to call their bonds, holding callable debt until maturity. This dynamic influences not only firms' leverage structures but also their risk-taking and investment decisions, which respond to aggregate shocks and firm-specific factors.

An important finding of this paper is that firms with low credit ratings tend to rely more heavily on callable bonds but call them less frequently. This behavior can be attributed to refinancing costs associated with potential rollover risk. For low-credit firms, the cost of refinancing callable bonds—especially the premium paid upon calling—often outweighs the benefits of reduced debt costs, particularly if market conditions fail to improve. Moreover, these firms face greater constraints in accessing favorable terms for new debt issuance than larger firms with better credit ratings. As a result, low-credit firms are more likely to retain callable bonds in their portfolios for extended periods, reflecting a choice to preserve liquidity and mitigate refinancing risks. This finding shows the nuanced relationship between credit risk, callable debt usage, call decisions, and firm's decisions.

Related Literature. My paper is anchored into the broad literature that looks for the causes and consequences of firm heterogeneity and the various implications of financial frictions for the macroeconomy. As shown in Brunnermeier and Krishnamurthy (2020), corporate debt and its aggregate implications for the macroeconomy started receiving great attention in the theoretical and empirical literature. Here, I present and discuss how my work is connected to several works in this literature, mostly works that are in the strand of the literature around the heterogeneity in corporate debt (structures) and its macroeconomic implications. I subdivide the following literature into two parts.

First, an exciting part of my related literature considers macroeconomic models in which corporate debt choices are incorporated, and then aggregate shocks are studied. This part starts from DSGE models with the famous works of Kiyotaki and Moore (1997) and Bernanke et al. (1999). More recent works in the literature, especially those on monetary policy transmission and financial crisis, use heterogeneous firms' models with financial frictions. De Fiore and Uhlig (2015) use a DSGE model to explain the importance of firms' ability to substitute between bank loans and bond debts and the banks' flexibility in mitigating adverse real effects of the financial crisis. In their model, firms choose either uniquely bonds or uniquely loans to be financed. Beyond this consideration, the paper of Crouzet (2018) considers firms with access to public debt markets that chose a mix of their external funds. Among others, prominent examples that develop interesting theoretical approaches work on the interactions between the composition of corporate debt and the firm's investment decisions. These examples include Martellini et al. (2018), Crouzet (2018), Salomao and Varela (2022), Arellano et al. (2019), Jeenas (2024), Arellano et al. (2020) Jungherr and Schott (2021), Ottonello and Winberry (2020).[8] My paper steps out in this part of the literature by adding the endogenous choice of debt callability structure.

Second, I complement works that document the role and the importance of bond callability in firm dynamics from a corporate finance view. This literature studies the reasons for issuing callable bonds; see, for example, Elsaify and Roussanov (2016); Chen et al. (2010); Banko and Zhou (2010). Elsaify and Roussanov (2016) used the same data as the present paper and found that firms use callable bonds to handle refinancing risk. Chen et al. (2010) provide empirical evidence suggesting that callable bond issuers tend to call back their bonds when they have better performance and the highest investment activity. Banko and Zhou (2010) found that firms issue callable bonds with under-investment problems and information asymmetry. Their paper focused on a decade before the Global Financial Crises (2008 - 2009). The literature has developed models incorporating factors influencing bond pricing and firms' issuance decisions. For instance, Acharya and Carpenter (2002) provides a theoretical framework for corporate bond valuation and examines the

[8]These papers provide attractive framework to quantify the transmission of financial shocks on aggregate investment through corporate debt choices or debt heterogeneity.

effects of default and call risk on durations using an option-based valuation model for coupon-bearing callable, defaultable bonds. They show that default risk reduces bond duration, and a call option reduces bond duration without default risk. These results are empirically tested and confirmed in Xie et al. (2009). Their framework inspired my work by introducing the endogenous choice of callable debt structures to examine their implications for firm dynamics and investment decisions.

A paper by Xu (2018), close to mine, states that early refinancing adjusts firms' maturity structure and reduces interest payments. The paper finds that speculative-grade firms are more likely to have and use callable bonds contrarily to investment-grade firms because of their higher exposure to refinancing risks. Becker et al. (2024) offer new insights on the callability features of bonds. They show that longer maturity and lower quality bonds will likely be issued with a callable feature. Call features reduce debt overhang by decreasing the value gains to the lenders. They also explain that callable bonds have significantly higher yields and lower secondary market prices than non-callable ones: the cost of callability. In the same logic, Flor et al. (2023) examines the choice between these callable debts and the convertible ones.[9] By incorporating an investment choice in a dynamic capital structure theory model, they show that firms that are more exposed to debt overhang issue callable rather than convertible bonds, and also if bonds have covenants attached. I extend this above literature with two additional ingredients: the endogenous default risk of the issuer and a more extensive set of information on both issuers and issuance characteristics.

Lastly, Gilchrist and Zakrajšek (2012) examines the influence of interest rate changes, term structure shape, and interest rate volatility on credit spreads of callable bonds. They find that an increase in general interest rates and a steeper Treasury term structure narrow the spreads of callable bonds. At the same time, higher volatility in long-term Treasury yields widens these spreads. Their results indicate that callable bond spreads are significantly affected by the shape of the term structure and interest rate volatility, with call options attenuating the impact of default risk compared to non-callable bonds.

While existing studies emphasize the significance of considering the role of callable debt in corporate financing (e.g., Elsaify and Roussanov, 2016; Becker et al., 2024), there is a notable gap in understanding how the endogenous decision to issue and call such debt interacts with firms' investment behavior and default risk. Specifically, the joint effect of callability and default risk on firm dynamics and their macroeconomic implications remains underexplored. This paper aims to address this gap by theoretically and empirically investigating these interactions. Since the changes in the risk-free rates can occasion additional uncertainty in the bond pricing in the pres-

[9] These bonds offer investors the possibility to convert the bonds into a predetermined number of common stock or equity.

ence of callable bonds, many (almost all) of the above studies usually confine their focus to the analysis of non-callable bonds.[10] Also, because the callable bonds accounted for an average of at least 50% of the corporate debt for the last decades, it is too much reducing and limiting to shed light on the firm heterogeneity implications for the macroeconomy by using just non-callable bonds. In that sense, I contribute to this relatively overlooked part of the literature by revealing the callability of corporate debt as a new potential source of interactions between corporate debt heterogeneity and the macroeconomy.

Layout. The rest of the current paper is organized as follows. Section 2 describes the data, the growing interest for callability in the growing importance of bond financing, and facts on callability linked to firms' characteristics. Section 3 presents a three-period model, allowing me to characterize the mechanisms under the role of callable bonds in firm decisions. Section 4 sets up a general equilibrium model with firm investment, borrowing, callability choice, default, and call decision. Section 5 presents the model parametrization, its quantitative results, and the role of debt callability in cross-sectional and aggregate investment dynamics. I conclude in section 6.

2 Data and Empirical Facts

This section describes the dataset used in my empirical analysis, the data sources, and the methodology employed to construct the sample. I also present empirical facts on the growing importance of callable bonds in corporate financing and the relationship between callability and firm characteristics.

2.1 Data Sources and Sample Construction

I use three different data sources to investigate the debt callability of non-financial firms in the U.S. The data sources and sample construction procedures are detailed below.

2.1.1 Mergent Fixed Income Securities Database (Mergent FISD)

The Mergent Fixed Income Securities Database (Mergent FISD) serves as the primary data source for this study. This comprehensive database provides detailed information on various corporate bonds, including issuance date, original issuance amount, callability, convertibility, presence of covenants, and other bond-specific attributes. I focus on data from January 1990 to May 2018 to match the data availability from other sources used in this research.

[10] The literature also recognizes that callability structure adds some new complexity to the -already complex- analysis of corporate debt structure and its implications for the macroeconomy (See, e.g., Duffee, 1998; Gilchrist and Zakrajšek, 2012).

Mergent FISD also includes historical data on changes that occurred throughout the life of each bond, enabling me to track the outstanding amount and the callable status of each bond at any given time through the protection period.[11] By filtering the data to include only U.S. corporate bonds issued by non-financial firms, I obtained a relevant sample for analyzing debt callability in this context.

2.1.2 Compustat and CRSP

To supplement the bond-level data, I use Compustat, which offers financial information on publicly listed firms in the U.S. Compustat provides quarterly data on over 60 firm-specific variables, including financial positions, balance sheet information, and other relevant attributes.

Following standard practice in the literature, I exclude utilities (Standard Industrial Classification - SIC codes 4900–4999), financial firms (SIC codes 6000–6999), and other firms with SIC codes greater than 8999 from the sample. Additionally, I require firms in the final sample to have total debt representing at least 5% of their assets and winsorize all variables at the 1% and 99% level to mitigate the influence of extreme values.

Lastly, I supplement the dataset with stock price data from the Center for Research in Security Prices (CRSP). This data source provides valuable information on stock prices and other stock-related variables for the firms in the sample.

2.1.3 Merging Data Sources

I merge the bond-level data from Mergent FISD with the firm-level data from Compustat and CRSP using the CUSIP identifier, uniquely identifying each firm or issuer. The resulting dataset offers insights into the relationship between firm characteristics and the degree of callability of their debts, providing a solid foundation for the empirical analysis of debt callability.

By combining these three data sources and constructing a comprehensive dataset, this research seeks to understand better the role of debt callability in the financial landscape of non-financial U.S. firms. This rich dataset enables a thorough examination of the factors that influence firms' decisions to issue callable debt and the impact of callability on firm behavior and investment.

2.2 Empirical Facts

In this subsection, I explore the empirical facts related to callable bonds and their role in corporate financing and firm dynamics. I provide a more detailed analysis of the prevalence of callable bonds,

[11]The period between issuance of the bond and the (first) potential call date. It is also called the lockout period.

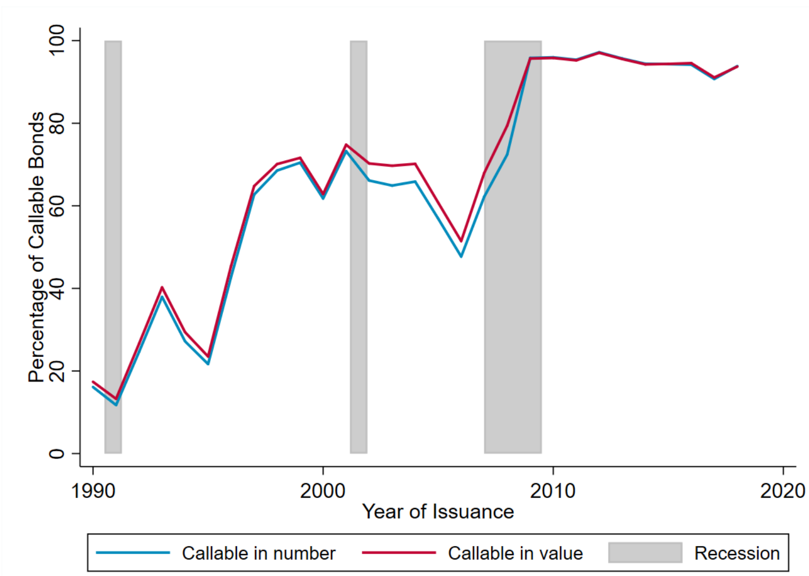
firm characteristics associated with callable debt, and the exercise of call options.

2.2.1 The prevalence of callable bonds in the U.S.

Over the last two decades, bond financing, particularly callable bonds, has gained increasing importance in firms' funding strategies. My analysis of the data reveals that 62.11% of the bonds issued are callable. Before the Global Financial Crisis (GFC) of 2007-2009, this percentage stood at 49.54%. During the GFC, the percentage of callable bonds increased to 78.22%, and after the GFC, the prevalence of callable bonds surged to an impressive 95.54%.

Furthermore, when considering the total value of the bonds issued, callable bonds comprise 66.1% of the overall amount. Before the GFC, this percentage was 54.01%. During the GFC, the percentage of callable bonds in terms of value skyrocketed to 85.12%. After the GFC, this percentage only marginally decreased to 95.48%.

Figure 2: Prevalence of Callable Bonds over Time



Notes: This figure displays the percentage of bonds issued with a call option, regarding the number of bonds issued (blue) and the value issued (red). For each year of issuance, I compute these two prevalences. The shading bands indicate recession periods. The sample period is 1990 Jan - 2018 May. Source: Mergent FISD; Fed Data.

I calculated the prevalence of callable bonds by year, computing the percentage of callable bonds concerning both the number of bonds issued and the total value of bonds issued. The resulting graph, Figure 2, displays the prevalence of callable bonds over time, with the area representing the recession periods (1990-1991, 2001, and 2007-2009). The graph shows that the percentage of callable bonds has generally increased over time, particularly after the GFC. In addition, the prevalence of callable bonds in terms of value is consistently higher than the prevalence in terms of the number of bonds issued, suggesting that callable bonds tend to be larger (See the last facts of this section.).

This fact reveals that callable bonds are prevalent in the corporate bond market, increasing over time, particularly after the GFC. The higher prevalence of callable bonds in terms of value compared to the number of bonds issued suggests that callable bonds are more common among larger bond issuance. The shift from bank loans to the bond market during the crisis, as documented by Crouzet (2018), has led to continued growth in the issuance of callable bonds, reflecting their attractiveness as a funding option for firms in the post-crisis environment.

The analysis also reveals that callable bond issuance varies across industries (See Figure A1.). The use of callable bonds is more prevalent in certain industries, such as Services; Agriculture, Forestry, and Fishing; Manufacturing; and Transportation, Communications, and Utilities. These industries tend to have higher capital expenditure requirements and more volatile cash flows, which may explain their preference for callable debt, as it provides greater flexibility in managing their financial obligations.

2.2.2 Callability and Firm characteristics

My analysis reveals several key insights about the characteristics of firms that issue callable bonds. The majority of the variables exhibit statistically significant differences in means, which indicates a clear distinction between firms that issue more than 50% of their bonds as callable and those that issue less than 50% as callable. The exceptions are leverage and market-to-book ratio, which did not show a significant difference between the two groups of firms.

Firms with a majority of callable bonds tend to have smaller assets than firms with less than half of their bonds issued as callable. This suggests that smaller firms may be more inclined to issue callable bonds to take advantage of the flexibility in managing their debt and refinancing options. These firms are presented as those with the most intensive need for capital structure management. Interestingly, these firms exhibit lower total investment levels, perhaps reflecting a more cautious approach to capital investments. While callable bonds can allow firms to refinance their debt at a lower interest rate if market conditions improve, it appears that firms with a higher proportion of callable bonds have a lower investment rate. This suggests that they invest a smaller proportion of their assets than firms with less than half of their bonds issued as callable. These firms may value the financial flexibility that callable bonds provide, potentially using them as a tool to manage their financial risk and sustain their operations rather than aggressively pursuing new investments or expansion. This is supported by the Figure 3.

Return On Assets (Net Income/Assets): Firms that issue most callable bonds have higher prof-

Table 1: Firms Characteristics over Callability Profile

	(1)	(2)	(3)	(4)
	All	Callable Profile	Ncallable Profile	p-value (2) = (3)
Size (Log of Real Total Assets)	8.98	8.86 (8.87)	9.26 (9.23)	0.00 (0.00)
Investment rate (%)	2.24	2.31 (2.31)	2.06 (2.06)	0.00 (0.00)
Leverage (% debt/assets)	32.89	34.09 (34.0)	30.23 (30.37)	0.00 (0.00)
Return On Assets (%)	1.58	1.60 (1.60)	1.55 (1.54)	0.01 (0.02)
O. Return On Assets (%)	8.98	9.18 (9.17)	8.54 (8.55)	0.00 (0.00)
Cash (% Assets)	11.1	10.98 (11.02)	11.36 (11.28)	0.00 (0.00)
Sales (Log - Real)	7.32	7.21 (7.22)	7.55 (7.53)	0.00 (0.00)
Market to Book Ratio	3.46	3.71 (3.70)	2.90 (2.92)	0.59 (0.61)
Market Capitalization	9.85	9.72 (9.73)	10.16 (10.13)	0.00 (0.00)
Tobin's Q	1.94	1.95 (1.95)	1.93 (1.94)	0.00 (0.14)
Credit Rating (Firm level)	14.26	13.66 (13.7)	15.40 (15.37)	0.00 (0.00)
Interest Coverage Ratio	16.03	15.38 (15.39)	17.44 (17.45)	0.02 (0.02)
Share of callable (% bonds)	84.68	96.08 (95.95)	59.31 (58.94)	0.00 (0.00)

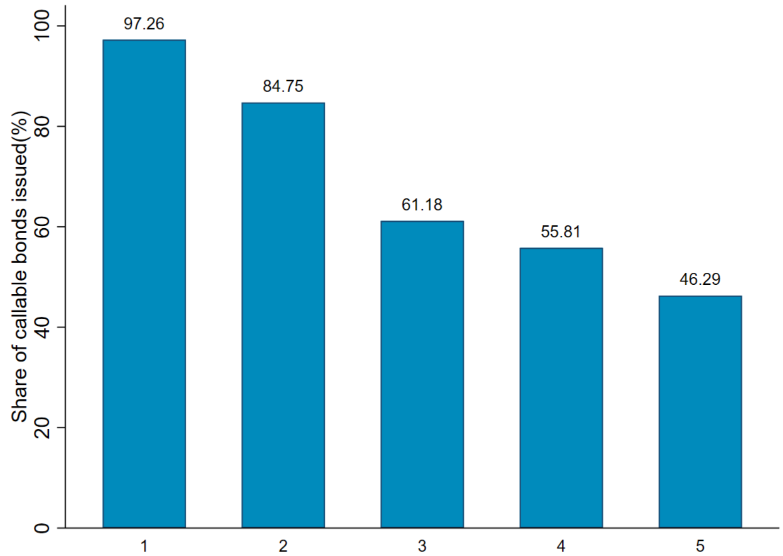
Notes: This table reports means of key firm characteristics for all non-financial firms and with Callable Profile and Non-Callable Profile, respectively. A firm has the Callable Profile if its share of callable bonds is >=0.5. The sample period is 1990 Jan - 2018 May. The last column presents the p-value for tests of equality of means (columns 2 and 3). Credit Downgrade is a dummy variable which is one if, on average, the credit rating of the firm degrades, is zero otherwise. Source: Mergent FISD & CRSP-Compustat.

itability than those that issue less than half of their bonds as callable. This suggests that the decision to issue callable bonds can be driven by better management practices and more efficient production processes, especially risk exposure management, which allow callable majority firms to achieve better financial performance. Firms that predominantly issue callable bonds exhibit a higher operating return on assets (OROA - EBIT/Assets) than non-callable majority firms because of their financing flexibility. Callable bonds allow more flexibility, resulting in lower financing costs and interest expenses for callable majority firms, ultimately improving their operating efficiency and increasing their OROA. In contrast, firms with a small share of callable bonds do not have this flexibility. They may be locked into higher interest rates over the life of their bonds, which can negatively impact their OROA.

Firms with high shares of callable bonds have greater cash holdings, which might indicate a more conservative approach to managing liquidity risks. Callable bonds allow firms to refinance their debt when favorable market conditions enable them to maintain higher cash reserves.

Credit Downgrade and Interest Coverage: Firms that issue a majority of callable bonds are more likely to experience credit rating downgrades, as evidenced by the higher percentage of credit downgrades in this group. Additionally, these firms have a higher interest coverage ratio, which suggests they face more significant refinancing risks. This finding indicates that callable

Figure 3: Share of callable bonds by firms credit risk



Notes: This figure displays the share of callable bonds issued by class of firm size, class 1 is for the 20% low-ranked firms, and class 5 represents the top quintile of higher-ranked and large firms. The sample period is 1990 Jan - 2018 May.
Source: Mergent FISD.

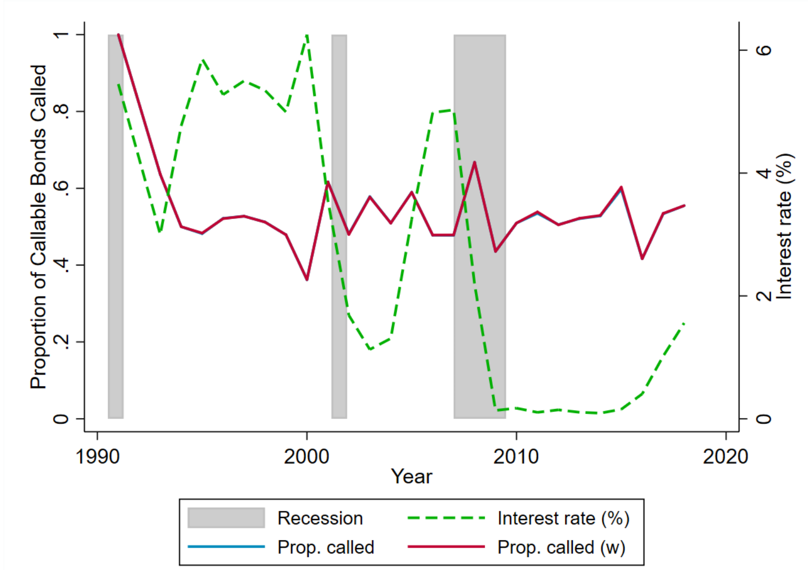
bonds may be a risk management tool for firms with higher credit risk or more volatile cash flows.

In summary, the results from the table reveal several key insights about the characteristics of firms that issue callable bonds. Firms with the most shares of callable bonds tend to have smaller assets, smaller investment levels, and greater profitability than firms with less than half of their bonds issued as callable. These firms may be more inclined to issue callable bonds due to their flexibility in managing debt and refinancing options. Additionally, firms with high share callable bonds have higher cash holdings and face a higher likelihood of credit downgrades and refinancing risks, suggesting that callable bonds serve as a risk management tool for these firms. This also suggests that these firms will face difficulty calling until their credit profile improves significantly; see the following subsection.

2.2.3 Callable bonds effectively called

A significant proportion of callable bonds are effectively called overall, with 55.69% of callable bonds called before their maturity date, which represents 48.1% of the total amount of callable bonds. Callable bonds that are called tend to have smaller offering amounts compared to those that are not called. The strategic decision to call a bond may be driven by factors such as changes in interest rates, credit risk, and financial performance. Figure 4 illustrates the decreasing proportion

Figure 4: Proportion of Callable Bonds Called vs Interest rates



Notes: This figure displays the proportions of callable bonds called, with shading to indicate recession periods and interest rates in green. The Prop. called (w) is the proportion calculated when weighing the bonds called by the amount called. The sample period is 1990 Jan - 2018 May.
Source: Mergent FISD & CRSP-Compustat; Fed Data.

of bonds called over time, with significant shifts occurring around 1993 and during the financial crisis of 2007-2008. Throughout the 1990s, the average proportion of called bonds was 57.92% (49.88% when weighted by the amount). This figure dropped to 47.6% (39.57% weighted, so the value of callable bonds called during the crises significantly dropped relative to the number of bonds) during the crisis period, which spanned from December 2007 to June 2009. In the post-crisis period (after June 2009), the proportion of bonds called decreased further to 44.21% (38.62% weighted on average). Additionally, on average, at the moment they are called, callable bonds have lived 48.56% of their lifespan; see the trend in Appendix A2.

The fluctuating proportion of called bonds over time can be attributed to several factors. One such factor is the influence of higher interest rates, which make it less appealing for firms to call bonds and refinance at increased borrowing costs. The peak in the proportion of called bonds in 1993 is likely due to a combination of factors, including economic conditions, changes in interest rates, and market behavior at the time. In the early 1990s, the U.S. economy underwent a period of growth following a recession in the early part of the decade. This economic recovery may have prompted companies to refinance their debt by calling existing bonds and issuing new ones at lower interest rates. Interest rates were relatively high during the early 1990s but began to decline in subsequent years. This decrease in interest rates made it more attractive for companies to call their bonds, as they could refinance their debt at lower costs. In 1993, the Federal Reserve initiated a reduction in interest rates, which may have contributed to the peak in the proportion of called bonds that year. Decreasing interest rates increased demand for companies to refinance their outstanding debt, resulting in increased bond calls.

Additionally, market behavior and regulatory changes may have affected the fluctuating proportion of called bonds. The bond market in the early 1990s was characterized by high volatility and uncertainty, which could have influenced companies to call their bonds to manage risks or capitalize on favorable market conditions. The peak in 1993 could also result from specific events or trends in the bond market that led to an increase in the proportion of called bonds.

The decreased proportion of called bonds during the financial crisis can be attributed to the turmoil in financial markets and tighter credit conditions. Companies were more cautious about refinancing their debt during this period, facing higher borrowing costs and greater uncertainty. In the post-crisis period, central banks worldwide implemented various monetary policies to stimulate economic growth, leading to a prolonged period of low interest rates. This environment made it less attractive for companies to call their bonds, as the potential benefits of refinancing their debt were diminished due to the historically low interest rates. This shift in monetary policy and market conditions contributed to the decline in the proportion of bonds called during the post-crisis period.

