Low Interest Rates, Debt Heterogeneity, and Firm Dynamics*

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Abstract

This paper sets up a quantitative model of firm dynamics with debt heterogeneity to study the implications of changes in real interest rates for the firm size distribution and firm dynamics. It shows that the decline in long-term real interest rates since the early 1980s can account for a significant fraction of the shift in employment shares to large firms as well as the decline in firms per capita and firm entry rates experienced in the U.S. over the same period. In the model, firms endogenously choose financial intermediaries issuing debt with either earnings-based (EBC) or asset-based (ABC) borrowing constraints. The two types of constraints arise naturally from the imperfect enforceability of debt contracts and are in line with recent empirical findings. A decline in real interest rates benefits firms with EBC more because they are not constrained by their assets and can expand more due to increased earnings. Since firms with higher earnings optimally choose earnings-based lending, the decline in real interest rates shifts employment shares to larger firms. Moreover, the growth of large firms crowds out smaller firms and firm entry through general equilibrium effects. The paper tests the mechanism in cross-country data from the OECD and finds a stronger association between the decline in real interest rates and changes in firm dynamics, especially in countries with deeper credit markets.

Keywords: Real Interest Rate, Firm Dynamics, Firm Size Distribution, Debt Heterogeneity

JEL Codes: E24, E43, E44

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1 Introduction

The long-term real interest rate in the U.S. has declined from about 6 percent in the early 1980s to around zero by 2019. This decline has coincided with various notable changes in the U.S. firm size distribution and firm dynamics: (i) employment has shifted from small to larger firms, thereby increasing employment shares of large firms; (ii) the number of firms per capita has declined steadily; (iii) the firm entry rate has trended down.[1]

This paper argues that the decline in long-term real interest rates can account for these changes in firm dynamics through its asymmetric effects on small and large firms. The source of the asymmetric effects is debt heterogeneity by firm size. As is well known, large firms tend to rely on the credit market for debt financing, while small firms tend to borrow from traditional financial intermediaries such as banks (e.g., Colla et al., 2020). The type of debt financing is shown to have important implications for firms' borrowing limits (Lian and Ma, 2021). Specifically, large public firms that borrow from the credit market issue debt against their earnings (earnings-based borrowing limits), while others tend to borrow against their physical assets or collateral, also known as asset-based or collateral borrowing limits.

This paper studies the effects of long-term real interest rates on the firm size distribution and firm dynamics in a general equilibrium model in the spirit of Hopenhayn (1992). A key deviation from the Hopenhayn model is that firms need to finance the capital used in production by choosing between two types of financial instruments, which I refer to as credit market lending and traditional bank lending. The two types of financial instruments differ in their cost structures and contract enforceability. In equilibrium, those differences yield heterogeneous debt financing by firm size, and the difference in contract enforceability, in particular, leads to earnings-based borrowing limits in market lending and asset-based borrowing limits in bank lending.[2]

The equilibrium results of the model demonstrate asymmetric effects: the decline in real interest rates can shift employment shares to large firms and generate crowding-out effects for smaller firms. Quantitatively, the decline in real interest rates can explain all the changes in employment shares, 40% of the decline in firms per capita, and 10% of the decline in the firm entry rate experienced in the U.S. since the early 1980s. In addition, the model can explain 80% of the changes in the U.S. debt composition over the same

[1] See Davis et al. (2006) and Haltiwanger et al. (2011, 2012), who noted the decline in the startup rate. Fort et al. (2013) documented that employment shares at large/mature firms has risen since the early 1980s.

[2] One thing to note here is that this paper emphasizes the distinction between earnings-based and asset-based covenants, rather than the difference between financial intermediaries, i.e., the credit market and banks.

period, resulting from an increasing credit market debt financing relative to bank loans (Berg et al., 2021).

Debt heterogeneity is crucial for understanding those impacts. The results suggest that the secular trends of the firm size distribution and dynamics are closely linked to the decline in real interest rates. The linkage is novel, and the results are interesting as the decline in real interest rates, conventionally considered a positive financial development, can actually generate pervasive and significant negative effects on the employment and profitability of small firms.

Why can the decline in real interest rates generate asymmetric effects at all? Ultimately, firms' profitability and saving decisions can respond to changes in real interest rates differently. When the interest rate goes down, ceteris paribus, firms' profits increase, and interest expenses decrease. In contrast, the return on saving assets decreases at lower rates, reducing firms' incentives to save. Therefore, compared to firms that borrow against assets, the borrowing capacities of those firms that borrow against earnings increase more so that they can produce more efficiently. Since firms with higher earnings optimally choose earnings-based lending, the decline in real interest rates shifts employment shares to larger firms. Moreover, the growth of large firms crowds out smaller firms and firm entry through general equilibrium effects. More intuitively, low interest rates lower the return rate of assets but increase firms' profits. This effectively shifts the composition of firm values from the part generated by physical assets to the part yielded by firm production technology. As a result, firms that can leverage against their production benefit disproportionately from this shift.[3]

The model economy has three types of agents: entrepreneurs who run the firms, financial intermediaries, and the representative household. Each period, an exogenous number of potential entrants draw their initial productivity and asset levels. After observing their productivity levels and assets, they decide whether to enter. Incumbent firms face an exogenous exit shock at the beginning of each period. Surviving incumbent firms then decide whether to continue operating or exit. All firms can own illiquid assets and use labor and capital as inputs of production. Firms finance capital by raising debt from financial intermediaries. Exiting entrepreneurs keep their assets.

There are two types of financial intermediaries from which firms can borrow capital:

[3]Similar intuitions can be found in Kiyotaki et al. (2021). One thing to note here is that the effects of changes in real interest rates on asset prices play no additional role in the long-term stationary equilibrium. In fact, in such an equilibrium, only the rate of return matters for agents' decisions. This contrasts with the role of asset prices played in the amplification of short-run business cycle fluctuations. Intuitively, while a higher asset price allows an agent to leverage more, it also makes it more costly to acquire and afford the asset. In the long run, these two effects cancel each other out in the model.

banks and credit market lenders. Two critical distinctions between the two financial intermediaries are contract enforceability and cost structures. If firms choose to borrow from banks, they will pay lower fixed costs. I assume the bank debt has *one-period* imperfect enforceability: firms can renege on their debt contracts with the penalty of losing a fraction of one period's earnings as well as some equity (Buera et al., 2011). Doing so can save their interest expenses and retain some borrowed capital. Firms will resume normal operation and have the same access to financial intermediaries after one period. The notion of one-period imperfect contract enforceability can be considered liquidation bankruptcy. After the liquidation process, the entrepreneurs can resume or restart their businesses.

In contrast, firms can borrow from credit market lenders. When doing so, they need to pay higher fixed costs. In line with the literature, I further assume the interest spread charged by the credit market is weakly less than the spread charged by banks (e.g., Saunders and Steffen, 2011). I assume that the debt issued by the credit market lenders has *N-period* imperfect enforceability.[4] In detail, similar to bank borrowers, credit market borrowers can renege on their debt contracts. However, when they default, they will incur the penalty of losing some equity as well as a fraction of earnings for the following multiple (N) periods. In return, they can save interest expenses and retain some borrowed capital. After N periods, the firms will resume normal operation and regain the same access to the financial intermediaries. Higher fixed costs here represent the costs of disclosing information to enter the public market or make prolonged contract enforcement feasible. N-period imperfect enforceability of debt contracts is similar to the concept of restructuring bankruptcy. After defaulting on the contract, firms will enter a prolonged restructuring process, where firms will lose some earnings but keep producing outputs.

The model reveals important insights into two types of borrowing limits. It can be shown that the difference in the duration of the penalty, or the strength of contract enforceability, yields different types of borrowing limits. The incentive compatibility of one-period imperfect enforceability leads to an asset-based/collateral borrowing limit, whereas the incentive compatibility of N-period imperfect enforceability shifts a firm's borrowing limit toward an earnings-based constraint. In other words, the longer the penalty periods last, the more weight the borrowing limit puts on earnings. The intuition behind the discrepancies is that as the (penalty) duration increases, earnings for more extended periods will be reduced when a firm defaults, so earnings become more important when

[4] See similar examples in Albuquerque and Hopenhayn (2004) and Clementi and Hopenhayn (2006). My approach differs from the previous literature by focusing on debt heterogeneity. Particularly, my model reveals that whether firms can leverage over future values is the key distinction between earnings-based and asset-based constraints. Moreover, the model can match the empirical facts that collateral types vary by firm size.

the financial intermediaries decide on the borrowing limit. Therefore, earnings-based borrowing limits, compared with borrowing against assets, represent stronger enforceability of debt contracts.

The difference in the importance of future earnings in the borrowing limits matches recent empirical findings. The recent paper by Caglio et al. (2021) extended the work of Lian and Ma (2021) by showing that mostly small-medium-size enterprises (SMEs) also use their enterprise's continuation values as collateral, but the weight of each type of collateral varies by firm size. My theory is consistent with the findings in that with one-period imperfect enforceability, firms will still collateralize their values, but the values are limited to the current period. Therefore, the current income, capital, and illiquid assets, of which the empirical counterparts can be considered cash/accounts receivable, inventory, and fixed assets/real estate, play a major role in determining their borrowing limits, as shown in Caglio et al. (2021). In contrast, unsecured debt and blanket lien, which can be considered the future values of firms, play an important role for large firms that are subject to N-period imperfect enforceability.

In the model, the difference in cost structures then leads to sorting. Firms with greater profit margins and greater needs for capital will borrow through the credit market, and the others will choose banks. As a result, the model also provides a new interpretation for firms to borrow against earnings: firms decide to pay higher fixed costs to enter the credit market and thus increase the enforceability of debt contracts. By doing so, they will benefit from expanding production due to relaxed borrowing limits.

Finally, there is a representative household living in the economy. Each period, the household provides labor, earns wages, consumes goods, and saves in the financial intermediaries. I assume that the household is patient enough to be the pure lender, and entrepreneurs will be the borrowers in the economy. The changes in household preference will drive the changes in the real interest rate.[5]

Based on the model, I study the effects of the exogenous decline in real interest rates on the firm size distribution and firm dynamics. To explore the mechanism of the heterogeneous effects of the decline in real interest rates on firms, I first study a simplified version of the model by shutting down firm entry/exit and endogenous debt choices. The simplification allows me to provide analytical solutions to reveal the mechanism. The solutions show that a lower interest rate can shift employment shares to firms using earnings-based borrowing limits at the steady-state equilibrium. The shifts in employment shares can

[5] The assumption can be thought of as the decline in the real interest rate as the result of factors that are exogenous to firm owners, such as the saving glut (Bernanke, 2005) or increasing wealth inequality of households (Dynan et al., 2004). Similar assumptions are commonly made in other papers (e.g., Liu et al., 2019; Chatterjee and Eyigungor, 2020; Hall, 2016).

then generate crowding-out effects. The intuition is that when the real interest rate goes down, the borrowing capacities of those firms that borrow against earnings increase due to increased earnings and decreased interest expenses, and their limits are less restricted by their assets. As a result, they can borrow and produce more efficiently than others so that the competition can reduce the employment and profits of smaller firms.

Two additional channels related to debt heterogeneity can further amplify the shifts in employment shares and the crowding-out effects. First, if debt issued by the credit market is cheaper than bank loans (e.g., Saunders and Steffen, 2011), similar effects can be shown as the relative cost of debt decreases for market-financed firms. While this is a realistic channel, the interest rate differential is not necessary for the main results in the paper. Secondly, the endogenous debt choices, which are absent in the simplified model, can play an additional role. As the real interest rate goes down, marginal firms that borrow against assets when the rate is high can switch to the credit market and extend their borrowing limits to fulfill their increasing needs for capital.

Then, I use the full model to perform quantitative experiments and analyze the impacts of the exogenous decline in long-term real interest rates on the firm size distribution and firm dynamics. I calibrate the model to fit a rich set of moments of the U.S. firm dynamics, size distribution, and debt composition in the 1980s. The model generates an excellent fit for the moments. I then perform quantitative experiments in the model. In particular, I subject the economy to a decline in the real interest rate, as in the U.S. from 1980 to 2020. The experiment shows that the calibrated model can explain a significant fraction of long-term changes in the U.S. economy in terms of stationary equilibrium and transitional dynamics. Specifically, in the model, the decline in real interest rates can explain all the changes in the shifts in employment shares and 80% of the changes in debt composition experienced in the U.S. for the past forty years. The model can also explain about 40% of the decline in firms per capita and more than 10 % of the decline in the firm entry rate. Counterfactual analysis shows that debt heterogeneity plays an important role in generating the effects.

Finally, cross-country evidence based on an unbalanced panel of OECD countries from 1996 to 2019 is presented. The decline in real interest rates has been a global phenomenon affecting many advanced economies. This allows me to test the implications of my model in the cross-country data. Using real interest rates and credit market debt usage, I test the effects of the proposed mechanism on firm entry rates and large firm employment shares. The results show that after controlling for country and year fixed effects as well as potential confounding factors, the decline in real interest rates is positively associated with the increase of large firm employment shares and the decrease in firm entry. In addition, such effects are stronger for counties with higher credit market debt usage. A back-of-envelope calculation shows that comparing high market debt usage countries (90th percentile) with low usage countries (10th percentile), a 5 percent decrease in real interest rates implies that shifts in firm employment shares increase 3.9% (of total employment) more and entry rates decline 1.1% (of total firms) more, on average.

Related Literature. This paper is related to several recent important literatures. First, my article concerns the changes in the firm size distribution and firm dynamics in the U.S. A recent literature has shown the decline in the firm entry in the U.S., first documented by Haltiwanger et al. (2011) and Haltiwanger et al. (2012). Since then, a large body of literature has explored the reasons and impacts of this trend. Proposed possible explanations include population growth (Hathaway and Litan, 2014; Pugsley and Șahin, 2019; Karahan et al., 2019), business consolidation (Hathaway and Litan, 2014), import competition(Pugsley and Șahin, 2019), as well as aging (Bornstein et al., 2018) and technological changes (Poschke, 2018; Salgado, 2020). In addition, the firm size distribution has become more dispersed, and employment shares have shifted to large firms. Poschke (2018) explores the potential impacts of skill-based technology changes on systematic changes in firm size distribution globally, including the U.S., and Begenau et al. (2018) argue that big data disproportionately benefit large firms and may result in changes in firm size distribution. This paper contributes to this strand of literature by first linking the decline in the long-term real interest rate to those secular trends and proposing a novel and powerful channel, debt heterogeneity, which can account for both the changes in the firm size distribution and the decline in business dynamics.

Secondly, my work is related to the studies regarding the causes and consequences of heterogeneous debt financing. In particular, Lian and Ma (2021) show that firms are heterogeneous in their types of borrowing limits. Large firms, especially those who borrow from the credit market, issue mostly debt under earnings-based borrowing limits. In contrast, smaller firms borrow predominately against assets. Caglio et al. (2021) extended their studies to small-medium-size-enterprises. The following studies that adopt the ideas mostly focus on the effects of the earnings-based constraints and model them in reduced-forms (e.g., Greenwald et al., 2019; Drechsel, 2022). The contribution of this paper is that I propose a new theory that rationalizes and unifies the two types of constraints through the imperfect enforceability of debt contracts and provides a novel interpretation for earnings-based borrowing limits, which represents stronger enforceability of debt contracts. Moreover, this paper focuses on the interaction of two types of constraints in a general equilibrium economy instead of focusing on one type of constraint.

In addition, another recent literature focuses on the potential impacts of the decline in the long-term real interest rate on market concentration. Theoretically, Liu et al. (2019) emphasize the strategic effect of lower rates between market leaders and followers using a model with Bertrand-type competition. Chatterjee and Eyigungor (2020) pay attention to the link between firm size and firm leverage, and they emphasize the ability to leverage more due to the lower volatility of big firms, which can generate a lower entry rate and a concentration of sales through the acquisition of new businesses. Asriyan et al. (2021) argue that lower interest rates can cause capital misallocation and increase the investment of lower-productivity firms. Kroen et al. (2021) provide empirical evidence for that falling nominal rates can disproportionately benefit industry leaders, especially when the initial interest rate is already low.

I consider this paper to complement the above studies and differ from them in three aspects. First, my work proposes a novel channel, debt financing heterogeneity, through which the decline in long-term interest rates can affect the economy. Moreover, in contrast to the strategic effect that works between market leaders and followers or the acquisition of new businesses, the mechanism that I proposed shows that the decline in real interest rates can generate asymmetric effects in the absence of those specific market structures. Finally, my work provides novel empirical validation from OECD countries that supports the proposed mechanism and channels.

