

Debt Callability and Firm Dynamics ^{*}

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Abstract

Callable debt - a bond that allows the issuer to "redeem" or "call" the bond before its maturity - has reached a high proportion of corporate finance structures in the U.S. debt market. This paper examines the dynamic interaction between corporate bond callability and firm decisions. Using a comprehensive dataset spanning firm- and bond-level data, this study reveals that firms with higher credit risk tend to issue more callable bonds. To capture these dynamics, I develop a heterogeneous firm model with investment decisions, non-callable and callable debt, call options, and default risk. The model shows that smaller firms and those with higher credit risk are more inclined to issue callable bonds but tend to call them less frequently. However, when they do call, these firms show a relatively higher level of investment driven by favorable investment opportunities. Lastly, the share of callable bonds amplifies the firm's optimal behavior - investment and refinancing - particularly when calling in response to declining interest rates. This highlights callable debt's dual and asymmetric role in risk management and investment decisions.

Keywords: Debt Callability, Financial Frictions, Firm Investment, Firm Heterogeneity

JEL Codes: E22, E44, G31, G32, O16, E44

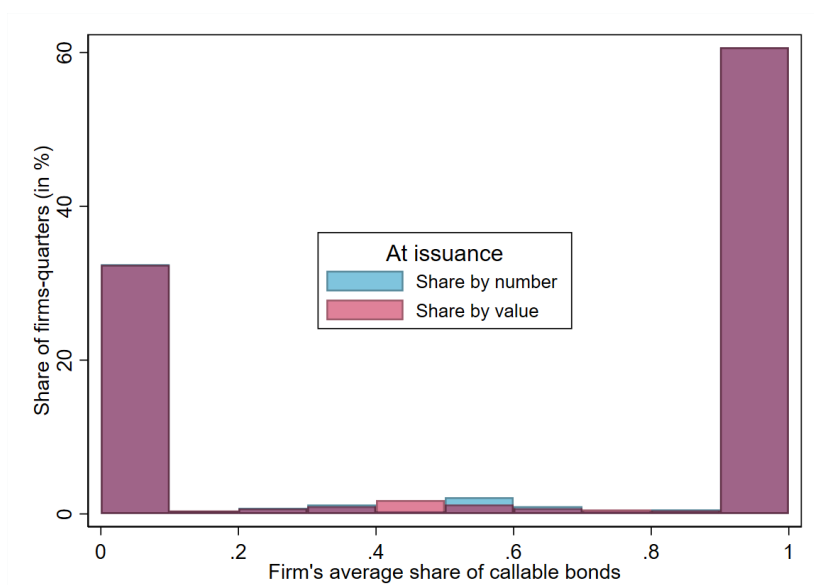
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1 Introduction

Firms' balance sheets are crucial in explaining business cycles and investment dynamics.¹ Additionally, firms' capital structure is of considerable importance to studying firm dynamics and investment and has long been the subject of significant research attention in the literature.² While more and more papers focus on the composition of firms' debts, particularly on the dynamic structure of their debt maturity, most ignore an essential characteristic of these debts: the callable nature of the debt, i.e., a callable debt that the issuing firm can repurchase before its maturity.³ As shown in Figure 1, the callable share of firms' debt is neither identical across firms nor negligible.

Figure 1: DISTRIBUTION OF CALLABLE SHARE IN U.S.



Notes: This figure overlays two distributions of the firms' average share of callable bonds. The first (in blue) concerns the number of callable bonds, and the second (in red) shows the offering amount. The sample period is 1990Jan - 2018May.

Source: Mergent FISD & CRSP-Compustat.

In this paper, I examine how the callable structure of a firm's debt interacts with its optimal choices by posing the following questions: Under what conditions do firms choose to issue callable debt? Does the option to redeem the debt before maturity impact the firm's optimal borrowing and

¹An extensive macroeconomic literature starting from [Bernanke and Gertler \(1989\)](#); [Kiyotaki and Moore \(1997\)](#) has shown the importance of firms' balance sheets for understanding macroeconomic business cycles. [Gomes \(2001\)](#); [Cooley and Quadrini \(2001\)](#) has found that financial frictions firms face account for their investment behavior and dynamics.

²[Leland and Toft \(1996\)](#); [Covas and Haan \(2011\)](#); [Jermann and Quadrini \(2012\)](#) are among seminal papers that stress the importance of the dynamic choice of firms' capital structure.

³The literature in macroeconomics and finance research about firms' debt maturity includes: [Demirgüç-Kunt and Maksimovic \(1999\)](#); [Gomes et al. \(2016\)](#); [Gürkaynak et al. \(2022\)](#); [Fabiani et al. \(2022\)](#).

investment decision? What factors influence a firm's decision to redeem debt before maturity? I answer the above questions both empirically and theoretically. First, I document facts about callability: small firms tend to issue relatively more callable bonds but are less likely to call them on average. Second, to explain these facts, my paper incorporates a choice of debt callability into a dynamic firm financing and investment model.

First, to address the research questions empirically, I employ balance sheet data from publicly listed firms in the United States from January 1990 to May 2018, combined with bond-level data.⁴ My empirical analysis reveals the following facts. (i) On average, 66 % of the value of corporate bonds offered on the U.S. bond market are callable; the prevalence of callability in bond issuance has increased from 49.54 % before the Great Financial Crisis to 95.5 % post-recession. (ii) Sorting issuers based on their credit rating positions into five size categories, I document that issuers in the bottom quintile of credit rating distribution issue relatively more callable bonds. These firms tend to have smaller assets than firms that issue less than half of their bonds as callable. (iii) across firms, 55.44 % of these callable bonds are effectively called, indicating that the exercise of the call option significantly impacts the role played by maturity in the standard model of debt maturity. (iv) Interest rate movements, changes in credit risk, financial performance, firm size, and liquidity ratios influence the decision of a firm to call a bond.