2.2.4 The decision to call

Now, to investigate the probability of a bond being called, I estimate a panel logistic regression model to analyze the likelihood of its issuer calling the bond while accounting for unobserved firm-specific effects.

The logit regression model can be specified as follows:

$$\log\left(\frac{\mathbb{P}(\text{called}_{it}=1)}{1-\mathbb{P}(\text{called}_{it}=1)}\right) = \beta_0 + \beta_1 \cdot \Delta rate_t + \theta' \cdot \Gamma_{it} + \mu_i + \varepsilon_{it} \tag{1}$$

My dependent variable is a binary indicator, called_{it}, which equals one if the bond i is called in the quarter t and 0 otherwise. The independent variables include factors that may influence the probability of a bond being called, such as interest rate changes in the quarter t, $\Delta rate_t$, and the vector Γ_{it} of variables relative to the issuer of the bond i, such as credit risk improvements, financial performance improvements, firm size increases, leverage, liquidity ratio increases, and offering amount. The interest rate changes represent fluctuations in macroeconomic conditions. I also consider TFP improvement for idiosyncratic productivity. The leverage reflects the impact of the issuer's debt level on the likelihood of a bond being called. The liquidity ratio measures the issuer's ability to meet short-term obligations using its most liquid assets.

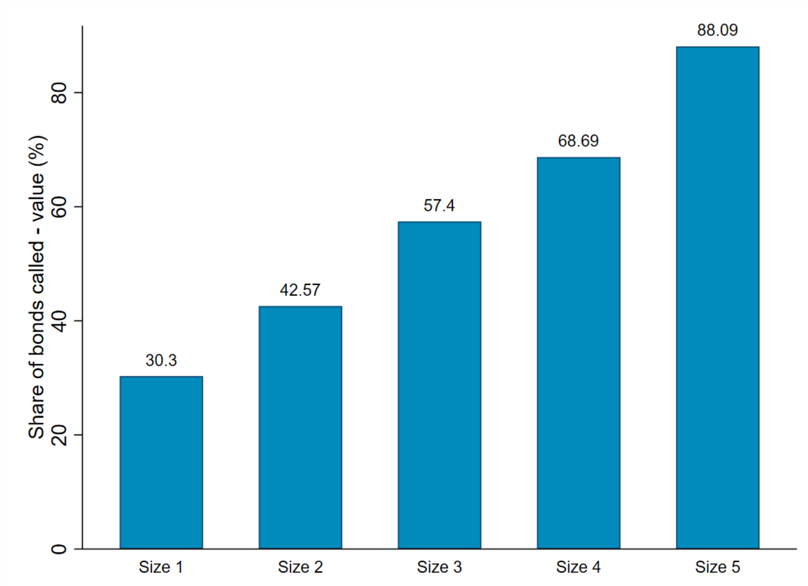
Table 2: PROBABILITY OF BONDS TO BE CALLED - LOGISTIC REGRESSION

	Likelihood to be called				
VARIABLES	(1)	(2)	(3)	(4)	(5)
$\Delta rate_t$	-0.409***	-0.409***	-0.405***		-0.405***
	(0.0719)	(0.0719)	(0.0719)		(0.0719)
Credit r. imp.		0.0748**		0.0713**	0.0714**
		(0.0304)		(0.0304)	(0.0305)
TFP imp.			0.414***	0.415***	0.404***
			(0.120)	(0.120)	(0.120)
Performance inc.	0.105	0.103	0.0975	0.0947	0.0950
	(0.0718)	(0.0718)	(0.0718)	(0.0718)	(0.0718)
Firm size inc.	-0.0747	-0.0757	-0.0687	-0.0714	-0.0698
	(0.0675)	(0.0675)	(0.0675)	(0.0674)	(0.0675)
Leverage	0.00347	0.00329	0.00301	0.00482	0.00284
	(0.00516)	(0.00516)	(0.00516)	(0.00515)	(0.00516)
Liquidity ratio inc.	-0.230**	-0.229**	-0.222**	-0.226**	-0.220**
	(0.102)	(0.102)	(0.102)	(0.102)	(0.102)
Amount issued	-8.36e-07***	-8.35e-07***	-8.32e-07***	-8.32e-07***	-8.32e-07***
	(4.01e-08)	(4.01e-08)	(4.01e-08)	(3.99e-08)	(4.01e-08)
F.E.	✓	✓	✓	✓	✓
Observations	62,666	62,666	62,666	62,666	62,666

Notes: This table shows estimates if Equation (1). Credit r. imp. and TFP imp. indicate improvement in the firm's credit risk and TFP, which is exercising the call option. The author estimates TFP. Performance Inc., Firm Size Inc., and Liquidity Ratio Inc. represent dummies for increasing in financial performance (ROA), firm size, and liquidity ratio. The sample period is 1990 Jan - 2018 May. Standard errors are reported in parentheses. ***, ** and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.
Source: Mergent FISD & CRSP-Compustat; Fed Data.

The results presented in Table 2 show that when interest rates rise, the probability of a bond being called decreases. This is consistent with the trends in Figure 4 and supports the idea that firms are less likely to call their bonds when interest rates are higher. The results suggest that when a firm's credit rating improves, the likelihood of a bond being called increases. This finding is also consistent with that improved credit ratings incentivize firms to call bonds and refinance at better terms. When a firm's financial performance improves, the probability of a bond being called increases. This aligns with the principle suggesting that better financial performance allows firms to manage their debt more effectively and potentially refinance by calling bonds. However, the likelihood of a bond being called decreases when a firm's size increases. This result is consistent with the idea that larger firms may have access to more diversified funding sources and better access to capital markets than smaller firms, reducing their need to call bonds for refinancing purposes. The leverage ratio does not significantly impact the probability of a bond being called. When the liquidity ratio rises, the probability of a bond being called lowers. Firms with higher liquidity ratios have a stronger financial position and may not need to call bonds to manage their debt obligations or improve their financial standing. Conversely, firms with a smaller ability to meet short-term obligations using their most liquid assets may be more likely to call their debt. Finally, concerning the original issuance size, the negative coefficient indicates that larger bond offerings may have more stringent covenants or greater market scrutiny, which might discourage firms from calling bonds.

Figure 5: SHARE OF CALLED BONDS BY FIRMS CREDIT RISK



Notes: This figure displays the share of callable bonds that are called by class of firm size (at the moment of call), class 1 is for the 20% low-ranked firms, and class 5 represents the top quintile of higher-ranked and large firms. The sample period is 1990 Jan - 2018 May.
Source: Mergent FISD.

High-ranked firms use callable bonds to actively manage their debt costs and capitalize on favorable market conditions, while low-ranked firms issue callable bonds more as a hopeful option for future flexibility and as a potential signal of positive prospects, even though they may not call these bonds as frequently due to less favorable market conditions and limited refinancing opportunities. Thus, while callable debt offers flexibility, the costs and market perceptions associated with high-default-risk firms make them less likely to call their debts early. Firms with high default risk may suffer from debt overhang, meaning they have significant outstanding debt obligations. Therefore, calling debt early could further tighten their financial resources, making it less attractive. This mechanism is emphasized by Figure 5.

These findings reinforce the importance of interest rate variations, credit risk, financial performance, firm size, and the firm-level share of callable bonds in the decision to call a bond.

2.2.5 Bonds characteristics

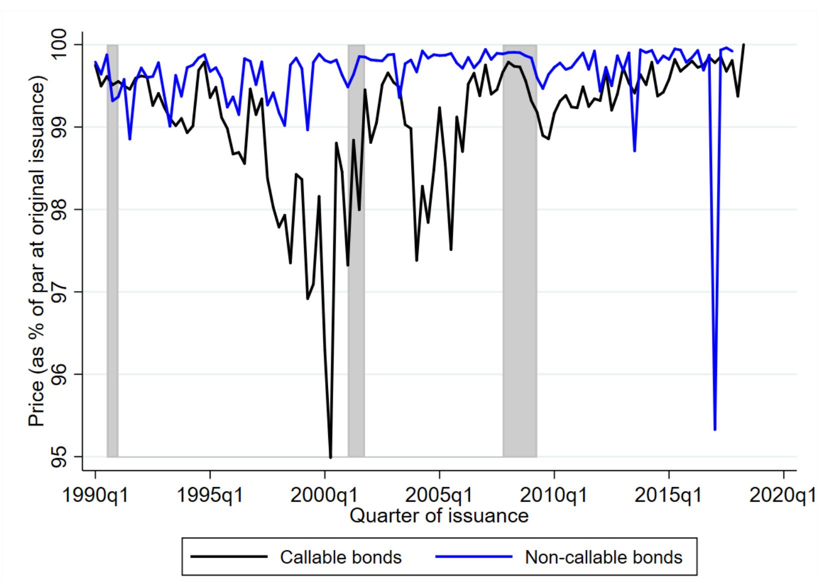
I observe in data the following trends showing that callable bond issuance is associated with specific bond characteristics (more detailed in Appendix B).

Maturity: Callable bonds exhibit a longer (statistically) average maturity (13 years) compared to non-callable bonds (12 years). This could be because issuing firms prefer to have the option to call back bonds when interest rates change or their credit quality improves, which is more likely to happen over a longer period.

Issue Size: Callable bonds have larger average issue sizes (49.11 $ mn) than non-callable bonds (24.77 $ mn). This could suggest that firms issuing callable bonds require more significant financing, and having the option to call back bonds provides them greater financial flexibility.

Coupon Rate: Callable bonds have higher average coupon rates compared to non-callable bonds. This trend can be explained by the additional risk investors take when investing in callable bonds. Since investors face the risk of early redemption and the potential loss of future interest payments, they require higher coupon rates as compensation for this risk.

Figure 6: Bonds prices over time



Notes: This figure plots the average bond price by firms for each type of bond (callable bonds in black). The sample period is 1990 Jan - 2018 May. Source: Mergent FISD.

Offering Price: On average, the offering price of callable bonds is 98.20, while for non-callable bonds, it is 99.56. This difference in offering prices can be attributed to the fact that callable bonds come with a call option, which gives the issuer the right to redeem the bond before its maturity. Recognizing this additional risk, investors may require a lower offering price for callable bonds as compensation for the potential loss of future interest payments if the bond is called. This results in a lower average offering price for callable bonds than non-callable bonds, as shown in Figure 6. Becker et al. (2024) found that the difference between the callable bonds' average yield and the non-callable bonds one, at issuance, is 2.67 %.

These empirical facts underscore the significance of callable debt in corporate financing and provide strong motivation for incorporating callability in the analysis of firm dynamics and macroeconomic outcomes. In the following sections, I build on these empirical findings and develop a theoretical framework to analyze the impact of debt callability on firm dynamics and the macroeconomy.

3 Three-Period Model

This section presents the framework that depicts the interaction mechanism between debt callability and firm behavior. I consider the following five assumptions in the model. First, firms issue the two types of bonds. Second, firms only differ in productivity and face the same operations costs and aggregate shocks. Third, there is no initial outstanding debt, but it is compensated by the second period. Fourth, I allow the firms to issue new debt after the initial callable debt has been redeemed. Fifth, exits follow endogenous default on the debts. These ingredients help characterize firms' optimal decisions (capital, prices, the decision to call, and the refinancing).

Economy. The economy lasts three periods, $t = 0, 1, 2$. Risk-neutral shareholders own the firms. I consider a firm characterized by its productivity, which produces with only capital and is exposed to idiosyncratic capital shocks. Each firm is financed through debt (bond) and equity payout. Equity is initially costless issued in the first period $t = 0$, e_0 and second period $t = 1$, e_1. Equity is positive in this setup ($e > 0$), representing shareholder payout. The firm raises its debt by issuing two-period defaultable non-callable and callable bonds.

Technology. The production process occurs during the periods $t = 1, 2$. The firm produces goods y with the production function:

$$y = zk^{\alpha}, \text{ with } \alpha \in (0, 1)$$

where z is a persistent total factor productivity shock realized in periods $t = 0, 1, 2$. The initial productivity z_0 defines the type of the firm. The productivity shock evolves following a log-AR(1) process $logz_t = \rho logz_{t-1} + \epsilon_{zt}$, where $\epsilon_{zt} \sim^{i.i.d.} \mathcal{N}(0, {\sigma_z}^2)$.

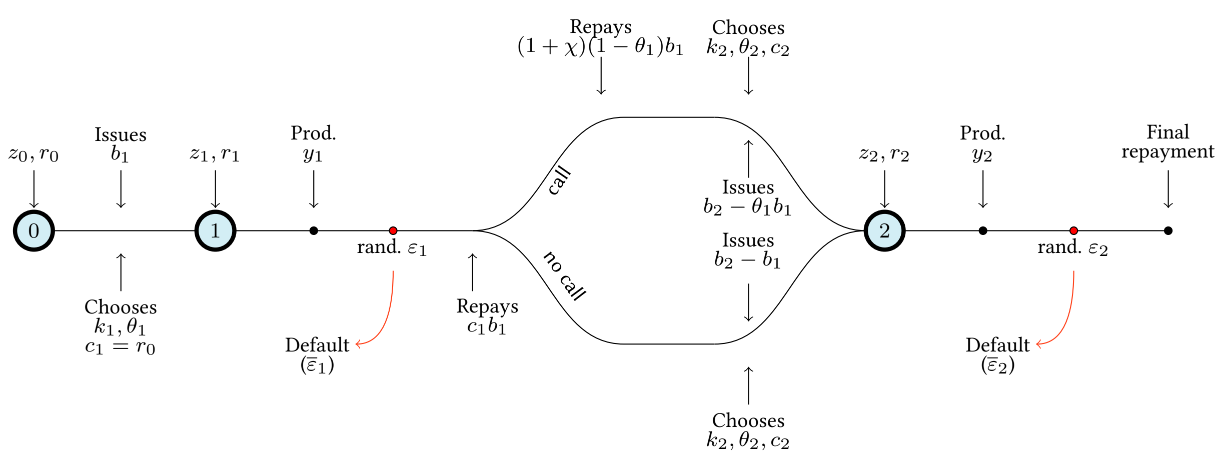
Capital Quality Shocks. The firm receives, after production, an idiosyncratic capital quality shock ε i.i.d. across time and firms. It is drawn from a normal distribution $\mathcal{N}(0, \sigma_{\varepsilon}^2)$. It influences the un-depreciated capital of the firm after production. The capital quality shocks allow the model to generate the default risk and match the default rates observed in the data. It can be viewed as an unforeseen force in the efficiency of capital, such as an unmodeled technological decline that reduces the value of the firm's capital.

3.1 Setup

A non-callable debt issued in the period $t = 0$ is a promise to pay the fixed coupon c_1 in period $t = 1$ and to repay the principal of the debt to the bondholder together with the fixed coupon c_1, in period $t = 2$. The market price of such a non-callable bond is set at p_0^{nc} in period $t = 0$.

A callable debt issued in the period $t = 0$ with initial maturity in period $t = 2$. It has embedded in it, *a call option*, which gives the right to the issuer to call or redeem the bond in period $t = 1$. The callable bond carries two promises. If the issuer exercises the call option, it repays the debt to the bondholder with a premium χ. The other promise is to pay the fixed coupon of c_1 and continue as a non-callable bond if it is not called. The market price of the callable bond is denoted p_0^c and set in period $t = 0$.

Figure 7: Timing for the three-period setup



The timing of the three-period model is depicted in Figure 7 and explained in the following lines. At the beginning of the period $t = 0$, the firm gets the productivity shock z_0 and the information on the interest rate r_0 prevailing. It issues equity e_0 and chooses its capital k_1 by issuing a fixed-coupon bond b_1, composed of non-callable bond in share θ_1 and callable bond in share $1 - \theta_1$. Both bonds mature in period $t = 2$. I allow the exercise of the call option to happen in period $t = 1$. I assume that issuance is costly for each type of bond, and I adopt the specification of a quadratic form.

$$\eta_b = \left(\eta_{nc}\theta_1^2 + \eta_c(1-\theta_1)^2\right) \cdot [\max(0, b)]^2$$

The capital constituted for production in the period $t = 1$ is:

$$k_1 = e_0 + p_0^{nc}\theta_1 b_1 + p_0^{c}(1 - \theta_1)b_1 \tag{2}$$

At the beginning of period $t = 1$, the firm has a new productivity z_1 conditional on its previous one and observes the interest rate r_1. The firm produces and receives the capital quality shock ε_1. For simplicity, there is no depreciation of capital. The firm decides to default when its value is less than zero or to repay its debt obligations otherwise. It depends on the realization of capital quality, and when it is under a certain threshold $\bar{\varepsilon}_1$, the firm defaults. Conditional on surviving, the firm decides whether to call the bond $(1 - \theta_1)b_1$ or not. Whether the firm calls or not, it has to pay the interest on debts. The stock of assets of the firm in the period $t = 1$, when:

- it does not call the bond is:

$$q_1^{nocall} = k_1 + z_1(k_1)^{\alpha} + \varepsilon_1 k_1 - c_1 b_1 \tag{3}$$

- it calls the bond is:

$$q_1^{call} = k_1 + z_1(k_1)^{\alpha} + \varepsilon_1 k_1 - c_1 b_1 - (1 + \chi)(1 - \theta_1)b_1 \tag{4}$$

The term $-c_1 b_1$ is the periodic payment of coupon on the outstanding debt b_1 when the firm chooses not to call during the period $t = 1$, while χ represents the call premium paid additionally to the principal of the callable bond $(1 - \theta_1)b_1$. χ is an amount over the bond's face value paid to the bondholder if the bond is called early. This call premium compensates for the bondholder's loss of future income.

The firm considers all the state variables and new capital quality shock to decide whether to call or not $(1 - \theta_1)b_1$. Following this decision, it chooses its new level of capital k_2 by choosing the amount of debt b_2 for the next period $t = 2$. The firm chooses the non-callable composition (θ_2) of the next period debt by choosing the non-callable share θ_2^{new} in the newly issued debt. I consider interest payment as set in a weighted average coupon c_2 to be paid on the next period debt b_2.[12]

[12] This weighted average coupon is essential for accurately tracking the firm's debt servicing costs over time. Ippolito et al. (2018) use a similar formulation to model the share of hedged floating-rate debt. The coupon rate determines the periodic interest payments the firm needs to make on its debt. The firm's decision to call or not call its bonds impacts future coupon payments. By incorporating the coupon rate formulas, the model captures the cost of debt accurately, influencing the firm's investment and financing decisions. The formulas also reflect changes in market interest rates over time and ensure that when the firm issues new debt, the coupon rate aligns with prevailing market conditions.

- when the firm does not call the bond, the non-callable fraction, and the periodic coupon c_2 evolve as follows:

$$\begin{cases} \theta_2 & = \theta_1 \dfrac{b_1}{b_2} + \theta_2^{new} \dfrac{b_2 - b_1}{b_2} \\ c_2 & = c_1 \dfrac{b_1}{b_2} + r_1 \dfrac{b_2 - b_1}{b_2} \end{cases} \tag{5}$$

where $\theta_2^{new} = 1$, as the new debt is a non-callable one-period bond.

- when the firm calls its callable bond, the non-callable fraction, and the periodic coupon c_2 evolve as follows:

$$\begin{cases} \theta_2 & = \theta_1 \dfrac{b_1}{b_2} + \theta_2^{new} \dfrac{b_2 - \theta_1 b_1}{b_2} \\ c_2 & = c_1 \dfrac{\theta_1 b_1}{b_2} + r_1 \dfrac{b_2 - \theta_1 b_1}{b_2} \end{cases} \tag{6}$$

In period $t = 2$, the firm gets its new productivity z_2 and observes the interest rate r_2.[13] The stock of assets of the firm in the period $t = 2$, after producing, would be:

$$q_2 = k_2 + z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2)b_2 \tag{7}$$

where $(1 + c_2)b_2$ is the final debt reimbursement.

3.2 Firm Problem

In period $t = 0$, the firm chooses the capital k_1, the debt b_1, the non-callable fraction of debt θ_1, and the coupon rate c_1. In the period $t = 1$, the firm produces with k_1, and there is a threshold of capital quality shock $\overline{\varepsilon}_1$ under which the firm's value is null. I denote by $s_1 = \{z_1, r_1\}$ the state variables vector at the beginning of the period $t = 1$ and by $x_1 = \{k_1, b_1, \theta_1^{new}, c_1\}$ the policy vector.[14] The firm maximizes its present value, which is the value of the shareholders, by solving the following:

$$V_0(s_0) = \max_{x_1 = \{k_1, b_1, \theta_1, c_1\}} \left\{ -k_1 + p_0 b_1 - \eta_b + \beta_0 \mathbb{E}_{s_1|s_0} \int_{\overline{\varepsilon}_1}^{\infty} \overline{q}_1 + \overline{V}_1(s_1, x_1)\varphi(\varepsilon_1) d\varepsilon_1 \right\} \tag{8}$$

[13]The interest rate r_2 is irrelevant since the period $t = 2$ is the final period, and there is no new bond issuance.
[14]In the period $t = 0$, the non-callable fraction $\theta_1 = \theta_1^{new}$, because there is no outstanding bond at the beginning of this period.

$$
\begin{aligned}
\text{subject to: } \overline{q}_1 &= k_1 + z_1(k_1)^\alpha + \varepsilon_1 k_1 \\
\overline{V}_1(s_1, x_1) &= \max_{\mathbb{1}_{\text{call}}(s_1,x_1)} \left\{ \overline{V}_1^{call}(s_1, x_1), \overline{V}_1^{nocall}(s_1, x_1) \right\} \\
\overline{\varepsilon}_1 : 0 &= \overline{q}_1 + \overline{V}_1(s_1, x_1) \\
p_0 &= \theta_1 p_0^{nc} + (1 - \theta_1) p_0^c \\
\eta_b &= \left(\eta_{nc} \theta_1^2 + \eta_c (1 - \theta_1)^2 \right) \cdot [\max(0, b)]^2
\end{aligned}
$$

The following indicator states the decision to call $(1 - \theta_1)b_1$ in $t = 1$:

$$
\mathbb{1}_{\text{call}}(s_1, x_1) = \begin{cases} 1, & \text{if} \quad \overline{V}_1^{call}(s_1, x_1) > \overline{V}_1^{nocall}(s_1, x_1) \\ 0, & \text{otherwise} \end{cases} \tag{9}
$$

Where $\overline{V}_1^{call}$ ($\overline{V}_1^{nocall}$) is the firm's value when $(1 - \theta_1)b_1$ is called (not called) in $t = 1$.

When the firm does not call its callable debt in period $t = 1$, it adjusts its capital by choosing k_2 and decides the new period debt by issuing $b_2 - b_1$. In period $t = 2$, there exists a threshold $\overline{\varepsilon}_2^{nocall}$ such that under this value, $V_2^{nocall} = 0$. The firm obtains its maximal value by solving the following:

$$
\overline{V}_1^{nocall}(s_1, x_1) = \max_{k_2, b_2, \theta_2^{new}, c_2} \left\{ -e + \beta_1 \mathbb{E}_{z_2|z_1} \int_{\overline{\varepsilon}_2^{nocall}}^{\infty} V_2^{nocall}(s_2, x_2) \varphi(\varepsilon_2) d\varepsilon_2 \right\} \tag{10}
$$

$$
\begin{aligned}
\text{subject to: } k_2 &= e - c_1 b_1 + p_1(b_2 - b_1) \\
V_2^{nocall}(s_2, x_2) &= \max_{\overline{\varepsilon}_2^{nocall}} \left\{ 0, k_2 + z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2) b_2 \right\} \\
\overline{\varepsilon}_2^{nocall} : 0 &= V_2^{nocall}(s_2, x_2) \\
\theta_2 &= \theta_1 \frac{b_1}{b_2} + \theta_2^{new} \frac{b_2 - b_1}{b_2} \\
c_2 &= c_1 \frac{b_1}{b_2} + r_1 \frac{b_2 - b_1}{b_2} \\
p_1 &= \theta_2^{new} p_1^{nc} + (1 - \theta_2^{new}) p_1^c
\end{aligned}
$$

Where p_1 is the weighted average price of the new debt, composed by the price of the non-callable (callable) new bond $p_1^{nc}(p_1^c)$. These prices are given by:

$$
p_1^{nc} = p_1^c = \frac{1}{(1 + r_1)} \mathbb{E}_{z_2|z_1} \int_{\overline{\varepsilon}_2^{nocall}}^{\infty} (1 + r_1) \varphi(\varepsilon_2) d\varepsilon_2 = \mathbb{E}_{z_2|z_1} \left(1 - \Phi(\overline{\varepsilon}_2^{nocall}) \right) \tag{11}
$$

$\left(1 - \Phi(\overline{\varepsilon}_2^{nocall})\right)$ is the probability of not defaulting in the period $t = 2$ after the capital quality shock when the firm does not call in period $t = 1$.