More broadly, my article fits into the literature regarding the effects of financial constraints on entrepreneurship (Buera et al., 2011, 2015) and firm dynamics (Cooley and Quadrini, 2001; Albuquerque and Hopenhayn, 2004; Clementi and Hopenhayn, 2006). My work contributes to the literature in that I incorporate the feature of heterogeneous borrowing constraints into a firm dynamics model and study the effects of the features on the secular trends of the U.S.

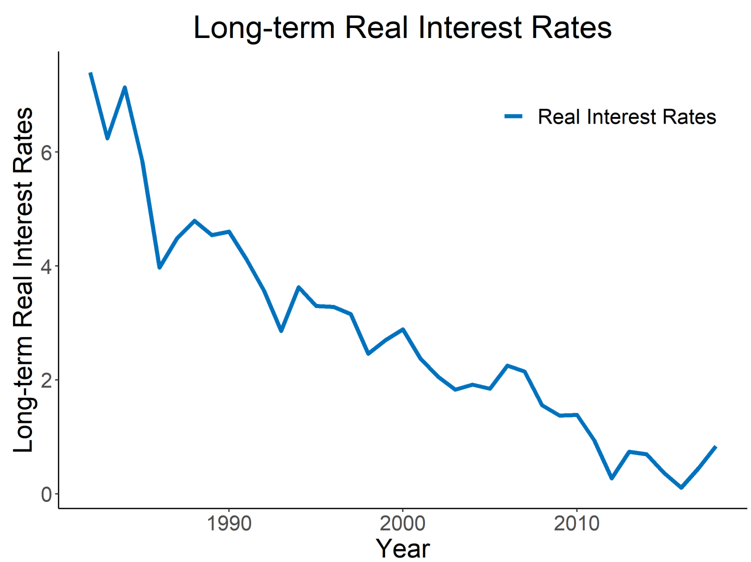
The paper is organized as follows. Section 2 documents the secular trends of the U.S. economy. Section 3 sets up a quantitative model of firm dynamics. Section 4 derives the analytical solutions of a simplified version of the model to provide intuitions. Section 5 reports the results from quantitative experiments. Section 6 provides the empirical evidence to validate the proposed mechanism. Section 7 concludes.

2 Secular Trends of the U.S. Economy

In this section, I present a number of stylized facts on secular trends in the U.S. economy. In particular, I show the changes in the long-term real interest rate in the U.S. and the evidence of heterogeneous debt financing at both the firm and aggregate levels. Then, I document the changes in the U.S. firm size distribution and firm dynamics.

2.1 The decline in Real Interest Rates and Debt Heterogeneity

Figure 1: Real Interest Rates over Years



Note: This figure depicts the changes in the long-term real interest rate in the U.S. The plot is created based on Federal Reserve Economic Data (FRED) using the 10-year treasury rates minus the 10-year inflation expectation.

The decline in real interest rates. Figure 1 shows the changes in long-term real interest rates in the U.S. over the last 40 years. The plot is created based on Federal Reserve Economic Data (FRED) using the 10-year treasury rate minus the 10-year inflation expectation. It shows a sizable decrement in the real interest rate, which has declined from around 6% to below 0% since the early 1980s.

It is helpful to discuss some potential explanations for the trend. The debates over the causes of the decline are enormous and remain controversial. The proposed explanations include changes in economic growth, demographic factors, government savings at national and global levels, and increasing demand for safe assets, especially in emerging-market economies. Kiley (2020) and Gamber (2020) provide excellent reviews of the empirical evidence and potential causes. For this paper, I assume the decline of the real interest rate solely comes from changes in the preference of lenders or foreign factors so that the changes in the real interest rate are exogenous to those firms. Theoretically, the proposed mechanism can work through the real interest rate channel regardless of the causes.[6]

[6]This assumption is made for both empirical and theoretical reasons. Empirically, as noted in Krueger and Ludwig (2007) and Kiley (2020), global factors, especially impacts from foreign countries, can play an essential role in the decline in the U.S. real interest rates. Moreover, Del Negro et al. (2017) show that the premium for safety and liquidity has increased related to the saving glut in emerging markets, first proposed by Bernanke (2005), dominates other factors. Theoretically, this assumption can make me alter

Debt heterogeneity. The existing literature has shown that firms are heterogeneous in terms of their debt types. Colla et al. (2020) review the literature on debt structure and present empirical evidence and potential economic mechanisms regarding discrepancies between firm debt types and financing costs. As they point out, most firms specialize in one type of debt. Following the dichotomy similar to Lian and Ma (2021), debt types can be broadly categorized as traditional bank debt and credit market financing.

Different types of debt have several implications. The first implication, which is also the main focus of this paper, is that the most recent work by Lian and Ma (2021) has shown that different types of debt differ in the types of borrowing constraints. Public financing typically faces an earnings-based borrowing constraint; the borrowing limits cannot be greater than a fraction of the measure of earnings. Private debt uses more asset-based borrowing constraints. In addition, different types of debt imply distinct cost structures. Public debt is associated with lower interest rates but higher fixed costs, whereas bank loans typically charge higher interest rates but lower fixed costs (Saunders and Steffen, 2011; Ewens and Farre-Mensa, 2021).[7]

Figure 2 shows the fraction of different loan types used by Compustat firms as well as the aggregate composition in the U.S. The left panel replicates the calculation of Lian and Ma (2021) regarding the fraction of earnings-based loans used by each Compustat company. I classify big firms as those whose employment levels are above the median for each 2-digit sector, and the rest are small firms. Consistent with Lian and Ma (2021), large firms use a substantial fraction of earnings-based loans, which is more than three-quarters on average. In contrast, on average, small firms use only roughly 25% of earnings-based loans. In addition, the patterns hold outside the Compustat firms. Benmelech et al. (2020) calculate the different types of loans used by small businesses over the years and find that about 70% of the loans are secured, which can be reasonably thought of as asset-based loans. The rest of them are mostly lines of credit or credit cards.

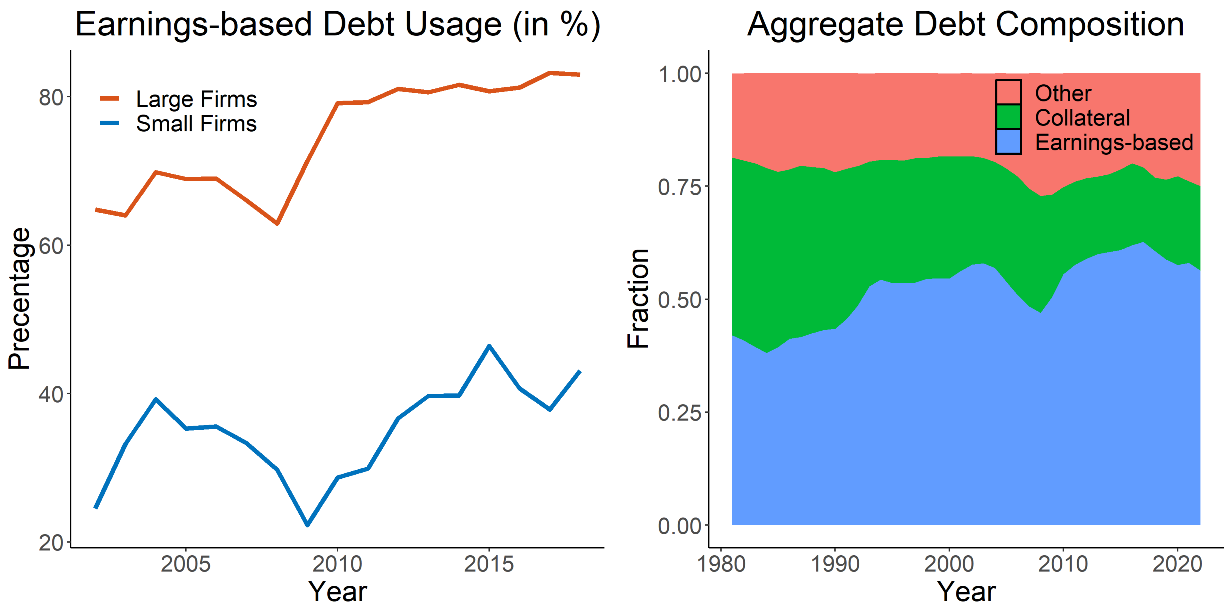
Regarding the aggregate composition, the right panel uses data from the Federal Reserve Board and categorizes the U.S. non-financial corporate debt into earnings-based and asset-based debt. It shows the heterogeneity of debt types in the U.S. economy, where earnings-based loans weigh more than 40% of the U.S. debt. In addition, firms increasingly borrow against earnings, representing the changes in debt composition.

In addition to borrowing limits, different debt financing also implies different cost

the real interest rate without changing other parameters so that the impacts from debt heterogeneity can be isolated from other elements.

[7]Higher rates of bank loans can be the result of bank monitoring costs, the regulatory constraint on bank leverage, etc. Higher fixed costs of public debt often arise as flotation costs of debt, the requirement of disclosure of firm information, etc.

Figure 2: Debt Heterogeneity



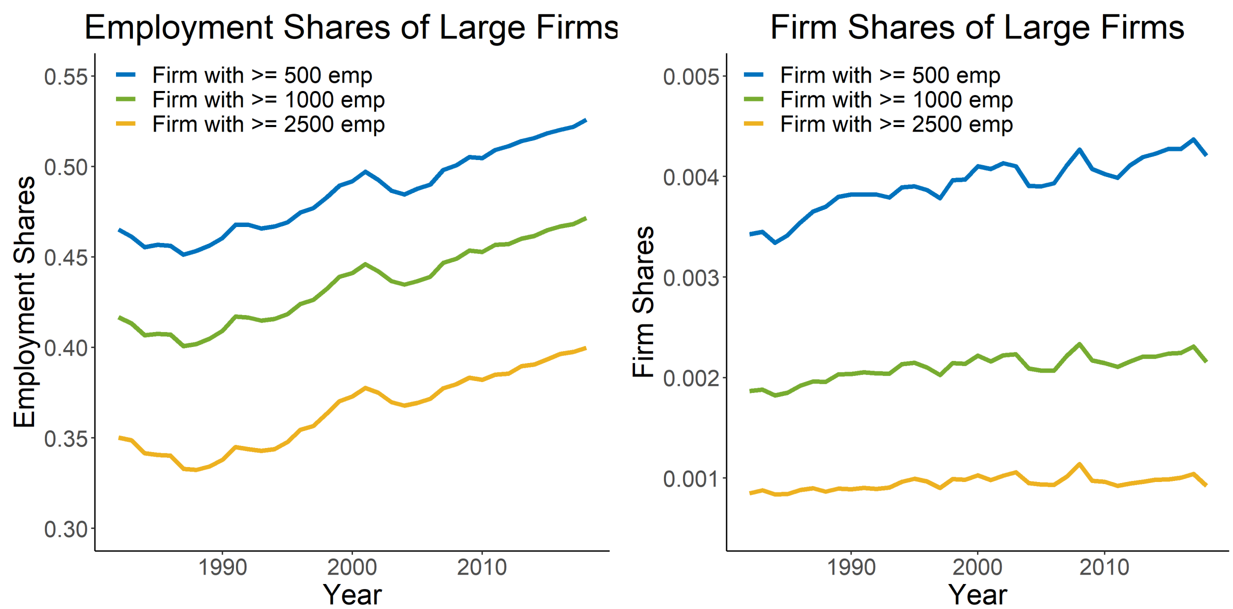
Note: This figure shows the fraction of different loan types used by Compustat firms and the aggregate composition. The left figure plots the median percentage of earnings-based loans of big and small Compustat firms. The percentages of earnings-based loans are calculated based on methods used by Lian and Ma (2021). The right panel categorizes the aggregate composition of different debt types. To keep conservative, I classify all U.S. bonds into earnings-based loans and all bank loans and mortgages into asset-based loans.

structures. Credit market financing, including instruments such as corporate bonds and syndicated loans, typically has lower costs (Krishnaswami et al., 1999). It requires public disclosure of information (Li et al., 2019). In contrast, bank loans tend to use asset-based collateral as well as monitoring to reduce the risks. Therefore, credit market financing tends to be cheaper than traditional bank loans in terms of interest rates but requires higher fixed costs.

2.2 The U.S. Firm Size Distribution and Firm Dynamics over the Years

Shifts in employment shares. The U.S. firm size distribution has changed dramatically over the past four decades. The firm size distribution has become increasingly dispersed, as documented in other papers (see Poschke, 2018, for example). As shown in Figure 3, employment shares have shifted to large firms. The share of firms that employ at least 500 employees has increased from 45% to more than 50%. Similar patterns are observed for firms with at least 1000 or 2500 employees. Each category has had a net increase of about 5% in employment share over the past four decades. The number of large firms among the total firms has also increased slightly since the 1980s. For example, the number

Figure 3: Employment Shares and Firm Shares of Large Firms over the Years



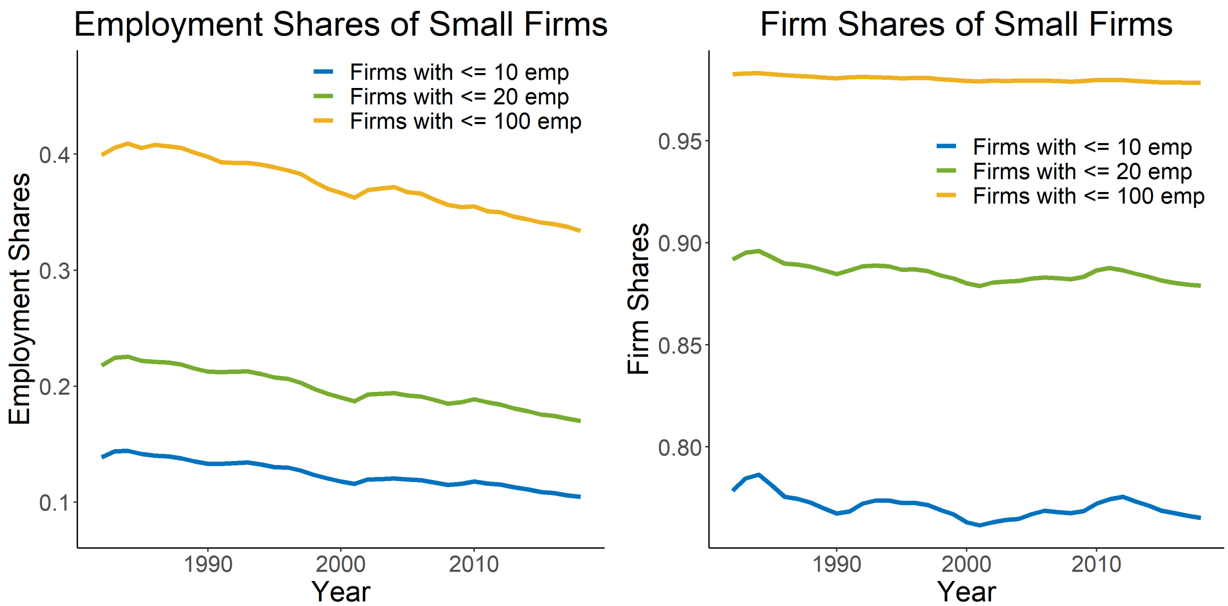
Note: This figure depicts long-term changes in the U.S. firm size distribution. The left panel shows the employment shares of firms with more than 500, 1000, and 2500 employees, respectively. The right panel shows the firm shares of firms with more than 500, 1000, and 2500 employees, respectively. All calculations are based on BDS data.

of companies with 500 or more employees was about 0.35% in the early 1980s. By the end of 2018, the share had become 0.4%, which has increased by more than 10%. Similar percentage changes can also be observed if one looks at firms with at least 1000 or 2500 workers.

In contrast, small firms exhibit a distinct pattern. Figure 4 displays small firms' employment shares and firm shares. Firms with less than 10, 20, or 100 employees have less and less employment share, even though those firms' shares have changed a little. More specifically, for example, the number of employees for firms with fewer than 20 workers was approximately 22.5% of total employment in the early 1980s. In 2018, the share declined to below 17.5%. The percentage of the firms has only changed by about 2%. In other words, small firms are getting smaller and smaller over time.