In a theoretical framework, I first use a three-period model with an endogenous choice of investment, default, callable share, and call decision (in the intermediate period) to derive analytical solutions and illustrate intuitions. As in data, the model predictions say that less productive firms with higher credit risk issue more callable debt. These are partially in line with findings in the literature (e.g. [Gilchrist and Zakrajšek \(2012\)](#); [Clymo and Rozsypal \(2023\)](#); [Poeschl \(2023\)](#); [Kochen \(2023\)](#)).⁵ In a quantitative model of firm dynamics, I examine optimal decisions and implications of debt callability for firms and in aggregate.

I build the quantitative model with several innovative features. First, I allow firms to issue non-callable and callable debt simultaneously, whereas the existing literature generally treats debt as uniquely non-callable. Second, debt callability is treated as an endogenous choice, thus the types of debt are not issued in fixed proportions. Third, the exercise of the call option is also an endogenous decision. This means the probability of calling callable bonds each period is not exogenously fixed. By doing so, my model allows this probability to dynamically influence the

⁴During economic hard times, loan contracts may be more easily renegotiated or restructured for firms than bonds. As this analysis focuses exclusively on bond financing, the callability feature parallels the rollover risk concept, which falls beyond the scope of this study and is well documented by [Jungherr and Schott \(2021\)](#).

⁵Other papers show that small firms rely less on debt than equity, but during economic booms, these firms increase both debt and equity financing, reflecting their need for external funds to fuel growth opportunities, see e.g., [Begenau and Salomao \(2019\)](#).

pricing of both callable and non-callable bonds.⁶ With this feature, I argue realistically that the share of bonds that can be called at every period changes because of firm-level and aggregate shocks, while the choice of debt's callable structure reflects the trade-off between default risk and refinancing flexibility (e.g. [Guntay et al. \(2004\)](#)).

An additional contribution lies in the distinct approach to debt pricing in my model. Specifically, the continuation price of new debt considers the current interest rate, directly affecting future coupon payments. Conversely, the continuation price of outstanding debt is anchored to the fixed coupon rate set at issuance. This distinction accurately reflects the contractual obligations for each debt class, capturing creditors' expectations under different interest rate environments. By distinguishing these pricing characteristics, my model provides a more realistic depiction of firm debt dynamics, accommodating creditor heterogeneity that arises from separate pricing rules for new and existing debt.⁷ Lastly, instead of treating net worth as a firm-specific variable alongside productivity, I distinguish both types of bonds separately in the structure of firms' capital as they have neither the same law of motion nor the same pricing characteristics.⁸ While non-callable bonds continue their path until maturity, callable bonds face the potential of being called within the allowed period. Then, I incorporate periodic stochastic coupons into firms' state variables to account for changing interest rates carried by new debt issued. These elements capture the complexity and dynamics of firms' capital structure decisions, but the downside of these innovations is the increase in computational complexity (e.g., [Berndt \(2004\)](#); [Jarrow et al. \(2010\)](#)).

Over the years, advancements in the financial industry have led to a proliferation of financial instruments designed to facilitate financing for firms and households. Typically, funding external with bonds has grown significantly in the last decade ([Jungherr and Schott \(2021\)](#); [Darmouni et al. \(2022\)](#)). As such, it is of particular interest to examine the potential impacts and implications of this growth on our understanding of the transmission of policies, especially monetary policy. One notable feature of bonds is the call option, which allows for the issuer's early redemption of a bond. To enrich our knowledge of the implications of firms' debt heterogeneity, we need to study key characteristics of firms' debt. Then, my paper shows why it may be essential to consider debt callability, which significantly impacts the behavior of firms and their optimal decisions. Callable bonds differ from short-term bonds because they typically have a more extended maturity

⁶The firm's call policy has been previously modeled from the market perspective through a stochastic intensity process (see [Guntay \(2002\)](#); [Jarrow et al. \(2010\)](#)), where the call intensity does not result from an endogenous firm's decision. In [Leland and Toft \(1996\)](#); [Goldstein et al. \(2001\)](#); [Chen et al. \(2021\)](#), firms can restructure their capital by calling entirely the existing debt.

⁷This approach contrasts with standard models that assume a unified continuation price, which may overlook contractual rigidity in outstanding debt.

⁸In the model of callable bond valuation from seminal paper [Duffie and Singleton \(1999\)](#), the firm's balance sheet is not considered in the decision to call. See [Jarrow et al. \(2010\)](#) for an extension of their approach.

period. Additionally, callable bonds offer a higher yield to compensate for the added risk of early redemption by the issuer. In contrast, short-term bonds typically provide lower yields due to their shorter maturity and lower risk. On the other hand, callable bonds differ from long-term bonds in that the issuer can redeem them before maturity, whereas long-term bonds are held until their maturity date. This early redemption feature of callable bonds adds an element of uncertainty, making them distinct from traditional long-term bonds. Callable bonds are a middle ground between short-term and long-term bonds, offering a particular combination of yield and risk that sets them apart from both.

The bonds' callability, as defined, presents different trade-offs. Callable bonds represent, for the investors, a source of prepayment risk. Still, this risk is compensated by considering the call option's expected value in the callable bond's pricing. On the firms' side, the relatively lower price of callable bonds may attract investors and improve these bond issuers' access to external capital. These options on the bonds give a (future) opportunity for deleveraging and refinancing for issuers when the interest rates drop. In other words, the callability of bonds allows a potential reduction of the cost of debt for the firm when it decides to call. By issuing callable bonds, firms tend to replace the rollover risk with refinancing risk.⁹ The uncertainty about the future interest rate values influences the bonds' prices. It explains mainly the difference between callable and non-callable bonds since a firm that uses exclusively non-callable bonds cannot adjust its debt. The callable bonds are subject to be more responsive to this uncertainty in the term structure and volatility of interest rates than the non-callable ones (Duffee (1998)).