In period $t = 1$, when the firm calls its callable debt $(1-\theta_1)b_1$, it returns the debt and issues a new one $b_2 - \theta_1 b_1$. In period $t = 2$, there exists a threshold $\bar{\varepsilon}_2^{call}$ such that under this value, $V_2^{call} = 0$. The firm then solves:

$$\overline{V}_1^{call}(s_1, x_1) = \max_{k_2, b_2, \theta_2^{new}, c_2} \left\{ -e + \beta_1 \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{call}}^{\infty} V_2^{call}(s_2, x_2)\varphi(\varepsilon_2) d\varepsilon_2 \right\} \tag{12}$$

$$\begin{aligned}
\text{subject to: } k_2 &= e - c_1 b_1 - (1+\chi)(1-\theta_1)b_1 + p_1(b_2 - \theta_1 b_1) \\
V_2^{call}(s_2, x_2) &= \max_{\bar{\varepsilon}_2^{call}} \{0, k_2 + z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1+c_2)b_2\} \\
\bar{\varepsilon}_2^{call} &: 0 = V_2^{call}(s_2, x_2) \\
\theta_2 &= \theta_1 \frac{b_1}{b_2} + \theta_2^{new} \frac{b_2 - \theta_1 b_1}{b_2} \\
c_2 &= c_1 \frac{\theta_1 b_1}{b_2} + r_1 \frac{b_2 - \theta_1 b_1}{b_2} \\
p_1 &= \theta_2^{new} p_1^{nc} + (1-\theta_2^{new}) p_1^c
\end{aligned}$$

where χ is the call premium, the additional amount over the face value the issuer pays to redeem the callable bond $(1-\theta_1)b_1$ early. p_1 is the weighted average price of the new debt, composed by the price of the non-callable (callable) new bond $p_1^{nc}(p_1^c)$. These prices are given by:

$$p_1^{nc} = p_1^c = \frac{1}{(1+r_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{call}}^{\infty} (1+r_1)\varphi(\varepsilon_2) d\varepsilon_2 = \mathbb{E}_{z_2|z_1} \left(1 - \Phi(\bar{\varepsilon}_2^{call})\right) \tag{13}$$

$\left(1 - \Phi(\bar{\varepsilon}_2^{call})\right)$ is the probability of not defaulting in the period $t = 2$ after the capital quality shock when the firm calls in period $t = 1$.

3.3 Creditors' Problem

Creditors are risk neutral and discount the future at the same rate $\beta_0 = 1/(1+r_0)$ in period $t = 0$ and $\beta_1 = 1/(1+r_1)$ in period $t = 1$ as shareholders. This part follows the structure of creditors' problem in Jungherr and Schott (2021, 2022).

3.3.1 Non-callable bond pricing

After producing, when the firm decides to default, in $t = 1$, the creditors of the non-callable bonds recover the remaining fraction $(1-\xi)$ of the firm's liquidation value $\underline{q}_1$, with the fraction ξ being lost and:

$$\underline{q}_1 \equiv k_1 + y_1 + \varepsilon_1 k_1 \tag{14}$$

In this three-period model, for simplicity, I consider that the whole value of the firm is lost when it defaults on its debt, so $\xi = 1$. The coupon received by the debt holders in period $t = 1$ is r_0, then the price of the non-callable debt $\theta_1 b_1$ is:

$$p_0^{nc} = \frac{1}{(1+r_0)} \mathbb{E}_{s_1|s_0} \int_{\bar{\varepsilon}_1}^{\infty} \left(r_0 + \bar{p}_1^{nc}\right) \varphi(\varepsilon_1) d\varepsilon_1 \tag{15}$$

with $\bar{p}_1^{nc}$ the average expected continuation price of $\theta_1 b_1$ in $t = 1$:

$$\bar{p}_1^{nc} = \mathbb{1}_{\text{call}} \times \bar{p}_1^{nc\_call}(s_1, x_1) + (1 - \mathbb{1}_{\text{call}}) \times \bar{p}_1^{nc\_nocall}(s_1, x_1)$$

where $\mathbb{1}_{\text{call}}$ is the dummy that indicates the firm decision to call or not, $\bar{p}_1^{nc\_call}$ ($\bar{p}_1^{nc\_nocall}$) is the continuation price of $\theta_1 b_1$ when $(1-\theta_1)b_1$ is called (not called) in $t = 1$:

$$\bar{p}_1^{nc\_call}(s_1, x_1) = \frac{1}{(1+r_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{call}}^{\infty} (1+c_1)\varphi(\varepsilon_2) d\varepsilon_2 = \beta_1(1+c_1)\mathbb{E}_{z_2|z_1} \left(1 - \Phi(\bar{\varepsilon}_2^{call})\right)$$

and

$$\bar{p}_1^{nc\_nocall}(s_1, x_1) = \frac{1}{(1+r_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{nocall}}^{\infty} (1+c_1)\varphi(\varepsilon_2) d\varepsilon_2 = \beta_1(1+c_1)\mathbb{E}_{z_2|z_1} \left(1 - \Phi(\bar{\varepsilon}_2^{nocall})\right)$$

3.3.2 Callable bond pricing

The price of the callable debt $(1-\theta_1)b_1$ is generally set relative to a similar non-callable debt, such that the option is not exercised in the period $t = 1$. The issuer has to consider the minimum between the non-callable bond price and the exercise price (here at par), using a callable bond premium.[15] I use the following formula to model the callable bond pricing at the issuance in the period $t = 0$:

$$p_0^c = \frac{1}{(1+r_0)} \mathbb{E}_{s_1|s_0} \int_{\bar{\varepsilon}_1}^{\infty} \left[r_0 + (1 - \mathbb{1}_{\text{call}})\, \bar{p}_1^{c\_nocall} + \mathbb{1}_{\text{call}}\, (1+\chi)\right] \varphi(\varepsilon_1) d\varepsilon_1 \tag{16}$$

The term $(1+\chi)$ represents the bondholder's potential payoff, ensuring that they do not receive more than the fixed call price even if the market price $\bar{p}_1^{c\_call}$ is higher at the time of the call, this aligns with the fact that the call scenario occurs when interest rates drop, increasing the continuation value of the bond. This cap protects the issuer and defines the financial outcome of the early redemption. The call premium $\chi \in (0, 1)$ is typically a small percentage of the face

[15] The pricing of callable bonds is similar to the extension of defaultable callable bonds valuation' model in Jarrow et al. (2010) which extended the model of defaultable callable bond in Duffie and Singleton (1999). But I consider endogenous decisions to default and to call the callable bonds. In Duffie and Singleton (1999), the exercise price at the call period is set at par. Ultimately, the firm internalizes the minimal payment in the call scenario.

value to compensate bondholders for the early redemption.[16]

$\overline{p}_1^{c\_call}$ ($\overline{p}_1^{c\_nocall}$) is the market price of $(1-\theta_1)b_1$ when $(1-\theta_1)b_1$ is called (not called) in $t=1$. Because of the issuance of both bonds in the same period by the same issuer facing the same conditions, and due to the fact there is only one period left at this stage of the timing, a callable bond that is not called is priced as a similar one-period non-callable one: $\overline{p}_1^{c\_call} = \overline{p}_1^{nc\_call}$ and $\overline{p}_1^{c\_nocall} = \overline{p}_1^{nc\_nocall}$.

3.4 Characterization of firm optimal policies

According to this framework within a partial equilibrium setting, the model is solved by backward induction, considering the problems in (10), (12), and (8), alongside the creditors' problems through (15), (16). The subsection discusses the first-order conditions regarding capital choices for both initial and intermediate periods ($t=0,1$), the central decision to call a callable bond in period $t=1$, and the dynamics of debt issuance across these time frames. The feature of calling the callable bond $(1-\theta_1)b_1$ at $t=1$ serves to differentiate those optimal choices. I don't use superscripts "nocall" and "call" when there is no ambiguity in the notation.

3.4.1 Refinancing decision in the intermediate period $t=1$

Now, I analyze the optimal decisions on issuing new debt after not calling and after calling $(1-\theta_1)b_1$ in this period $t=1$. The first-order conditions with respect to b_2-b_1 ($b_2-\theta_1 b_1$) when the firm does not call (calls) $(1-\theta_1)b_1$ are the following:

nocall

$$[b_2-b_1] : -(1+c_2)\frac{b_2-b_1}{k_2}\mathbb{E}_{z_2|z_1}\left(\varphi(\overline{\varepsilon}_2)\right) - \beta_1(c_1-r_1)\frac{b_1}{b_2}\mathbb{E}_{z_2|z_1}\left[1-\Phi\left(\overline{\varepsilon}_2\right)\right] = 0 \tag{17}$$

call

$$[b_2-\theta_1 b_1] : -(1+c_2)\frac{b_2-\theta_1 b_1}{k_2}\mathbb{E}_{z_2|z_1}\left(\varphi(\overline{\varepsilon}_2)\right) - \beta_1(c_1-r_1)\frac{\theta_1 b_1}{b_2}\mathbb{E}_{z_2|z_1}\left[1-\Phi\left(\overline{\varepsilon}_2\right)\right] = 0 \tag{18}$$

The proof of these optimal decisions can be found in Appendix C.1. These conditions present the net cost of refinancing and indicate that the optimal refinancing after the call decision should

[16] Setting the call premium as a percentage of the face value aligns with the idea that the premium compensates for the risk relative to the prevailing interest rate environment. This ensures that the premium is proportionate to market conditions during issuance. Callable bonds usually offer a higher coupon rate than non-callable bonds to offset call risk. However, the call premium is generally lower than this higher coupon rate, as it is designed to offer a buffer rather than full compensation.

account for the new interest rate (r_1), the level of the periodic coupon (c_1), and the expected effect on default risk ($\bar{\varepsilon}_2$).

High level b_2 (or $b_2 - b_1$) increases the default probability, as $\dfrac{\partial \bar{\varepsilon}_2}{\partial (b_2 - b_1)} = \dfrac{1 + c_2}{k_2} > 0$. It reduces the revenue $p_1(b_2 - b_1)$ from selling the new bond. In period $t = 1$, the optimal refinancing b_2 increases in the stock of existing debt b_1.[17] In the second term, the denominator is the next period stock of non-callable debt in both cases. However, for the no-call scenario, the uncalled bond in the same proportions at both the numerator and the denominator levels cancels out the effects of θ_1. The implication is that θ_1 does not influence this optimal refinancing choice when the firm does not call the fraction $(1 - \theta_1)$ of its debt b_1. When the firm calls the share $(1 - \theta_1)$ of its debt, the optimal new debt $b_2 - \theta_1 b_1$ also increases the default probability $\dfrac{\partial \bar{\varepsilon}_2}{\partial (b_2 - \theta_1 b_1)} = \dfrac{1 + c_2}{k_2} > 0$, since:

$$c_2 = \begin{cases} r_1 + (c_1 - r_1)\dfrac{b_1}{b_2} & \text{when no call} \\[2ex] r_1 + (c_1 - r_1)\dfrac{\theta_1 b_1}{b_2} & \text{when call} \end{cases} \tag{19}$$

Then, θ_1 emphasizes the role of the outstanding debt b_1 at the beginning of the period $t = 1$. It amplifies the positive impact of the stock of debt on the choice of optimal refinancing b_2 when the firm calls. Considering firms with default threshold at the right of the distribution of capital quality shock ($\bar{\varepsilon}_1 < \bar{\varepsilon}$), φ increases in $\bar{\varepsilon}_1$, so firms with high default risk face the more severe cost of refinancing (roll-over risk). I derive the following properties of debt callability in refinancing.

Proposition 1. *In period $t = 1$,*

- *θ_1 plays an (asymmetric) role in refinancing only when the callable debt is called.*
- *If the interest rate does not change, i.e., is set s.t. $r_1 = c_1$, θ_1 (or the share of callable bond) has no effect in refinancing, whether the firm decides to call or not.*

The proof of Proposition 1 can be found in Appendix C.4. Proposition 1 shows that the callable bonds are important in the amplitude of refinancing when the firm decides to call it. It also shows that the interest rate prevailing at the moment of call decision matters for this role. The asymmetry means that the share of callable bonds significantly impacts refinancing decisions when interest rates are falling, allowing firms to take advantage of lower borrowing costs. In contrast, when interest rates rise, the share of callable bonds has a less significant impact, as firms are less likely to refinance at higher rates.

[17] This result is consistent with the literature. See the slow debt property in Jungherr and Schott (2022).

3.4.2 Optimal capital choice in the intermediate period $t = 1$

I consider the optimal choices of capital under two scenarios: when the firm doesn't call the bond (k_2^{nocall}) and when it does (k_2^{call}). The first-order conditions to k_2 for these cases are respectively the following:

nocall

$$[k_2] : -1 - \frac{\partial \bar{\varepsilon}_2}{\partial k_2}\left[\beta_1 k_2 \left[1 - \Phi\left(\bar{\varepsilon}_2\right)\right] - (b_2 - b_1)\frac{\partial p_1}{\partial \bar{\varepsilon}_2}\right] + \beta_1 \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} \Big(\varepsilon_2 - \bar{\varepsilon}_2\Big)\varphi(\varepsilon_2) d\varepsilon_2 = 0 \tag{20}$$

call

$$[k_2] : -1 - \frac{\partial \bar{\varepsilon}_2}{\partial k_2}\left[\beta_1 k_2 \left[1 - \Phi\left(\bar{\varepsilon}_2\right)\right] - (b_2 - \theta_1 b_1)\frac{\partial p_1}{\partial \bar{\varepsilon}_2}\right] + \beta_1 \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} \Big(\varepsilon_2 - \bar{\varepsilon}_2\Big)\varphi(\varepsilon_2) d\varepsilon_2 = 0 \tag{21}$$

Where the capital effect on default risk is:

$$\frac{\partial \bar{\varepsilon}_2}{\partial k_2} = -\frac{1 + \alpha z_2 k_2^{\alpha-1} + \bar{\varepsilon}_2}{k_2} = \frac{(1-\alpha) z_2 k_2^{\alpha} - (1 + c_2) b_1}{(k_2)^2} \tag{22}$$

Despite the positive impact of additional capital on production, the diminishing return to capital (due to $\alpha < 1$) reduces the firm's ability to absorb shocks, contributing to a high risk of default. I then consider that capital reduces the default risk ($\partial \bar{\varepsilon}_2 / \partial k_2 < 0$). In this circumstance, a high debt stock b_1 decreases the optimal choice k_2. θ_1 does not influence the optimal investment when the firm does not call $(1 - \theta_1) b_1$ in $t = 1$.

As c_2 increases, the default risk induced by capital is supposed to be more important. But this effect depends on the composition of c_2, which depends in turn on θ_1 and the relative level of the interest rate r_1 to the average coupon c_1.

Corollary 3.0.1. *θ_1 (or callable share) reduces (or amplifies) the firm's optimal investment when it exercises the option to call its callable debt.*

1. *The callable fraction of bonds does not influence the optimal investment decision when the firm does not call its debt.*

2. *Following an easing of monetary policy (i.e., a decrease in r_1), firms with a high share of callable bonds (i.e., low θ_1) increase their investment more when they call these bonds compared to firms*

with a low share of callable bonds.

3. *Following a tightening of monetary policy (i.e., an increase in r_1), firms with a high share of callable bonds (low θ_1) decrease their investment more when they call these bonds compared to firms with a low share of callable bonds.*

The proof of Corollary 3.0.1 can be found in Appendix C.5. Corollary 3.0.1 aligns with Becker et al. (2024) results by highlighting how callable bonds enhance a firm's flexibility in managing debt and investment, particularly under varying monetary policy conditions and for firms with high-yield ratings.

3.4.3 The decision to call (in the intermediate period $t = 1$)

The firm decides to call its callable bond $(1-\theta_1)b_1$ **iff**:

$$\overline{V}_1^{nocall}(s_1, x_1) < \overline{V}_1^{call}(s_1, x_1) \tag{23}$$

Proposition 2. *In period $t = 1$, the firm's decision to call its callable bond $(1-\theta_1)b_1$ is determined by the following inequality:*

$$\begin{aligned} &\mathbb{E}_{z_2|z_1}\left\{\left(-b_1 + \frac{(1-\alpha)y_2^{nocall}}{1+c_2}\right)\left[1-\Phi\left(\bar{\varepsilon}_2^{nocall}\right)\right]\right\} \\ < -(1+\chi)(1-\theta_1)b_1 + &\mathbb{E}_{z_2|z_1}\left\{\left(-\theta_1 b_1 + \frac{(1-\alpha)y_2^{call}}{1+c_2}\right)\left[1-\Phi\left(\bar{\varepsilon}_2^{call}\right)\right]\right\} \end{aligned} \tag{24}$$

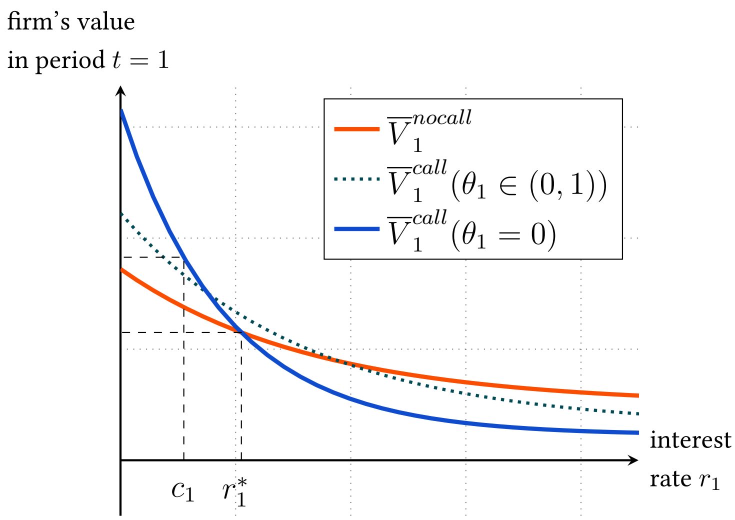
The proof of Proposition 2 can be found in Appendix C.6. Proposition 2 presents, in its simplest form, the firm's decision to call the bond based on comparing the expected net returns under the no-call and call scenarios. The firm calls its callable bond if the expected net return from calling the bond (RHS of (23)) exceeds the expected net return from not calling the bond (LHS of (23)).

The term $-(1+\chi)(1-\theta_1)b_1$ represents the immediate cost of calling the bond for the firm in period $t = 1$. Conditional to not default, the term in the expectation consists of the minimal value of the capital return of the firm after refinancing through $b_2 - b_1$ ($b_2 - \theta_1 b_1$ after calling).

Proposition 3. *For a given state s_1, an average coupon c_1 on existing debt b_1 which has a callable bond share ($1-\theta_1 \in [0,1]$), $\exists!\, r_1^* > c_1$, such that: $\forall r_1 < r_1^*$, $\overline{V}_1^{nocall}(s_1, x_1) < \overline{V}_1^{call}(s_1, x_1)$.*

The proof of Proposition 3 can be found in Appendix C.7. The main object of interest in Proposition 3 is the level of the interest rate set for the period $t = 1$. This rate influences the value of the firm's repayment in both periods $t = 1, 2$ in the call and non-call scenarios. Therefore, depending on the level of the share of non-callable bonds θ_1, these value can change their relative position along interest rates. I derive the following Corollary from the Proposition. Both Proposition 3 and Corollary 3.0.2 are illustrated in Figure 8.

Figure 8: Illustration of call decision in period $t = 1$



Corollary 3.0.2. *A decrease in the interest rate in the period $t = 1$:*

1. *improves the credit quality of the firm;*
2. *increases the probability of calling the callable debt in this period;*
3. *these effects are stronger for firms with a smaller share of callable bonds (i.e., high θ_1) and weaker for firms with a high share of callable bonds.*

Corollary 3.0.2 explores the conditions behind the decision to call for firms. The first point indicates that low interest rates improve the firm's credit quality by reducing its future default probability. To understand that, let's consider that the new low interest rate improves the future cash-flow of the firm, making it more able to repay any (new) debt. The expected future default probability of the firm is then reduced, which represents an improvement of its credit quality. Through this corollary, I am showing that the credit upgrade is common and -on average consequence- for all firms, and that does not make firms with a low credit rating become high credit rating. Therefore, the degree of callability is at the origin of the incentive to call after a decline in the interest rate. Though, a decline in interest rates can be associated with favorable opportunities that encourage firms to call.[18] This is because the default threshold is positively

[18] Becker et al. (2024) found that calls are more likely when issuer credit quality improves, but they ignored in their analysis the level of callable debt, which exposes a firm to the need for new financing. The more the firm has callable debt, the more it will create space to fill with new debt if it calls.

sensitive to changes in the periodic coupon. So, a decrease in interest rate reduces this periodic coupon, then minimizes the cutoff capital quality in the future (see Appendix C.8 for the proof.). Second, when the original coupon rate c_1 is sufficiently high (due to a high interest rate r_0 in period $t = 0$), the cost of continuing to pay these high-interest rates (reflected in the left-hand side of the (23)) may exceed the cost of facing new debt payments through a potentially lower rate r_1. Then, a decrease in the interest rate in $t = 1$ raises this benefit of calling. This demonstrates what is theoretically behind the purpose of callable bonds. Third, a higher share of callable bonds indicates a higher debt burden, which leads firms to internalize and endogenize the fact that they will face difficulties refinancing their debt once they call.

3.4.4 The optimal choice of callable share $(1-\theta_1)$ in period $t = 0$

The FOCs for the share of non-callable bond in the period $t = 0$ is given by:

$$[\theta_1] : b_1 \frac{\partial p_0}{\partial \theta_1} + p_0 \frac{\partial b_1}{\partial \theta_1} - \frac{\partial \eta_b}{\partial \theta_1} \leq -\beta_0 \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \theta_1} \tag{25}$$

where I denote V_1 the firm's continuation value in the period $t = 1$, given the realization of new shocks and when default is avoided:

$$V_1(s_1, x_1) = \int_{\bar{\varepsilon}_1}^{\infty} \left[\bar{q}_1 + \mathbb{1}_{\text{call}} \times \overline{V}_1^{call}(s_1, x_1) + (1 - \mathbb{1}_{\text{call}}) \times \overline{V}_1^{nocall} \right] \varphi(\varepsilon_1) d\varepsilon_1 \tag{26}$$

The optimal share of non-callable bonds (θ_1) is defined by the following expression:

$$\begin{aligned} &\beta_0 \mathbb{E}_{s_1|s_0} \Bigg\{ \Bigg[\underbrace{\left[r_0 + \theta_1 \overline{p}_1^{nc\_call} + (1-\theta_1)(1+\chi) \right] \frac{b_1}{k_1}}_{\text{Effective value of firm's leverage in } t=1} \varphi(\bar{\varepsilon}_1) + \underbrace{[1 - \Phi(\bar{\varepsilon}_1)]}_{\text{Repayment probability in } t=1} \Bigg] \\ &\times \frac{\partial}{\partial \theta_1} \underbrace{\left[\mathbb{1}_{\text{call}} \times (\overline{V}_1^{call} - \overline{V}_1^{nocall}) \right]}_{\text{net future benefit from calling in } t=1} \Bigg\} - 2\left[(\eta_{nc} + \eta_c)\theta_1 - \eta_c \right] b_1^2 = 0. \end{aligned} \tag{27}$$

The optimal share of non-callable bonds, θ_1, is defined in (27) and reflects the interplay of two forces:

1. **Call Decision (Qualitative) Effect:** Determines the likelihood of calling ($\mathbb{1}_{\text{call}}$) based on θ_1.
2. **Payoff (Quantitative) Effect:** Captures how the economic benefit of calling $(\overline{V}_1^{call} - \overline{V}_1^{nocall})$ evolves with θ_1.

Together, these forces explain the trade-offs firms face in their capital structure decisions.

Callable Bonds and Future Conditions

The presence of callable bonds allows firms to respond flexibly to changes in future firm-specific and aggregate conditions. The proposition (2) shows that:

- When the interest rate r_1 in $t = 1$ is below the threshold r_1^*, the call decision ($\mathbb{1}_{\text{call}} = 1$) is activated, but the net payoff ($\overline{V}_1^{call} - \overline{V}_1^{nocall}$) which is positive, decreases with θ_1.
- In contrast, when $r_1 > r_1^*$, the calling decision is inactive ($\mathbb{1}_{\text{call}} = 0$), and the net payoff of the calling is negative and increases with θ_1.

The last term in (27), which incorporates issuance costs, also plays a role in determining the optimal share θ_1.[19] The following distinctions arise based on the relative issuance costs (η_c and η_{nc}):

- If $\eta_c \ll \eta_{nc}$, callable bonds dominate due to their low cost.
- If $\eta_c \gg \eta_{nc}$, callable bonds are less favorable due to their high cost.
- If $\eta_c \sim \eta_{nc}$, the issuance costs have minimal impact and θ_1 centers on 50%. θ_1 amplifies the issuance cost effect of callable bonds.