Changes in the employment size distribution. In addition, firms' employment sizes at the top of the distribution grow faster than the others. The left-hand side of figure 5 plots the ratio of the average Compustat firm size to the average firm size of all firms over the years. From the graph, it is clear that public firms outgrow other firms on average. While all firms, on average, are growing in size over the past years, public firms have grown almost twice faster as average firms, whereas those with assets above the sector median have grown even faster. As a comparison, I plot the ratio of the average size of small firms

Figure 4: Employment Shares and Firm Shares of Small Firms over the Years



Note: This figure depicts long-term changes in U.S. firm size distribution. The left panel shows the employment shares of firms with less than 10, 20, and 100 employees, respectively. The right panel shows the firm shares of firms with less than 10, 20, and 100 employees, respectively. All calculations are based on BDS data.

to the average size of all firms on the right-hand side. The increase in firm size dispersion is evident, as the size patterns for big and small firms are significantly different.

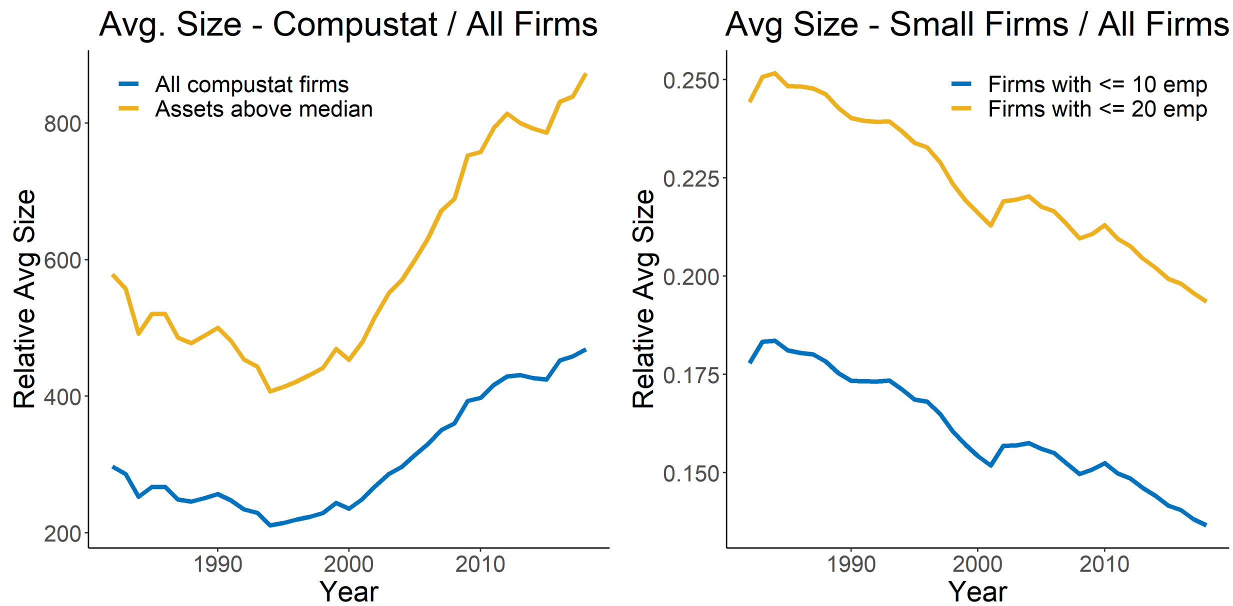
The decline in the firm entry rate and firms per capita. The start-up rate, measured as the number of new firms divided by the number of total incumbents, dropped from about 13% to 8%, as shown in the left panel of Figure 6. A relevant measure is the number of firms per capita, defined as the number of total firms divided by the number of total employees, which has declined by about 15% since its peak around the 1980s. The above patterns show a decline in new business activities as fewer firms are created, and fewer people are engaged in operating businesses.

3 A Model of Firm Dynamics with Debt Heterogeneity

This section sets up a quantitative model of firm dynamics with endogenous firm entry/exit and firm size distribution similar to Hopenhayn (1992). Compared with the previous literature, the key distinctions are that I enrich the model with endogenous choices of debt types that differ in the strength of contract enforceability and cost structures.

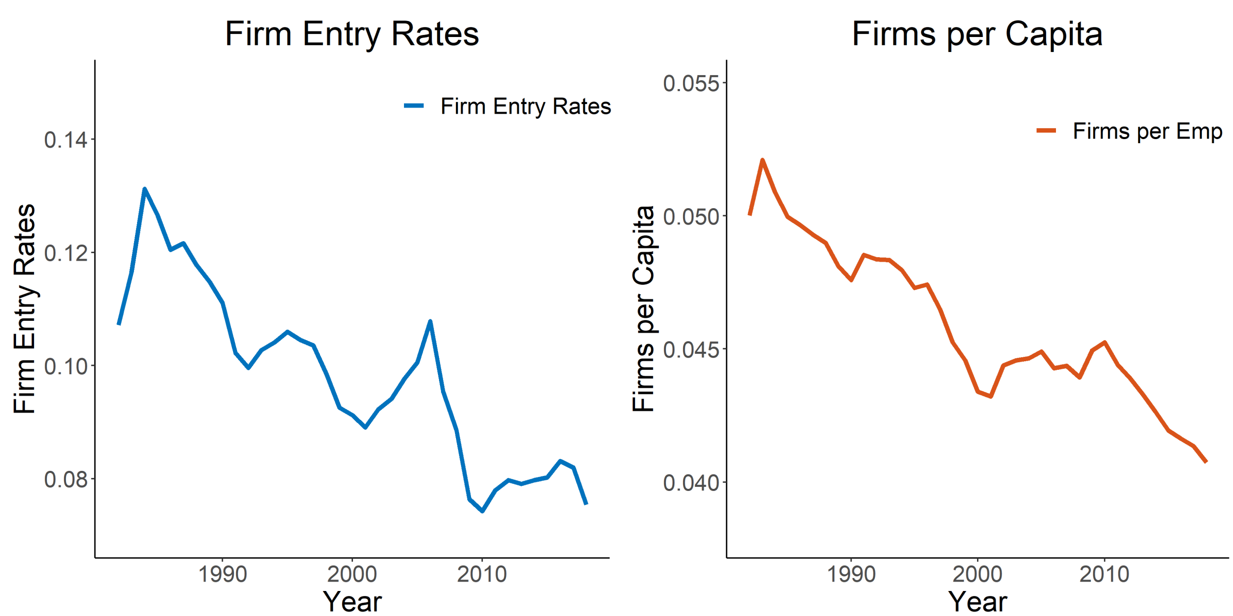
In the following subsections, I will present the economic environment in detail, outline the model timing, then describe the firm, financial intermediaries, and household prob-

Figure 5: Average Firm Sizes on the Top and Bottom over the Years



Note: This figure depicts long-term changes in the average size of firms at the top and bottom of the distribution in the U.S. The left panel shows the average employment of Compustat firms and those assets above the median compared with the average size of all firms. The right panel shows the average employment of firms with less than 10 and 20 employees, compared with the average size of all firms.

Figure 6: The Decline in Firms per capita and the Firm Entry Rate



Note: This figure depicts long-term changes in business dynamism in the U.S. The left panel shows the decline in the firm entry rate calculated based on BDS data. The right panel shows firms per capita, defined as the number of total firms divided by the total number of employees.

lems. Finally, I define a stationary equilibrium for the aggregate economy.

3.1 Environment

Time is discrete, and the horizon is infinite. Three types of agents populate the economy: entrepreneurs, households, and financial intermediaries. The economy is on a balanced growth path.

Entrepreneurs (firms). Entrepreneurs own firms. There is an exogenous measure of $m_0(1+\eta)^t$ potential entrants each period and an endogenous measure m_t of the incumbent firms. Firms are heterogeneous in their productivity, composed of a permanent component z and a stochastic component s_t. Firms operate a decreasing-returns-to-scale (DRS) production technology $y(z, s_t, k_t, l_t)$ that uses inputs of labor $l_t \in L$ and capital $k_t \in K$. The output of production is a homogeneous final good whose competitive price is the numéraire of the economy.

All potential entrants receive an initial equity draw a_0 from the exogenous asset distribution $\Gamma_{a_0}(a_0)$. Next, they draw a value of z from the initial distribution $\Gamma_z(z)$ and a value of s_0 from the initial distribution $\Gamma_{s_0}(s_0)$. Conditional on this draw, they decide whether to enter and become an incumbent by paying the set-up cost c_e.

For each period t, the incumbents can exit exogenously or endogenously. With probability ξ, a destruction shock hits an incumbent firm, forcing it to exit. Surviving firms observe their new values of s_t, drawn from the conditional distribution $\Gamma_s(ds, s_{t-1})$, and choose whether to exit or continue production. Under either exogenous or endogenous exit, the firm pays out its positive net worth to entrepreneurs.

Firms that continue production will choose the levels of labor and capital inputs and a source to finance their capital. All firms hire in a perfect, competitive, and homogeneous labor market. Firms can also accumulate illiquid assets, which can be used as collateral and generate interest revenue. In addition, they need to borrow capital from financial intermediaries.

Financial intermediaries. There are two types of financial intermediaries: banks and credit market lenders. The two sources of debt financing differ in terms of their cost structures and the strength of enforceability of contracts. Firms that borrow from the credit market need to pay a relatively higher fixed cost per period to represent self-disclosure of their financing information or expenses to build their reputation. The debt issued through the credit market has N-period imperfect enforceability of debt contracts: firms can renege on the contracts to keep a fraction of borrowed capital and save interest expenses but suffer multiple periods of loss of earnings and some illiquid assets.

Firms can also choose to borrow from banks. If firms decide to borrow from banks, they do not need to pay for the self-disclosure of their financing information and therefore pay a lower fixed cost per period. The debt contracts written by banks have one-period imperfect enforceability: firms can renege on the contracts to keep a fraction of borrowed capital and save interest expenses but lose a fraction of earnings for one period and some illiquid assets. Furthermore, I allow the lending rate to be different from the saving rate, and the interest spread of the credit debt is equal to or lower than the spread charged by banks.

Households. There is a measure of $H(1+\eta)^t$ representative households with $\bar{L}$ family members. The household is risk-neutral with a discount factor $\sigma_h \in (0,1)$. The household provides the labor and earns the wage payments that firms make to employed family members. The household also puts savings in financial intermediaries.

It is helpful to outline the timing of the model before introducing the details of the firm's problem. For each period, the events occur in the following order: (i) realization of the productivity shocks for incumbent firms; (ii) endogenous and exogenous exit of incumbents; (iii) realization of initial productivity and asset levels, and entry decision of potential entrants; (iv) financing decisions by all firms; (v) production and revenues from sales; (vii) payment of wage bill, costs of capital, hiring and operating expenses; saving decisions, and household consumption.

3.2 Firm Problem

Exit. At the beginning of each period, incumbent firms will face an exogenous shock to exit. After the shock, firms will observe their new productivity and decide whether to operate, as well as which type of debt they would like to borrow. If they exit, firms will pay out their current net worth. Let $x(z,s,a) \in \{0,1\}$ be the exit decision. A firm with permanent productivity component z and idiosyncratic component s_t and equity level a_t has a value V before the destruction shock, which equals

$$V(z, s_t, a_t) = (1-\xi) \cdot V^c(z, s_t, a_t) + \xi a_t(1+r_t),$$

where V^c is the expected value from continuing operation. The firms then choose whether to exit endogenously, and they will only choose to do so if the value offered by the outside option exceeds the value of being incumbent. Therefore, the value from continuing operation V^c satisfies

$$V^c(z, s_t, a_t) = \max\{V^i(z, s_t, a_t), a_t(1 + r_t)\},$$

where V^i represents the value of being an incumbent firm.

Entry. In each period, a measure of $m_0(1+\eta)^t$ potential entrants draw productivity and equity injections and decide whether to enter after paying an entry cost c_e. Let $i_0(z, s, a) \in \{0, 1\}$ denote the entry decision rule, which depends only on the initial productivity draw and equity injection. A firm will decide to enter if its expected value upon entry is greater than its equity injection.

$$V^i(z, s_0, a_0 - c_e) \geq a_0(1 + r_t)$$

Debt Financing Choices. An incumbent firm with permanent productivity component z and idiosyncratic component s_t and equity level a_t will decide the source of financing they would use, and its expected value V^i solves

$$V^i(z, s_t, a_t) = \max\{V^m(z, s_t, a_t), V^b(z, s_t, a_t)\}$$

Two value functions V^m and V^b associated with borrowing from the market (m) and the bank (b) are described below.

Firm Problem. Firms that borrow from the financial intermediary $j \in \{m, b\}$ solve the following problem:

$$\begin{aligned}
& V^j(z, s_t, a_t) = \max_{d_t, k_t, l_t, a_{t+1}} d_t + \sigma \mathrm{E_t} V(z, s_{t+1}, a_{t+1}) \\
\text{s.t.} \quad & d_t + a_{t+1} = e^{(z+s_t)} k_t^\alpha l_t^\beta - w_t l_t - \delta k_t - r_t^j k_t + a_t(1 + r_t) - f_j \\
& r_t^j = r_t + \rho^j \\
& k_t \leq \overline{K}^j(z, s_t, a_t) \\
& 0 \leq a_t \\
& 0 \leq d_t
\end{aligned}$$

Firms maximize the value and use σ_f as their discount factor. Firms will decide the next period's equity holding and current dividends by maximizing the revenues from the production net of interests on the rental, the wage bill, depreciation δ, and operating costs f_j, which depends on the debt types. The interest rate is set up as the real interest rate r_t plus a market-specific interest spread ρ_j. The last three equations reiterate that firms face endogenous borrowing limits set by the financial intermediaries and a non-negativity constraint on dividends and equity holdings.

3.3 Financial Intermediaries

There are two types of financial intermediaries in the economy: banks and credit market lenders. Two critical distinctions between the two financial intermediaries are the strength of enforceability of debt contracts and the cost structures.

Banks. Borrowing and capital rental by firms are limited by one-period imperfect enforceability of contracts (Buera et al., 2011). In particular, I assume that firms may renege on the contracts. In such cases, firms can keep 1 - ϕ_k of the borrowed capital and will not pay the interest expenses. The punishment is the garnishment of their financial assets deposited with the financial intermediary, a_t, and a fraction ϕ_e of one-period earnings. In the following period, the entrepreneurs in default regain access to financial markets and are not treated any differently despite the default history.

I consider equilibria where the borrowing and capital rental contracts are incentive-compatible and are hence fulfilled. As shown by Buera et al. (2011), firms will abide by the contract if and only if

$$\overbrace{y_t^* - w_t l_t^* - \delta k_t^*}^{\text{earnings (EBIT)}} - \overbrace{r_t^b k_t^*}^{\text{interest expenses}} + \overbrace{(1+r)a_t}^{\text{illiquid assets}} \\ \geq \\ (1-\phi_e) \times (\underbrace{y_t^* - w_t l_t^* - \delta k_t^*}_{\text{earnings (EBIT)}}) + (1-\phi_k) \times \underbrace{k_t^*}_{\text{capital}}$$

Simple algebra yields

$$\phi_e(\overbrace{y_t^* - w_t l_t^* - \delta k_t^*}^{\text{earnings (EBIT)}}) + \overbrace{a_t}^{\text{illiquid assets}} \geq (1-\phi_k) \times \overbrace{k_t^*}^{\text{capital}} + \overbrace{r_t^b k_t^* - r a_t}^{\text{net interest expenses}}$$

, where y_t^*, l_t^*, and k_t^* are the optimal output, labor input, and capital input, respectively. I argue that the inequality above is essentially an asset-based constraint. It can be easily seen when ϕ_e is small, or earnings are small. More generally, since the capital needed for production at the full scale is much larger compared with earnings, the above inequality can be approximated by

$$\overbrace{a_t}^{\text{assets}} \geq (1-\phi_k) \times \overbrace{k_t^*}^{\text{debt}}$$

That is, the borrowed capital cannot exceed a proportion of assets.

Credit Market Lenders. Debt contracts offered by credit market lenders are assumed to have N-period imperfect enforceability. More precisely, once the firms renege on the contract, they will experience a reduction in earnings for multiple (N) periods. Meanwhile, they cannot access financial intermediaries. By doing so, the firms will not pay back the borrowed capital nor the interest expenses generated. Let $V^i(z, s_t, a_t; \overline{K})$ be the value of the firms that abide by the contract and $O(z, s_t, a_t; \overline{K})$ be the value of the firms that renege on the contract. For the incentive compatibility constraint to hold, the credit market lenders will set borrowing limits $\overline{K}$ such that $V^i(z, s_t, a_t; \overline{K}) \geq O(z, s_t, a_t; \overline{K})$. To simplify the problem, I assume the values will stay the same after the penalty periods.[8] If $1/\sigma_f - 1 = r_t$, it can be shown that the incentive compatibility constraints imply

Proposition 1 *$V^i(z, s_t, a_t; \overline{K}) \geq O(z, s_t, a_t; \overline{K})$ IFF*

$$\phi(z, s_t, a_t; N)\pi^*_{i,t} \geq r k^*_{i,t+s} + \frac{\phi_k \sigma_f^N k^*_{i,t+s} - \phi_k a_{i,t}}{\Pi(z, s_t, a_t; , N)},$$

where $\Pi(z, s_t, a_t; N)$ increases monotonically in N conditional on $\{z, s_t, a_t\}$.