Depending on the callability of their debts, firms' behavior may not be the same. Firms with call provisions in their debts have more incentives to invest because this option provides a hedging tool in case of hiking interest costs of debt. These types of firms have more present value in their dividend payments. In other words, when having more callable bonds than non-callable in their debt portfolio, firms will tend to get the opportunity to call back their bonds during episodes of decreasing interest rates (or for other reasons like a decrease of their credit risk, e.g., Becker et al. (2024)). Still, in the opposite case (when interest rates increase), the opposite behavior will not be observed, e.g., they will just let the callable bonds and even issue non-callable bonds to insure themselves against a long period of high rates. So basically, the number (the value) of debts that remain callable in the debt portfolio of firms shall modify their new issuance of debt, change the leverage structure, the risk-taking, and then the investment in reaction to and depending on the nature of changes in aggregates shocks and firm-level changes. In this paper, I show that firms with low credit ratings seek more for callable bonds but tend to call their bonds less because of refinancing costs due to potential rollover risk.

⁹Guntay et al. (2004) found that the issuance of callable bonds addresses firms' interest-rate risk management needs.

Related Literature. My paper is anchored into the broad literature that looks for the causes and consequences of firm heterogeneity and the various implications of financial frictions for the macroeconomy. As revealed in [Brunnermeier and Krishnamurthy \(2020\)](#), corporate debt and its aggregate implications for the macroeconomy started receiving great attention in the theoretical and empirical literature. Here, I present and discuss how my work has any connection with several works in this literature, mostly works that are in the strand of the literature around the heterogeneity in corporate debt (structures) and its macroeconomic implications. I subdivide the following literature into two parts.

First, an exciting part of my related literature considers macroeconomic models in which corporate debt choices are incorporated, and then aggregate shocks are studied. This part starts from DSGE models with the famous works of [Kiyotaki and Moore \(1997\)](#) and [Bernanke et al. \(1999\)](#). More recent works in the literature, especially those on monetary policy transmission and financial crisis, use heterogeneous firms' models with financial frictions. [De Fiore and Uhlig \(2015\)](#) use a DSGE model to explain the importance of firms' ability to substitute between bank loans and bond debts and the banks' flexibility in mitigating adverse real effects of the financial crisis. In their model, firms choose either uniquely bonds or uniquely loans to be financed. Beyond this consideration, the paper of [Crouzet \(2018\)](#) considers firms with access to public debt markets that chose a mix of their external funds. Among others, prominent examples that develop interesting theoretical approaches work on the interactions between the composition of corporate debt and the firm's investment decisions. These examples include [Martellini et al. \(2018\)](#), [Crouzet \(2018\)](#), [Salomao and Varela \(2022\)](#), [Arellano et al. \(2019\)](#), [Jeenas \(2024\)](#), [Arellano et al. \(2020\)](#), [Jungherr and Schott \(2021\)](#), [Ottonello and Winberry \(2020\)](#).¹⁰ My paper steps out in this part of the literature by adding the endogenous choice of debt callability structure.

Second, I complement works that document the role and the importance of bond callability in firm dynamics from a corporate finance view. This literature studies the reasons for issuing callable bonds; see, for example, [Elsaify and Roussanov \(2016\)](#); [Chen et al. \(2010\)](#); [Banko and Zhou \(2010\)](#). [Elsaify and Roussanov \(2016\)](#) used the same data as the present paper and found that firms use callable bonds to handle refinancing risk. [Chen et al. \(2010\)](#) provide empirical evidence suggesting that callable bond issuers tend to call back their bonds when they have better performance and the highest investment activity. [Banko and Zhou \(2010\)](#) found that firms issue callable bonds with under-investment problems and information asymmetry. Their paper focused on a decade before the Global Financial Crises (2008 - 2009). The literature has developed models incorporating factors influencing bond pricing and firms' issuance decisions. For instance, [Acharya and Carpenter \(2002\)](#) provides a theoretical framework for corporate bond valuation and examines the

¹⁰These papers provide attractive framework to quantify the transmission of financial shocks on aggregate investment through corporate debt choices or debt heterogeneity.

effects of default and call risk on durations using an option-based valuation model for coupon-bearing callable, defaultable bonds. They show that default risk reduces bond duration, and a call option reduces bond duration without default risk. These results are empirically tested and confirmed in [Xie et al. \(2009\)](#). Their framework inspired my work by introducing the endogenous choice of callable debt structures to examine their implications for firm dynamics and investment decisions.

A paper by [Xu \(2018\)](#), close to mine, states that early refinancing adjusts firms' maturity structure and reduces interest payments. The paper finds that speculative-grade firms are more likely to have and use callable bonds contrarily to investment-grade firms because of their higher exposure to refinancing risks. [Becker et al. \(2024\)](#) offer new insights on the callability features of bonds. They show that longer maturity and lower quality bonds will likely be issued with a callable feature. Call features reduce debt overhang by decreasing the value gains to the lenders. They also explain that callable bonds have significantly higher yields and lower secondary market prices than non-callable ones: the cost of callability. In the same logic, [Flor et al. \(2023\)](#) examines the choice between these callable debts and the convertible ones.¹¹ By incorporating an investment choice in a dynamic capital structure theory model, they show that firms that are more exposed to debt overhang issue callable rather than convertible bonds, and also if bonds have covenants attached. I extend this above literature with two additional ingredients: the endogenous default risk of the issuer and a more extensive set of information on both issuers and issuance characteristics.

Lastly, [Gilchrist and Zakrajšek \(2012\)](#) examines the influence of interest rate changes, term structure shape, and interest rate volatility on credit spreads of callable bonds. They find that an increase in general interest rates and a steeper Treasury term structure narrow the spreads of callable bonds. At the same time, higher volatility in long-term Treasury yields widens these spreads. Their results indicate that callable bond spreads are significantly affected by the shape of the term structure and interest rate volatility, with call options attenuating the impact of default risk compared to non-callable bonds.