This term generates intensive margins due to the composition of the debt issued. Moreover, when the firm issues a perfect 50/50 mix of callable and non-callable bonds, the issuance cost falls by 50%, penalizing debt concentration. It also adds curvature in the θ_1 dimension.

High Initial Coupon (c_0) and Callable Bond Dynamics

When the initial coupon c_0 is high, the probability of calling in $t = 1$ increases because $r_1 < r_1^*$. In this case, we can distinguish:

- the **Call Decision (Qualitative) Effect** is dominant ($\mathbb{1}_{\text{call}} \to 1$), driving the decision to call callable bonds;
- however, the **Payoff (Quantitative) Effect** diminishes as $\partial(\overline{V}_1^{call} - \overline{V}_1^{nocall})/\partial\theta_1 < 0$, indicating reduced economic benefits from callable bonds as θ_1 increases.

Firms with lower default risk ($\overline{\varepsilon}_1 < \overline{\varepsilon}$) gain less from callable bonds because they are more likely to survive and repay, reducing the net benefit value of the call.

[19]This term shows the role of considering separate issuance costs for both types of debt. It raises another wedge of a higher-up front fee on callable issues because of differences in registration fees or legal underwriting spreads. In the full model, allowing these separate issuance costs may cause an identification problem since, in practice, issuing non-contingent and contingent debt often comes with different transaction costs due to factors like credit risk and investor preferences. I then consider a unique issuance cost for both types of debt in the full model.

Low Initial Coupon (c_0) and Callable Bond Dynamics

When the initial coupon c_0 is low, the probability of calling callable bonds in $t = 1$ is minimal ($\mathbb{1}_{\text{call}} \to 0$). Here we have:

- the **Payoff (Quantitative) Effect** dominates ($\partial(\overline{V}_1^{call} - \overline{V}_1^{nocall})/\partial\theta_1 > 0$), as the economic advantage of callable bonds increases with θ_1;
- high-risk firms ($\overline{\varepsilon}_1 > \overline{\varepsilon}$) tend to issue more callable bonds to hedge against the potential for default, as calling provides a valuable net benefit.

The three-period model highlights the core trade-offs firms face when issuing callable and non-callable debt. Firms weigh the **Call Decision (Qualitative) Effect** against the **Payoff (Quantitative) Effect**, balancing the likelihood of exercising the call option against its economic value. High-risk firms with greater default probabilities are more inclined to issue callable bonds due to the flexibility they provide. In contrast, low-risk firms prefer non-callable bonds to minimize costs. Although the three-period model provides valuable insight, it cannot capture the full dynamic interactions between bond issuance, call decisions, and investment. The following sections extend this analysis to incorporate firm heterogeneity, aggregate dynamics, and multi-period decision-making, quantifying the macroeconomic implications of callable debt and firm-level behaviors in shaping broader economic outcomes.

4 Full Model

I develop a heterogeneous firm general equilibrium model with risky debt and debt callability. The model has the following ingredients: (1) firms use labor and capital as factors of production; (2) to finance their capital investments, they combine equity and debt issuance decisions; (3) debts are sold as long-term defaultable bonds; (4) debt consists of non-callable bond and callable bond following a composition law of the share of non-callable; (5) upon non-default, firms can call their callable debt before the new optimal decisions.[20] Callable bond offers higher yields to investors in compensation for the risk of early redemption during the bond's duration. Still, it is supposed to save costs of interest payments (when it is called) and provide greater flexibility to its issuer. The model economy has four types of agents: firms, creditors, government, and households. Firms with different persistent productivity first hire labor, produce, and then receive capital quality shock upon which they decide to default or not. They decide whether to call their callable bond and then invest in the capital by dividend and debt issuance. There is a continuum of risk-neutral creditors who price the bonds; a government decides the tax and interest rates level; a represen-

[20] I assume that only public firms can access defaultable debts with a call option. They issue only long-term debts (See Karabarbounis and Macnamara (2021).)

tative household completes the model. Time is discrete, and as we set in a period, I use the prime symbol ($'$) to denote the future values.

4.1 Firms

4.1.1 Technology and Productivity

Firms are perfectly competitive and produce a single, unique, homogeneous final good. Each firm produces by combining capital k and labor l in a decreasing returns-to-scale technology and using a Cobb-Douglas production function:

$$y = z\left(k^{\psi} l^{1-\psi}\right)^{\nu}$$

where $\psi, \nu \in (0,1)$. z is the total factor productivity following a persistent shock learned from the previous period. The idiosyncratic productivity z follows an AR(1) process:

$$logz' = \rho_z logz + \epsilon'_z, \quad \epsilon'_z \sim^{i.i.d.} \mathcal{N}\left(0, {\sigma_z}^2\right), \quad \rho_z \in (0,1).$$

The firms pays every period a fixed cost of operation f. The firm receives, after production, an idiosyncratic capital quality shock ε i.i.d. across time and firms. It is drawn from a normal distribution $\mathcal{N}(0, \sigma_\varepsilon^2)$. This shock defines the threshold which influences the firm's decision to default after production.

4.1.2 Financing

Now, I define the types of bonds and present how the firm issues and combines them in financing.

Non-callable debt. A long-term debt issued in the period t promises to pay a fixed coupon c each period. I assume that a fraction γ of the outstanding principal matures each period, following Hatchondo and Martinez (2009); Chatterjee and Eyigungor (2012); Gomes et al. (2016). It means that the firm pays back γ additionally to the coupon, and the debt is of finite maturity. The market price of such a non-callable bond is set at p^{nc} at the issuance period.

Callable debt. A long-term debt issued in the period t promises to pay a fixed coupon c each period. It has embedded in it, *a call option*, which gives the right to the issuer to call back or redeem the bond early to its initial maturity. The callable bond carries two promises. If the issuer decides to exercise the call option, it repays the debt to the bondholder with a premium set at the coupon c. The other promise is to pay the fixed coupon of $c + \gamma$ on the outstanding debt and continue as a non-callable bond if it is not called. The market price of the callable bond is denoted p^c and set at the issuance period.

Share of bond not to call. Considering the quantity of the outstanding debt b at the beginning of a period, I denote by θ the bond share not be called in this period. Then, if the firm exercises its call option, it will call $(1-\theta)b$. Also, if the firm issues new debt, the share of bond not to call is denoted $\tilde{\theta}$.

The firm can only issue new debt if $b' \geq (1-\gamma)b$ when there is no call and $b' \geq (1-\gamma)\theta b$ when there is a call. I denote by $\tilde{b}$ the new debt in each scenario when there is no ambiguity about the call decision of the firm.

Bond issuance cost. I assume that retiring all the outstanding debt $\left(\tilde{b} \geq 0\right)$ is costless for the firm. I adopt the specification of a quadratic form as is done in the literature (see Jungherr and Schott, 2021).[21] The cost of issuing new debt is:

$$\eta_b = \eta \cdot \left[\max(0, \tilde{b})\right]^2$$

The firm can also issue equity but with a lower bound $e \geq -\underline{e}$ where $\underline{e} > 0$, avoiding the possibility of financing constantly through equity Ponzi games.

Equity issuance cost. I assume that the firm incurs a cost when it issues external equity ($e \geq 0$), and no cost when it distributes dividends ($e < 0$). I adopt again the specification of a quadratic form as in Jungherr and Schott (2021). The cost of issuing new external equity is:

$$\eta_e = \eta_e \cdot [\max(0, e)]^2$$

4.1.3 Callability structure and Call decision

At the call decision, the amount of debt the firm can call is $(1-\theta)b$. Whatever its decision, it pays the fraction γ on the non-callable share of its debt. Then, depending on its decision, the fraction of debt not to call in the next period in the remaining debt is different. The direct implication is that the composition of the new debt issued, denoted by $\tilde{b}$, and the periodic coupon payment are not constant. This justifies the introduction of the firm idiosyncratic variables θ, and c, and also the index for the call decision, $j \in \{\text{nocall}, \text{call}\}$. As the debt chosen for the next period is

[21] I set a single debt issuance cost η to let the debt composition be driven solely by the endogenous defaultable-bond price wedge and the call premium channel. It allows me to isolate the macro role of the call option. By doing so, I argue that all cross-sectional variation in the cost of issuing callable versus non-callable debt comes exclusively from the call premium and the option value embedded in prices. The quadratic cost still dictates the quantity of new debt and helps discipline the calibration without affecting the model's mechanism and implications.

denoted by b', the new debt issued $\tilde{b}$ is described as follows:

$$\forall j \in \{\text{nocall}, \text{call}\}, \tilde{b}_j = \begin{cases} b'_j - (1-\gamma)b, & \text{if } b'_j > (1-\gamma)b, \quad \text{when no call}, \\ b'_j - (1-\gamma)\theta b, & \text{if } b'_j > (1-\gamma)\theta b, \quad \text{when call}. \end{cases} \tag{28}$$

When deciding on the new debt, the non-callable composition (θ') of the next period debt is the weighted average of the remaining non-callable share (which is not to be called) θb in the outstanding debt $(1-\gamma)b$, and the non-callable share $\tilde{\theta}$ in the new debt issued $\tilde{b}$. It has the following law of motion: $\forall j \in \{\text{nocall}, \text{call}\}$,

$$\theta'_j = \begin{cases} \theta \dfrac{(1-\gamma)b}{b'} + \tilde{\theta}_j \dfrac{\tilde{b}_j}{b'}, & \text{if } b'_j > (1-\gamma)b, \text{ when no call } \left[b'_j > (1-\gamma)\theta b, \text{ when call} \right] \\ \theta, & \text{otherwise}. \end{cases} \tag{29}$$

The periodic coupon c' of the firm is set as the weighted average of the current coupon c on the outstanding debt and the current interest rate r which is the coupon on the new debt issued. The new interest rate r is the coupon attached to the new bond $\tilde{b}$, while the not-matured-yet and uncalled bond left in the next period bond b' will have to pay the same coupon c. This formulation allows me to track the coupon associated with the not-to-call part of the bond in the next period.

For the nocall scenario, the remaining debt after repaying the fraction γ is $(1-\gamma)b$, so we have:

$$c'_{\text{nocall}} = \begin{cases} c\dfrac{(1-\gamma)b}{b'} + r\dfrac{\tilde{b}_j}{b'}, & \text{if } b'_j > (1-\gamma)b, \\ c, & \text{otherwise}. \end{cases} \tag{30}$$

For the call scenario, the remaining debt after repaying the fraction γ is $(1-\gamma)\theta b$, since the callable debt $(1-\theta)b$ is called, so we have:

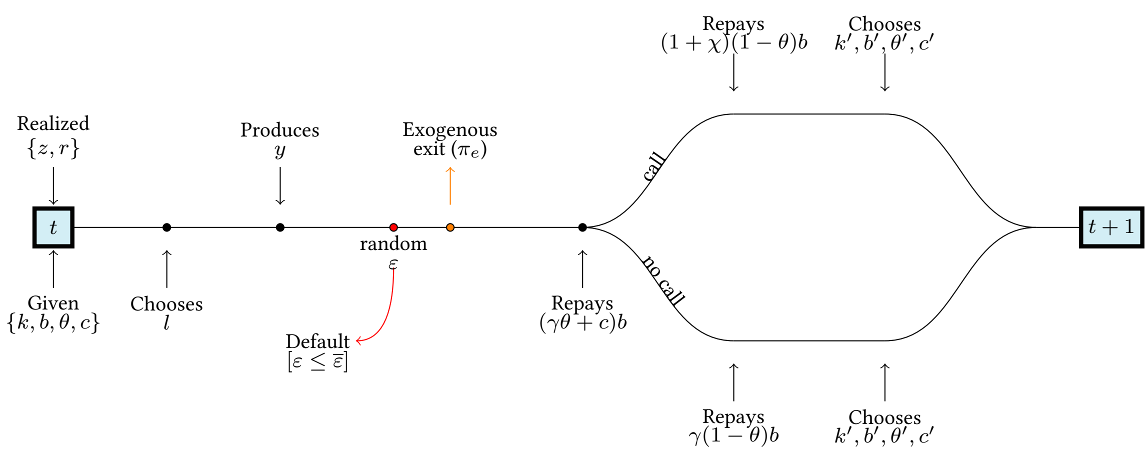
$$c'_{\text{call}} = \begin{cases} c\dfrac{(1-\gamma)\theta b}{b'} + r\dfrac{\tilde{b}_j}{b'}, & \text{if } b'_j > (1-\gamma)\theta b, \\ c, & \text{otherwise}. \end{cases} \tag{31}$$

Now, I define the firm's internal funds to be the net worth after production, tax, depreciation, and interest payment:

$$n_j(s,x) = \begin{cases} k - \gamma b + (1-\tau)\left[y + \varepsilon k - \delta k - wl - cb - f\right], & \text{when no call} \\ k - \left[\gamma\theta + (1+\chi)(1-\theta)\right] b + (1-\tau)\left[y + \varepsilon k - \delta k - wl - cb - f\right], & \text{when call}. \end{cases}$$

The call premium $\chi \in (0, 1)$, over the face value, is provided as compensation for the potential loss of future interest payments due to the early redemption of the bond. The premium might be predetermined or calculated as a percentage of the face value of the outstanding callable bond. It is a penalty during the call protection period and gradually declines as the maturity date approaches. This formulation ensures that the call premium decreases as the bond matures, aligning with the intuition that the cost of calling the bond should reduce over time.[22] Although endogenously fixed, it allows tractability in the model.[23]

Figure 9: Timing for the firm's problem



Notes: The first blue indicates the beginning of period t and the last marks the beginning of the next period $t + 1$. The firm decides to continue operations or to default endogenously (red arrow) after the capital quality shock ε. The exogenous exit is illustrated in orange and happens with a probability π_e.

[22]The call premium increases with the prevailing interest rate, reflecting the higher cost of calling the bond early when interest rates are high. It is proportional to the remaining principal of the callable bond. The call premium decreases as the bond principal decreases over time due to repayments. This relationship is consistent with the model's incorporation of the call decision indicator and default condition, which adjust the bond's value based on whether the bond is called or not. The call premium also shows an inverse relationship with the bond's original maturity. A longer original maturity typically results in a lower call premium for a given remaining principal and interest rate, as the longer duration allows more opportunity for rates to change, affecting the likelihood of calling. Note that with each period, repayment of the bond at a fraction γ leads to a geometric decrease in the outstanding principal. This geometric decay ensures that the remaining unmatured and uncalled principal of callable bonds decreases over time, which aligns with the model's treatment of the call premium and its adjustment based on the bond's callability and default conditions.

[23]The call premium adjusts dynamically with changes in the prevailing interest rate, ensuring that the model remains responsive to economic conditions. The formulation aligns with important papers in macro-finance and term structure models (refer to Joslin et al. (2014); Cochrane (2017)).

4.1.4 Value Functions

Each period, the firm has its new productivity z, it gets the information on the latest interest rate r, and based on its vector of state variable $x = \{k, b, \theta, c\}$ decided in the previous period, it chooses its labor to produce. I denote the vector of shock variables by $s = \{z, r\}$. The firm maximizes the shareholder value and discounts its future cash flows at the current interest rate. Without any ambiguity, the set of state variables is (s, x) to simplify the notations. Let $V(s, x)$ be the value of the solvent firm making the finance-investment decision.

The timing within a period t is illustrated in Figure 9 and is as follows. At the beginning of a period t, the economy is characterized by the interest rate r. A firm carries capital k, debt b, the share of the not-to-call bond θ, and its weighted average coupon rate c. Given its realized productivity z, it chooses the labor l to produce y—the decision to call interacts with the firm's optimal decisions. Knowing the distribution of the capital quality shock, the firm decides on the cutoff $\bar{\varepsilon}$. Upon non-default, the firm chooses to call or not the callable share $1-\theta$ of its outstanding bond b. Thus, it can choose k', b', θ', c'. Then, at the beginning of the period, the value of the firm is given by:

$$V(s, x) = \max \left\{ \underbrace{V^r(s, x)}_{\text{continue}}, \underbrace{0}_{\text{default}} \right\}. \tag{32}$$

I assume that in default, the firm exists and receives 0, which implies the following definition of the default cutoff $\bar{\varepsilon}$:

$$\bar{\varepsilon} : 0 = V^r(s, x) \tag{33}$$

Upon non-default, the value received by the firm is $V^r(s, x)$, defined by:

$$V^r(s, x) = (1 - \pi_e)\overline{V}(s, x) + \pi_e V^{ex}(s, x). \tag{34}$$

V^r represents the repayment value of the firm when it decides to continue. In this case, a probability π_e still exists to exit exogenously; then, with its earnings after production, the firm has to liquidate its capital and debt. It pays back its outstanding debt and pays out the dividend to shareholders. Its value in exogenous exit is:

$$V^{ex}(s, x) = k - \gamma b + (1 - \tau)\left[y + \varepsilon k - \delta k - wl - cb - f\right] - (1 - \gamma)bp. \tag{35}$$

Then, the firm continues operations when it survives default and exogenous exit by choosing whether to call or not the callable fraction $1 - \theta$ of its outstanding bond b. Based on its choice, it chooses equity to complement the internal fund from its asset n. I now define the firm's value of continuation.

$$\overline{V}(s,x) = \max_{\mathbb{1}_{\text{call}}(s,x)} \left\{ \overline{V}_{\text{nocall}}(s,x), \overline{V}_{\text{call}}(s,x) \right\} \tag{36}$$

The indicator function $\mathbb{1}_{\text{call}}(s,x)$ is one if and only the firm decides to exercise the option to call. I define the continuation price of each type of bond as their expected continuation prices:

$$\begin{cases} \overline{p}^{nc} & = (1-\mathbb{1}_{\text{call}}(s,x)) \cdot \overline{p}^{nc}_{\text{nocall}}(s,x) + \mathbb{1}_{\text{call}}(s,x) \cdot \overline{p}^{nc}_{\text{call}}(s,x) \\ \overline{p}^{c} & = (1-\mathbb{1}_{\text{call}}(s,x)) \cdot \overline{p}^{c}_{\text{nocall}}(s,x) + \mathbb{1}_{\text{call}}(s,x) \cdot \overline{p}^{c}_{\text{call}}(s,x) \end{cases} \tag{37}$$

When the firm takes the decision $j \in \{\text{nocall}, \text{call}\}$ on the callable share $1-\theta$ of its bond b, it receives:

$$\overline{V}_j(s,x) = \max_{e,k'_j,b'_j,\theta'_j,c'_j} \left\{ -e - \eta_e(e) + \frac{1}{1+r} \mathbb{E}_{s'|s} \int_{\varepsilon'} V(s',x') d\Phi(\varepsilon') \right\} \tag{38}$$

subject to:

$$\begin{aligned} k'_j &= e + n_j + p\tilde{b}_j - \eta_b(\tilde{b}_j), \\ V(s',x') &= \max \left\{ V^r(s',x'), 0 \right\}, \\ V^r(s',x') &= (1-\pi_e)\overline{V}(s',x') + \pi_e V^{ex}(s',x'), \\ \overline{V}(s',x') &= \max_{\mathbb{1}_{\text{call}}(s',x')} \left\{ \overline{V}_{\text{nocall}}(s',x'), \overline{V}_{\text{call}}(s',x') \right\}, \\ p &= \tilde{\theta}_j p^{nc} + (1-\tilde{\theta}_j) p^c. \end{aligned}$$

Where p is the weighted average price of the new debt, composed by the price of the non-callable (callable) new bond $p^{nc}(p^c)$.

Firms Entry

In each period, there is free entry. I follow Hopenhayn (1992), I assume that a potential entrant draws an initial level of productivity z^e from an invariant distribution after paying the entry cost. A constant mass M of potential entrants enters the economy without any initial capital $k = 0$, any initial debt $b = 0$. The free entry condition is the following:

$$\int V(z_e, r, 0, 0, \theta, c) = c_e.$$

4.1.5 Bond Pricing

When the firm decides to default on its debts, to liquidate, and to exit, the creditors recover a fraction $1-\xi$ of the firm's liquidation $\underline{n}$ defined by:

$$\underline{n}(\varepsilon) = \max\Big\{0, k + (1-\tau)\Big[y + \varepsilon k - \delta k - wl - f\Big]\Big\} \tag{39}$$

ξ is the parameter that indicates the fraction loosed when the firm is liquidated, and $(1-\xi)$ is, then, the rate of recovery of capital for the creditor. I assume that all creditors have the same seniority over the firm's liquidation value claim.

I first present the pricing schedule for the non-callable and callable components of the new bond $\tilde{b}$, and second, I define the continuation price of the two types of bonds in cases of no-call and call.

Non-callable bond pricing

For the non-callable $\tilde{\theta}$ in the new debt $\tilde{b}$, the coupon rate attached to it is the current interest rate r, so next period, when the firm does not default, it has to pay the fraction γ and the interest r. After this payment, the remaining fraction of the debt will be $1-\gamma$, valued by creditors at $\overline{p}^{nc}$. But the current interest rate of r has the weight $\tilde{b}/b'$ in the composition of the new average periodic coupon for the next period c', defined in (30) and (31), and the continuation price $\overline{p}^{nc}$ contains information on this future average coupon c' instead of direct value of the current rate r.

$$\begin{aligned} p^{nc}(z,r,k',b',\tilde{\theta},r) = \frac{1}{1+r}\mathbb{E}_{s'|s}\Bigg\{ \int_{\overline{\varepsilon}'}^{\infty} \Big[\gamma + r + (1-\gamma)\overline{p}^{nc}(z',r',\hat{x}^k,\hat{x}^b,\tilde{\theta},r)\Big]\, d\Phi(\varepsilon') \\ + \int_{-\infty}^{\overline{\varepsilon}'} \frac{1-\xi}{b'}\underline{n}' d\Phi(\varepsilon')\Bigg\} \quad , \end{aligned} \tag{40}$$

where $\overline{p}^{nc}(z',r',\hat{x}^k,\hat{x}^b,\tilde{\theta},r)$ is the continuation price of this non-callable bond ($\tilde{\theta}\tilde{b}$) in the next period.[24] I define it as the continuation price of the previous non-callable debt θb, so $\overline{p}^{nc}$ instead as shown in (37). Its formulation does not change depending on the call decision in the current period. It is important to notice that the prevailing non-callable bond θb has to pay the periodic

[24]Notice that $\overline{p}^{nc}$ is a function, and by keeping the arguments $k', b', \tilde{\theta}, r$ in the price, I am sure to keep the current interest rate r on the non-callable bond $\tilde{\theta}\tilde{b}$. See detailed explanations in Appendix (D.1.1).

coupon c set at its issuance. Then, it is set as:

$$\overline{p}_j^{nc}(z,r,k',b',\theta,c) = \frac{1}{1+r}\mathbb{E}_{s'|s}\Bigg\{\int_{\overline{\varepsilon}'}^{\infty}\left[\gamma + c + (1-\gamma)\overline{p}^{nc}(z',r',\hat{x}_j(s',x'))\right] d\Phi(\varepsilon') + \int_{-\infty}^{\overline{\varepsilon}'}\frac{1-\xi}{b'}\underline{n}' d\Phi(\varepsilon')\Bigg\} \quad , \quad \forall j \in \{\text{nocall}, \text{call}\} \tag{41}$$

where $\hat{x}_j(s,x) = \left[\hat{x}_j^k(s,x), \hat{x}_j^b(s,x), \hat{x}_j^\theta(s,x), \hat{x}_j^c(s,x)\right]$ is the vector of policy functions for k', b', θ', and c' in the scenario j.