Proof. See the appendix. ■

$\pi^*_{i,t}$ is the earnings before interest expenses (EBIT) commonly used to measure earnings by the creditors. $\phi(z, s_t, a_t; N)$ is a state-contingent parameter that measures the loss of earnings. $\Pi(z, s_t, a_t; N)$ measures the multiplier that maps the current income to the total future discounted earnings (EBIT).[9] Since $\Pi(z, s_t, a_t; N)$ increases monotonically in N conditional on $\{z, s_t, a_t\}$, it is trivial to see that the longer the penalty periods last, the less (more) weight the borrowing limits depend on illiquid assets (earnings). The debt limits put more weight on earnings as N gets large. It is easier to see the point and compare it with the one-period case if I further assume the firm is at the steady-state equilibrium without future shocks. If $N \to \infty$, then

[8] In the appendix, I show that similar propositions can be derived if I relax the assumptions.

[9] For a simple example, at the steady state, $\Pi(z, s_t, a_t; N)$ is just the sum of discounted factors of finite periods $\frac{1-\sigma_f^N}{1-\sigma_f}$

Corollary 1 *At the steady-state equilibrium, if $N \to \infty$, $V^i(z, s_t, a_t) \geq O(z, s_t, a_t)$ IFF*

$$\phi_e \overbrace{y_i^* - w_t l_i^* - \delta k_i^*}^{\text{earnings (EBIT)}} \quad \geq \quad \overbrace{r^m k_i^* - r a_i}^{\text{net interest expenses}}$$

Proof. See the appendix. ■

Here, y_i^*, l_i^*, and k_i^* are the optimal output, labor input, and capital input, respectively. The borrowing limit now is the interest coverage ratio, one of the most commonly used earnings-based borrowing limits (Greenwald et al., 2019).

Consistent with the empirical facts, I further assume that, in general, $\rho_b \geq \rho_m$ and $f_b < f_m$ so that firms will trade-off between cost structures and debt enforceability (Saunders and Steffen, 2011; Ewens and Farre-Mensa, 2021). As a result, firms with greater financial needs will choose to be market-financed, whereas firms with sufficient self-financing or less need of external funding will borrow from the bank. I argue this simple mechanism captures the merit of the trade-offs between the two different types. Earnings-based borrowing limits give advantages in terms of greater borrowing capacities to firms that borrow from the credit market. To generate debt heterogeneity, the higher fixed cost broadly represents the costs of the benefits.[10]

Discussion. The above propositions show that the difference in the strength of the imperfect enforceability of debt contracts can yield different types of borrowing limits. It is still helpful to provide possible interpretations for such differences. One way to interpret one-period imperfect enforceability, among others, is that firms have options to directly liquidate all tangible assets and quickly resume operations somewhere else. It is especially feasible if the production technology is easy to adopt or the technology is the private information of the entrepreneurs so that creditors cannot seize the technology. In contrast, multi-period imperfect enforceability here represents that firms cannot resume operation quickly after defaulting. Instead, they have to go through the restructuring process, where their profits will be reduced for prolonged periods. It can be the consequence of involuntary restructuring bankruptcy, or firms cannot easily resume the operation, both of which become feasible after firms disclose the information to the public. In the former case, detailed information makes restructuring feasible for external creditors, whereas in the latter case, the creditors can seize the production technology. Either way, firms choose to pay higher fixed costs and voluntarily increase the enforceability of debt contracts to satisfy their debt financing and production needs. Based on those insights, earnings-based bor-

[10]Similar mechanisms are used on Chang et al. (2017) and Russ and Valderrama (2012), who built a model with a similar intra-industry selection of financing sources mechanism to show that the development of both the bond market and the banking sector can increase aggregate productivity.

rowing limits, or choice of the credit market, has a new interpretation: firms voluntarily leverage over their future values by paying additional costs to increase the enforceability of their debt contracts. By doing so, they can leverage more and produce more efficiently.

It is worthwhile to point out that the outside options for both cases can be rationalized through a Nash bargaining game. This is straightforward for the one-period enforceability. It is easy to see that a one-period case is equivalent to that firms can renege on the contract and bargain with financial intermediaries over one period of earnings and borrowed capital. In other words, firms have the threat to quit bargaining to keep all possible future values. In contrast, a multi-period case can be generated when they can bargain over multiple periods of earnings and borrowed capital. Therefore, the essential difference between the two types of borrowing limits is to what extent the future values firms can keep out of the bargaining game.

The difference regarding whether future earnings matter for the borrowing limits matches recent empirical findings. In fact, the recent paper by Caglio et al. (2021) extended the work of Lian and Ma (2021) by showing that mostly small-medium-size enterprises (SMEs) also use their enterprise's continuation values as collateral, but the weight of each type of collateral is different for firms with different sizes. In particular, a greater fraction of loans to SMEs is collateralized by cash/accounts receivable, inventory, and fixed assets/real estate, whereas the majority of loans to large firms are collateralized by blanket lien or unsecured. My theory is consistent with the findings in that with one-period imperfect enforceability, firms will still collateralize their values, but the values are limited to the current period. Therefore, the current income, capital, and illiquid assets, of which the empirical counterparts can be considered cash/accounts receivable, inventory, and fixed assets/real estate, play a major role in determining their borrowing limits. In contrast, unsecured debt and blanket lien, which can be considered the future values of firms, play an important role for large firms subject to N-period imperfect enforceability.

In fact, one-period imperfect enforceability of debt contracts is essentially embedded in various types of "moral hazard" issues featured with static or one-period incentive compatibility constraints (e.g., Chang et al., 2017). Once the deviation does not punish agents' future values, the setups tend to generate the minimum requirements for the current internal funding, or illiquid assets in my case. The assumption of N-period imperfect enforceability of debt contracts is frequently used in many dynamic contracting problems when defaulting impacts future values of firms (e.g., Albuquerque and Hopenhayn, 2004). This paper rationalizes the two types of constraints in a unified framework, highlighting the mechanism behind their distinctions.

3.4 Household Problem

At time t, the representative household with a measure of $H(1+\eta)^t$ provides the total of $\bar{L}H(1+\eta)^t$ units of labor and receives wages.

$$
\begin{aligned}
W(B_t) &= \max_{C_t, B_{t+1}} C_t + \sigma_h(1+\eta)W(B_{t+1}) \\
\text{s.t.} \quad C_t + B_{t+1} &= w_t\bar{L} + \frac{(1+r_t)}{1+\eta}B_t,
\end{aligned}
$$

where C_t is the household consumption, $\bar{L}$ is the labor supply for one unit of household, and B_t is the savings to the financial intermediaries.

3.5 Stationary Equilibrium

Let $m_t(Z,S,A)$ be the measure of incumbent firms with permanent productivity $z \in Z$, idiosyncratic productivity $s \in S$, and previous equity holding $a \in A$ at the beginning of the period t, following the draw of firm-level productivity, before the exogenous exit shock.

Let $\bar{m_t} = \frac{m_t}{H(1+\eta)^t}$ and $\bar{m_0} = \frac{m_0(1+\eta^t)}{H(1+\eta^t)}$ denote these objects per capita, then the law of motion of the distribution is

$$
\begin{aligned}
\bar{m_{t+1}}(Z,S,A) = &\iint \frac{(1-\xi)}{1+\eta} Q(z, s_{t+1}, a_{t+1}|z, s_t, a_t)(1 - X(z, a_t, s_t))\bar{m_t}(dz, ds, da) \\
&+ \bar{m_0} \iiint Q_0(z, s_{t+1}, a_{t+1}|z, s_t, a_t) I(z, s_t, a_t) d\Gamma_{a_0}(a_0) d\Gamma_z(z) d\Gamma_{s_0}(s_0),
\end{aligned}
$$

where $Q_0(z, s_{t+1}, a_{t+1}|z, s_t, a_t)$ and $Q(z, s_{t+1}, a_{t+1}|z, s_t, a_t)$ are the resulting transition matrix based on the selection of equity holding and evolution of productivity shocks for entry and incumbent firms respectively.

Closing the Model. Finally, the labor market will clear such that

$$
\bar{L} = \iiint l(z, s_t, a_t)(1 - X(z, a_t, s_t))\bar{m_t}(dz, ds, da) + \iiint l(z, a_t, s_t) I_0(z, s_t, a_t)\bar{m_0}(dz, ds, da),
$$

where $l(z, s_t, a_t)$ is the policy function of labor demand for firms with state variables (z, s_t, a_t). The capital market will clear such that

$$
B_t = \iiint k(z, s_t, a_t)(1 - X(z, a_t, s_t))\bar{m_t}(dz, ds, da) + \iiint k(z, a_t, s_t) I_0(z, s_t, a_t)\bar{m_0}(dz, ds, da),
$$

where $k(z, s_t, a_t)$ is the policy function of capital demand for firms.

4 Lessons from A Stylized Example

This section uses a simplified version of the full model to illustrate the mechanism that links the decline in the real interest rate to changes in the firm size distribution and the decline in firm dynamics.

4.1 A Stylized Example

To facilitate the analysis, I assume, for now, there is no entry and exit and no balanced growth. Instead of endogenous choices of debt types, I assume that the economy exits two types of firms exogenously. One kind of firm will borrow from the credit market, and the other type will borrow from banks. There will be no productivity difference between the two types of firms, and for simplicity, I further assume there is no depreciation, which will not affect the results if there is one. In addition, I assume that debt issued by banks faces asset-based constraints, and borrowing from the credit market has an infinite period of imperfect enforceability.

The analysis here focuses on the steady-state equilibrium. As shown in Corollary 1, the infinite period imperfect enforceability collapses to the interest coverage ratio. Therefore, in the economy, an exogenous measure of μ_m market-financed firms borrow from the credit market and face the following firm problem

$$
\begin{aligned}
& \max_{d_t, k_t, l_t, a_{t+1}} \sum_{t=0}^{\infty} \sigma^t d_t \\
\text{s.t.} \quad & d_t + a_{t+1} = k_t^{\alpha} l_t^{\beta} - w_t l_t - r_t^m k_t + r_t a_t + a_t \\
& r_t^m k_t - r_t a_t \leq \phi_e \cdot [k_t^{\alpha} l_t^{\beta} - w_t l_t] \\
& 0 \leq a_{t+1} \\
& r_t^m = r_t + \rho^m
\end{aligned}
$$

Firms aim to maximize each period's dividend and are subject to budget and borrowing constraints. At the beginning of each period, firms will start with some asset level a_t. They will produce goods using a Cobb-Douglas function, where I assume $\alpha + \beta < 1$ to achieve decreasing returns to scale. Firms will hire labor l_t and pay the wage w_t. In addition, firms decide how much to, a_t, and borrowing from debt issuers, k_t. After paying the net financing cost $r_t^m k_t - r_t a_t$, firms will distribute retained earnings to either current dividends or savings for the next period, which need to be positive. As the last equation shows, the interest r_t^m faced by market-financed firms equals the real interest rate r_t plus a time-invariant firm type-specific interest spread ρ^m.

In contrast, there is a measure of μ_b "bank-financed" firms borrow bank loans and face the following firm problem:

$$
\begin{aligned}
& \max_{d_t,k_t,l_t,a_{t+1}} \sum_{t=0}^{\infty} \sigma^t d_t \\
\text{s.t.} \qquad & d_t + a_{t+1} = k_t^{\alpha} l_t^{\beta} - w_t l_t - r_t^b k_t + r_t a_t + a_t \\
& k_t \leq \frac{1}{1-\phi_k} a_t \\
& 0 \leq a_{t+1} \\
& r_t^b = r_t + \rho^b
\end{aligned}
$$

Similarly, the firms aim to maximize each period's dividend and are subject to budget and borrowing constraints. The budget constraint is similar to that of market-financed firms. The only difference is that the interest r_t^b faced equals the real interest rate r_t plus a different interest spread ρ^b. Bank-financed firms face an asset-based borrowing constraint, shown in the second equation, that is, the debt level, k_t, cannot be more than a fraction, $\frac{1}{1-\phi_k}$, of savings a_t.

For simplicity, the measures of both types of firms, ν_m and ν_b, are given exogenously. In addition, there is $\bar{L}$ unit of inelastic labor supply in the economy; then the market-clearing condition for the labor market at time t is

$$\bar{L} = \nu_m \cdot l_t^m + \nu_b \cdot l_t^b$$

, where l_t^m and l_t^b are the equilibrium labor demand for market-financed firms and bank-financed firms, respectively.

4.2 The Effects of Decline in the Real Interest Rates

First, a statistic should be introduced to see the effects of the decline in the real interest rate. With a simple algebra, the labor market clearing condition becomes

$$\bar{L} = l_t^b \cdot (\nu_m \cdot \frac{l_t^m}{l_t^b} + \nu_b)$$

With this simple framework, it is helpful to look at $\frac{l_t^m}{l_t^b}$ (hereafter, the labor ratio) to inform us about the firm size distribution and firm dynamics. It is trivial that the labor ratio is proportional to the employment shares. For instance, if $\frac{l_t^m}{l_t^b}$ increases, then the employment shares shift to market-financed firms. Meanwhile, the labor demand for bank-financed firms will decrease due to fixed firm measures and inelastic labor supply. Moreover, since labor shares can inform firms' profitability, the labor ratio can also be used to analyze the firm dynamics.

Therefore, I will focus on the impact of the decline in real interest rates on the labor ratio to reveal their connections as well as potential mechanisms. The effects of the decline in long-run real interest rates through heterogeneous borrowing limits are summarized in the following proposition.

Proposition 2 *If $\rho^m = \rho^b$, ϕ_e,ϕ_k are positive, $\frac{1}{\sigma} > 1 + r_t + \rho_t^b$, and $\phi_e < \frac{\alpha}{1-\beta}$, then:*

- *if $\phi_k \leq \phi_e$, the labor ratio $\frac{l^m}{l^b}$ increases as r decreases.*
- *if $\phi_k > \phi_e$, the labor ratio $\frac{l^m}{l^b}$ and r are "U-shaped": $\exists r^* \in (0, \frac{1}{\sigma} - 1)$ such that when $r > r^*$, the labor ratio decreases as r decreases, and when $r < r^*$, the labor ratio increases as r decreases.*

Proof. See the appendix. ∎

The proposition states that if both types of firms pay the same interest rate, then the change in the labor ratio with respect to r depends on ϕ_k and ϕ_e, which govern the leverage ratio of the two types of firms separately. If bank-financed firms cannot leverage enough, then the decline in real interest rates will cause the labor ratio to increase monotonically. If, however, bank-financed firms can get enough leverage, then the relationship will be "U-shaped." At a high real interest rate time, the decline in the real interest rate will cause the labor ratio to decrease, and the relationship will reverse at a lower real interest rate time.

More intuitively, two channels work here. The first one is the borrowing limit channel. As the real interest rate decreases, the interest expense decreases accordingly. Governed by the interest coverage ratio, the financial constraint of market-financed firms is relaxed directly. The second one is the saving incentive channel. As the interest rate decreases, the direct return of savings, or the interest income, decreases. Meanwhile, bank-financed firms are less willing to save assets, directly affecting their borrowing constraints. In contrast, the borrowing constraint of market-financed firms is not restricted by their savings. The coefficients ϕ_k and ϕ_e jointly determine the magnitudes of the impacts of two channels.

There are other factors related to debt heterogeneity that can amplify the effects. The following proposition illustrates the effects of heterogeneous interest expenses on the labor ratio when the interest rates decline. Let l^m and l^b be the steady-state equilibrium labor demand for firms with access to the credit market and bank loans, respectively, and r is the real interest rate:

Proposition 3 *Assuming $\frac{1}{\sigma} > 1 + r_t + \rho_t^b$, if $\rho_m < \rho_b$ and ϕ,$\lambda \to 1$, then the steady-state labor ratio $\frac{l^m}{l^b}$ increases monotonically as r decreases.*

Proof. See the appendix. ■

The proposition says that if firms are unconstrained, and market-financed firms pay constant lower interest rates than bank-financed firms, the same decline in the real interest rate will result in an increase in the labor ratio so that bank-financed firms will shrink, and market-financed firms will expand. The fundamental reason for this result is that the profit functions of the firms are convex and downward-sloping in r. Therefore, the same reduction in r starting from a lower interest rate can cause a higher increase in profits. Moreover, since both types of firms compete in the same market, market-financed firms, which outperform the others, will expand, and bank-financed firms will shrink.