While existing studies emphasize the significance of considering the role of callable debt in corporate financing (e.g., [Elsaify and Roussanov \(2016\)](#); [Becker et al. \(2024\)](#)), there is a notable gap in understanding how the endogenous decision to issue and call such debt interacts with firms' investment behavior and default risk. Specifically, the joint effect of callability and default risk on firm dynamics and their macroeconomic implications remains underexplored. This paper aims to address this gap by theoretically and empirically investigating these interactions. Since the changes in the risk-free rates can occasion additional uncertainty in the bond pricing in the

¹¹These bonds offer investors the possibility to convert the bonds into a predetermined number of common stock or equity.

presence of callable bonds, many (almost all) of the above studies usually confine their focus to the analysis of non-callable bonds.¹² Also, because the callable bonds accounted for an average of at least 50 % of the corporate debt for the last decades, it is too much reducing and limiting to shed light on the firm heterogeneity implications for the macroeconomy by using just non-callable bonds. In that sense, I contribute to this relatively overlooked part of the literature by revealing the callability of corporate debt as a new potential source of interactions between corporate debt heterogeneity and the macroeconomy.

Layout. The rest of the current paper is organized as follows. Section 2 describes the data, the growing interest for callability in the growing importance of bond financing, and facts on callability linked to firms' characteristics. Section 3 presents a three-period model allowing me to characterize the mechanisms under the role of the callable bonds in firm decisions, quantitatively illustrated in Section 4. Section 5 sets up a general equilibrium model with firm investment, borrowing, callability choice, default, and call decision. Section 6 presents the model parametrization, its quantitative results, and the role of debt callability in cross-sectional and aggregate investment dynamics. I conclude in section 7.

2 Data and Empirical Facts

In this section, I describe the dataset used in my empirical analysis, the sources of the data, and the methodology employed to construct the sample. I also present empirical facts on the growing importance of callable bonds in corporate financing and the relationship between callability and firm characteristics.

2.1 Data Sources and Sample Construction

I use three different sources of data to investigate the debt callability of non-financial firms in the U.S. The data sources and sample construction procedures are detailed below.

2.1.1 Mergent Fixed Income Securities Database (Mergent FISD)

The Mergent Fixed Income Securities Database (Mergent FISD) serves as the primary data source for this study. This comprehensive database provides detailed information on a wide range of corporate bonds, including issuance date, original issuance amount, callability, convertibility, presence of covenants, and other bond-specific attributes. I focus on data spanning from January 1990 to May 2018 to match the availability of data from other sources used in this research.

¹²The literature also recognizes that callability structure adds some new complexity to the -already complex- analysis of corporate debt structure and its implications for the macroeconomy (See, e.g., [Duffee \(1998\)](#); [Gilchrist and Zakrajšek \(2012\)](#)).

Mergent FISD also includes historical data on changes that occurred throughout the life of each bond, enabling me to track the outstanding amount and the callable status of each bond at any given time, through the protection period.¹³ By filtering the data to include only U.S. corporate bonds issued by non-financial firms, I obtain a sample that is relevant for the analysis of debt callability in this context.

2.1.2 Compustat and CRSP

To supplement the bond-level data, I use Compustat, which offers financial information on publicly listed firms in the U.S. Compustat provides quarterly data on over 60 firm-specific variables, including financial positions, balance sheet information, and other relevant attributes.

Following standard practice in the literature, I exclude utilities (Standard Industrial Classification - SIC codes 4900–4999), financial firms (SIC codes 6000–6999), and other firms with SIC codes greater than 8999 from the sample. Additionally, I require firms in the final sample to have total debt representing at least 5% of their assets and winsorize all variables at the 1% and 99% level to mitigate the influence of extreme values.

Lastly, I supplement the dataset with stock price data from the Center for Research in Security Prices (CRSP). This data source provides valuable information on stock prices and other stock-related variables for the firms in the sample.

2.1.3 Merging Data Sources

I merge the bond-level data from Mergent FISD with the firm-level data from Compustat and CRSP using the CUSIP identifier, which uniquely identifies each firm or issuer. The resulting dataset offers insights into the relationship between firm characteristics and the degree of callability of their debts, providing a solid foundation for the empirical analysis of debt callability.

By combining these three data sources and constructing a comprehensive dataset, this research seeks to better understand the role of debt callability in the financial landscape of non-financial U.S. firms. This rich dataset enables a thorough examination of the various factors that influence firms' decisions to issue callable debt and the impact of callability on firm behavior and investment.

2.2 Empirical Facts

In this subsection, I explore the empirical facts related to callable bonds and their role in corporate financing and firm dynamics. I provide a more detailed analysis of the prevalence of callable bonds,

¹³The period between issuance of the bond and the (first) potential call date. It is called also the lockout period.

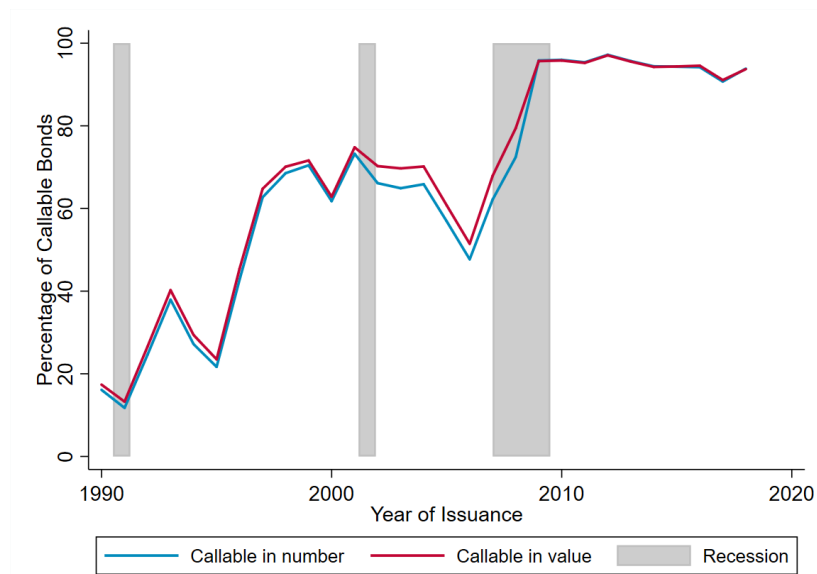
firm characteristics associated with callable debt, and the exercise of call options.