Callable bond pricing

The price of a callable bond follows three steps: (i) a similar non-callable bond is priced; (ii) the endogenous probability to call the callable bond in the next period; (iii) the potential loss of return for the bondholder in case of call. The callable bond is in fraction $(1-\tilde{\theta})$ in the new debt $\tilde{b}$ issued in the current period. I provide a pricing model that incorporates the expected average loss of the creditor in case of the call option exercise. Based on the pricing of the non-callable bond above, in (40), the callable bond is priced in $t=0$, at:

$$p^c(z,r,k',b',\tilde{\theta},r) = \frac{1}{1+r}\mathbb{E}_{s'|s}\Bigg\{\int_{\overline{\varepsilon}'}^{\infty}\Big[r + (1-\mathbb{1}'_{\text{call}})\left(\gamma + (1-\gamma)\overline{p}^c_{\text{nocall}}(z',r',\hat{x}^k,\hat{x}^b,\tilde{\theta},r)\right) + \mathbb{1}'_{\text{call}}\left(1+\chi(s')\right)\Big] d\Phi(\varepsilon') + \int_{-\infty}^{\overline{\varepsilon}'}\frac{1-\xi}{b'}\underline{n}' d\Phi(\varepsilon')\Bigg\} \tag{42}$$

where $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}\left(s',\hat{x}(s,x)\right)$ is the call decision in the next period based on $(s',\hat{x}(s,x))$, $\overline{p}_j^c(z,r,k',b',\tilde{\theta},r)$ is the continuation price of the callable bond $(1-\tilde{\theta})\tilde{b}$ in the next period in the scenario $j \in \{\text{nocall}, \text{call}\}$. The formulation in (42) considers the potential loss to the bondholder due to the issuer's option to call the bond. For simplicity, instead of $\overline{p}^c$, I present the continuation price of the current callable bond $(1-\theta)b$.

$$\overline{p}^c(z,r,k',b',\theta,c) = (1-\mathbb{1}_{\text{call}}(s,x))\cdot\overline{p}^c_{\text{nocall}}(s,x) + \mathbb{1}_{\text{call}}(s,x)\cdot\overline{p}^c_{\text{call}}(s,x) \tag{43}$$

Contrarily to non-callable bonds, the pricing of the continuation value of callable bonds should differ depending on the call option exercise. While for the no-call scenario, an outstanding callable bond is priced similarly to a non-callable bond, in (41), in the case of calling, its pricing should stop at the final repayment, which is the principal and the call premium. This difference is then reconsidered in the continuation prices as follows.

$$\begin{aligned}\overline{p}_j^c = \frac{1}{1+r}\mathbb{E}_{s'|s}\Bigg\{ \int_{\overline{\varepsilon}'}^{\infty} \Big[c + (1-\mathbb{1}_{\text{call}}')\left(\gamma + (1-\gamma)\overline{p}_{\text{nocall}}^{c\prime}\right) \\ + \mathbb{1}_{\text{call}}'\left(1+\chi\right)\Big]d\Phi(\varepsilon') + \int_{-\infty}^{\overline{\varepsilon}'} \frac{1-\xi}{b'}\underline{n}'d\Phi(\varepsilon')\Bigg\}\end{aligned} \tag{44}$$

where $\overline{p}_j^{c\prime} = \overline{p}_j^c(z', r', \hat{x}(s', x')), \forall j \in \{\text{nocall}, \text{call}\}$. The price above is the callable bond's market value in scenario j. The bond is priced like it is newly issued with the current market conditions and the firm's characteristics. I provide further discussion of the pricing of callable bonds in Appendix (D.1.2).

4.2 Households

I consider a representative infinitely-lived household that owns all firms and receives the income in the economy. The household consumes, works, and invests its savings in equity and debt. Government revenue from taxation is returned to the household as a lump-sum transfer. The household has preferences over consumption C_t and labor supply L_t. The utility function is:

$$U(C_t, L_t) = ln(C_t) - \frac{L_t^{1+\vartheta}}{1+\vartheta}$$

The household discounts the future by the discount factor β (which corresponds to the average interest rate) and makes optimal choices through the marginal rate of substitution and inter-temporal substitution, i.e.

$$\begin{aligned}1 =& \mathbb{E}\Big[\beta\frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)}\Big] \\ w =& -\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)}\end{aligned} \tag{45}$$

4.3 Policy Authority

The government sets a fixed corporate tax τ on corporate periodic earnings. Concerning the monetary policy, I consider a simple framework where the authority decides on the real interest rate exogenous path. As in some works using New Keynesian Models (Jeenas, 2018; Ottonello and Winberry, 2020; Jungherr et al., 2024) or using a simple exogenous interest rate setting (e.g., Ippolito et al., 2018; Deng and Fang, 2022), I assume an AR(1) process:

$$logr' = \mu_r + \rho_r logr + \epsilon_r', \quad \epsilon_r' \sim^{i.i.d.} \mathcal{N}\left(0, {\sigma_r}^2\right), \quad \mu_r, \rho_r \in (0,1) \tag{46}$$

4.4 Equilibrium

Now, I define the competitive recursive equilibrium of the model, considering the economy's steady state. First, I describe the law of motion of the firms' distribution and define the stationary equilibrium.

Definition 4.1 (Law of motion of the firms' distribution)**.** *Let Γ be the distribution of incumbent firms at the beginning of the period, and Ω be the distribution of entrants firms. The distribution of firms that will produce in the next period is determined as follows:*

$$\begin{aligned}\Gamma'(z',k',b') = &\int_0^\infty \int_0^\infty \Gamma(z,k,b)\; \pi_e(z'|z)\Big[1 - \Phi\left(\overline{\varepsilon}'(z,r,k,b,\theta,c)\right)\Big] \cdot \mathbb{1}_{B_{inc}}\; dz\; db \\ &+ \Omega'(z',k',b')\end{aligned} \tag{47}$$

where $B_{inc} = \Big\{(z,k,b,k',b') \mid b' = (1-\gamma)\cdot b(z,r,k,b,\theta,c)\Big\}$*; and the distribution function of future entrants* $\Omega'(z', b'_{nc}, b'_c)$ *is defined as:*

$$\Omega'(z',k',b') = M \cdot \mathbb{1}_{B_{entr}}$$

where $B_{entr} = \Big\{(z',k',b') \mid k'=0, b'=0\Big\}$.

Definition 4.2 (Stationary Equilibrium)**.** *A stationary equilibrium in this economy consists of a set of: (i) value functions* $V(z,r,k,b,\theta,c)$, $V^r(z,r,k,b,\theta,c)$, $V^x(z,r,k,b,\theta,c)$, $\overline{V}(z,r,k,b,\theta,c)$, $\overline{V}_{nocall}(z,r,k,b,\theta,c)$*, and* $\overline{V}_{call}(z,r,k,b,\theta,c)$*; (ii) a vector of policy functions* $\hat{x}(z,r,k,b,\theta,c) = \{k',b',\theta',c'\}$*; (iii) bond pricing functions* $p^{nc}(z,r,k,b,\theta,c)$ *and* $p^c(z,r,k,b,\theta,c)$ *given by* (40) *and* (42)*; (iv) a mass of entrants* M^* *and a stationary distribution* Γ^* *; (v) household consumption* C^* *and aggregate labor supply* L^**; and (vi) a wage* w^* *and an interest rate* r^**, such that:*

1. *Given the bond price functions* $p^{nc}(z,r,k,b,\theta,c)$ *and* $p^c(z,r,k,b,\theta,c)$*, the policy vector* $\hat{x}(z,r,k,b,\theta,c)$*, the value function* $V(z,r,k,b,\theta,c)$*, and the default decision* $\overline{\varepsilon}$ *solve the firm's optimization problem* (38)*.*

2. *The free entry condition holds:* $V(z_e,0,0,\theta,c) = 0$*.*

3. *The bond price functions* $p^{nc}(z,r,k,b,\theta,c)$ *and* $p^c(z,r,k,b,\theta,c)$ *are consistent with the zero*

expected profit condition for the investors and the default probabilities and expected recovery rates satisfy the repayment policy;

4. *The representative household chooses optimally C^* and L^*, consistent with (45).*

5. *The goods market clears:*

$$
\begin{aligned}
Y \equiv & \int_0^\infty \int_0^\infty \Big[y - f - H\left(\tilde{b}(z,r,k,b,\theta,c), b, \theta, c\right) \\
& - \xi \int_{-\infty}^{\bar{\varepsilon}(z,r,k,b,\theta,c)} \underline{n} d\Phi(\varepsilon) \Big] \, \Gamma^* \left(z, b\right) \; dz \; db \\
= & \; C^* + I^*
\end{aligned}
\tag{48}
$$

with C^*, the household optimal consumption and I^*, the aggregate investment, defined as:

$$
I^* \equiv \delta \int_0^\infty \int_0^\infty k(z, b_{nc}, b_c) \; \Gamma^*(z, b_{nc}, b_c) \; dz \; db \tag{49}
$$

5. *The labor market clears*

$$
L^* \equiv \int_0^\infty \int_0^\infty l(z, b_{nc}, b_c) \; \Gamma^*(z, b_{nc}, b_c) \; dz \; db \tag{50}
$$

I follow the solution methods in Hatchondo et al. (2016) and Jungherr and Schott (2021) to solve the model equilibrium; see details in Appendix E.2.

5 Quantitative Analysis

This section presents the quantitative analysis of the model, including the calibration strategy, model validation, dynamic analysis, and counterfactual experiments. After calibrating the model to match key empirical moments, I assess its ability to replicate observed firm behavior and macroeconomic dynamics, focusing on the role of callable debt in firm financing decisions and its implications for the broader economy.

5.1 Calibration

The calibration process is divided into two parts: externally fixed parameters, which are chosen based on values found in the literature or empirically observed data, and internally calibrated parameters, which are estimated by fitting the model to match key empirical moments.

5.1.1 Externally fixed parameters

The externally fixed parameters are drawn from existing studies or empirical data and represent well-established or relatively stable aspects of the economy across different models. These parameters are not estimated within the model but are held constant throughout the calibration.

Table 3: Externally fixed parameters

Params.	Description	Value	Source/Target
ϑ	Inverted Frisch elasticity	0.5	Arellano et al. (2019)
τ	Corporate tax rate	0.4	Gomes et al. (2016)
ψ	Capital share	0.33	Bloom et al. (2018)
ζ	Decreasing returns-to-scale	0.75	Bloom et al. (2018)
δ	Quarterly depreciation rate	0.025	Standard (BEA)
π_e	Exogenous exit rate	0.01	Ottonello and Winberry (2020)
γ_{nc}	Non-callable debt repayment rate	0.05	Jungherr and Schott (2021)
ρ_z	Productivity shock persistence	0.9	Ottonello and Winberry (2020)

These externally fixed parameters are the model's foundation, ensuring the baseline environment reflects the broader macroeconomic and firms' average-level context. By grounding these parameters in empirical evidence and the literature, I ensure that the model's dynamics are consistent with well-established economic relationships. For the parameters of the production function, I take the capital share ζ and the decreasing returns-to-scale ψ from Bloom et al. (2018). The capital quarterly depreciation $\delta = 0.025$ is set to fit estimates from BEA. I use the persistence of the productivity process, $\sigma_z = 0.03$, estimated by Ottonello and Winberry (2020). Following Gomes et al. (2016), I fix the corporate tax rate τ to 0.4.

5.1.2 Internally fitted parameters

The internally fitted parameters are calibrated by matching the model to essential empirical moments. These parameters are adjusted to capture not only firm-specific financial behaviors but also aggregate dynamics. The calibration process is based on minimizing the distance between the model-implied moments and their empirical counterparts. Below, I detail the ten internally calibrated parameters, their roles within the model, their target empirical moments, and the data sources used accordingly for calibration.

The issuance cost parameters (η_{nc}, η_c) are primarily identified through their impact on the leverage ratio and the share of callable debt. Higher issuance costs for callable debt (η_c) decrease its prevalence, directly affecting the share of callable debt in the capital structure. The importance of these targets is emphasized with observations and intuitions on investment dynamics and refinancing flexibility, documented by Covas and Haan (2011), Crouzet (2018), Begenau and Salomao (2019), and Becker et al. (2024). The default cost parameter (ξ) is identified through its influence

Table 4: Internally fitted parameters

Params.	Description	Value	Target	Data	Model
η_{nc}	Non-callable issuance cost	0.0110	Leverage ratio	33%	29%
η_c	Callable issuance cost	0.0107	Share of callable debt	62%	64%
ξ	Default cost	0.6941	Avg credit spread (non-callable)	2.9%	2.9%
χ	Call premium	0.0082	Avg credit spread (callable)	3.2%	3.6%
f	Fixed operation cost	0.4772	Investment rate	22%	13%
σ_ε	Capital quality shock volatility	0.8874	Average call rate	47%	26%
σ_z	Productivity shock volatility	0.0180	Average exit rate	8.7%	8.6%
ρ_r	Interest rate persistence	0.8652	Average callable bond lifespan	48.6%	44.1%
σ_r	Interest rate volatility	0.0216	Average long-run interest rate	2.94%	2.91%
γ_c	Callable debt repayment rate	0.0408	Callable bond duration (years)	6.47	3.66

on credit spreads. Higher default costs lead to wider credit spreads, aligning with the observed average credit spreads across debt types. Acharya and Carpenter (2002), who emphasize the role of default costs in shaping corporate debt dynamics.

The call premium is central to matching the average credit spread on callable debt. By adjusting χ, we ensure that the model accurately reflects the market's additional risk assessment associated with callable features. A higher call premium discourages the issuance of callable debt, influencing its relative pricing compared to non-callable debt. This approach is consistent with Duffie and Singleton (1999), who explore the pricing implications of defaultable bonds with callable features.

Fixed operating costs influence the profitability and the investment rate. By calibrating f to match the average investment rate, we ensure that firms' investment behaviors in the model reflect empirical observations. High fixed operating costs constrain firms' ability to undertake growth-enhancing investments, highlighting the role of operating expenses as an essential determinant of firm-level decisions, as discussed in the framework of Hopenhayn (1992).

Capital quality shock volatility measures the uncertainty in firms' capital effectiveness, directly influencing default risk. Higher volatility implies increased earnings uncertainty and a greater likelihood of default, leading to wider credit spreads. The calibration of σ_ε targets the average call rate, capturing the overall risk premium demanded by financial markets. It allows the model to be in the sense of essential works in the literature. (e.g., Gilchrist and Zakrajšek, 2012 on credit spreads as a reflection of firm-specific risks).

Productivity shock volatility represents the unpredictability in firms' productivity levels, affecting their operational efficiency and survival prospects. This parameter is calibrated to match the average exit rate, ensuring that the model accurately reflects the impact of productivity fluctuations on firm dynamics and macroeconomic outcomes. By capturing the variability in productivity, σ_z plays an essential role in determining firms' investment and exit decisions. On the interest rate process, its persistence ρ_r is identified through its effect on the lifespan of callable bonds. Its effect, combined with the volatility of the process σ_r, also determines the intensity of the call decision. Higher persistence leads to longer-lived callable bonds, aligning the model with empirical observations of bond lifespans.

My calibration relies on firm-level data sources, primarily from Compustat and the Financial Information System Dataset (FISD). I use FRED data for the long-run interest rate. For instance, the leverage ratios, share of callable debt, and investment rates are sourced from Compustat. Moreover, secondary, some moments are taken coherently from key works in the literature (e.g., Arellano et al., 2019; Ottonello and Winberry, 2020).

5.2 Dynamic effects of callability

In this section, we explore the dynamics of the calibrated model, focusing on the implications of callable debt on firm behavior and macroeconomic outcomes. We analyze bond prices, capital and debt decisions, and the response of key variables to shocks.

5.2.1 Bond Prices

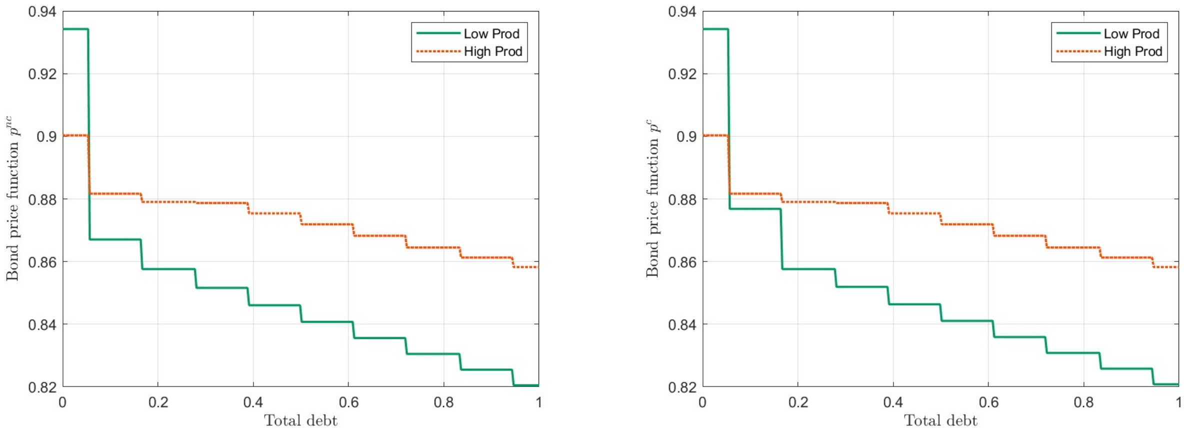
This subsection investigates how bond prices vary with the firm's debt levels, capital stock, and productivity. Understanding the relationship between these variables and bond prices is crucial for analyzing the cost of financing and the risk premium investors require. I examine the price of non-callable and callable bonds as functions of the firm's state variables: total debt b, capital k, and productivity z. The bond price reflects the present value of future coupon payments, adjusted for the probability of default and the probability that the bond will be called in the case of callable bonds.

Higher debt levels: As debt increases, bond prices decline, reflecting the increased default risk associated with higher leverage. This effect is more pronounced for callable bonds, where the call option introduces additional pricing considerations.

Productivity influence: Higher productivity levels z are associated with higher bond prices, as firms are less likely to default when productivity is strong. This relationship is consistent across both callable and non-callable bonds.

Capital stock: A larger capital stock k generally leads to higher bond prices, which signals a stronger balance sheet and lower default risk. These findings highlight the link between a firm's financial position and the cost of debt financing, particularly when callable debt is involved.

Figure 10: Bonds prices as function of debt



(a) Average non-callable bond prices

(b) Average callable bond prices

Notes: This figure plots the prices of non-callable and callable bonds across different levels of total debt for two distinct productivity levels: low ($z = z_{low}$), and high ($z = z_{high}$).

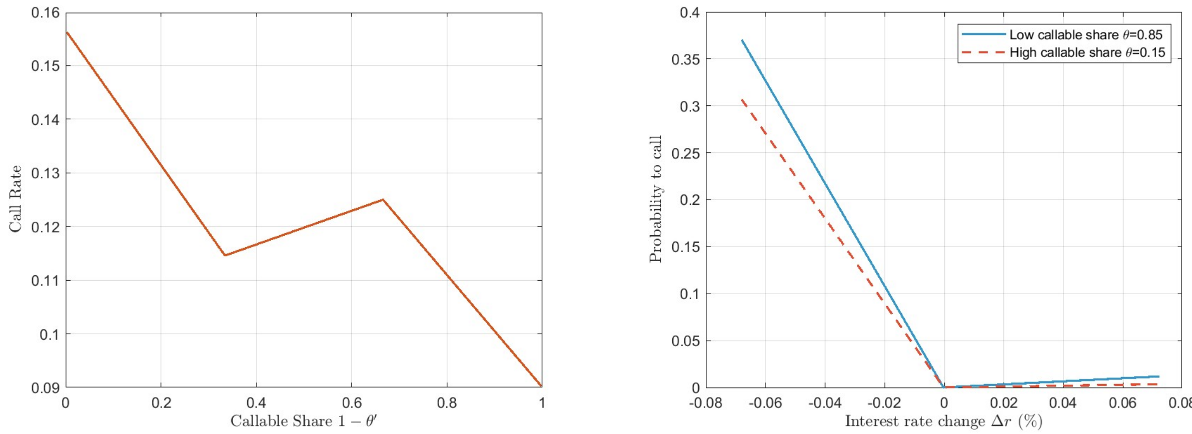
5.2.2 Heterogeneous effects of callability

The decision to call a bond is influenced by interest rate fluctuations, the firm's financial position, and the callable share of its debt portfolio. As shown in Figure 11a, firms with a higher share of callable bonds are less likely to call. Theoretically, the firm's decision to call depends on the trade-off between the savings from refinancing at lower rates and the premium paid to exercise the call option. For firms with a high callable share ($\theta = 0.15$), the average probability of calling is relatively lower across interest rate changes, reflecting limited refinancing flexibility. Conversely, firms with a low callable share ($\theta = 0.85$) exhibit a steeper response to rate declines, as refinancing significantly reduces their cost of capital and may appear more accessible. The call option allows them to benefit from declining rates. Specifically, a lower interest rate environment incentivizes firms to refinance their debt at reduced costs. In contrast, the cost of maintaining callable bonds may deter firms from exercising the option when rates rise.

Figure 12 shows the firms' investment and financing response to interest rate changes in the two cases of low callable share (i.e., $\theta = 0.85$) and high share (i.e., $\theta = 0.15$) debt. The values of the variables are normalized to 1 when the rate $r_t = 1.01\%$.

Figures 12a and 12b illustrate the behavior of firms' next-period capital (k') and next-period debt (b') in response to interest rate changes (Δr), depending on their share of non-callable debt (θ). These figures reveal how callability affects firms' investment and financing strategies, with significant differences between firms with low callable shares ($\theta = 0.85$) and high callable shares ($\theta = 0.15$). Firms with a high callable share ($1 - \theta = 0.85$)—typically smaller firms—demonstrate

Figure 11: Call decision

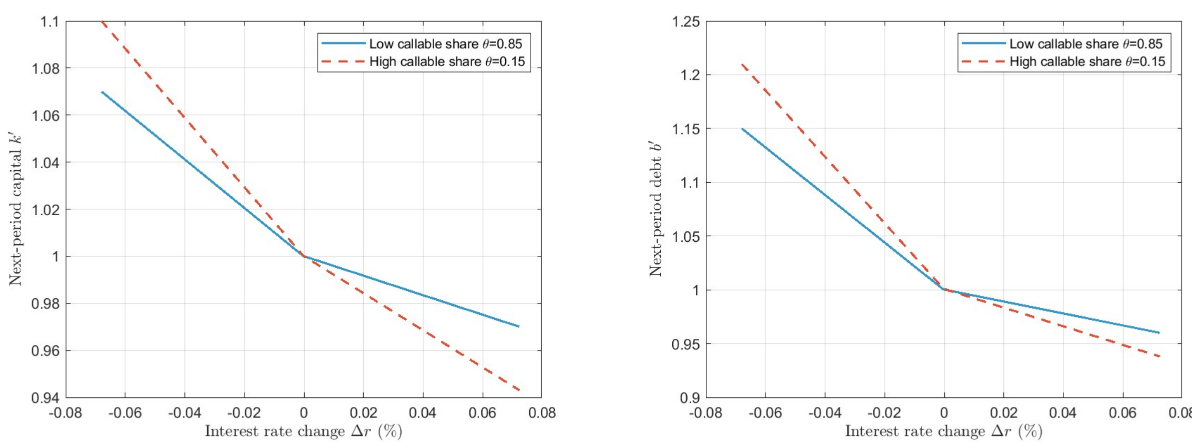


(a) Call rate

(b) Decision rule to call

Notes: This figure plots the average call rate depending on the share of callable debt (left panel) and the response of firms' optimal decision to call (right panel). The left panel shows the average call rate of a generic firm simulated for the average level of productivity, capital, debt, and periodic coupon. The right panel displays two generic firms simulated for the average level of productivity, capital, debt, share of non-callable debt, and periodic coupon. The firms are blue for the low share of callable debt ($\theta = 0.15$) and red for the high share ($\theta = 0.85$). The right panel represents the average call rate for different values of the interest rate realization $\{0.93\%, ..., 1.09\%\}$.

Figure 12: The effect of callability on firm behavior



(a) Investment

(b) Next period debt

Notes: This figure plots the response of firms' optimal choice of investment (left panel) and next-period debt (right panel) to an interest rate change. The panels display two generic firms simulated for the average level of productivity, capital, debt, share of non-callable debt, and periodic coupon. The firms are blue for the low share of callable debt ($\theta = 0.15$) and red for the high share ($\theta = 0.85$). The panels represent the values of the variables of interest for different values of the interest rate realization $\{0.93\%, ..., 1.09\%\}$.

a sharper response to declining interest rates. Callable bonds provide these firms with refinancing flexibility, enabling them to redirect savings from lower borrowing costs into investment. This heightened sensitivity mirrors traditional models of monetary policy, where smaller firms react more strongly due to their reliance on external financing. In contrast, firms with a low callable share ($1 - \theta = 0.15$)—often larger firms—exhibit more stable investment responses. While they call less frequently, these firms adjust their investment moderately, benefiting less from the refinancing flexibility offered by callable bonds. Their smoother adjustment highlights their reliance on non-callable debt, which does not allow immediate cost reductions when rates decline.

The financing behavior, reflected in next-period debt (b'), aligns with these patterns. High-callable-share firms ($1 - \theta = 0.85$) increase their borrowing significantly when interest rates fall, leveraging callable debt's refinancing benefits when they call existing callable debts. However, they reduce debt more when rates rise, reflecting the higher cost burden of callable bonds in such environments.

As a quantitative exercise, I compare an economy with callable debt ($\theta = 0$) to a baseline economy without callable debt ($\theta = 1$). The results demonstrate the critical role of callable debt in amplifying firms' investment responses to interest rate changes. Specifically, for a 6.8 percentage point decline in interest rates, the optimal investment in the callable economy reaches $k' = 1.285$, compared to $k' = 1.012$ in the baseline economy. This represents a 26.98% higher investment level in the callable economy. When normalized, this implies that within the investment response to a change of interest rates, callable debt contributes to 21%. These findings confirm that callable debt introduces additional flexibility in firms' capital allocation, significantly enhancing their sensitivity to monetary policy. This amplification underscores the broader macroeconomic implications of callable bonds and their potential to reshape monetary policy transmission across firms.