It is also intuitive to look at the analytical solution of the equilibrium labor ratio. At the steady-state equilibrium, if both types of firms are unconstrained, the labor ratio

$$\frac{l^m}{l^b} = (\frac{r + \rho^b}{r + \rho^m})^{\frac{\alpha}{1-\alpha-\beta}},$$

which is equal to the relative cost of interest (if taking the rate of bank-financed firms as the base) with a power of some coefficients. That is, as the interest rate declines, the relative financing cost of market-financed firms decreases, so they expand relative to bank-financed firms.

In addition to the borrowing limits and different interest spreads, endogenous debt choices, absent in this simple model, can also enhance the effects. As the real interest rates go down, marginal firms who borrow from banks at the high interest rate period can switch to the credit market to borrow and produce more efficiently.

When effects are large enough, bank-financed firms' profits can also decline.[11] Suppose that an economy has a selection on the firm values at entry. In that case, the entry rate can also fall if more entering firms rely on bank loans.

Another interesting lesson from the equation is that if there is no heterogeneity in debt financing, the labor ratio and the equilibrium labor demand will not change with respect

[11] In the appendix, I show that for the simple case where both types of firms are unconstrained. If market-financed firms have sufficient productivity advantages and firms respond to the decline in real interest rates sufficiently, bank-financed firms will reduce their profits. For more complicated cases, I refer to the quantitative analysis introduced in Section 5, as the analytical solution can be complicated and less intuitive.

Table 1: Externally Calibrated Parameters

Parameter	Name	Value	Source
α	capital share	0.3	Poschke (2018)
β	employment share	0.6	Poschke (2018)
σ_f	discounting rate	0.94	Interest Rate at 1980s
δ	depreciation	0.10	Poschke (2018)
ρ_b	interest spread	0.025	Compustat
ρ_s	interest spread	0.040	Compustat
η	population growth	0.025	Karahan et al. (2019)
$\bar{L}$	avg employment	19.8	BDS

to the real interest rate. Moreover, since the cost of production (interest rates) are declining for all firms, the profitability will increase so that the decline in the real interest rates will boom the entry, as shown in the quantitative exercise of Röhe and Stähler (2020).

5 Quantitative Experiments

This section reports the results of the quantitative experiments. I calibrate the model to match a rich set of statistics in the U.S. Then, I show the effects of the exogenous changes in the real interest rates on the firm dynamics and firm size distribution at the stationary equilibrium. Furthermore, the transient dynamics for a sequence of perfect foresight changes on the real interest rate will be studied. Finally, I compare the benchmark economy with a counterfactual economy with no debt heterogeneity to demonstrate the role of debt heterogeneity.

5.1 Parameterization

Externally calibrated parameters. I begin with the subset of parameters that are calibrated externally. Table 1 shows these parameters. The model is calibrated at an annual frequency. I set σ equal to 0.94 to replicate the annualized risk-free rate (6%) at the beginning of the 1980s. Since the measure of potential entrants $\bar{m}_0$ scales $\bar{\mu}_t$, I choose $\bar{m}_0$ to normalize the total measure of incumbent firms to one. I normalize the size of the labor force $\bar{L}$ so that, given a measure of one of the firms, the average firm size will be 19.8, consistent with Business Dynamics Statistics (BDS) data over the period 1982 - 1986.

Consistent with empirical facts and literature, I calibrate the magnitude of decreasing returns to scale to 0.90 and the ratio of the output elasticity of capital and labor to be 1/2 so that the resulting value of $\alpha = 0.30$ and $\beta = 0.60$ (Poschke, 2018). Additionally,

Table 2: Internally Calibrated Parameters

Parameter	Name	Value
μ_{s_0}	mean of s_0	-0.283
σ_{s_0}	std of log s_0	0.302
μ_{a_0}	mean of a_0	0.463
σ_{a_0}	std of log a_0	0.802
σ_{z_0}	std of log z_0	0.100
σ_ϵ	std of shocks	0.044
ρ	persistence of shocks	0.981
f_b/y	fixed costs / output	0.013
c_e	entry cost	0.001
ϕ_k	leverage ratio of asset	0.280
ϕ_e	leverage ratio of earning	0.764
$F(N)$	restructure term	9.026
f_m/y	fixed cost / output	0.025

I calibrate the interest spread for market-financed firms $\rho_m = 0.025$ and that for bank-financed firms $\rho_b = 0.040$ to match the difference between the imputed interest rates paid by those Compustat firms and the real interest rate. Note that I classify those firms with an asset level above the 2-digit sector median for each year as market-financed firms, whereas the others are assumed to be bank-financed.

Finally, the depreciation rate is calibrated to be equal to 10%, similar to the value commonly used in the literature. The population growth rate η is calibrated to 2.5% to match the U.S. population growth rate, similar to Karahan et al. (2019)

Internally calibrated parameters. The rest of the parameters are calibrated internally. Table 2 lists those parameters that are set by minimizing the weighted distance between an equal number of empirical moments and their equilibrium counterparts in the model. In what follows, I will discuss the moments used to calibrate the model and the reasons for choosing them.

The first set of parameters is mostly related to the initial productivity and equity distributions, the productivity shock, and the shock to exit. σ_{z_0} governs the initial distribution of permanent productivity z, which is assumed to be a log-normal distribution with $z_0 \sim N(-\sigma_{z_0}^2/2, \sigma_{z_0})$. μ_{s_0} and σ_{s_0} governs the initial distribution of the stochastic component of productivity s_0, which is assumed to be a log-normal distribution with $s_0 \sim N(\mu_{s_0}, \sigma_{s_0})$. I assume that s_t evolves according to an AR(1) process such that $s_t = (1-\rho)s_{ss} + \rho s_{t-1} + \sigma_\epsilon \epsilon_t$, where s_{ss} is set so that $E(s_t) = 0$ and $\epsilon_t \sim N(0,1)$. μ_{a_0} and σ_{a_0} governs the initial distribution of equity draw a_0, which is assumed to be a log-normal distribution with $log(a_0) \sim N(\mu_{a_0}, \sigma_{a_0})$. The initial distributions and the productivity process are important for the equilibrium firm size distribution as well as firm entry/exit dynamics. In addition, the exogenous shock of exit ξ is particularly important for the exit rate of firms, especially for those older firms.

I calibrate those parameters to match some key annual statistics of firm size distribution and firm dynamics averaged over the 1982 - 1986 period. Precisely, for the firm size distribution, the model matches the firm shares and employment shares for those who hire more than 500, 1000, and 2500 employees. Also, the overall exit rate and the firm entry rate are selected. I also target the exit rate by firm age to better match the dynamics. In addition, I also match the standard deviation of employment growth.

Another set of parameters is mainly related to the financial environment. Fixed costs f_m and f_b are important in terms of the selections of firms on financing sources and their characteristics, especially sizes, within each financing option. In terms of the borrowing limits, I assume the parameters that govern the earnings-based borrowing constraint are state-independent to facilitate the process of solving the model. As shown in the appendix, it can be achieved by making additional assumptions on the defaulting process. Then, three parameters regulate two types of financial constraints: ϕ_k governs the ability of the assets to leverage. Since the majority of firms in the economy use asset-based loans, this parameter is pinned down by matching the debt-to-output ratio. ϕ_e governs the ability of earnings to leverage, which is calibrated by targeting the interest coverage ratio of market-financed firms. F(N) regulates the weights of assets relative to earnings. I choose to match the ratio of market debt and bank debt, calculated using data from the Flow of Funds[12] In addition, the average size of market-financed firms in Compustat (those firms with assets above the median) is targeted to match the characteristics of firms that choose the credit market.

In short, there are 16 moments to identify 13 parameters. Table 3 shows some moments, and Figure 7 shows the rest. Overall, the moments are well-matched, and the model generates a good fit for the data. In the appendix, Figure 13 shows the moments of the firm age $\times$ size distribution, which are non-targeted. The figure shows that those non-targeted moments are also fitted well into the data.

5.2 Equilibrium Effects of the Decline in Real Interest Rates

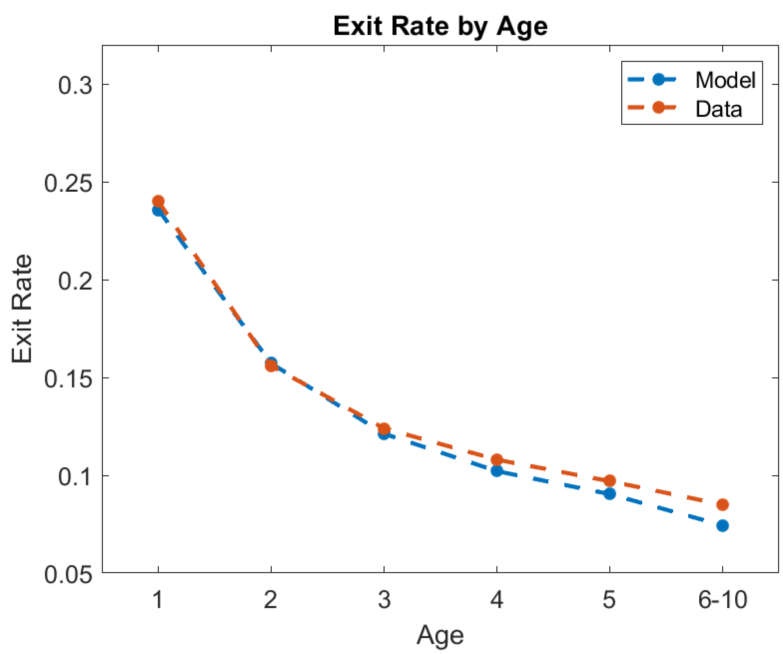
This subsection shows the stationary equilibrium outcomes of the economy for different values of the real interest rate. I use the calibrated model to evaluate the effects of declining

[12]To classify the credit market loans, I include the level of commercial paper, corporate bonds, and other loans from a finance company. I classify all mortgages and bank loans not elsewhere classified as bank loans using the Flow of Funds table L.102 from 1982-1986.

Table 3: Targeted Moments

Moment	Model	Data	Source
Firm share $\geq$ 500	0.0034	0.0034	BDS
Employment share $\geq$ 500	0.462	0.460	BDS
Employment share $\geq$ 1000	0.403	0.410	BDS
Employment share $\geq$ 2500	0.327	0.340	BDS
Exit rate (Entry Rate)	0.093 (0.113)	0.097 (0.116)	BDS
S.T.D. of emp growth	0.292	0.420	Gavazza et al. (2018)
Avg. size for large firm	9937	10000	Compustat
Market/bank debt ratio	1.26	1.30	Flow of Funds
Interest coverage ratio	0.328	0.30	Lian and Ma (2021)
Debt-to-output ratio	0.381	0.350	Flow of Funds

Figure 7: Targeted Firm Exit Rates

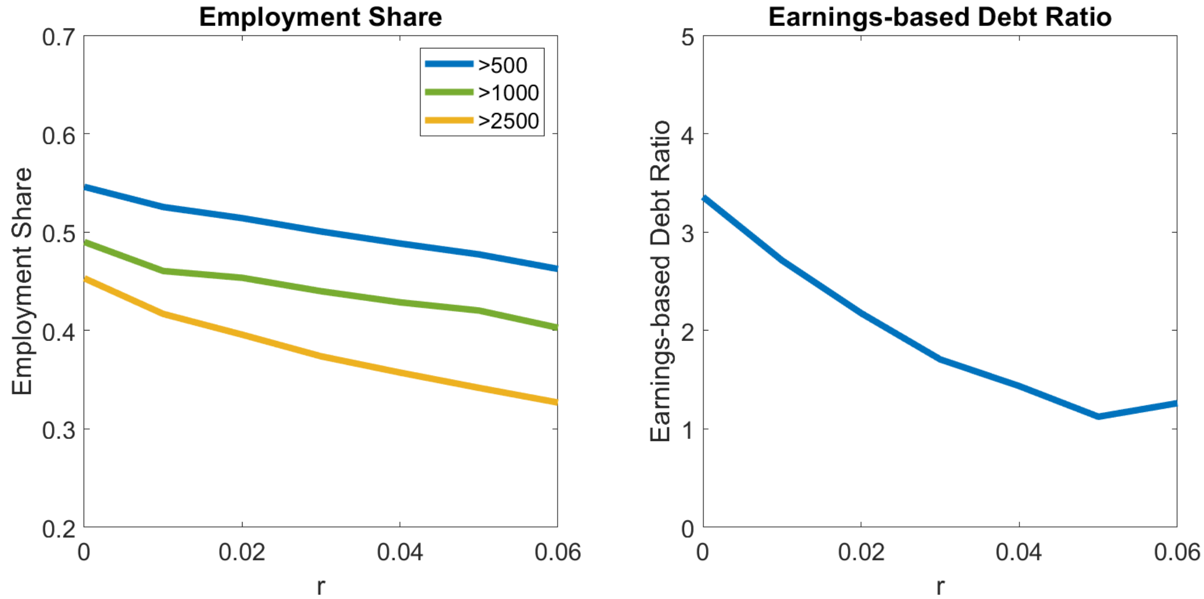


Note: This figure shows the targeted firm exit rates. The firm exit rates are calculated as the number of total exiting firms for each age divided by total incumbents at the same age.

interest rates on the economy. In particular, I analyze its asymmetric effects through debt heterogeneity. I find that the decline in the real interest rate can explain all the changes in the shifts in employment shares and 80% of the changes in debt composition experienced in the U.S. over the past forty years. The model can also explain about 40% of the decline in firms per capita and more than 10 % of the decline in the firm entry rate. Counterfactual analysis shows that debt heterogeneity plays an important role in generating the effects.

In terms of employment shares of large firms, the left panel of Figure 8 shows the

Figure 8: Shifts in Employment Shares and Debt Composition



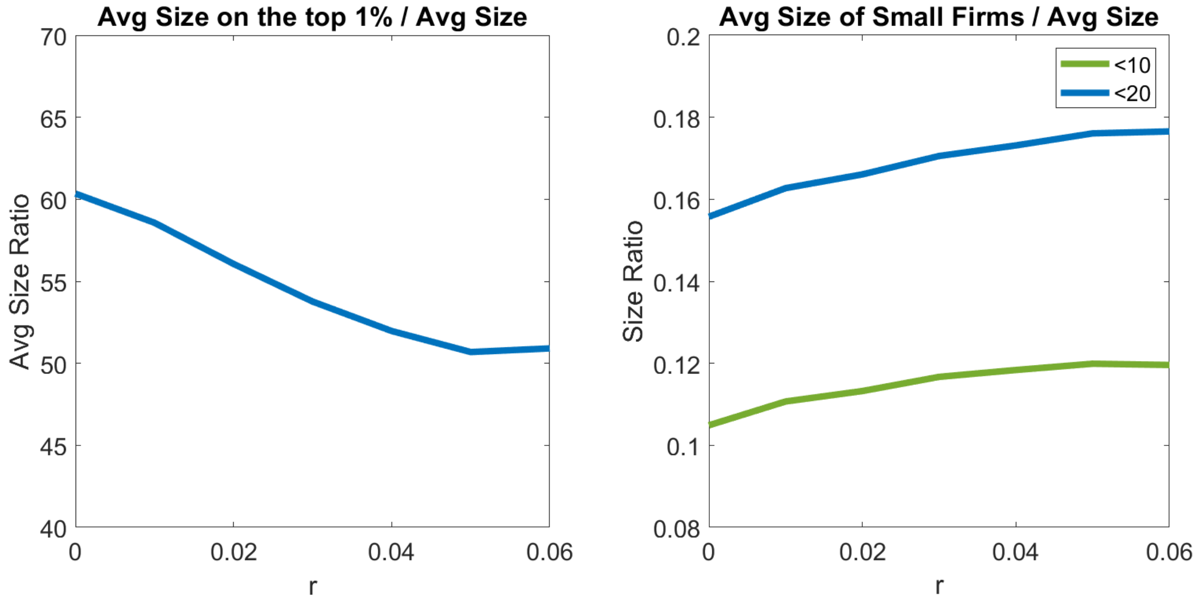
Note: This figure shows the model predicted employment shares and debt compositions at the stationary equilibrium for each level of r.The left panel shows the employment shares of firms with more than 500, 1000, and 2500 employees, respectively. The right panel shows the aggregate earnings-based to asset-based debt ratio.

model-predicted employment shares and firm shares of large firms. Since most firms that use market debt are the larger companies in the model, the employment shares of large firms increase. Consistent with the empirical patterns, the right panel of Figure 8 shows how the debt composition also shifts to earnings-based debt.