2.2.1 The prevalence of callable bonds in the U.S.

Over the last two decades, bond financing, particularly callable bonds, has gained increasing importance in firms' funding strategies. My analysis of the data reveals that 62.11% of the bonds issued are callable. Before the Global Financial Crisis (GFC) of 2007-2009, this percentage stood at 49.54%. During the GFC, the percentage of callable bonds increased to 78.22%, and after the GFC, the prevalence of callable bonds surged to an impressive 95.54%.

Furthermore, when considering the total value of the bonds issued, callable bonds comprise 66.1% of the overall amount. Before the GFC, this percentage was 54.01%. During the GFC, the percentage of callable bonds in terms of value skyrocketed to 85.12%. After the GFC, this percentage only marginally decreased to 95.48%.

Figure 2: PREVALENCE OF CALLABLE BONDS OVER TIME



Notes: This figure displays the percentage of bonds issued with a call option, regarding the number of bonds issued (blue) and the value issued (red). For each year of issuance, I compute these two prevalences. The shading bands indicate recession periods. The sample period is 1990Jan - 2018May. Source: Mergent FISD; Fed Data.

I calculated the prevalence of callable bonds by year, computing the percentage of callable bonds concerning both the number of bonds issued and the total value of bonds issued. The resulting graph, Figure 2 displays the prevalence of callable bonds over time, with the area representing the recession periods (1990-1991, 2001, and 2007-2009). The graph shows that the percentage of callable bonds has generally increased over time, particularly after the GFC. In addition, the preva-

lence of callable bonds in terms of value is consistently higher than the prevalence in terms of the number of bonds issued, suggesting that callable bonds tend to be larger in size (See the last facts of this section.).

This fact reveals that callable bonds are prevalent in the corporate bond market, with their prevalence increasing over time particularly after the GFC. The higher prevalence of callable bonds in terms of value compared to the number of bonds issued suggests that callable bonds are more common among larger bond issuance. The shift from bank loans to the bond market during the crisis, as documented by [Crouzet \(2018\)](#), has led to continued growth in the issuance of callable bonds, reflecting their attractiveness as a funding option for firms in the post-crisis environment.

The analysis reveals also that callable bond issuance varies across industries (See Figure [A1](#)). The use of callable bonds is more prevalent in certain industries, such as Services; Agriculture, Forestry, and Fishing; Manufacturing; and Transportation, Communications, and Utilities. These industries tend to have higher capital expenditure requirements and more volatile cash flows, which may explain their preference for callable debt as it provides greater flexibility in managing their financial obligations.

2.2.2 Callability and Firm characteristics

The results of my analysis reveal several key insights about the characteristics of firms that issue callable bonds. The majority of the variables exhibit statistically significant differences in means, which indicates a clear distinction between firms that issue more than 50% of their bonds as callable compared to those that issue less than 50% as callable. The exceptions to this are leverage and market-to-book ratio, which did not show a significant difference between the two groups of firms.

Firms with a majority of callable bonds tend to have smaller assets compared to firms with less than half of their bonds issued as callable. This suggests that smaller firms may be more inclined to issue callable bonds, potentially to take advantage of the flexibility in managing their debt and refinancing options. These firms are presented as those with the most intensive need for capital structure management. Interestingly, these firms exhibit lower total investment levels, perhaps reflecting a more cautious approach to capital investments. While callable bonds can provide firms with an opportunity to refinance their debt at a lower interest rate if market conditions improve, it appears that firms with a higher proportion of callable bonds have a lower investment rate. This suggests that they invest a smaller proportion of their assets compared to firms with less than half of their bonds issued as callable. These firms may value the financial flexibility that callable bonds provide, potentially using them as a tool to manage their financial risk and sustain their

Table 1: FIRMS CHARACTERISTICS OVER CALLABILITY PROFILE

	(1)	(2)	(3)	(4)
	All	Callable Profile	Ncallable Profile	p-value (2) = (3)
Size (Log of Real Total Assets)	8.98	8.86 (8.87)	9.26 (9.23)	0.00 (0.00)
Investment rate (%)	2.24	2.31 (2.31)	2.06 (2.06)	0.00 (0.00)
Leverage (% debt/assets)	32.89	34.09 (34.0)	30.23 (30.37)	0.00 (0.00)
Return On Assets (%)	1.58	1.60 (1.60)	1.55 (1.54)	0.01 (0.02)
O. Return On Assets (%)	8.98	9.18 (9.17)	8.54 (8.55)	0.00 (0.00)
Cash (% Assets)	11.1	10.98 (11.02)	11.36 (11.28)	0.00 (0.00)
Sales (Log - Real)	7.32	7.21 (7.22)	7.55 (7.53)	0.00 (0.00)
Market to Book Ratio	3.46	3.71 (3.70)	2.90 (2.92)	0.59 (0.61)
Market Capitalization	9.85	9.72 (9.73)	10.16 (10.13)	0.00 (0.00)
Tobin's Q	1.94	1.95 (1.95)	1.93 (1.94)	0.00 (0.14)
Credit Rating (Firm level)	14.26	13.66 (13.7)	15.40 (15.37)	0.00 (0.00)
Interest Coverage Ratio	16.03	15.38 (15.39)	17.44 (17.45)	0.02 (0.02)
Share of callable (% bonds)	84.68	96.08 (95.95)	59.31 (58.94)	0.00 (0.00)

Notes: This table reports means of key firm characteristics for all non-financial firms and with Callable Profile and Non-Callable Profile, respectively. A firm has the Callable Profile if its share of callable bonds is ≥ 0.5 . The sample period is 1990Jan - 2018May. The last column 4 presents the p-value for tests of equality of means (columns 2 and 3). Credit Downgrade is a dummy variable which is 1 if on average the credit rating of the firm degrades, is 0 otherwise.