6 Conclusion

In this paper, I investigated the relationship between callable debt and firm decisions, grounded in three key stylized facts regarding the callability structure of corporate bonds among U.S. non-financial firms from 1990 to 2018. My analysis revealed that the use of callable bonds has risen significantly over the past decades, with 62% of corporate bonds now issued with a call option. Additionally, I found that firms holding a larger share of callable bonds tend to have more assets and higher investment levels, though significant heterogeneity exists across firms.

Empirical evidence indicates that larger firms with higher creditworthiness are more likely to call their callable bonds, underscoring the importance of firm size and credit quality in callability decisions. These firms benefit from more favorable borrowing conditions, enabling them to strategically call their bonds to reduce borrowing costs. In contrast, smaller, higher-risk firms are more constrained in exercising their call options due to the rollover risks associated with refinancing, which incentivizes them to retain callable bonds as a way to secure long-term financing.

To further explore the dynamics of callable debt, I developed a quantitative model that incorporates firm decisions on bond issuance, call options, and default risk, with particular attention to the role of credit quality. The results suggest that firms face a trade-off when choosing callable over non-callable debt: the potential to refinance under better terms versus the exposure to greater debt obligations and default risk. Callable bonds offer firms flexibility to adjust their debt burden in response to productivity shocks and changes in interest rates, thereby mitigating default risk. However, the effectiveness of this flexibility is heavily contingent on both market conditions and the firm's credit quality. Firms with higher credit ratings are better equipped to leverage callable debt as a refinancing tool, whereas lower-rated firms may struggle to do so due to heightened refinancing risks and limited access to favorable terms.

Consistent with the findings of Gilchrist and Zakrajšek (2012), my model shows that callable bonds generally reduce default risk for issuers by allowing them to refinance at more favorable terms during periods of declining interest rates. However, the impact of callability on default risk is asymmetric: smaller, higher-risk firms tend to call their bonds less frequently than larger, more creditworthy firms, reflecting the former's greater exposure to refinancing risk and limited flexibility.

The model also highlights significant asymmetries in callable bond issuance across different interest rate environments and credit quality distributions. In a low-interest-rate environment, where firms expect future rates to rise, high-risk firms with lower credit ratings are more likely to favor non-callable bonds, as callable bonds become less advantageous in the context of rising rates. Conversely, in a high-interest-rate environment, where firms anticipate stable or declining future rates, callable bonds become more attractive—especially for smaller firms—since they offer the potential for refinancing at lower rates, despite the increased default risk. In this scenario, firms with stronger credit quality are better positioned to manage the trade-offs between flexibility and debt servicing costs.

Large, low-risk firms approach callable debt issuance more strategically, carefully balancing the benefits of future refinancing flexibility against the higher interest rates associated with callable bonds. These firms are generally better able to manage the trade-offs between the debt revenue effect (from future refinancing) and the repayment effect (reflecting current debt obligations), optimizing their debt structure in response to economic conditions.

In conclusion, this study underscores the important role of callable debt in managing firm debt obligations and in responding to productivity and interest rate shocks. Credit quality plays a pivotal role in determining how firms utilize callable bonds, influencing their ability to refinance and mitigate default risk. Firms optimize their debt structure based on productivity levels, creditworthiness, volatility in productivity shocks, and prevailing economic conditions such as interest rates. The findings suggest significant heterogeneity in how firms of different sizes and credit qualities utilize callable debt, impacting their investment and financing decisions.

Moreover, the framework developed in this paper provides an insightful tool for future research on the macroeconomic implications of debt callability. It offers a structured approach for examining how callable bonds influence firm behavior in different economic contexts, and how this behavior interacts with broader macroeconomic forces such as monetary policy and aggregate demand fluctuations. Future studies could build on this model to explore the aggregate effects of callable debt across business cycles and assess the financial stability implications of widespread callability structures in corporate debt markets.

Looking forward, further research could explore the aggregate implications of debt callability, particularly its role in the transmission of monetary policy shocks. The rise of callable debt in corporate financing raises important questions about how the structure of debt callability and the distribution of credit quality influence firms' responsiveness to monetary policy and macroeconomic fluctuations. These questions present promising avenues for further investigation into the broader macroeconomic impact of callable bonds on financial stability and economic growth.

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Appendix to "Debt Callability and Firm Dynamics"

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July 22, 2025

A Data

In this paper, I use two datasets: bond-level data on bonds' characteristics from Mergent FISD, and firm-level data on investment and financial information from CRSP-Compustat. In this section, I describe the sample selection and the variables used in the empirical analysis.

A.1 Sample Selection

The sample used in this study consists of non-financial publicly listed firms in the United States from January 1990 to May 2018. The primary data sources include:

Mergent Fixed Income Securities Database (Mergent FISD): This database provides detailed information on corporate bonds, including issuance date, original issuance amount, callability, convertibility, covenants, and other bond-specific attributes. The sample is filtered to include only U.S. corporate bonds issued by non-financial firms, excluding utilities (SIC codes 4900–4999) and financial firms (SIC codes 6000–6999).

Compustat: Compustat offers quarterly data on firm-specific variables, including financial positions and balance sheet information. Firms in the final sample are required to have total debt representing at least 5% of their assets. All variables are winsorized at the 1% and 99% levels to mitigate the influence of extreme values.

Center for Research in Security Prices (CRSP): Stock price data and other stock-related variables are obtained from CRSP.

These data sources are merged using the CUSIP identifier, which uniquely identifies each firm or issuer, a la Jungherr and Schott (2021).

A.2 Variable Construction

The variables below provide insights into the characteristics of firms that issue callable bonds and their impact on firm behavior and financing decisions.

Callable Bonds Indicator:
A bond is classified as callable if the issuer has the option to redeem the bond before its maturity. The proportion of callable bonds issued by a firm is calculated as the share of callable bonds relative to the total bonds issued.

Firm Size (Log of Real Total Assets):
Firm size is measured as the logarithm of the firm's total assets, using Compustat. I also define

firm size as the size of its average stock of debt. In the callable structure analysis, I use the credit rating as a proxy for the firm size.

Investment Rate (%):
The investment rate is computed as the ratio of capital expenditures to total assets.

Leverage (% Debt/Assets):
Leverage is calculated as the ratio of total debt to total assets.

Return on Assets (ROA, %):
ROA is measured as the ratio of net income to total assets.

Market-to-Book Ratio:
This ratio is calculated as the market value of equity divided by the book value of equity.

Credit Rating:
Firms are sorted into credit rating categories based on their long-term credit ratings, with lower scores indicating higher credit risk.

Interest Coverage Ratio:
This ratio is computed as earnings before interest and taxes (EBIT) divided by interest expenses.

Cash Holdings (% Assets):
Cash holdings are measured as the ratio of cash and cash equivalents to total assets.

Debt Maturity:
Debt maturity is calculated as the proportion of long-term debt (debt maturing in more than one year) relative to total debt. This variable captures the firm's debt structure and potential refinancing risks. In Mergent FISD, I use the "action" variable to ensure that the bond is still unmatured and have its status of callability.

Call Date and Call Dummy:
The call date is recorded as the date when a callable bond is effectively called by the issuer. It excludes then the unmatured bonds which disappear for other reasons. Based on this definition, the call dummy indicates 1 if the bond is callable and called, and 0 if the bond is callable, still alive, and not matured yet. I use the "action" variable in Mergent FISD and the redemption file to identify properly the firm's decision of call.

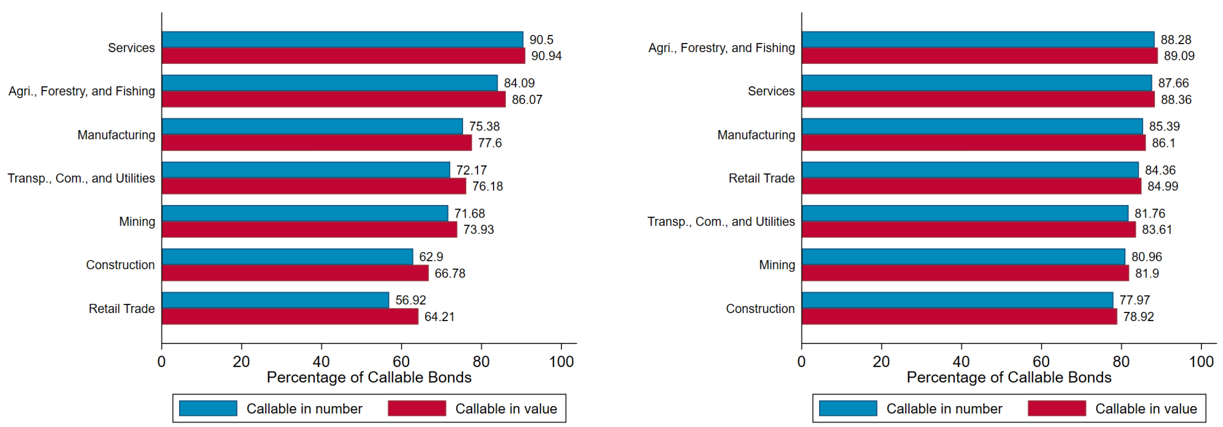
B Additional Empirical Results

This subsection includes additional empirical exercises that support differences brought by callable bond issuance.

B.1 Callable Bond prevalence across Industry

Using SIC codes, I categorized bonds into different industries and calculated the number of callable and total bonds issued by each industry. I also computed the percentage of callable bonds in terms of both the number of bonds issued and the total value of bonds issued for each industry. Industries like services, agriculture, forestry, fishing, and manufacturing tend to have higher shares of callable debts. These industries often experience significant fluctuations in cash flows due to seasonal demand, market conditions, debt overhangs, and other external factors. These are relevant reasons for the high callable share for small -high risky- firms.

Figure A1: Callable Bonds by Industry



(a) At the issuance (b) Outstanding bonds

Notes: I compute the share of callable bonds for the case of the issuance and the case of outstanding bonds. Data are from Mergent FISD & CRSP-Compustat.

B.2 Bonds Characteristics

Callable bonds are generally issued at lower prices compared to non-callable bonds. Their coupon rate is, on average, higher because it makes them more attractive and justifies the risk of reinvesting for the holder of the bonds. The characteristics are shown in Table A1.

Table A1: Callable bonds vs Non Callable bonds

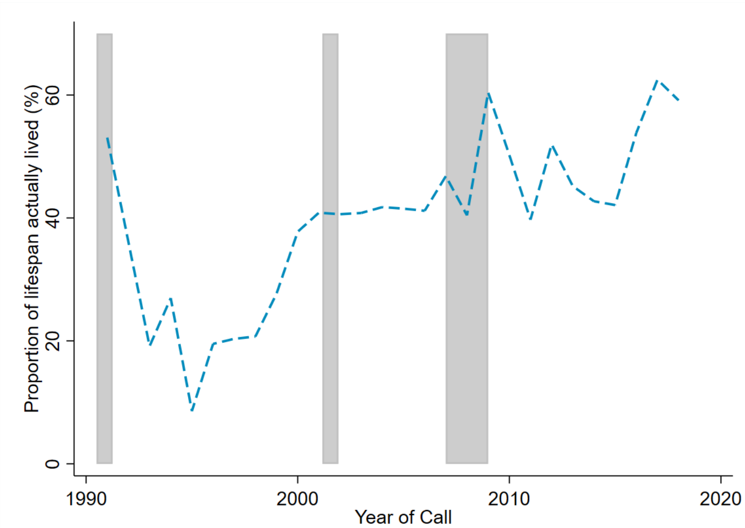
	Callable bonds	N Callable bonds
Maturity	12.9657 years	12.4684 years
Issue size ($ mn)	49.1148	24.7659
Coupon rate (%)	6.8351	6.61763
Offering price	98.2012	99.5552

Notes: The differences between the two columns are significant at 5%. For the bond prices, I consider the average prices across data at the firm level. Data are from Mergent FISD.

B.3 Time of calling

This section adds empirical evidence about the average timing of the call option exercise. The figure shows that callable bonds are typically called on average at the middle of their initial maturity.

Figure A2: After which percentage of their lives, bonds are called?



Notes: I identify what percentage of the initial maturity of the debt has elapsed between the issuance and the redemption of the debt. This percentage is computed as the annual average over our sample, with the call option exercise happening from 1991 to 2018, in our data. Data are from Mergent FISD.

C Three-period Model Appendix

This subsection provides details on theoretical derivations and proofs of propositions, corollaries, and other optimal characteristics of the three-period model presented in the section (3).

C.1 Proof of the refinancing decision

In this proof of (17) and (18), I provide the details of the optimal decision on refinancing in period $t = 1$ after respectively, not calling and calling the callable debt $(1 - \theta_1)b_1$. The new debt chosen is denoted by b_2. The first order condition with respect to the debt issued, respectively $b_2 - b_1$ and $b_2 - \theta_1 b_1$, is described in the following lines.

For the no-call scenario, we have:

$$[b_2 - b_1] : p_1 + \frac{\partial p_1}{\partial (b_2 - b_1)}(b_2 - b_1) + \beta_1 \frac{\partial}{\partial (b_2 - b_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{nocall}}^{\infty} V_2^{nocall}(s_2, x_2)\varphi(\varepsilon_2)d\varepsilon_2 \quad \leq 0 \tag{A1}$$

Using the definition of the price of the new debt in (10) and its components in (11), I have the derivatives of the price and the expected future value of the firm with respect to the default risk:

$$\frac{\partial p_1}{\partial \bar{\varepsilon}_2} = \theta_2^{new} \frac{\partial p_1^{nc}}{\partial \bar{\varepsilon}_2} + (1 - \theta_2^{new}) \frac{\partial p_1^{c}}{\partial \bar{\varepsilon}_2} = -\mathbb{E}_{z_2|z_1}\left[\varphi(\bar{\varepsilon}_2)\right] \tag{A2}$$

$$\frac{\partial}{\partial \bar{\varepsilon}_2} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} V_2(s_2, x_2)\varphi(\varepsilon_2)d\varepsilon_2 = -k_2 \mathbb{E}_{z_2|z_1}\left[1 - \Phi(\bar{\varepsilon}_2)\right] \tag{A3}$$

A high level of of next period debt b_2 increases the default risk, shown by the following:

$$\frac{\partial \bar{\varepsilon}_2}{\partial (b_2 - b_1)} = \frac{1 + c_2}{k_2} \tag{A4}$$

When combining (A2), (A3), and (A4), I have the variations in the new debt, of the price of the new debt and the expected future value of the firm:

$$\frac{\partial p_1}{\partial (b_2 - b_1)} = -\frac{1 + c_2}{k_2} \mathbb{E}_{z_2|z_1}\left[\varphi(\bar{\varepsilon}_2)\right] \tag{A5}$$

$$\frac{\partial}{\partial (b_2 - b_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} V_2(s_2, x_2)\varphi(\varepsilon_2)d\varepsilon_2 = -(1 + c_2) \mathbb{E}_{z_2|z_1}\left[1 - \Phi(\bar{\varepsilon}_2)\right] \tag{A6}$$

I rewrite the FOC of the new debt $b_2 - b_1$:

$$[b_2 - b_1] : -(1 + c_2)\frac{b_2 - b_1}{k_2}\mathbb{E}_{z_2|z_1}\left(\varphi(\overline{\varepsilon}_2)\right) + \left(1 - \beta_1(1 + c_2)\right)\mathbb{E}_{z_2|z_1}\left[1 - \Phi\left(\overline{\varepsilon}_2\right)\right] = 0 \qquad \text{(A7)}$$

I can deduce the form shown in (17) by using the composition of the average coupon defined in (19).

Concerning the call scenario, the method is the same, except for the debt b_1 which is replaced by $\theta_1 b_1$ in the outstanding debt. I write then the FOC of the new debt $b_2 - \theta_1 b_1$:

$$[b_2 - \theta_1 b_1] : -(1 + c_2)\frac{b_2 - \theta_1 b_1}{k_2}\mathbb{E}_{z_2|z_1}\left(\varphi(\overline{\varepsilon}_2)\right) + \left(1 - \beta_1(1 + c_2)\right)\mathbb{E}_{z_2|z_1}\left[1 - \Phi\left(\overline{\varepsilon}_2\right)\right] = 0 \qquad \text{(A8)}$$

In turn, I consider the composition of the average coupon when the firm exercises its call option, to obtain finally the optimal refinancing in the call scenario in (18).

■

C.2 Optimal values when $(1 - \theta_1)b_1$ is not called in $t = 1$

The default threshold $\overline{\varepsilon}_2^{nocall}$ in $t = 2$, when the callable bond is not called in $t = 1$, is defined as follows:

$$\begin{aligned}
\overline{\varepsilon}_2^{nocall} &: 0 = V_2^{nocall}(s_2, x_2) \\
&: 0 = z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2)b_2
\end{aligned}$$

I derive the threshold $\overline{\varepsilon}_2^{nocall}$:

$$\overline{\varepsilon}_2^{nocall} = -\frac{k_2 + y_2 - (1 + c_2)b_2}{k_2} \qquad \text{(A9)}$$

I reformulate the optimal value of the firm (in (10)) when it doesn't call its callable bond:

$$V_1^{nocall}(s_1, x_1) = -c_1 b_1 - k_2 + p_1(b_2 - b_1) + \beta_1 \cdot k_2 \cdot \mathbb{E}_{z_2|z_1}\int_{\overline{\varepsilon}_2}^{\infty}(\varepsilon_2 - \overline{\varepsilon}_2)\varphi(\varepsilon_2)d\varepsilon_2 \qquad \text{(A10)}$$

The variation in the value function above in (A10), induced by changes in the default threshold $\overline{\varepsilon}_2^{nocall}$ is defined as follows:

$$\Delta\overline{\varepsilon}_2^{nocall} = -(b_2 - b_1)(1 + c_2)\mathbb{E}_{z_2|z_1}\left[\varphi\left(\overline{\varepsilon}_2\right)\right] - \beta_1 \cdot k_2 \cdot \mathbb{E}_{z_2|z_1}\Big[1 - \Phi(\overline{\varepsilon}_2)\Big] \qquad \text{(A11)}$$

The first-order condition with respect to k_2 in the period $t = 1$ is explained in:

$$-1 + \frac{\partial \overline{\varepsilon}_2}{\partial k_2} \cdot \Delta \overline{\varepsilon}_2^{nocall} + \beta_1 \cdot \mathbb{E}_{z_2|z_1} \int_{\overline{\varepsilon}_2}^{\infty} \Big(\varepsilon_2 - \overline{\varepsilon}_2\Big) \varphi(\varepsilon_2) d\varepsilon_2 = 0 \tag{A12}$$

with

$$\frac{\partial \overline{\varepsilon}_2}{\partial k_2} = -\frac{1 + mpk_2 \ + \ \overline{\varepsilon}_2}{k_2} = \frac{(1-\alpha)y_2 - (1+c_2)b_2}{k_2^2} \tag{A13}$$

Where $mpk_2 = \alpha z_2 (k_2)^{\alpha-1}$ is the marginal productivity of the capital k_2 in the period $t = 2$. I use the optimal condition in (A12) multiplied by k_2 to write the optimal value of the function in (A10):

$$V_1^{nocall}(s_1, x_1) = -c_1 b_1 + p_1(b_2 - b_1) - \frac{\partial \overline{\varepsilon}}{\partial k_2} \cdot \Delta \overline{\varepsilon}_2^{nocall} \cdot k_2 \tag{A14}$$

Now, I replace $\dfrac{\partial \overline{\varepsilon}_2}{\partial k_2}$ and $\Delta \overline{\varepsilon}_2^{nocall}$ by their respective expressions in (A13) and (A11), and I use the optimal refinancing formula in (17). I obtain the expression of the optimal value of the firm used in the proposition (2):

$$V_1^{nocall}(s_1, x_1) \ = \ - c_1 b_1 + \mathbb{E}_{z_2|z_1} \left\{ \left[-b_1 + \frac{(1-\alpha)y_2}{1+c_2} \right] \times \left[1 - \Phi(\overline{\varepsilon}_2^{nocall})\right] \right\} \tag{A15}$$

■

C.3 Optimal values when $(1-\theta_1)b_1$ is called in $t = 1$

When the firm calls its the callable bond in $t = 1$, the default threshold $\overline{\varepsilon}_2^{call}$ in $t = 2$, is obtained as follows:

$$\begin{aligned} \overline{\varepsilon}_2^{call} &: 0 = V_2^{call}(s_2, x_2) \\ &: 0 = z_2(k_2)^{\alpha} + \varepsilon_2 k_2 - (1+c_2)b_2 \end{aligned}$$

I derive the threshold $\overline{\varepsilon}_2^{call}$:

$$\overline{\varepsilon}_2^{call} \ = - \ \frac{k_2 + y_2 - (1+c_2)b_2}{k_2} \tag{A16}$$

I reformulate the optimal value of the firm (in (12)) when it doesn't call its callable bond:

$$\begin{aligned} V_1^{call}(s_1, x_1) = -(1+\chi)(1-\theta_1)b_1 - c_1 b_1 - k_2^{call} + p_1(b_2 - \theta_1 b_1) \\ + \beta_1 \cdot k_2^{call} \cdot \mathbb{E}_{z_2|z_1} \int_{\overline{\varepsilon}_2^{call}}^{\infty} (\varepsilon_2 - \overline{\varepsilon}_2^{call}) \varphi(\varepsilon_2) d\varepsilon_2 \end{aligned} \tag{A17}$$

I derive here the variation in the value function above in (A17), induced by changes in the default threshold $\overline{\varepsilon}_2^{call}$ defined as:

$$\Delta\overline{\varepsilon}_2^{call} = -(b_2 - \theta_1 b_1)(1+c_2)\mathbb{E}_{z_2|z_1}\left[\varphi\left(\overline{\varepsilon}_2^{call}\right)\right] - \beta_1 \cdot k_2^{call} \cdot \mathbb{E}_{z_2|z_1}\Big[1 - \Phi(\overline{\varepsilon}_2^{call})\Big] \tag{A18}$$

The first-order condition with respect to k_2 in the period $t = 1$ is shown in:

$$-1 + \frac{\partial\overline{\varepsilon}_2}{\partial k_2} \cdot \Delta\overline{\varepsilon}_2^{call} + \beta_1 \cdot \mathbb{E}_{z_2|z_1} \int_{\overline{\varepsilon}_2}^{\infty} \Big(\varepsilon_2 - \overline{\varepsilon}_2\Big)\varphi(\varepsilon_2)d\varepsilon_2 = 0 \tag{A19}$$

with

$$\frac{\partial\overline{\varepsilon}_2}{\partial k_2} = -\frac{1 + mpk_2 \ + \ \overline{\varepsilon}_2}{k_2} = \frac{(1-\alpha)y_2 - (1+c_2)b_2}{k_2^2} \tag{A20}$$

I multiply by k_2 the optimal condition in (A19) to write the optimal value of the function in (A17):

$$V_1^{call}(s_1, x_1) = -(1+\chi)(1-\theta_1)b_1 - c_1 b_1 + p_1(b_2 - \theta_1 b_1) - \frac{\partial\overline{\varepsilon}_2}{\partial k_2} \cdot \Delta\overline{\varepsilon}_2^{call} \cdot k_2 \tag{A21}$$

Now, I replace $\frac{\partial\overline{\varepsilon}_2}{\partial k_2}$ and $\Delta\overline{\varepsilon}_2^{call}$ by their respective expressions in (A20) and (A18), and I consider the final expression of the optimal refinancing in (18). I get the expression of the optimal value of the firm used in the proposition 2:

$$V_1^{call}(s_1, x_1) = -(1+\chi)(1-\theta_1)b_1 - c_1 b_1 + \mathbb{E}_{z_2|z_1}\left\{\left[-\theta_1 b_1 + \frac{(1-\alpha)y_2}{1+c_2}\right] \times \left[1 - \Phi(\overline{\varepsilon}_2^{call})\right]\right\} \tag{A22}$$

■

C.4 Proof of Proposition 1

Proof. First, let's recall the expression of the optimal refinancing in the no-call scenario.

$$[b_2 - b_1] : -(1+c_2)\frac{b_2 - b_1}{k_2}\mathbb{E}_{z_2|z_1}\left(\varphi(\overline{\varepsilon}_2)\right) - \beta_1(c_1 - r_1)\frac{b_1}{b_2}\mathbb{E}_{z_2|z_1}\left[1 - \Phi\left(\overline{\varepsilon}_2\right)\right] = 0 \tag{A23}$$

θ_1 does not affect the optimal refinancing when the callable part is not called. This is due to the same impact of θ_1 in the second fraction in front of the expectation term.