Regarding the changes in the employment size distribution, the left panel of Figure 9 shows that the average firm size at the top grows faster than the average size of all firms. In contrast, the average size of smaller firms, those with less than 10 or 20 employees, shrinks relative to the average size of all firms, as shown in the right-hand side of Figure 9.

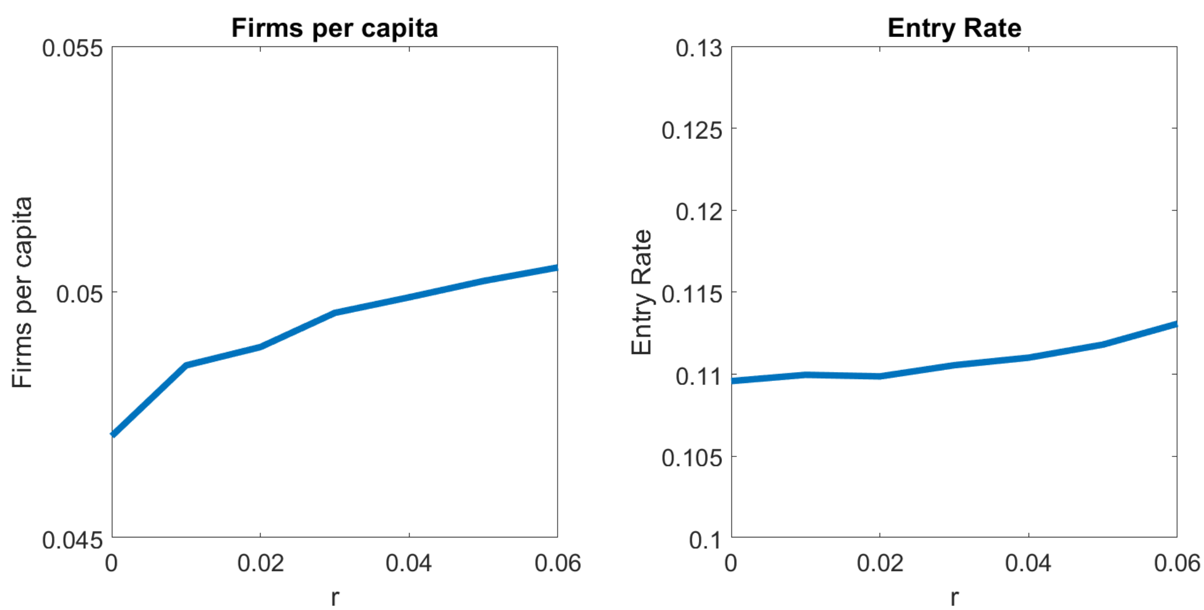
Figure 10 depicts changes in firm entry and firms per capita. For a lower interest rate r_t, market-financed firms, mainly large firms in the model, expand their employment demands. Meanwhile, some firms that initially borrow from banks switch to the credit market and expand their production. As a result, smaller firms, mostly still borrowing from banks, shrink or exit. In terms of entry margin, since most entry firms are small and rely on bank borrowing, entering the economy is no longer profitable. Therefore, a lower real interest rate is associated with lower firms per capita. Less profitable firms opt out of the market due to the surge in labor costs caused by the faster growth of market-financed firms. In the meantime, as shown in the upper right panel, the firm entry rate also de-

Figure 9: Employment Size Distribution



Note: This figure shows the model predicted changes in employment size distribution at the stationary equilibrium for each level of r. The left panel shows the ratio of the average employment size at 1% percentile to the average size of all firms. The right panel shows the ratio of average employment size with less than 10 or 20 employees to the average size of all firms.

Figure 10: The Decline in Firms per capita and the Firm Entry Rate



Note: This figure shows the model predicted levels of firms per capita and firm entry rates at the stationary equilibrium for each level of r. The left panel shows the levels of firms per capita, and the right panel depicts the firm entry rates.

Table 4: Effects of the Real Interest Rate on the Economy

	Emp Share ≥ 500		Emp Share ≥ 1000		Emp Share ≥ 2500	
	Data	Model	Data	Model	Data	Model
1982 - 1986	0.458	0.462	0.410	0.403	0.344	0.326
2013 - 2017	0.518	0.546	0.464	0.490	0.394	0.453
Changes	0.060	0.082	0.054	0.087	0.050	0.127

	Firms per capita		Entry Rate		Market/Bank Debt	
	Data	Model	Data	Model	Data	Model
1982 - 1986	0.051	0.051	0.116	0.113	1.30	1.26
2013 - 2017	0.042	0.047	0.081	0.109	4.10	3.36
Changes	-0.009	-0.004	-0.035	-0.004	2.70	2.10

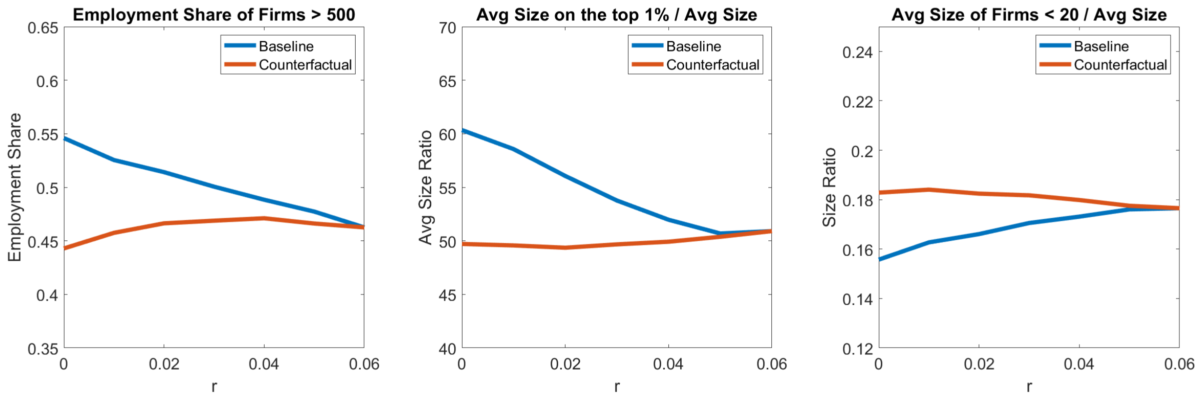
creases for a lower level of r_t.

For the stationary equilibrium of the firm dynamics model similar to Hopenhayn (1992), the entry rate is always proportional to the exit rate. In fact, when the number of entering firms increases, if the exit rate stays the same, the firm entry rate will also stay the same regardless of increases in new entering firms since the number of total firms also increases. Therefore, looking at the transition path of the firm entry rate is more helpful. In the appendix, Figure 14 shows the equilibrium results on the perfect-foresight transitional path. The results show similar effects of the decline in long-term real interest rates on the firm size distribution and firm dynamics. The results show that the decline in real interest rates also significantly discourages the firms' entry along the transition path.[13]

Finally, Table 4 summarizes the comparison of the secular changes in the U.S. and the model's prediction. The results show that the proposed channel alone can explain a significant fraction of secular trends. In particular, the decline in the real interest rate can explain all the changes in the shifts in employment shares and 80% of the changes in debt composition experienced in the U.S. over the past forty years. The model can also explain about 40% of the decline in firms per capita and more than 10 % of the decline in the firm entry rate.

[13]To solve the transitional path, I assume that the economy was in stationary equilibrium at the beginning of the 1980s. Then, the economy perfectly sees the path of the real interest rate in the U.S. from 1982 to 2019. Then, I assume that the real interest rate stays at 0 afterward, and the economy gradually reaches the new stationary equilibrium. The model is solved using the extended function path (EFP) framework (Maliar et al., 2020).

Figure 11: Changes in the Employment Size Distribution



Note: This figure compares the benchmark economy with a counterfactual economy without debt heterogeneity: all firms borrow from banks. The results are the outcomes at the stationary equilibrium for each level of r. The left panel compares the shifts in employment shares, and the middle and right panels compare the average employment sizes at the top and the bottom of the distribution.

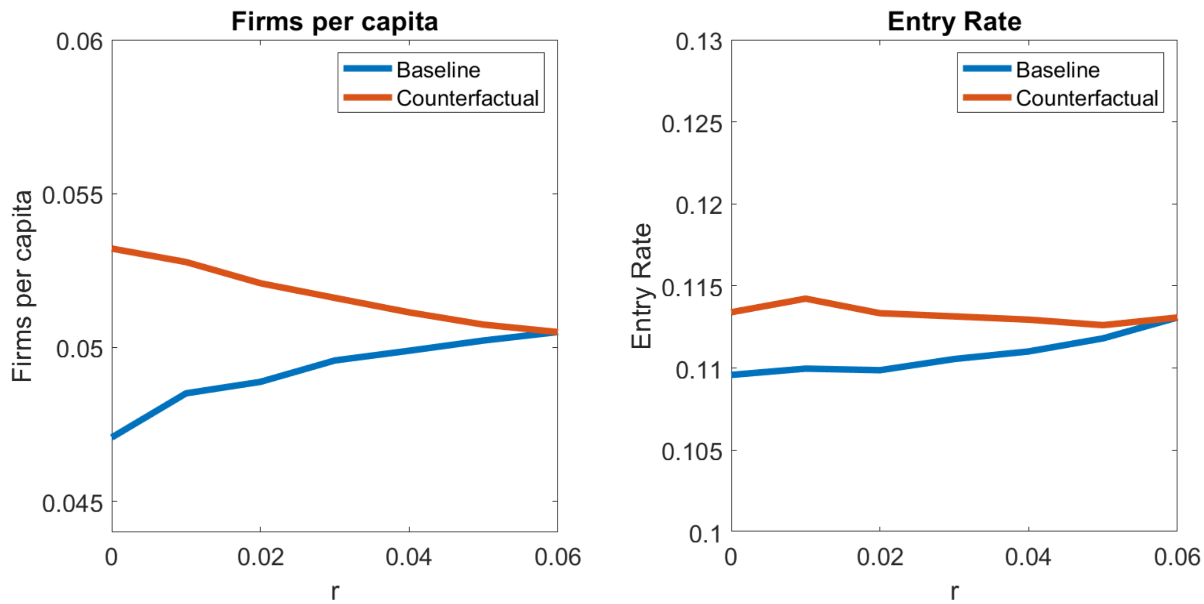
5.3 The Role of Debt Heterogeneity

In this section, I compare the quantitative results from the benchmark model with a counterfactual economy. In the counterfactual economy, I assume there is no debt heterogeneity. In other words, all firms will borrow from banks that issue one-period enforceability debt. Similarly, I calibrate the model to match the same U.S. statistics and conduct the same quantitative experiments as the benchmark model. The purpose of this exercise is to study the role of debt heterogeneity.

Figure 11 compares the changes in the firm size distribution of the benchmark model with the results of the counterfactual model. The left panel shows that without debt heterogeneity, the decline in real interest rates will not significantly shift employment shares, and the middle panel shows that firms at the top grow slightly slower than the average in the counterfactual economy. In contrast, the right panel shows that smaller firms are growing faster than the average without debt heterogeneity.

Figure 12 compares the changes in firm dynamics in the benchmark model with the outcomes of the counterfactual model. The left panel shows distinct patterns between the economy with and without debt heterogeneity. The decline in real interest rates will increase firms per capita without debt heterogeneity. Therefore, debt heterogeneity is crucial for the effects of real interest rates on firm dynamics. I also plot the comparison of the firm entry rate. Consistent with the results of firms per capita, debt heterogeneity is essential in generating the decline in the entry rates of the economy. As mentioned above, it is also helpful to see distinctions between the two in transitional paths. In the appendix,

Figure 12: The Decline in Firm Entry Rates and Firms per capita



Note: This figure compares the benchmark economy with a counterfactual economy without debt heterogeneity: all firms borrow from banks. The results are the outcomes at the stationary equilibrium for each level of r. The left panel compares firms per capita, and the right panel compares the firm entry rate.

Figure 15 compares the firm entry rates for two models on the transitional path. It is clear that debt heterogeneity is the key channel that results in the decline in firm entry rates.

Therefore, it is evident that debt heterogeneity plays a crucial role in connecting the decline in long-term real interest rates with the changes in the firm size distribution and firm dynamics.

6 Evaluating the Mechanism based on OECD Countries

The decline in real interest rates has been a global phenomenon affecting many advanced economies. This allows me to test the implications of my model in the cross-country data. This section utilizes the cross-country variations in real interest rates and credit market usage to investigate the effects of the real interest rate on employment shares and firm entry rates.

6.1 Empirical Strategy

To evaluate the prediction of the model, the empirical analysis compares the responses of large firm employment shares and firm entry rates to real interest rates of different OECD countries using the following regression:

$$y_{c,t} = \alpha_c + \gamma_t + \beta \cdot r_{c,t} + X_{c,t} + \epsilon_{c,t}$$

, where $y_{c,t}$ is the large firm employment shares or the firm entry rates for country c in year t. α_c and γ_t are country and year fixed effects, $X_{c,t}$ is a vector consisting of controls for each country at each year.

Furthermore, besides the average effect of the real interest rate, the following regression is analyzed to evaluate the impacts of credit market usage:

$$y_{c,t} = \alpha_c + \gamma_t + \beta_1 \cdot r_{c,t} + \beta_2 \cdot \text{Market}_{c,t} + \beta_3 \cdot r_{c,t} \cdot \text{Market}_{c,t} + X_{c,t} + \epsilon_{c,t}$$

m, where $\text{Market}_{c,t}$ represents the ratio of the amount of credit market debt to bank debt for the country c in year t.

The first implication of the model is that employment shares of large firms should be negatively associated with the real interest rate, and the firm entry rate should be positively associated with the real interest rate. Therefore, one should expect β in the first regression to be negative when $y_{c,t}$ is employment shares of large firms and positive when $y_{c,t}$ is the firm entry rate.

The second implication of the model is that countries with a more developed public credit market, measured as a greater fraction of the usage of credit market debt, should see stronger effects of the real interest rates. Therefore, one should expect β_3 in the second regression to be negative when $y_{c,t}$ is the employment share of large firms and positive when $y_{c,t}$ is the firm entry rate.

6.2 Data

The regressions are applied to an unbalanced panel of OECD countries from 1996 to 2019. Employment shares of large firms are measured by the percentage of employment of firms with 250 employees in the economy. The entry rate is measured by the fraction of new firms of the total number of firms. The real interest rate is constructed using the annual intermediate interest rate minus the inflation.

One key measure in the regression is credit market debt usage. I use the amount of "Securities other than shares" and "Loans" stated in the liabilities of Financial Balance Sheets.[14]

[14] According to the definitions of OECD statistics, "Securities other than shares" consist of bills, bonds, certificates of deposit, commercial paper, debentures, and similar instruments normally traded in the financial markets, where "Loans" are financial assets that are created when creditors lend funds directly to debtors, that are evidenced by non-negotiable documents, or for which the lender receives no security evidencing

Table 5: Regression Results: OECD Countries

	Dependent variable:			
	Emp Share $\geq$ 250	Entry Rate	Emp Share $\geq$ 250	Entry Rate
	(1)	(2)	(3)	(4)
r_t	-0.004**	0.040	-0.001	-0.053
	(0.002)	(0.072)	(0.002)	(0.084)
Market	0.012	3.293*	0.033	2.255
	(0.040)	(1.725)	(0.040)	(1.785)
$r_t \times$ Market			-0.019***	0.596**
			(0.007)	(0.281)
Year FE	Y	Y	Y	Y
Country FE	Y	Y	Y	Y
Controls	Y	Y	Y	Y
Observations	472	456	472	456
R^2	0.891	0.794	0.893	0.796

Notes: *p<0.1; **p<0.05; ***p<0.01. Controls include log level of GDP, working-age population, old population, debt, and government spending, as well as the level of unemployment rate, female share of employment, unit cost of labor, and GDP growth.

I further restrict the amounts to those only from non-financial corporations.