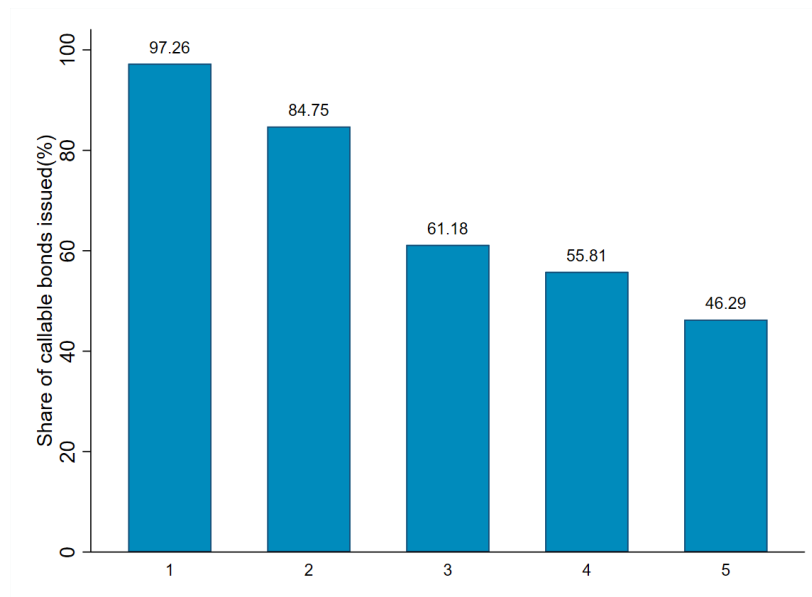
Source: Mergent FISD & CRSP-Compustat.

operations, rather than aggressively pursuing new investments or expansion. This is supported by the Figure 3.

Return On Assets (Net Income/Assets): Firms that issue a majority of callable bonds have higher profitability compared to those that issue less than half of their bonds as callable. This suggests that the decision to issue callable bonds can be driven by better management practices, and more efficient production processes, especially risk exposure management that allow callable majority firms to achieve better financial performance. Firms that predominantly issue callable bonds, exhibit a higher operating return on assets (OROA - EBIT/Assets) compared to non-callable majority firms because of their financing flexibility. Callable bonds allow more flexibility which can result in lower financing costs and interest expenses for callable majority firms, ultimately improving their operating efficiency and increasing their OROA. In contrast, non-callable majority firms do not have this flexibility and may be locked into higher interest rates over the life of their bonds, which can negatively impact their OROA.

Firms with a majority of callable bonds have greater cash holdings, which might indicate a more conservative approach to managing liquidity risks. Callable bonds allow firms to refinance their debt when market conditions are favorable, enabling them to maintain higher cash reserves.

Figure 3: SHARE OF CALLABLE BONDS BY FIRMS CREDIT RISK



Notes: This figure displays the share of callable bonds issued by class of firm size, class 1 is for the 20% low-ranked firms, and class 5 represents the top quintile of higher-ranked and large firms. The sample period is 1990Jan - 2018May.

Source: Mergent FISD.

Credit Downgrade and Interest Coverage: Firms that issue a majority of callable bonds have a higher likelihood of experiencing credit rating downgrades, as evidenced by the higher percentage of credit downgrades in this group. Additionally, these firms have a higher interest coverage ratio, which suggests that they face more significant refinancing risks. This finding indicates that callable bonds may be used as a risk management tool for firms with higher credit risk or more volatile cash flows.

In summary, the results from the table reveal several key insights about the characteristics of firms that issue callable bonds. Firms with a majority of callable bonds tend to have smaller assets, smaller investment levels, and greater profitability compared to firms with less than half of their bonds issued as callable. These firms may be more inclined to issue callable bonds due to the flexibility they provide in managing debt and refinancing options. Additionally, firms with a majority of callable bonds have higher cash holdings and face a higher likelihood of credit downgrades and refinancing risks, suggesting that callable bonds serve as a risk management tool for these firms.

2.2.3 Callable bonds effectively called

A significant proportion of callable bonds are effectively called overall, with 55.69% of callable bonds called before their maturity date, which represents 48.1% of the total amount of callable bonds. Callable bonds that are called tend to have smaller offering amounts compared to those that are not called. The strategic decision to call a bond may be driven by factors such as changes in interest rates, credit risk, and financial performance. Figure 4 illustrates the decreasing proportion

Figure 4: PROPORTION OF CALLABLE BONDS CALLED VS INTEREST RATES



Notes: This figure displays the proportions of callable bonds called, with shading to indicate recession periods and interest rates in green. The Prop. called (w) is the proportion calculated when weighing the bonds called by the amount called. The sample period is 1990Jan - 2018May. Source: Mergent FISD & CRSP-Compustat; Fed Data.

of bonds called over time, with significant shifts occurring around 1993 and during the financial crisis of 2007-2008. Throughout the 1990s, the average proportion of called bonds was 57.92% (49.88% when weighted by the amount). This figure dropped to 47.6% (39.57% weighted, so the value of callable bonds called during the crises significantly dropped relative to the number of bonds) during the crisis period, which spanned from December 2007 to June 2009. In the post-crisis period (after June 2009), the proportion of bonds called decreased further to 44.21% (38.62% weighted on average). Additionally, on average, at the moment they are called, callable bonds have lived 48.56% of their lifespan, see the trend in Appendix A2.

The fluctuating proportion of called bonds over time can be attributed to several factors. One such factor is the influence of higher interest rates, which make it less appealing for firms to call bonds and refinance at increased borrowing costs. The peak in the proportion of called bonds in

1993 is likely due to a combination of factors, including economic conditions, changes in interest rates, and market behavior at the time. In the early 1990s, the U.S. economy underwent a period of growth following a recession in the early part of the decade. This economic recovery may have prompted companies to refinance their debt by calling existing bonds and issuing new ones at lower interest rates. Interest rates were relatively high during the early 1990s but began to decline in subsequent years. This decrease in interest rates made it more attractive for companies to call their bonds, as they could refinance their debt at lower costs. In 1993, the Federal Reserve initiated a reduction in interest rates, which may have contributed to the peak in the proportion of called bonds that year. This period of decreasing interest rates led to a higher demand for companies to refinance their outstanding debt, resulting in the observed increase in bond calls.