To complete the proof, I discuss the relative levels of the coupon rate c_1 on existing debt and the new interest rate r_1.

If $c_1 < r_1$, refinancing is not so beneficial for the firm, whatever its level of debt (same for calling). A high interest rate of r_1 is not ideal for refinancing in the no-call scenario. But if the firm calls its debt despite the interest rate level, the share θ_1 of non-callable will reduce the debt cost since it carries smaller interest expenses. So firms with high θ_1 are more likely to refinance after calling when $c_1 < r_1$ through the reduction in the cost of their new total debt. θ_1 is important for cheaper refinancing when a bond is called in a $c_1 < r_1$ period.

If $c_1 > r_1$, it is the right time to refinance (eventually to call), and the refinancing cost can be reduced. Moreover, firms with small θ_1 will be more encouraged to refinance because more weight is put on the interest rate (which is smaller) for future debt costs. Even if refinancing is more supportable in the future when the new interest rate is smaller, firms with high θ_1 are stuck with the relatively high-interest payment, discouraging the refinancing. θ_1 is then detrimental for lower-cost refinancing in the call scenario in a $c_1 > r_1$ period. ■

C.5 Proof of Corollary 3.0.1

Proof. I recall the definition of the firm-specific periodic coupon in (19) after the call decision. The change in this coupon concerning the change in the interest rate is given by:

$$\frac{\partial c_2}{\partial r_1} = \begin{cases} 1 - \dfrac{\theta_1 b_1}{b_2} & \text{when no call} \\ 1 - \dfrac{\theta_1 b_1}{b_2} & \text{when call} \end{cases} \tag{A24}$$

If $c_1 < r_1$, when the firm does not call, having existing debt b_1 will attenuate the rise in the cost of capital, whatever the stock of (non)callable bond. When the firm calls, having high θ_1 will serve to drag down the rising cost of capital c_2. So firms with a high θ_1 face a relatively low new cost of capital in c_2 than firms with low θ_1. Then, their investment decreases less when they call. This is because they have a high share of existing callable bonds, so they will face more fully the new rate r_1 after calling it.

Suppose $c_1 > r_1$, when the firm decides not to call its callable debt, having a small stock of debt b_1 contributes to the reduction of the new cost of capital c_2. This is independent of the share of callable in this existing debt. In the case of calling, the part that is not called $\theta_1 b_1$ has to pay the differential interest $c_1 - r_1$. So firms with a high share of callable bonds (which means low θ_1) increase the optimal capital more when they call than firms with lower callable shares.

This completes the proof of the corollary. ■

C.6 Proof of Proposition 2

Proof. To prove Proposition 2, I first recall the price formulas of new bonds in period $t = 1$:

$$p_1 = \begin{cases} \mathbb{E}_{z_2|z_1}\left[1 - \Phi(\bar{\varepsilon}_2^{nocall})\right] & \text{when no call} \\ \mathbb{E}_{z_2|z_1}\left[1 - \Phi(\bar{\varepsilon}_2^{call})\right] & \text{when call} \end{cases} \tag{A25}$$

I consider the inequality with the transformed formulations of the firm's optimal value in the cases of not calling ((A15)) and calling((A22)), to establish the inequality:

$$\begin{aligned} &\cancel{-c_1 b_1} + \mathbb{E}_{z_2|z_1}\left\{\left(-b_1 + \frac{(1-\alpha)y_2^{nocall}}{1+c_2}\right)\left[1 - \Phi\left(\bar{\varepsilon}_2^{nocall}\right)\right]\right\} \\ &\quad < -(1+\chi)(1-\theta_1)b_1 \cancel{-c_1 b_1} + \mathbb{E}_{z_2|z_1}\left\{\left(-\theta_1 b_1 + \frac{(1-\alpha)y_2^{call}}{1+c_2}\right)\left[1 - \Phi\left(\bar{\varepsilon}_2^{call}\right)\right]\right\} \end{aligned} \tag{A26}$$

To complete the proof, we make the term $-c_1 b_1$ disappear at both sides of the inequality. ■

C.7 Proof of Proposition 3

Proof. The proposition concerns the possibility of crossing for the firm's value in both cases of not calling and calling. I prove the proposition through four points. It is important to note that the signs of both values do not matter because they are monotone and continue functions.

1. Based on the form of the firm's value in the no-call scenario, I have:

$$\frac{\partial \overline{V}_1^{nocall}(s_1, x_1)}{\partial \theta_1} = 0. \tag{A27}$$

2. Now, I derive the change in the firm's value in both scenarios, following a change in interest rate r_1.

$$\begin{aligned} \frac{\partial \overline{V}_1^{nocall}(s_1, x_1)}{\partial r_1} &= \frac{\partial}{\partial r_1}\mathbb{E}_{z_2|z_1}\left\{\left(-b_1 + \frac{(1-\alpha)y_2^{nocall}}{1+c_2}\right)\left[1 - \Phi\left(\bar{\varepsilon}_2^{nocall}\right)\right]\right\} \\ &= \mathbb{E}_{z_2|z_1}\left\{\left[1 - \Phi\left(\bar{\varepsilon}_2^{nocall}\right)\right](1-\alpha)y_2^{nocall}\frac{\partial}{\partial r_1}\left(\frac{1}{1+c_2}\right)\right\} \\ &= -\mathbb{E}_{z_2|z_1}\left\{\left[1 - \Phi\left(\bar{\varepsilon}_2^{nocall}\right)\right]\frac{(1-\alpha)y_2^{nocall}}{(1+c_2)^2}\frac{b_2 - b_1}{b_2}\right\} < 0 \end{aligned} \tag{A28}$$

$$
\begin{aligned}
\frac{\partial \overline{V}_1^{call}(s_1,x_1)}{\partial r_1} &= \frac{\partial}{\partial r_1}\mathbb{E}_{z_2|z_1}\left\{\left(-\theta_1 b_1 + \frac{(1-\alpha)y_2^{call}}{1+c_2}\right)\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\right\} \\
&= \mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right](1-\alpha)y_2^{call}\frac{\partial}{\partial r_1}\left(\frac{1}{1+c_2}\right)\right\} \\
&= -\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\frac{(1-\alpha)y_2^{call}}{(1+c_2)^2}\frac{b_2-\theta_1 b_1}{b_2}\right\} < 0
\end{aligned}
$$

3. At this step, I need to study the (the sign of) second derivative of $\overline{V}_1^{nocall}$ and $\overline{V}_1^{call}$ with respect to the interest rate r_1:

$$
\begin{aligned}
\frac{\partial^2 \overline{V}_1^{nocall}(s_1,x_1)}{\partial r_1^2} &= -\frac{\partial}{\partial r_1}\left(\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right]\frac{(1-\alpha)y_2^{nocall}}{(1+c_2)^2}\frac{b_2-b_1}{b_2}\right\}\right) \\
&= -\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right](1-\alpha)y_2^{nocall}\frac{b_2-b_1}{b_2}\frac{\partial}{\partial r_1}\left(\frac{1}{(1+c_2)^2}\right)\right\} \\
&= 2\cdot\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{nocall}\right)\right]\frac{(1-\alpha)y_2^{nocall}}{(1+c_2)^3}\left(\frac{b_2-b_1}{b_2}\right)^2\right\} > 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \overline{V}_1^{call}(s_1,x_1)}{\partial r_1^2} &= -\frac{\partial}{\partial r_1}\left(\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\frac{(1-\alpha)y_2^{call}}{(1+c_2)^2}\frac{b_2-\theta_1 b_1}{b_2}\right\}\right) \\
&= -\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right](1-\alpha)y_2^{call}\frac{b_2-\theta_1 b_1}{b_2}\frac{\partial}{\partial r_1}\left(\frac{1}{(1+c_2)^2}\right)\right\} \\
&= 2\cdot\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\frac{(1-\alpha)y_2^{call}}{(1+c_2)^3}\left(\frac{b_2-\theta_1 b_1}{b_2}\right)^2\right\} > 0
\end{aligned}
$$

Therefore, considering the two above inequalities, I arrive at both functions $\overline{V}_1^{nocall}$ and $\overline{V}_1^{call}$ are convex in r_1, with other four parameters fix.

4. Lastly, to know the relative positions of the slope of both curves, I derive their asymptotic behavior with respect to θ_1, as follows:

$$
\begin{aligned}
\frac{\partial^2 \overline{V}_1^{call}(s_1,x_1)}{\partial \theta_1 \partial r_1} &= -\frac{\partial}{\partial \theta_1}\left(\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\frac{(1-\alpha)y_2^{call}}{(1+c_2)^2}\frac{b_2-\theta_1 b_1}{b_2}\right\}\right) \\
&= -\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right](1-\alpha)y_2^{call}\frac{\partial}{\partial \theta_1}\left(\frac{b_2-\theta_1 b_1}{b_2}\frac{1}{(1+c_2)^2}\right)\right\} \\
&= 2\cdot\mathbb{E}_{z_2|z_1}\left\{\left[1-\Phi\left(\overline{\varepsilon}_2^{call}\right)\right]\frac{(1-\alpha)y_2^{call}}{(1+c_2)^2}\frac{b_1}{b_2}\left[1+2\cdot\frac{b_2-\theta_1 b_1}{b_2}\frac{c_1-r_1}{1+c_2}\right]\right\}
\end{aligned}
$$

The sign of the derivative in the previous line depends on the sign and the amplitude of $c_1 - r_1$. Considering the case of $r_1 \to 0$, the y-intercept of $\overline{V}_1^{call}(s_1,x_1)\big|_{r_1=0} > \overline{V}_1^{nocall}(s_1,x_1)\big|_{r_1=0}, \forall \theta_1 <$

1.

$$\left.\frac{\partial^2 \overline{V}_1^{call}(s_1, x_1)}{\partial\theta_1 \partial r_1}\right|_{r_1 \to 0} > 0 \tag{A29}$$

$$\left.\frac{\partial^2 \overline{V}_1^{call}(s_1, x_1)}{\partial\theta_1 \partial r_1}\right|_{r_1 \to +\infty} < 0 \tag{A30}$$

5. $\overline{V}_1^{nocall}$ coincides with $\overline{V}_1^{call}$ when $\theta_1 = 1$: $\overline{V}_1^{call}(s_1, x_1)|_{\theta_1=1} = \overline{V}_1^{nocall}(s_1, x_1), \forall c_1$ and $\forall r_1$.

When I combine the five points above, I can now deduce the proof of the existence of the interest rate r_1^*, such that $\overline{V}_1^{call}$ and $\overline{V}_1^{nocall}$ cross at r_1^*, based on the intermediate value theorem.

The existence of r_1^* is then proved, therefore through the signs described in (A29) and (A30), I state that when $r_1 < r_1^*$, the exercise of the call option is the unique optimal decision: $\overline{V}_1^{nocall}(s_1, x_1) < \overline{V}_1^{call}(s_1, x_1)$. This completes the proof of Proposition 3.

■

C.8 Proof of Corollary 3.0.2

Proof. To prove Corollary 3.0.2, I need two ingredients:

1. The last terms at both sides of the inequality in (24) concern the probability of not defaulting. This term decreases in the capital quality cutoff $\overline{\varepsilon}_2$. This threshold value, in turn, increases in the periodic coupon c_2:

$$\frac{\partial\overline{\varepsilon}_2}{\partial c_2} = \frac{b_2}{k_2} > 0 \tag{A31}$$

Within the decomposition of c_2, we showed through (A24) that θ_1 amplifies the sensitivity of the default threshold.

2. I recall the sign of $\frac{\partial^2 \overline{V}_1^{call}(s_1, x_1)}{\partial\theta_1 \partial r_1}$ in the the last line (4) and use what the signs in (A29) and (A30) say. This sign shows how strong the effect of the share of non-callable θ_1 (so that the share of callable) is in amplifying the decision to call regarding the variation in market interest rates.

$$\frac{\partial^2 \overline{V}_1^{call}(s_1, x_1)}{\partial\theta_1 \partial r_1} \begin{cases} > 0 & \text{if } r_1 < r_1^* \text{ (when call)} \\ < 0 & \text{if } r_1 > r_1^* \text{ (when no call)} \end{cases} \tag{A32}$$

With these inequalities, I have all elements illustrated in Figure 8.

■

C.9 Proof of the optimal callability in the condition (27)

Proof. To establish the expression of the optimal condition for the choice of θ_1 in (27), let's rewrite the F.O.C. expressed in (25) as an equality.

$$\begin{aligned} [\theta_1] : b_1 \frac{\partial p_0}{\partial \theta_1} + p_0 \frac{\partial b_1}{\partial \theta_1} - \frac{\partial \eta_b}{\partial \theta_1} + \beta_0 \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \theta_1} &= 0 \\ : \frac{\partial \bar{\varepsilon}_1}{\partial \theta_1} \left[b_1 \frac{\partial p_0}{\partial \bar{\varepsilon}_1} + p_0 \frac{\partial b_1}{\partial \bar{\varepsilon}_1} + \beta_0 \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \bar{\varepsilon}_1} \right] - \frac{\partial \eta_b}{\partial \theta_1} &= 0 \end{aligned} \tag{A33}$$

First, I derive the default cutoff $\bar{\varepsilon}_1$:

$$\bar{\varepsilon}_1 = - \frac{k_1 + y_1 + \mathbb{1}_{\text{call}} \times \overline{V}_1^{call}(s_1, x_1) + (1 - \mathbb{1}_{\text{call}}) \times \overline{V}_1^{nocall}(s_1, x_1)}{k_1} \tag{A34}$$

Now, I define the components of the above equation. I get the default risk derivative relative to the non-callable bond's share.

$$\frac{\partial \bar{\varepsilon}_1}{\partial \theta_1} = -\frac{1}{k_1} \times \frac{\partial}{\partial \theta_1} \left[\mathbb{1}_{\text{call}} \times (\overline{V}_1^{call} - \overline{V}_1^{nocall}) \right] \tag{A35}$$

I then consider the effects of the default risk on the price of the initial bond p_0 within both types of bond, and the expected firm's value in the intermediate period $\mathbb{E}_{s_1|s_0} V_1$.

$$\begin{aligned} \frac{\partial p_0^{nc}}{\partial \bar{\varepsilon}_1} &= -\beta_0 \mathbb{E}_{s_1|s_0} \left\{ (r_0 + \bar{p}_1^{nc})\, \varphi(\bar{\varepsilon}_1) \right\} \\ \frac{\partial p_0^{c}}{\partial \bar{\varepsilon}_1} &= -\beta_0 \mathbb{E}_{s_1|s_0} \left\{ (r_0 + 1 + \chi)\, \varphi(\bar{\varepsilon}_1) \right\} \\ \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \bar{\varepsilon}_1} &= -k_1 \mathbb{E}_{s_1|s_0} \left[1 - \Phi(\bar{\varepsilon}_1) \right] \end{aligned}$$

Concerning the issuance cost, its variation on the share of non-callable is described by:

$$\frac{\partial \eta_b}{\partial \theta_1} = 2 \left[(\eta_{nc} + \eta_c)\theta_1 - \eta_c \right] b_1^2 \tag{A36}$$

By replacing the five above derivatives in the expression (A33), I obtain the four following components of the optimal choice of non-callable share:

1. for the non-callable share of bond, I obtain:

$$\beta_0 \mathbb{E}_{s_1|s_0} \left\{ \left[(r_0 + \bar{p}_1^{nc}) \frac{b_1}{k_1} \varphi(\bar{\varepsilon}_1) \right] \times \frac{\partial}{\partial \theta_1} \left[\mathbb{1}_{\text{call}} \times (\overline{V}_1^{call} - \overline{V}_1^{nocall}) \right] \right\}$$

2. for the callable share of bond, I obtain:
$$\beta_0 \mathbb{E}_{s_1|s_0}\left\{\left[\mathbb{1}_{\text{call}}\left(r_0+1+\chi\right)\frac{b_1}{k_1}\varphi(\overline{\varepsilon}_1)\right]\times\frac{\partial}{\partial\theta_1}\left[\mathbb{1}_{\text{call}}\times(\overline{V}_1^{call}-\overline{V}_1^{nocall})\right]\right\}$$
3. for the continuation value in next period $t=1$, the component is:
$$\beta_0 \mathbb{E}_{s_1|s_0}\left\{[1-\Phi(\overline{\varepsilon}_1)]\times\frac{\partial}{\partial\theta_1}\left[\mathbb{1}_{\text{call}}\times(\overline{V}_1^{call}-\overline{V}_1^{nocall})\right]\right\}$$
4. for the issuance costs, the derivation is: $-2\left[(\eta_{nc}+\eta_c)\theta_1-\eta_c\right]b_1^2$

I recall the key property of any indicator variable which is idempotent, then I use the following for the rest of the proof:

$$\mathbb{1}_{\text{call}}\times\mathbb{1}_{\text{call}}=\mathbb{1}_{\text{call}} \tag{A37}$$

I use also the following simplification:

$$\mathbb{1}_{\text{call}}\times\overline{p}_1^{nc}=\overline{p}_1^{nc\_call} \tag{A38}$$

Back to the optimal equation on θ_1, we have:

$$\begin{aligned}\beta_0 \mathbb{E}_{s_1|s_0}&\left\{\left[\theta_1\left(r_0+\overline{p}_1^{nc\_call}\right)+(1-\theta_1)\left(r_0+1+\chi\right)\frac{b_1}{k_1}\varphi(\overline{\varepsilon}_1)+[1-\Phi(\overline{\varepsilon}_1)]\right]\right.\\&\left.\times\frac{\partial}{\partial\theta_1}\left[\mathbb{1}_{\text{call}}\times(\overline{V}_1^{call}-\overline{V}_1^{nocall})\right]\right\}-2\left[(\eta_{nc}+\eta_c)\theta_1-\eta_c\right]b_1^2=0.\end{aligned} \tag{A39}$$

Last, I bring the interest rate r_0 out of the continuation returns of the bond to derive the final expression:

$$\begin{aligned}\beta_0 \mathbb{E}_{s_1|s_0}&\left\{\left[\left[r_0+\theta_1\overline{p}_1^{nc\_call}+(1-\theta_1)\left(1+\chi\right)\right]\frac{b_1}{k_1}\varphi(\overline{\varepsilon}_1)+[1-\Phi(\overline{\varepsilon}_1)]\right]\right.\\&\left.\times\frac{\partial}{\partial\theta_1}\left[\mathbb{1}_{\text{call}}\times(\overline{V}_1^{call}-\overline{V}_1^{nocall})\right]\right\}-2\left[(\eta_{nc}+\eta_c)\theta_1-\eta_c\right]b_1^2=0.\end{aligned} \tag{A40}$$

■

D Full Model Appendix

D.1 Discussion of bonds pricing

D.1.1 Importance of distinct continuation prices for callable and non-callable bonds

The continuation price, denoted as $\overline{p}^{nc}$ (or $\overline{p}^{c}$), reflects the valuation of *outstanding debt* in the next period based on the initial terms established at issuance. Distinguishing between continuation prices for non-callable and callable bonds is essential due to differences in contractual terms and payment expectations between these bonds. Specifically, the continuation price for a non-callable bond, $\overline{p}^{nc}$, and for a callable bond, $\overline{p}^{c}$, serve distinct functions by reflecting either fixed or adjustable payment obligations.

- **Non-callable bond and fixed coupon rate:** The continuation price $\overline{p}^{nc}$ for a non-callable bond incorporates the original coupon rate c determined at issuance. This fixed coupon ensures that creditors holding non-callable debt are paid at a stable rate, unaffected by shifts in the current market interest rate r. Thus, $\overline{p}^{nc}$ accurately represents the creditor's expectation of receiving consistent payments as per the bond's initial terms, upholding the contractual integrity of non-callable debt.
- **Callable bond and call decision flexibility:** The continuation price $\overline{p}^{c}$ for a callable bond varies according to the firm's call decision. If the firm refrains from calling, $\overline{p}^{c}$ aligns with $\overline{p}^{nc}$, continuing under the original coupon payments. However, if the call option is exercised, the continuation price reflects only the principal repayment plus the call premium χ, omitting future coupon payments. This adjustment is crucial for callable bondholders, who face the potential for early repayment, including a compensatory premium for their investment. Therefore, $\overline{p}^{c}$ accounts for both no-call scenarios, where future coupons are expected, and call scenarios, where payment is limited to principal plus premium.

Can a single continuation price be used? Although it might seem appealing to simplify the model by using a single continuation price based on a weighted average of periodic coupon payments (as defined in (30) and (31)), this approach would introduce inaccuracies:

1. *Creditor composition differences:* Newly issued debt and outstanding debt are held by different creditor groups with distinct expectations. New creditors enter under current market conditions, while existing creditors rely on previously set coupon terms. A unified continuation price could obscure these differences, misrepresenting the expectations of each creditor class.[25]

[25] See e.g. Ippolito et al. (2018) for the weighted average periodic coupon, in the context of fixed vs floating rate debts.

2. *Contractual integrity of fixed coupons:* Outstanding debt is typically structured with fixed coupons that do not vary with interest rate changes. Averaging coupons across different debt types would imply contractual adjustments to these fixed payments, contradicting the standard fixed-rate structure in financial contracts.

Thus, maintaining distinct continuation prices—one based on the fixed coupon for outstanding debt and another on the current interest rate for newly issued debt—provides an accurate depiction of the firm's obligations and aligns with the varying expectations of each creditor group. For non-callable bonds, this distinction ensures stability through a fixed coupon rate, while for callable bonds, it allows for flexible adjustment based on the firm's optimal call decision. This dual pricing approach is particularly valuable for accurately modeling scenarios with mixed debt instruments, as it reflects the financial strategies of firms in fluctuating market conditions and their impact on bondholder returns.

D.1.2 Callable bond pricing

The callable bond pricing model must incorporate both the firm's endogenous call decision and the potential loss for bondholders when the call option is exercised. Equation (42) reflects this structure, providing a dynamic framework that captures how the firm optimizes its value under the possibility of calling the bond.

1. **Endogenous Call Decision:**

   In this framework, the firm decides whether to call the bond by comparing the **value of calling** with the **value of holding the bond until maturity**. The decision to call is endogenous, depending on the firm's state variables s, and policy functions $\hat{x}(s, x)$. This decision is modeled through the call indicator $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}\left(s', \hat{x}(s, x)\right)$, which determines whether the callable bond will be called or not. This allows for a more accurate representation of the firm's incentives and behavior, as opposed to assuming a fixed probability or exogenously determined call timing. The literature, such as Duffie and Singleton (1999); Jarrow et al. (2010), often relies on exogenous factors to model the call decision, which may not fully capture the issuer's strategic behavior in response to market conditions (See also Chen et al. (2010).).

2. **Min Function and Redundancy:**

   In traditional models, the **min function** is used to ensure that the bondholder's payoff does not exceed the call price, even if the market price of the bond exceeds it. However, in our extended model, this comparison is **internalized** in the firm's value-maximizing decision to call or not. Therefore, the **min function** becomes redundant since the firm already optimizes between calling the bond or not based on the comparison of its continuation value. The decision rule internalizes the market price dynamics and the call price, making the explicit use of **min** unnecessary (See Brennan and Schwartz (1977) for classical discussions on callable bond pricing.).

3. The model incorporates a dynamic continuation price for both callable and non-callable bonds, taking into account future state variables and policy functions, as is done in the dynamic debt model literature in macroeconomics (e.g. Crouzet (2017); Karabarbounis and Macnamara (2021); Jungherr and Schott (2021)). This contrasts with simpler models where the continuation value is often static or based on less granular assumptions. By considering the future coupon payments and the possibility of a call, the model provides a more comprehensive valuation that aligns with real market dynamics.

4. The call premium is intrinsically set as a function of the prevailing interest rate r, making the model sensitive to market conditions at issuance. This allows for the call premium to decrease as the bond matures without being called, reflecting the decreasing call risk. This feature provides a more accurate representation of the bond's pricing over its life, something not always addressed in the literature where call premiums may be assumed constant or independent of market rates.