To construct the measurement, I calculate the ratio of the amount of "Securities other than shares" to "Loans" as the measure of the credit market debt usage. The theoretical counterpart for this measure can be considered to represent the level of parameters ϕ_e, $F(N)$, and ϕ_k in the model conditional on that other features of the financial environment, for example, interest rates, remain unchanged. The summary statistics and the members of OECD countries are listed in Table 6 in the appendix.

6.3 Results

Table 5 shows the results of the regression. The first two columns report the results based on the first regression, while the last two columns report the results based on the second regression. The results of column (1) show that the real interest rate is negatively and statistically significantly associated with large firm employment shares. An increase of 100 basis points is associated with an increase of 0.4% (of total employment) in the shares of large firms on average.

The results of column (2) show that the real interest rate is positively associated with

the transaction. Although the classification can be in some sense coarse, it is reasonable to believe that it is a relatively accurate measure of credit market debt usage.

the firm entry rate. An increase of 100 basis points is associated with a decrease in the number of new companies for approximately 0.04% (of the total firms) on average.

The results from columns (3) and (4) show that the effects of the real interest rates are stronger within countries where the share of market-based debt is higher. Such effects through the credit market are strong and statistically significant. For the same amount of decline in the real interest rates, countries with higher utilization of credit market debt are associated with a stronger increase in employment shares of large firms. In contrast, with the same amount of decline, countries with higher utilization of credit market debt are associated with a more substantial decrease in firm entry rates.

A back-of-envelope calculation shows that comparing countries at 90th with 10th percentile of the market-bank debt ratio, a 5 percent decline in real interest rates implies that big firm employment shares increase 3.9% (of total employment) more on average and Entry rates decline 1.1% (of total firms) more on average.

7 Conclusion

Can the decline in the real interest rate cause the decline in firm dynamics and the increasing dispersed firm size distribution? This paper shows that the decline can generate asymmetric effects through debt heterogeneity and lead to those secular changes. Specifically, in the model, the decline in real interest rates can explain all the changes in the shifts in employment shares and 80% of the changes in debt composition experienced in the U.S. for the past forty years. The model can also explain about 40% of the decline in firms per capita and more than 10 % of the decline in the firm entry rate. Debt heterogeneity plays an important role in generating the effects. Through asymmetrically benefiting large firms, debt heterogeneity turns the lower interest rate, conventionally considered a positive financial development, into a detrimental factor that negatively impacts small firms.

Besides its contribution to understanding the effects of long-term real interests on the firm size distribution and firm dynamics, my paper has broader implications for macroeconomics and leaves some open questions. First, theoretically, the microfoundation of debt heterogeneity opens the door for possible policy interventions. In particular, one implication of the model is that increasing the transparency of firm information and/or reducing the barrier and costs of restructuring bankruptcy can improve the strength of debt enforceability so that firms can borrow more efficiently. Therefore, it is interesting to look at the empirical effects of those policy interventions.

Secondly, it is worth studying both the empirical and theoretical implications of debt heterogeneity in an economy with multiple sectors. In the model, the economy features only one sector. However, in reality, different sectors may have different accesses to the credit market, which could have possible impacts on structural transformation and the secular reallocation of employment across sectors (Dent et al., 2016). For example, the decline in employment shares of manufacturing could be the result of the reliance on asset-based debt and difficulties in accessing the public market due to already large fixed costs to set up plants.

Finally, there are some possible extensions of the current framework, which are beyond the scope of the current paper. One possible extension is to look at the possible impacts on or consequences of equity financing, which is another important financing option of the credit market. Another extension is to apply a similar analysis of the debt limits to the long-term debt contract and to see the potential implications.

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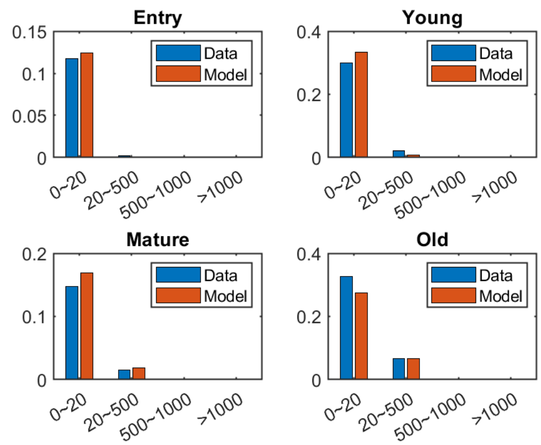
Appendix A. Additional Tables and Figures

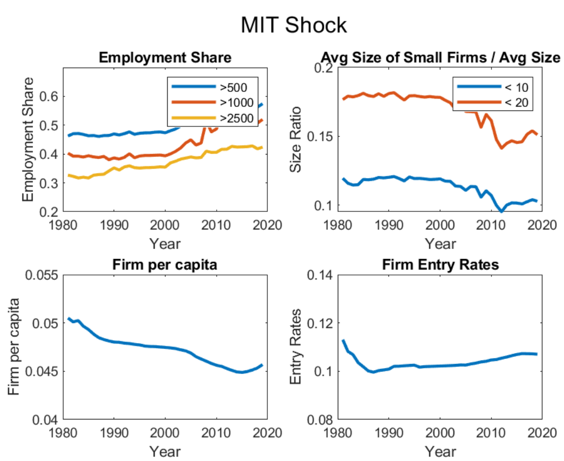
Figures

Figure 13: Non-targeted Moments



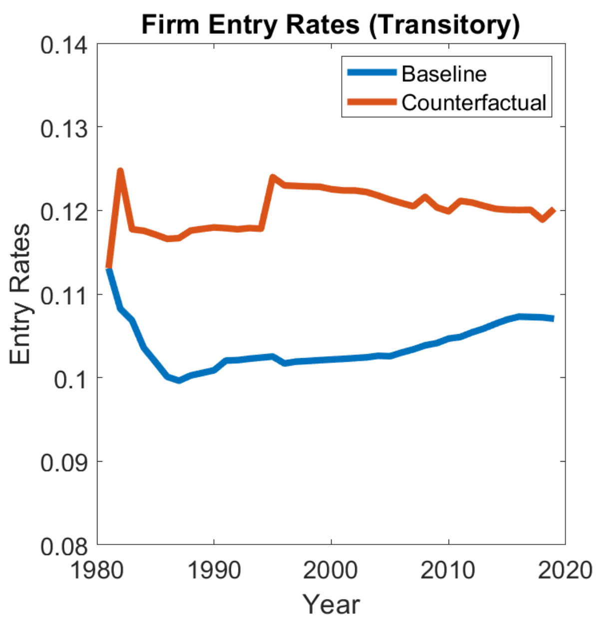
Note: This figure shows the non-targeted distributions of size × age bins. Entry is new firms. Young firms are those ages between 1 and 5. Mature firms are those ages between 6 and 10. Old firms are those with ages older than 10 years. All areas with the same color add up to one.

Figure 14: Transitional Dynamics



Note: This Figure shows the transitional equilibrium for the perfect foresight changes of real interest rates.

Figure 15: Transitional Dynamics



Note: This figure compares the benchmark economy with a counterfactual economy without debt heterogeneity: all firms borrow from banks. The results are firm entry rates at the transitional equilibrium.

Tables

Table 6: Summary Statistics: OECD Countries

	Avg. over time	Total Observations
Real interest rate	0.065	472
	(0.019)	
Market/bank debt	0.146	472
	(0.129)	
employment share >= 250	0.319	472
	(0.096)	
Entry rate	0.098	456
	(0.030)	
GDP	1180925	472
	(1624980)	
Unemployment rate	0.077	472
	(0.041)	
Working age population	19420	472
	(24841)	
Female employment rate	0.455	472
	(0.027)	
Account balance to GDP	0.019	472
	(0.043)	

The table shows the summary statistics for OECD countries. OECD countries include the 38 OECD members from Europe, which are Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Netherlands, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, and the United Kingdom, five countries from the Americas: Canada, Chile, Colombia, Mexico, Costa Rica, and the United States, and four Pacific members: Australia, Japan, Korea, and New Zealand. The two member countries from the Middle East are Israel and Turkey.

Appendix B. Proofs of Propositions

Proof of Proposition 1

Let $V^i(z, s_t, a_t)$ be the value function of the incumbent firms that adhere to their debt contracts, and $O(z, s_t, a_t)$ be the value function of the firms that default on their debt contracts. Let $\{d^*_{i,t}, a^*_{i,t+1}\}$ and $\{d^*_{o,t}, a^*_{o,t+1}\}$ be their optimal policy functions, respectively. Furthermore, without loss of generality, let $\{\pi^*_{i,t}, k^*_{i,t}, a^*_{i,t}\}$ and $\{\pi^*_{o,t}, k^*_{o,t}, a^*_{o,t}\}$ be the optimal earnings before interest expense (EBIT), borrowed capital, and holdings of illiquid assets for each period respectively. Note that firms choosing to exit implies future profits and holdings of illiquid assets equal to zero.

Therefore,

$$
\begin{aligned}
V^i(z, s_t, a_t) &= E_t \sum_{s=0}^{N-1} \sigma_f^s d^*_{i,t+s} + \sigma_f^N E_t V(z, s_{t+N}, a^*_{i,t+N}) \\
s.t. \quad & d^*_{i,t+s} + a^*_{i,t+s+1} = \pi^*_{i,t+s} - r^m k^*_{i,t+s} + a^*_{i,t+s}(1+r)
\end{aligned}
$$

Summing up the budget constraints with discounting factor for the first N periods yield

$$
E_t \sum_{s=0}^{N-1} \sigma_f^s d^*_{i,t+s} + E_t \sum_{s=0}^{N-1} \sigma_f^s a^*_{i,t+s+1} = E_t \sum_{s=0}^{N-1} \sigma_f^s (\pi^*_{i,t+s} - r^m k^*_{i,t+s}) + E_t \sum_{s=0}^{N-1} \sigma_f^s a^*_{i,t+s}(1+r)
$$

If $1/\sigma_f = 1 + r$, the equation above is equal to

$$
E_t \sum_{s=0}^{N-1} \sigma_f^s d^*_{i,t+s} = E_t \sum_{s=0}^{N-1} \sigma_f^s (\pi^*_{i,t+s} - r^m k^*_{i,t+s}) + a_{i,t}(1+r) - E_t \sigma_f^{N-1} a^*_{i,t+N}
$$

Therefore,

$$
V^i(z, s_t, a_t) = E_t \sum_{s=0}^{N-1} \sigma_f^s (\pi^*_{i,t+s} - r^m k^*_{i,t+s}) + a_{i,t}(1+r) - E_t \sigma_f^{N-1} a^*_{i,t+N} + \sigma_f^N E_t V(z, s_{t+N}, a^*_{i,t+N})
$$

When firms renege on the contracts, I assume they will not have access to the financial intermediaries but can save illiquid assets. Then,

$$
O(z, s_t, a_t) = E_t \sum_{s=0}^{N-1} \sigma_f^s \pi^*_{o,t+s} + \sigma_f^N k_{o,t} - E_t \sigma_f^{N-1} a^*_{o,t+N} + \sigma_f^N E_t V(z, s_{t+N}, a^*_{o,t+N})
$$

If $a^*_{i,t+N}$ and $a^*_{o,t+N}$ are either interior solutions or $a^*_{i,t+N} = a^*_{o,t+N} = 0$ for any possible state $\{z, s_{t+N-1}\}$, then

$$
-E_t \sigma_f^{N-1} a^*_{i,t+N} + \sigma_f^N E_t V(z, s_{t+N}, a^*_{i,t+N}) = -E_t \sigma_f^{N-1} a^*_{o,t+N} + \sigma_f^N E_t V(z, s_{t+N}, a^*_{o,t+N})
$$

due to linear utility.

Or if $N \to \infty$, both $-E_t\sigma_f^{N-1}a^*_{i,t+N}+\sigma_f^N E_t V(z, s_{t+N}, a^*_{i,t+N})$ and $-E_t\sigma_f^{N-1}a^*_{o,t+N}+\sigma_f^N E_t V(z, s_{t+N}, a^*_{o,t+N})$ are zero.

Then, $V^i(z, s_t, a_t) \geq O(z, s_t, a_t)$ IFF

$$E_t \sum_{s=0}^{N-1} \sigma_f^s(\pi^*_{i,t+s} - r^m k^*_{i,t+s}) + a_{i,t}(1 + r) \geq E_t \sum_{s=0}^{N-1} \sigma_f^s \pi^*_{o,t+s} + \sigma_f^N k_{o,t}$$

If assume $\pi^*_{i,t+s} > \pi^*_{o,t+s}$ for any $s \in [0, N]$, then the above equation can be rewritten as

$$\phi(z, s_t, a_t; N)\pi^*_{i,t} \geq r^m k^*_{i,t} + \frac{\phi_k \sigma_f^N k^*_{i,t} - \phi_a a_{i,t}}{I(z, s_t, a_t; , N)}$$

where $I(z, s_t, a_t; N) = E_t \sum_{s=0}^{N-1} \sigma_f^s k^*_{i,t+s}/k^*_{i,t}$ increases monotonically in N conditional on $\{z, s_t, a_t\}$. Moreover, $\phi(z, s_t, a_t; N) = \frac{E_t \sum_{s=0}^{N-1} \sigma_f^s(\pi^*_{i,t+s} - \pi^*_{o,t+s})}{E_t \sum_{s=0}^{N-1} k^*_{i,t+s}} \frac{k^*_{i,t}}{\pi^*_{i,t}}$. It is trivial that the term is bounded with the DRS Cobb-Douglas production function.

In general, let's assume that $1/\sigma_f \geq 1+r$. If $a^*_{i,t+N}$ and $a^*_{o,t+N}$ are either interior solutions or $a^*_{i,t+N} = a^*_{o,t+N} = 0$ for any possible state $\{z, s_{t+N-1}\}$, or if $N \to \infty$, then $V^i(z, s_t, a_t) \geq O(z, s_t, a_t)$ IFF

$$E_t \sum_{s=0}^{N-1} \sigma_f^s[\pi^*_{i,t+s} + (r + 1 - 1/\sigma_f)a^*_{i,t+s}] - E_t \sum_{s=0}^{N-1} \sigma_f^s r^m k^*_{i,t+s} + a_{i,t}/\sigma_f \geq E_t \sum_{s=0}^{N-1} \sigma_f^s[\pi^*_{o,t+s} + (r + 1 - 1/\sigma_f)a^*_{o,t+s}$$

If assume $\pi^*_{i,t+s} > \pi^*_{o,t+s}$ for any $s \in [0, N]$, then the above equation can be rewritten as

$$\phi(z, s_t, a_t; N)[\pi^*_{i,t} + (r + 1 - 1/\sigma_f)a^*_{i,t+s}] \geq r^m k^*_{i,t} + \frac{\phi_k \sigma_f^N k^*_{i,t} - \phi_a a_{i,t}}{I(z, s_t, a_t; , N)}$$

where $I(z, s_t, a_t; N) = E_t \sum_{s=0}^{N-1} \sigma_f^s k^*_{i,t+s}/k^*_{i,t}$ increases monotonically in N conditional on $\{z, s_t, a_t\}$. Moreover, $\phi(z, s_t, a_t; N) = \frac{E_t \sum_{s=0}^{N-1} \sigma_f^s[\pi^*_{i,t+s} - \pi^*_{o,t+s} + (r-1/\sigma_f-1)(a^*_{i,t+s} - a^*_{i,t+s})]}{E_t \sum_{s=0}^{N-1} k^*_{i,t+s}} \frac{k^*_{i,t}}{\pi^*_{i,t}}$. It is trivial that the term is bounded with the DRS Cobb-Douglas production function.