Additionally, market behavior and regulatory changes may have played a role in the fluctuating proportion of called bonds. The bond market in the early 1990s was characterized by high volatility and uncertainty, which could have influenced companies to call their bonds to manage risks or capitalize on favorable market conditions. The peak in 1993 could also be a result of specific events or trends in the bond market that led to an increase in the proportion of called bonds.

The decrease in the proportion of called bonds during the financial crisis can be attributed to the turmoil in financial markets and tighter credit conditions. Companies were more cautious about refinancing their debt during this period, as they faced higher borrowing costs and greater uncertainty. In the post-crisis period, central banks worldwide implemented various monetary policies to stimulate economic growth, leading to a prolonged period of low interest rates. This environment made it less attractive for companies to call their bonds, as the potential benefits of refinancing their debt were diminished due to the historically low interest rates. This shift in monetary policy and market conditions contributed to the further decline in the proportion of bonds called during the post-crisis period.

2.2.4 The decision to call

Now, to investigate the probability of a bond being called, I estimate a panel logistic regression model to analyze the likelihood of a bond being called by its issuer, while accounting for unobserved firm-specific effects.

The logit regression model can be specified as follows:

$$\log \left(\frac{\mathbb{P}(\text{called}_{it} = 1)}{1 - \mathbb{P}(\text{called}_{it} = 1)} \right) = \beta_0 + \beta_1 \cdot \Delta rate_t + \theta' \cdot \Gamma_{it} + \mu_i + \varepsilon_{it} \quad (1)$$

My dependent variable is a binary indicator, called_{it} , which equals 1 if the bond i is called

in the quarter t and 0 otherwise. The independent variables include factors that may influence the probability of a bond being called, such as interest rate changes in the quarter t , $\Delta rate_t$, and the vector Γ_{it} of variables relative to the issuer of the bond i , such as credit risk improvements, financial performance improvements, firm size increases, leverage, liquidity ratio increases, and offering amount. The interest rate changes represent fluctuations in macroeconomic conditions. I consider also TFP improvement for idiosyncratic productivity. The leverage reflects the impact of the issuer's debt level on the likelihood of a bond being called and the liquidity ratio is a measure of the issuer's ability to meet short-term obligations using its most liquid assets.

Table 2: PROBABILITY OF BONDS TO BE CALLED - LOGISTIC REGRESSION

VARIABLES	Likelihood to be called				
	(1)	(2)	(3)	(4)	(5)
$\Delta rate_t$	-0.409*** (0.0719)	-0.409*** (0.0719)	-0.405*** (0.0719)		-0.405*** (0.0719)
Credit r. imp.		0.0748** (0.0304)		0.0713** (0.0304)	0.0714** (0.0305)
TFP imp.			0.414*** (0.120)	0.415*** (0.120)	0.404*** (0.120)
Performance inc.	0.105 (0.0718)	0.103 (0.0718)	0.0975 (0.0718)	0.0947 (0.0718)	0.0950 (0.0718)
Firm size inc.	-0.0747 (0.0675)	-0.0757 (0.0675)	-0.0687 (0.0675)	-0.0714 (0.0674)	-0.0698 (0.0675)
Leverage	0.00347 (0.00516)	0.00329 (0.00516)	0.00301 (0.00516)	0.00482 (0.00515)	0.00284 (0.00516)
Liquidity ratio inc.	-0.230** (0.102)	-0.229** (0.102)	-0.222** (0.102)	-0.226** (0.102)	-0.220** (0.102)
Amount issued	-8.36e-07*** (4.01e-08)	-8.35e-07*** (4.01e-08)	-8.32e-07*** (4.01e-08)	-8.32e-07*** (3.99e-08)	-8.32e-07*** (4.01e-08)
F.E.	✓	✓	✓	✓	✓
Observations	62,666	62,666	62,666	62,666	62,666

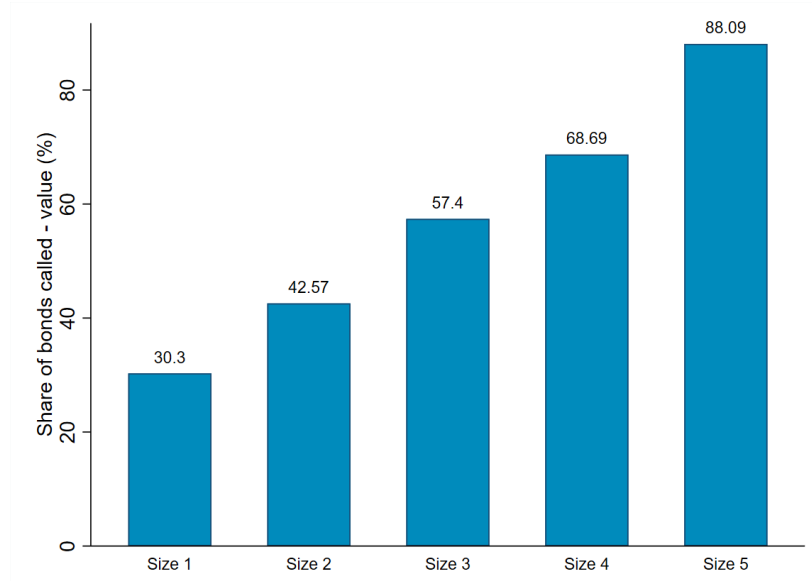
Notes: This table shows estimates if Equation (1). Credit r. imp. and TFP imp. indicate respectively improvement in credit risk and TFP of the firm which calls the bond. TFP is estimated by the author. Performance Inc., Firm Size Inc., and Liquidity Ratio Inc. represent dummies for increasing in financial performance (ROA), firm size, and liquidity ratio. The sample period is 1990Jan - 2018May. Standard errors are reported in parentheses. ***, ** and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.

Source: Mergent FISD & CRSP-Compustat; Fed Data.