D.2 First Order Conditions

In this subsection, I present the first-order conditions for the decision variables: the capital k', the debt b', and the share of non-callable θ'. Without any specificity, I consider the rewritten value function defined in (38), so for a given state variables $s = \{z, r\}, x = \{k, b, \theta, c\}$, for the case $j \in \{\text{nocall}, \text{call}\}$ we have:

$$\overline{V}_j(s, x) = \max_{e, k'_j, b'_j, \theta'_j, c'_j} \left\{ -k'_j + n_j + p\tilde{b}_j - \eta_b(\tilde{b}_j) - \eta_e(e) + \frac{1}{1+r} \mathbb{E}_{s'|s} \int_{\overline{\varepsilon}'}^{\infty} V^r(s', x') d\Phi(\varepsilon') \right\} \tag{A1}$$

The weighted average price of the new debt is: $p = \tilde{\theta}_j p^{nc} + (1 - \tilde{\theta}_j) p^c$. I define a new pseudo-value function of the firm which consider the part of its value function concerned by its optimal choices.

$$W_j(s', x') = -k'_j + p\tilde{b}_j - \eta_b(\tilde{b}_j) - \eta_e(e) + \frac{1}{1+r} \mathbb{E}_{s'|s} \int_{\overline{\varepsilon}'}^{\infty} V^r(s', x') d\Phi(\varepsilon') \tag{A2}$$

D.2.1 Optimal capital k'

The firm's first-order condition for capital k'_j follows:

$$\left(1+\frac{\partial\eta_e(e)}{\partial e}\right)\left\{-1+b'\left[\tilde{\theta}_j\frac{\partial p^{nc}}{\partial k'_j}+(1-\tilde{\theta}_j)\frac{\partial p^{c}}{\partial k'_j}\right]\right\}+\frac{1}{1+r}\mathbb{E}_{s'|s}\int_{\overline{\varepsilon}'}^{\infty}\frac{\partial V^r(s',x')}{\partial k'_j}d\Phi(\varepsilon')=0 \tag{A3}$$

Where the repayment value evolves depending on the capital as follows:

$$\begin{aligned}\frac{\partial V^r(s',x')}{\partial k'_j}=&\left[1-\mathbb{1}'_{\text{call}}\right]\left\{\frac{\partial n'_{\text{nocall}}}{\partial k'_j}\cdot\left[(1-\pi_e)\mathbb{E}_{s''|s'}\left(1+\frac{\partial\eta_e(e')}{\partial e'}\right)+\pi_e\left(1-(1-\gamma)b'\mathbb{E}_{s''|s'}\frac{\partial p'_{\text{nocall}}}{\partial n'_{\text{nocall}}}\right)\right]\right\}\\&+\mathbb{1}'_{\text{call}}\left\{\frac{\partial n'_{\text{call}}}{\partial k'_j}\cdot\left[(1-\pi_e)\mathbb{E}_{s''|s'}\left(1+\frac{\partial\eta_e(e')}{\partial e'}\right)+\pi_e\left(1-(1-\gamma)\theta'b'\mathbb{E}_{s''|s'}\frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}}\right)\right]\right\}\end{aligned}$$

where $\mathbb{1}'_{\text{call}}=\mathbb{1}_{\text{call}}(s',x')$ indicates whether the firm will choose to call the callable debt $(1-\theta')$ in the next period; $n'_{\text{nocall}}=n_{\text{nocall}}(s',x')$, and $n'_{\text{call}}=n_{\text{call}}(s',x')$ are respectively the internal fund of the firm after producing for both future choices. Then, we have:

$$\forall j\in\{\text{nocall},\text{call}\},\ \frac{\partial n'_j}{\partial k'_j}=\frac{\partial n_j(s',x')}{\partial k'_j}=1+(1-\tau)\left[z'\alpha k'^{\alpha-1}_j+\epsilon'-\delta\right] \tag{A4}$$

To obtain the derivatives of the prices relative to the next period capital, I need the effects on the default threshold, $\overline{\varepsilon}'$, which is defined implicitly in (33).

$$\frac{\partial\overline{\varepsilon}}{\partial k'_j}=-\frac{1+(1-\tau)\left(z'\alpha k'^{\alpha-1}_j+\epsilon'-\delta\right)}{(1-\tau)k'_j} \tag{A5}$$

I can then compute the non-callable bond price derivative with respect to k'_j:

$$\begin{aligned}\frac{\partial p^{nc}}{\partial k'_j}=&\beta\mathbb{E}_{s'|s}\left\{\int_{\overline{\varepsilon}'}^{\infty}(1-\gamma)\mathbb{E}_{s''|s'}\left[\left(1-\mathbb{1}'_{\text{call}}\right)\frac{\partial p'^{nc}_{\text{nocall}}}{\partial n'_{\text{nocall}}}\frac{\partial n'_{\text{nocall}}}{\partial k'_j}+\mathbb{1}'_{\text{call}}\frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}}\frac{\partial n'_{\text{call}}}{\partial k'_j}\right]d\Phi(\varepsilon')\right.\\&+\int_{-\infty}^{\overline{\varepsilon}'}\frac{1-\xi}{b'}\left[1+(1-\tau)\left(z'\alpha k'^{\alpha-1}_j+\epsilon'-\delta\right)\right]d\Phi(\varepsilon')\\&\left.+\varphi(\overline{\varepsilon}')\left[-\left(\gamma+r+(1-\gamma)\mathbb{E}_{s''|s'}p'^{nc}\right)+\frac{1-\xi}{b'}\underline{n}(\overline{\varepsilon}')\right]\frac{\partial\overline{\varepsilon}'}{\partial k'_j}\right\}\end{aligned}$$

I derive the variation of the callable bond price with respect to k_j':

$$\begin{aligned}\frac{\partial p^c}{\partial k_j'} =& \beta\mathbb{E}_{s'|s}\Bigg\{\int_{\bar{\varepsilon}'}^{\infty}(1-\gamma)\mathbb{E}_{s''|s'}\left[(1-\mathbb{1}_{\text{call}}')\frac{\partial p'^c}{\partial n'_{\text{nocall}}}\frac{\partial n'_{\text{nocall}}}{\partial k_j'}\right]d\Phi(\varepsilon')\\ &+\int_{-\infty}^{\bar{\varepsilon}'}\frac{1-\xi}{b'}\left[1+(1-\tau)\left(z'\alpha k_j'^{\alpha-1}+\epsilon'-\delta\right)\right]d\Phi(\varepsilon')\\ &+\varphi(\bar{\varepsilon}')\left[-\left[r+(1-\mathbb{1}_{\text{call}}')\left[\gamma+(1-\gamma)\,\mathbb{E}_{s''|s'}p'^c_{\text{call}}\right]+\mathbb{1}_{\text{call}}'(1+\chi)\right]+\frac{1-\xi}{b'}\underline{n}(\bar{\varepsilon}')\right]\frac{\partial\bar{\varepsilon}'}{\partial k_j'}\Bigg\}\end{aligned}$$

D.2.2 Optimal debt b'

Now, I derive the optimal condition for the choice of the next period level of debt b'.

$$\left(1+\frac{\partial\eta_e(e)}{\partial e}\right)\left\{p+b'\left[\tilde{\theta}_j\frac{\partial p^{nc}}{\partial b'}+(1-\tilde{\theta}_j)\frac{\partial p^c}{\partial b'}\right]-\frac{\partial\eta_b(b')}{\partial b'}\right\}+\frac{1}{1+r}\mathbb{E}_{s'|s}\int_{\bar{\varepsilon}'}^{\infty}\frac{\partial V^r(s',x')}{\partial b'}d\Phi(\varepsilon')=0 \tag{A6}$$

Where the derivative of the repayment value with respect to b' is:

$$\begin{aligned}\frac{\partial V^r(s',x')}{\partial b'}=&\left[1-\mathbb{1}_{\text{call}}'\right]\mathbb{E}_{s''|s'}\Bigg\{\quad\frac{\partial n'_{\text{nocall}}}{\partial b'}\cdot\left[(1-\pi_e)\left(1+\frac{\partial\eta_e(e')}{\partial e'}\right)+\pi_e\left(1-(1-\gamma)b'\frac{\partial p'_{\text{nocall}}}{\partial n'_{\text{nocall}}}\right)\right]\\ &+\frac{\partial\tilde{b}'_{\text{nocall}}}{\partial b'}\cdot\left[(1-\pi_e)\frac{\partial W(s'',x''_{\text{nocall}})}{\partial\tilde{b}'_{\text{nocall}}}-\pi_e\left(p'_{\text{nocall}}+b'\frac{\partial p'_{\text{nocall}}}{\partial n'_{\text{nocall}}}\right)\right]\Bigg\}\\ &+\mathbb{1}_{\text{call}}'\mathbb{E}_{s''|s'}\Bigg\{\quad\frac{\partial n'_{\text{call}}}{\partial b'}\cdot\left[(1-\pi_e)\left(1+\frac{\partial\eta_e(e')}{\partial e'}\right)+\pi_e\left(1-(1-\gamma)\theta'b'\frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}}\right)\right]\\ &+\frac{\partial\tilde{b}'_{\text{call}}}{\partial b'}\cdot\left[(1-\pi_e)\frac{\partial W(s'',x''_{\text{call}})}{\partial\tilde{b}'_{\text{call}}}-\pi_e\left(p'^{nc}_{\text{call}}+b'\frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}}\right)\right]\Bigg\}\end{aligned}$$

where the function $W(s'',x_j'')$ is the function defined in (A2) when the decision j will be taken the next period. $\mathbb{1}_{\text{call}}'=\mathbb{1}_{\text{call}}(s',x')$ indicates whether the firm will choose to call the callable debt $(1-\theta')$ in the next period; $n'_{\text{nocall}}=n_{\text{nocall}}(s',x')$, and $n'_{\text{call}}=n_{\text{call}}(s',x')$ are respectively the internal fund of the firm after producing for both future choices. Additionally, I define the following components of the above derivatives.

When the firm does not call in the next period, $\mathbb{1}_{\text{call}}'=0$, $\frac{\partial n'_{\text{nocall}}}{\partial b'}=-(\gamma+(1-\tau)c')$, and $\frac{\partial\tilde{b}'_{\text{nocall}}}{\partial b'}=1-\gamma$, while when the firm decides to call in the next period, $\mathbb{1}_{\text{call}}'=1$, we have $\frac{\partial n'_{\text{call}}}{\partial b'}=-\left[\gamma\theta'+(1+\chi)(1-\theta')+(1-\tau)c'\right]$, and $\frac{\partial\tilde{b}'_{\text{call}}}{\partial b'}=\theta'(1-\gamma)$.

I derive the variation of the pseudo-value functions. In the no-call scenario in the next period, we have:

$$\frac{\partial W(s'', x''_{\text{nocall}})}{\partial \tilde{b}'_{\text{nocall}}} = -p'_{\text{nocall}} \left(1 + \frac{\partial \eta_e(e')}{\partial e'}\right) - \frac{\partial \eta_b(\tilde{b}'_{\text{nocall}})}{\partial \tilde{b}'_{\text{nocall}}} \tag{A7}$$

and for the call scenario, we have:

$$\frac{\partial W(s'', x''_{\text{call}})}{\partial \tilde{b}'_{\text{call}}} = -p'^{nc}_{\text{call}} \left(1 + \frac{\partial \eta_e(e')}{\partial e'}\right) - \frac{\partial \eta_b(\tilde{b}'_{\text{call}})}{\partial \tilde{b}'_{\text{call}}} \tag{A8}$$

Lastly, to complete the optimal conditions components, I derive the prices sensitivities to the next period debt level.

I compute the non-callable bond price derivative with respect to b':

$$\begin{aligned}
\frac{\partial p^{nc}}{\partial b'} = \beta \mathbb{E}_{s'|s} \Bigg\{ \int_{\bar{\varepsilon}'}^{\infty} (1-\gamma) \mathbb{E}_{s''|s'} \Bigg[(1 - \mathbb{1}'_{\text{call}}) \left(\frac{\partial p'^{nc}_{\text{nocall}}}{\partial n'_{\text{nocall}}} \frac{\partial n'_{\text{nocall}}}{\partial b'} + \frac{\partial p'^{nc}_{\text{nocall}}}{\partial \tilde{b}'_{\text{nocall}}} \frac{\partial \tilde{b}'_{\text{nocall}}}{\partial b'} \right) \\
+ \mathbb{1}'_{\text{call}} \left(\frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}} \frac{\partial n'_{\text{call}}}{\partial b'} + \frac{\partial p'^{nc}_{\text{call}}}{\partial \tilde{b}'_{\text{call}}} \frac{\partial \tilde{b}'_{\text{call}}}{\partial b'} \right) \Bigg] d\Phi(\varepsilon') \\
- \int_{-\infty}^{\bar{\varepsilon}'} \frac{1-\xi}{(b')^2} \left[k' + (1-\tau)\left(z' k'^{\alpha} + \epsilon' k' - \delta k' - f \right) \right] d\Phi(\varepsilon') \\
+ \varphi(\bar{\varepsilon}') \left[- \left(\gamma + r + (1-\gamma) \mathbb{E}_{s''|s'} p'^{nc} \right) + \frac{1-\xi}{b'} \underline{n}(\bar{\varepsilon}') \right] \frac{\partial \bar{\varepsilon}'}{\partial b'} \Bigg\}
\end{aligned} \tag{A9}$$

For the callable bond, the difference stands from the call price in the call scenario. I compute the non-callable bond price derivative with respect to b':

$$\begin{aligned}
\frac{\partial p^{c}}{\partial b'} = \beta \mathbb{E}_{s'|s} \Bigg\{ \int_{\bar{\varepsilon}'}^{\infty} (1-\gamma) \mathbb{E}_{s''|s'} \Bigg[(1 - \mathbb{1}'_{\text{call}}) \left(\frac{\partial p'^{c}_{\text{nocall}}}{\partial n'_{\text{nocall}}} \frac{\partial n'_{\text{nocall}}}{\partial b'} + \frac{\partial p'^{c}_{\text{nocall}}}{\partial \tilde{b}'_{\text{nocall}}} \frac{\partial \tilde{b}'_{\text{nocall}}}{\partial b'} \right) \\
- \int_{-\infty}^{\bar{\varepsilon}'} \frac{1-\xi}{(b')^2} \left[k' + (1-\tau)\left(z' k'^{\alpha} + \epsilon' k' - \delta k' - f \right) \right] d\Phi(\varepsilon') \\
+ \varphi(\bar{\varepsilon}') \left[- \left[r + (1 - \mathbb{1}'_{\text{call}}) \left[\gamma + (1-\gamma) \mathbb{E}_{s''|s'} p'^{c}_{\text{call}} \right] + \mathbb{1}'_{\text{call}} (1+\chi) \right] + \frac{1-\xi}{b'} \underline{n}(\bar{\varepsilon}') \right] \frac{\partial \bar{\varepsilon}'}{\partial b'} \Bigg\}
\end{aligned} \tag{A10}$$

E Quantitative Appendix

In this section, I describe the model counterparts of key empirical moments for the full model, present the solution method, and discuss further quantitative results.

E.1 Model Moments

This subsection guides the connection between the theoretical framework and the empirical evidence presented earlier. The empirical moments we focus on include bond pricing, firm default, debt issuance, and investment dynamics, which are all essential for understanding the impact of debt callability on firm behavior and aggregate outcomes.

At the long-term steady-state equilibrium, several simplifying assumptions hold to facilitate the calculation of model moments. Specifically, the coupon rate (c^*) equals the risk-free rate (r^*), implying that the prices of riskless non-callable and callable debts are normalized to 1: $c^* = r^*$.

E.1.1 Bond Pricing and Credit Spreads

The pricing of callable and non-callable debt within the model is fundamental for understanding corporate financing strategies. The bond price is determined by evaluating the firm's decision to either call or not call the debt. This decision hinges on comparing the present value of callable debt, which includes the call premium, against the value of non-callable debt.

The price of callable debt in the model is determined by comparing the call and no-call decisions. The bond price depends on whether the firm exercises the call option and the associated future expected value of the callable and non-callable debt. The price computation is done for each state, accounting for firm productivity, interest rates, debt levels, and capital.

$$\begin{aligned}\textbf{average non-callable price} &= \hat{p}^{nc} \\ &= \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} (1 - \mathbb{1}_{\text{call}}) \cdot \overline{p}^{nc}_{\text{nocall}} + \mathbb{1}_{\text{call}} \cdot \overline{p}^{nc}_{\text{call}} \end{aligned} \tag{A1}$$

where $\overline{p}^{nc}_j = \overline{p}^{nc}_j(z, r, k', b', \theta, c)$ for $j \in \{\text{nocall}, \text{call}\}$ are defined in Equation (41). K, B, Θ, and $\bar{C}$ are the state spaces for respectively the state variables k, b, θ, and c, they are defined in the subsection below. The indicator $\mathbb{1}_{\text{call}}$ denotes whether the firm opts to call the debt in the current period.

Similarly, the price of callable debt is computed as:

$$\begin{aligned}\textbf{average callable price} &= \hat{p}^c \\ &= \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} (1 - \mathbb{1}_{\text{call}}) \cdot \overline{p}^c_{\text{nocall}} + \mathbb{1}_{\text{call}} \cdot \overline{p}^c_{\text{call}} \end{aligned} \tag{A2}$$

where $\overline{p}^c_j = \overline{p}^c_j(z, r, k', b', \theta, c)$ for $j \in \{\text{nocall}, \text{call}\}$ are defined in Equation (44). These prices are essential for calculating the average credit spreads, which reflect the compensation investors require for bearing credit risk.

Now, I derive the credit spreads. The credit spread measures the additional yield that investors demand to hold corporate debt over risk-free government bonds. It is the key indicator of the perceived riskiness of the firm's debt. At the steady state equilibrium, the prices have characteristics with fixed constant average call rate, and constant rate, and the price of riskless non-callable and callable debts is 1.

The credit spread on non-callable debt is calculated as:

$$\textbf{spread}^{nc} = \left[\frac{\gamma + c^*}{\hat{p}^{nc}} + (1-\gamma)\right]^4 - (1 + r^*)^4 \tag{A3}$$

Here, γ represents the quarterly rate of debt repayment, and c^* is the steady-state coupon rate on the firm's debts. The exponent 4 annualizes the quarterly returns, aligning with the typical frequency of debt repayments.

For callable debt, the spread is given by:

$$\textbf{spread}^{c} = \left\{\frac{1}{\hat{p}^c}\left[c^* + (1-q^*)\Big(\gamma + (1-\gamma)\,\hat{p}^c\Big) + q^*\,(1+\chi)\right]\right\}^4 - (1 + r^*)^4 \tag{A4}$$

In this equation, q^* denotes the average call rate, and χ is the call premium. The spread on callable debt accounts for the additional costs associated with the option to call, reflecting the increased risk for investors.

The overall average credit spread is a weighted average of the spreads on non-callable and callable debt:

$$\textbf{spread} = \theta^* \cdot \text{spread}^{nc} + (1 - \theta^*) \cdot \text{spread}^c \tag{A5}$$

where θ^* is the steady-state share of non-callable debt. This aggregation ensures that the model captures the combined effect of both debt types on the overall credit risk premium.

E.1.2 Rates of exit

First, the default rate reveals the probability that a firm will fail to meet its debt obligations, influenced by productivity shocks, debt levels, and most importantly capital quality shock. It is determined by the threshold $\overline{\varepsilon}$, below which a firm defaults.

$$\textbf{default rate} = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} \mathbb{P}[\varepsilon \leq \overline{\varepsilon}] = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} \Phi[\overline{\varepsilon}] = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} \int_{\overline{\varepsilon}}^{\infty} d\Phi(\varepsilon) \tag{A6}$$

where Φ is the cumulative distribution function of capital quality shocks, and $\overline{\varepsilon}$ is the default threshold in the current period. This expectation integrates over all possible states, providing an average default probability across the economy.

The exit rate accounts for the probability that a firm exits the market, either through default or exogenously. It is a combination of the default rate and the probability of an exogenous exit, π_e. As described on the timing of the model (9), the firm decides to default or not before the exogenous exit shock:

$$\textbf{exit rate} = \text{default rate} + (1 - \text{default rate}) \cdot \pi_e \tag{A7}$$

E.1.3 Callable bond lifespan

The average callable bond lifespan measures the expected duration that callable bonds remain outstanding before being redeemed by the firm. This moment captures the temporal aspect of debt callability, reflecting how often and under what conditions firms choose to exercise their call options.

$$\textbf{average callable bond lifespan} = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} \hat{T}(z,r,k,b,\theta,c) \cdot \mathbb{1}_{\text{callable}}(z,r,k,b,\theta,c) \tag{A8}$$

where $\hat{T}(z,r,k,b,\theta,c)$ represents the remaining lifespan of a callable bond given the current state. $(1 - \theta(z,r,k,b,\theta,c))$ denotes the share of callable debt in the current state, ensuring that only callable bonds contribute to the average lifespan. This moment constitutes one of this paper's contributions to the literature of macroeconomic implications of firms' debt heterogeneity. It helps for understanding the persistence of callable debt in the firm's capital structure and its implications for refinancing and financial flexibility.

E.1.4 Callability

The callable share rate measures the proportion of a firm's debt that is callable, reflecting the firm's reliance on callable debt instruments for financing flexibility. I consider the average value of the optimal share of callable within the solution of the model.

$$\textbf{callable share rate} = (1-\theta^*) = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} (1-\theta'(z,r,k,b,\theta,c)) \tag{A9}$$

where K, B, Θ, and $\bar{C}$ are the state spaces for respectively the state variables k, b, θ, and c.

The decision to call the bond is based on comparing the value of calling vs. not calling, considering the cost of paying the call premium and the potential gains from refinancing. Therefore, the average call rate quantifies the frequency with which firms exercise their option to call debt before maturity, influenced by factors such as interest rate movements, debt levels, and firm productivity.

$$\textbf{average call rate} = q^* = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} \mathbb{1}'_{\text{call}}(s', \hat{x}(s,x)) \cdot (1-\theta'(z,r,k,b,\theta,c)) \tag{A10}$$

where $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}(s', \hat{x}(s,x))$ is the indicator of the call decision the next period, and $\hat{x}(s,x)$ is the vector of policy functions. This formula ensures that the call rate reflects only those bonds eligible for calling.

E.1.5 Investment

The investment rate reflects the proportion of a firm's earnings allocated to capital investments, influenced by factors such as profitability, fixed operating costs, and available financing.

$$\textbf{Investment Rate} = \mathbb{E}_{z,r} \int_K \int_B \int_\Theta \int_{\bar{C}} I(z,r,k,b,\theta,c) \tag{A11}$$

Where $I(z,r,k,b,\theta,c)$ denotes the firm's investment decision function. This moment serves to control the aggregate investment behavior of firms, and the capital accumulation.

E.2 Solving the Model

The components of the model solution are the value functions, the bond price schedules, and the firm-level policies on k', b', θ', c'. To solve it, I use dynamic programming and numerical algorithms. The procedure iterates on the value functions and the price functions until the convergence. I start by discretizing the state space. Using the method in Tauchen (1986), I transform the AR(1) process of the productivity z into a 25-point grid, and the interest rate turns into a 10-point grid. For other state variables, I set a 25-point grid for bonds b, a 10-point grid for non-callable share θ, and a 10-point grid for the periodic coupon c.

At every step of the algorithm, I use interpolation on b, θ, c grids to approximate the policy functions. The main algorithm to solve the model works through the following steps.

1. I start with the finite-horizon (T periods) version of the model. I set the final value and price to be null, $V_{T+1} = 0, p = 0$.

2. I set the initial guesses for value and price functions, by solving the period $T-1$. In this case, I set the policy on new debt issued such as only non-callable bonds are issued $\tilde{\theta}_T = 1$), and the parameter of debt repayment at $\gamma = 1$, i.e. full repayment at the final period of non-callable bonds and unmatured and uncalled callable bonds. The following lines detail this step:

   (a) Guess of capital k_T, deduction of the labor supply, and using labor market clearing, get the wage. Guess future non-callable bonds share $\tilde{\theta}$ (for the period T, $\tilde{\theta}_T = 1$), and compute the periodic coupon for the next period, given by (30) and (31).

   (b) Using these values, for each point on the space state grid, solve the two optimization problems in (38), subsequently the call decision problem in (36), then in (34), and the optimal value in (32). To do it, first, given the guessed k_T and continuation value of decision variables, I compute the default $\bar{\varepsilon}$ given by (33) and the default probability. Second, I compute the prices p^{nc}, $\overline{p}_j^{nc}$, p^c, and $\overline{p}_j^c, j \in \{\text{nocall}, \text{call}\}$, using (40)-(44) through inner loop-outer loop procedure.

   (c) Based on these objects, I optimize the firm objective function to obtain the optimal firm's decision k, b, θ, c and the prices' schedules.

   (d) The continuation values and prices obtained here serve as the new guesses for the rest of the algorithm.

3. For each period $t < T-1$, I use these guesses as the continuation values $\overline{V}_j$ and prices p^{nc}, $\overline{p}_j^{nc}$, p^c, and $\overline{p}_j^c$ for the rest of the algorithm.

4. The solution is found if the distance between the new and previous continuation values $\overline{V}_j$, and the distance between the new and previous prices p^{nc}, p^c are below the tolerance level (set at 10^{-6}). If this criterion is not satisfied, I get back to step (2) using these new $\overline{V}_j$ and prices p^{nc}, $\overline{p}_j^{nc}$, p^c, and $\overline{p}_j^c$ as the new continuation values and prices.