Furthermore, I can make additional assumptions on the profits of defaulting process such that $\phi(z, s_t, a_t; N) = \phi_e$ is not state-dependent. Moreover, by assuming ϕ_k and ϕ_a are state-dependent such that $\phi_k(z, s_t, a_t; N)\sigma_f^N/I(z, s_t, a_t; N) = \phi_a(z, s_t, a_t; N)/I(z, s_t, a_t; N) = (1 - \phi_k)/F(N)$, the above equation will become state-independent borrowing limits

$$\phi_e[\pi^*_{i,t} + (r + 1 - 1/\sigma_f)a^*_{i,t+s}] \geq r^m k^*_{i,t} + \frac{(1 - \phi_k)(k^*_{i,t} - a_{i,t})}{F(N)}$$

Proof of Corollary 1

At the steady-state equilibrium, if $N \to \infty$, $V^i(z, s_t, a_t) \geq O(z, s_t, a_t)$ IFF

$$\sum_{s=0}^{\infty} \sigma_f^s [\pi_i^* + (1 + r - 1/\sigma_f) a_i^*] - \sum_{s=0}^{\infty} \sigma_f^s r^m k_i^* + a_i/\sigma_f \geq \sum_{s=0}^{\infty} \sigma_f^s [\pi_o^* + (1 + r - 1/\sigma_f) a_{o,t}^*]$$

, which is equivalent to

$$\frac{1}{1-\sigma_f}[\pi_i^* + (r - 1/\sigma_f - 1) a_i^*] - \frac{1}{1-\sigma_f} r^m k_i^* + a_i/\sigma_f \geq \frac{1}{1-\sigma_f}\pi_o^* + \sum_{s=0}^{\infty} \sigma_f^s (r - 1/\sigma_f - 1) a_{o,t+s}^*$$

Also note that $1 + r \leq 1/\sigma_f$, $a_{o,t}^* = 0$ for any s. Let $(1-\phi_e)\pi_o^* = \pi_i^*$Therefore,

$$\phi_e \pi_i^* \geq r^m k_i^* - r a_i^*$$

Proof of Proposition 2

The first order conditions of the market-financed firm problem give:

$$\begin{aligned}
&[l_t] \quad (1 + \mu_t \phi) \cdot (\beta k_t^{\alpha} l_t^{\beta - 1} - w_t) = 0 \\
&[a_{t+1}] \quad \delta[1 + r_{t+1} + r_{t+1}\mu_{t+1} + \eta_{t+1}] = 1 \\
&[k_t] \quad (1 + \phi\mu_t) \cdot (\alpha k_t^{\alpha-1} l_t^{\beta}) = r_t \mu_t + r_t
\end{aligned}$$

, where μ_t is the Lagrange multiplier of the borrowing constraint, and η_t is the Lagrange multiplier of the non-negative equity holding constraint. If I assume that the economy is at the steady-state equilibrium with interest rate r and wage w, the borrowing constraint is binding, and firms have positive equity holding, then the labor demand for the market-financed firm l_m is

$$l_m = \beta^{\frac{1}{1-\alpha-\beta}} w^{-\frac{1-\alpha}{1-\alpha-\beta}} \left(\frac{\alpha}{\beta} \frac{(1 + \phi\mu_m)}{r_m \mu_m + r_m}\right)^{\frac{\alpha}{1-\alpha-\beta}} \tag{1}$$

Similarly, the first order conditions of the bank-financed firm problem give:

$$\begin{aligned}
&[l_t] \quad \beta A_t k_t^{\alpha} l_t^{\beta-1} - w_t = 0 \\
&[a_{t+1}] \quad \delta[1 + r_{t+1} + (1 + \lambda)\mu_{t+1} + \eta_{t+1}] = 1 \\
&[k_t] \quad \alpha A_t k_t^{\alpha-1} l_t^{\beta} - r_t = \mu_t
\end{aligned}$$

Assume that the economy is at the steady-state equilibrium with interest rate r and wage w, the borrowing constraint is binding, and firms have positive equity holding, then the labor demand for the bank-financed firm l_b is

$$l_b = \beta^{\frac{1}{1-\alpha-\beta}} w^{-\frac{1-\alpha}{1-\alpha-\beta}} \left\{\frac{\alpha}{\beta} \frac{1}{r_b + \mu_b}\right\}^{\frac{\alpha}{1-\alpha-\beta}} \tag{2}$$

Then, the steady-state equilibrium labor ratio of the two is

$$\frac{l_m}{l_b} = (\frac{(r_b + \mu_b)(1 + \phi\mu_m)}{r_m + r_m\mu_m})^{\frac{\alpha}{1-\alpha-\beta}} \tag{3}$$

If ϕ and λ go to infinity, then it is trivial that $\mu_b = \mu_m = 0$. Therefore, the labor ratio

$$\frac{l_m}{l_b} = (\frac{r_b}{r_m})^{\frac{\alpha}{1-\alpha-\beta}} \tag{4}$$

$$= (\frac{r + \rho_b}{r + \rho_m})^{\frac{\alpha}{1-\alpha-\beta}} \tag{5}$$

Given that $\rho_b > \rho_m$, $\frac{l_m}{l_b}$ increases monotonically as r decreases.

Proof of Proposition 3

First, let's assume that the economy is at the steady-state equilibrium with interest rate r and wage w,the borrowing constraint is binding and firms have a positive equity holding If $\rho_b = \rho_m = 0$, then the equation 3 can be simplified to

$$\frac{l_m}{l_b} = (\frac{(r + \mu_b)(1 + \phi\mu_m)}{r + r\mu_m})^{\frac{\alpha}{1-\alpha-\beta}}$$

Since $\lambda\mu_b = \frac{1}{\sigma} - 1 - r = r\mu_m$,

$$\frac{l_m}{l_b} = (\frac{(1 + \frac{\mu_m}{1+\lambda})(1 + \phi\mu_m)}{1 + \mu_m})^{\frac{\alpha}{1-\alpha-\beta}}$$

Taking the derivative respect to μ yields

$$\frac{d\text{ratio}}{d\mu} = \frac{\phi\mu_m^2 + 2\phi\mu_m + (\phi - 1)(1 + \lambda) + 1}{(1 + \lambda)(\mu_m + 1)^2}$$

Since λ is positive, the denominator is always positive. Since ϕ is also positive, the numerator is a quadratic function opening upwards with the vertex at $\mu < 0$.

Therefore, if $\phi >= 1$ or $1 + \lambda < \frac{1}{1-\phi}$, the derivative will be positive for any $\mu_m > 0$. By assumption, $\phi < \frac{\alpha}{1-\beta} < 1$. Note that as r decreases from an arbitrary positive value to zero, μ_m is always positive and increases monotonically to infinity. Therefore, when $1+\lambda < \frac{1}{1-\phi}$, the labor ratio increases monotonically as r decreases until zero. When $1 + \lambda > \frac{1}{1-\phi}$, the relationship between the labor ratio and r will not be monotonic. Also, since $\mu_m = 0$ if $r = 1/\sigma - 1$, the turning point will fall within $(0, \frac{1}{\delta} - 1)$. Therefore, there $\exists r^* \in (0, \frac{1}{\delta} - 1)$ such that when $r > r^*$, the labor ratio decreases as r decreases, and when $r < r^*$, the labor ratio increases as r decreases.

Secondly, let's assume that the economy is at the steady-state equilibrium with interest rate r and wage w, the borrowing constraint is binding, and market-financed firms have zero equity holding. Then, the borrowing constraint for market-financed firms gives

$$\begin{aligned} rk &= \phi(k^{\alpha}l^{\beta} - wl) \\ &= \phi(\frac{1}{\beta} - 1)wl \end{aligned}$$

Therefore, a little algebra can show

$$\frac{l_m}{l_b} = (\frac{\beta\phi(\frac{1}{\beta} - 1)}{\alpha}\frac{r + \mu_b}{r})^{\frac{\alpha}{1-\alpha-\beta}}$$

Since $\mu_b = (\frac{1}{\sigma} - 1 - r)/\lambda$,

$$\frac{l_m}{l_b} = (\frac{\beta\phi(\frac{1}{\beta} - 1)}{\alpha}\frac{\lambda + \frac{\frac{1}{\sigma}-1}{r}}{1 + \lambda})^{\frac{\alpha}{1-\alpha-\beta}}$$

, and it is trivial that the labor ratio increases monotonically as r decreases.

Conditions for borrowing constraints and positive equity holdings

For the bank-financed firm problem, it is trivial to show that if $1/\sigma - 1 - r_b > 0$, then the borrowing constraint is binding and equity holding is positive.

However, it is less clear for the market-financed firm problem. Here will give a sufficient condition. Solve the equation without assuming that $\eta = 0$ and $\mu_m > 0$, and the borrowing constraint will give

$$k(1 - \frac{1-\beta}{\alpha}\frac{\phi + \phi\mu_m}{1 + \phi\mu_m}) \leq a$$

Assume that $\phi < \frac{\alpha}{1-\beta}$, and if $\mu_m = 0$, then $\eta > 0$ so that $a = 0$. Therefore,

$$k(1 - \frac{1-\beta}{\alpha}\phi) < 0$$

Therefore, $k < 0$, which is not feasible. In other words, $\mu_m > 0$. As a result,

$$k(1 - \frac{1-\beta}{\alpha}\frac{\phi + \phi\mu_m}{1 + \phi\mu_m}) = a$$

Note here as r gets close to zero, if $\eta = 0$, then $\mu_m \to \infty$, and there is no positive solution to k; therefore, when r is smaller enough, $\eta > 0$ and firms will have zero equity holdings.

Conditions of profit reduction of bank-financed firms

I will discuss the conditions that bank-financed firms will reduce the profits when all firms are unconstrained. First, It is useful to note that the profits from operating $\pi = \frac{1-\alpha-\beta}{\beta}wl$. Therefore, I need to only focus on how wl changes according to r.

Then, let's assume market-financed firms and bank-financed firms have productivity levels A_m and A_b, respectively. Combine equation (1), (2) and labor market clearing condition, one can easily have

$$\bar{L} = \nu_m A_m^{\frac{1}{1-\alpha-\beta}} l_m + \nu_b A_b^{\frac{1}{1-\alpha-\beta}} l_b$$

, where l_m and l_b are the equilibrium labor demand for both types of firms when productivity equals $\bar{L}$.

Moreover, one can have

$$wl_b = \frac{w}{\nu_m \frac{A_m}{A_b}^{\frac{1}{1-\alpha-\beta}} \frac{l_m}{l_b} + \nu_b}$$

The equilibrium wage w satisfies the following equation:

$$w = C \frac{1}{r_b}^{\frac{\alpha}{1-\alpha}} (\nu_m \frac{A_m}{A_b}^{\frac{1}{1-\alpha-\beta}} \frac{l_m}{l_b} + \nu_b)^{\frac{1-\alpha-\beta}{1-\alpha}}$$

, where C is a constant. Finally, the labor share is

$$wl_b = \frac{C \frac{1}{r_b}^{\frac{\alpha}{1-\alpha}}}{(\nu_m \frac{A_m}{A_b}^{\frac{1}{1-\alpha-\beta}} \frac{l_m}{l_b} + \nu_b)^{\frac{\beta}{1-\alpha}}}$$

Let $\nu'_m = \nu_m \frac{A_m}{A_b}$, then plugged into the labor ratio, and the derivative of $ln(wl_b)$ respect to r is the following:

$$\frac{d\ln(wl_b)}{dr} = \frac{\alpha}{(1-\alpha)(r+\rho_b)} \{ \frac{\beta \nu'_m (\frac{r+\rho_b}{r+\rho_m})^{\frac{\alpha}{1-\alpha-\beta}} (\frac{\rho_b - \rho_m}{r+\rho_m})}{(\nu'_m \frac{r+\rho_b}{r+\rho_m}^{\frac{\alpha}{1-\alpha-\beta}} + \nu_b)(1-\alpha-\beta)} - 1 \}$$

For $\frac{d\ln(wl_b)}{dr} > 0$, the following inequality needs to hold:

$$\nu'_m (\frac{r+\rho_b}{r+\rho_m})^{\frac{\alpha}{1-\alpha-\beta}} (\frac{\beta}{1-\alpha-\beta} (\frac{\rho_b - \rho_m}{r+\rho_m} - 1) > \nu_b$$

Two conditions need to be held here for the derivative to be positive: $\frac{\beta}{1-\alpha-\beta} \frac{\rho_b - \rho_m}{r+\rho_m} > 1$, and ν'_m compared to ν_b need to be large enough.

To make the derivative greater than zero, one need first $\frac{\beta}{1-\alpha-\beta} \frac{\rho_b - \rho_m}{r+\rho_m} > 1$ for a range of r. $\frac{\beta}{1-\alpha-\beta}$ indicates that the output elasticity of labor compared to the profit share should be high enough so that firm employment will respond sufficiently. Note that $\rho_b > \rho_m$, and $\frac{\rho_b - \rho_m}{r+\rho_m}$ represents the effects of the real interest rate. Therefore, this condition states that the firms need to respond to the decline in real interest rate sufficiently.

Conditional on the above condition, if the market-financed firms have sufficient productivity advantages, then the derivative will be positive.

Appendix C. Calibration of Counterfactual Economy

In the counterfactual economy, I assume all firms borrow from the banks. I keep externally calibrated parameters the same as the benchmark model. Also, I do not need to calibrate ρ_e , f_m, and $F(N)$ as there is no credit market lenders. Since there is no earnings-based debt, I set ϕ_k to be similar to the benchmark model. In terms of targeted moments, I follow the benchmark economy as close as possible. I target all the moments, except those related to the credit market.

Table 7 and Table 8 show the externally and internally calibrated parameters. Table 9 and Figure 16 show the targeted and non-targeted moments.

Table 7: Externally Calibrated Parameters: Counterfactual

Parameters	Name	Value	Source
α	capital share	0.3	Poschke (2018)
β	employment share	0.6	Poschke (2018)
σ_f	discounting rate	0.94	Interest Rate at 1980s
δ	depreciation	0.10	Poschke (2018)
ρ_s	interest spreads	0.040	Compustat
η	population growth	0.025	Karahan et al. (2019)
$\bar{L}$	avg employment	19.8	BDS

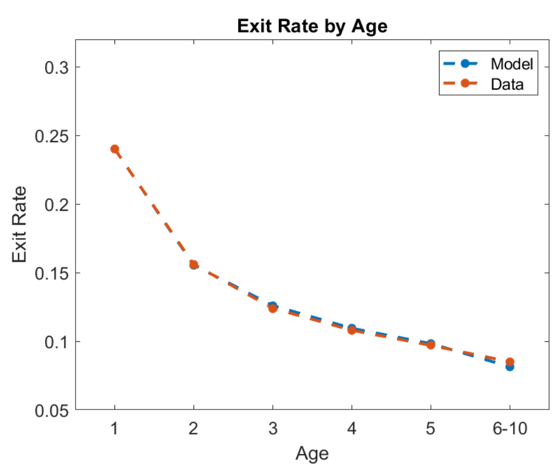
Table 8: Internally Calibrated Parameters: Counterfactual

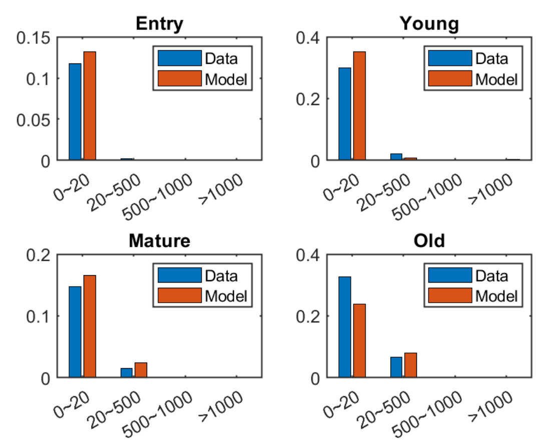
Parameters	Name	Value
μ_{s0}	mean of s_0	0.320
σ_{s0}	std of log s_0	0.200
μ_{a0}	mean of a_0	0.407
σ_{a0}	std of log a_0	0.650
σ_{z0}	std of log z_0	0.100
σ_ϵ	std of shocks	0.049
ρ	persistence of shocks	0.988
f_b/y	fixed costs / output	0.008
c_e	entry costs	0.001
ϕ_k	leverage ratio of assets	0.36728
ϕ_e	leverage ratio of earnings	0.700

Table 9: Targeted Moments: Counterfactual

	Model	Data	Source
Firm share ≥ 500	0.0042	0.0034	BDS
Employment share ≥ 500	0.463	0.460	BDS
Employment share ≥ 1000	0.378	0.410	BDS
Employment share ≥ 2500	0.300	0.340	BDS
Exit rate (Entry Rate)	0.099 (0.118)	0.097 (0.116)	BDS
S.T.D. of emp growth	0.267	0.420	Gavazza et al. (2018)
Debt to output ratio	0.283	0.350	Flow of Funds

Figure 16: Other Targeted and Non-targeted Moments: Counterfactual





Note: This Figure shows the targeted firm exit rates and non-targeted distribution of size $\times$ age bins. Firm exit rates are calculated as the number of total exiting firms for each age divided by total incumbents at the same age. For the right panel, entry is new firms. Young firms are those ages between 1 and 5. Mature firms are those ages between 6 and 10. Old firms are those with ages older than 10 years. All areas with the same color add up to one.