The results presented in Table 2, show that when interest rates rise, the probability of a bond being called decreases. This is consistent with the trends revealed in Figure 4 and supports the idea that firms are less likely to call their bonds when interest rates are higher. The results suggest that when a firm's credit rating improves, the likelihood of a bond being called increases. This

finding is also consistent with the fact that improved credit ratings incentivize firms to call bonds and refinance at better terms. When a firm's financial performance improves, the probability of a bond being called increases. This is in line with the principle suggesting that better financial performance allows firms to manage their debt more effectively and potentially refinance by calling bonds. However, the likelihood of a bond being called decreases when a firm's size increases. This result is consistent with the idea that larger firms may have access to more diversified funding sources and better access to capital markets compared to smaller firms, reducing their need to call bonds for refinancing purposes. The leverage ratio has no significant impact on the probability of a bond being called. Concerning the liquidity ratio rises, it lowers the probability of a bond being called. Firms with higher liquidity ratios have a stronger financial position and may not need to call bonds to manage their debt obligations or improve their financial standing. On the opposite, firms with smaller ability to meet short-term obligations using their most liquid assets may be more likely to call their debt. Finally, concerning the size of the original issuance size, the negative coefficient indicates that larger bond offerings may have more stringent covenants or greater market scrutiny, which might discourage firms from calling bonds.

Figure 5: SHARE OF CALLED BONDS BY FIRMS CREDIT RISK



Notes: This figure displays the share of callable bonds that are called by class of firm size (at the moment of call), class 1 is for the 20% low-ranked firms, and class 5 represents the top quintile of higher-ranked and large firms. The sample period is 1990Jan - 2018May.
Source: Mergent FISD.

High-ranked firms use callable bonds to actively manage their debt costs and capitalize on favorable market conditions, while low-ranked firms issue callable bonds more as a hopeful option for future flexibility and as a potential signal of positive prospects, even though they may not

call these bonds as frequently due to less favorable market conditions and limited refinancing opportunities. This mechanism is emphasized by Figure 5.

Overall, these findings reinforce the importance of interest rate variations, credit risk, financial performance, firm size, and the firm-level share of callable bonds in the decision to call a bond.

2.2.5 Bonds characteristics

I observe in data the following trends showing that callable bond issuance is associated with specific bond characteristics (more detailed in Appendix B).

Maturity: Callable bonds exhibit a longer (statistically) average maturity (13 years) compared to non-callable bonds (12 years). This could be because issuing firms prefer to have the option to call back bonds when interest rates change or their credit quality improves, which is more likely to happen over a longer period.

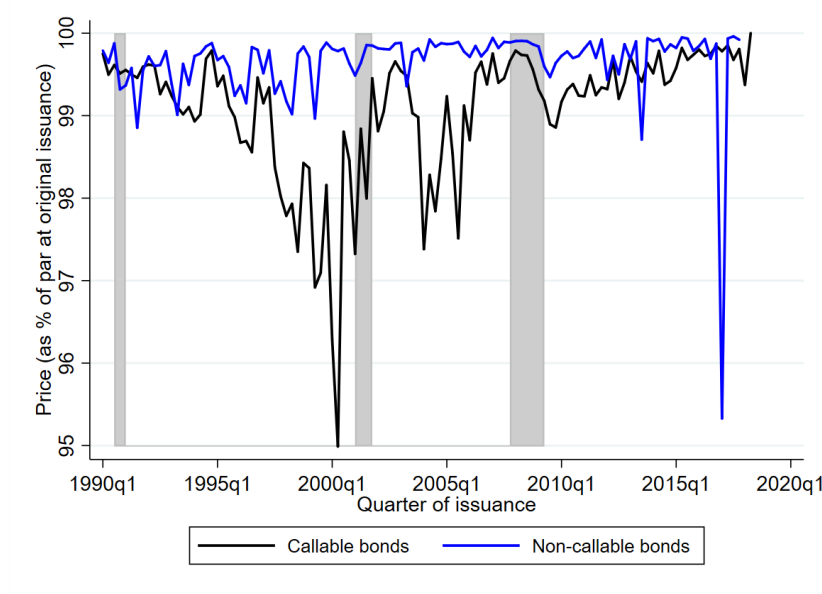
Issue Size: Callable bonds have larger average issue sizes (49.11 \$ mn) than non-callable bonds (24.77 \$ mn). This could suggest that firms issuing callable bonds require more significant financing, and having the option to call back bonds provides them with greater financial flexibility.

Coupon Rate: Callable bonds have higher average coupon rates compared to non-callable bonds. This trend can be explained by the additional risk that investors take when they invest in callable bonds. Since investors face the risk of early redemption and the potential loss of future interest payments, they require higher coupon rates as compensation for this risk.

Offering Price: On average, the offering price of callable bonds is 98.20, while for non-callable bonds, it is 99.56. This difference in offering prices can be attributed to the fact that callable bonds come with a call option, which gives the issuer the right to redeem the bond before its maturity. Investors, recognizing this additional risk, may require a lower offering price for callable bonds as compensation for the potential loss of future interest payments if the bond is called. This results in a lower average offering price for callable bonds compared to non-callable bonds as shown in Figure 6. Becker et al. (2024) found that the callable bonds average yield at issuance is 2.7 % higher than the non-callable bonds ones.

These empirical facts underscore the significance of callable debt in corporate financing and provide strong motivation for incorporating callability in the analysis of firm dynamics and macroeconomic outcomes. In the following sections, I build on these empirical findings and develop a theoretical framework to analyze the impact of debt callability on firm dynamics and the macroeconomy.

Figure 6: BONDS PRICES OVER TIME



Notes: This figure plots the average bond price by firms for each type of bond (callable bonds in black). The sample period is 1990Jan - 2018May.
Source: Mergent FISD.

3 Three-Period Model

This section presents the framework that depicts the interaction mechanism between debt callability and firm behavior. I consider the following five assumptions in the model. First, firms issue the two types of bonds. Second, firms only differ in productivity and face the same operations costs and aggregate shocks. Third, there is no initial outstanding debt, but it is compensated by the second period. Fourth, I allow the firms to issue new debt after the initial callable debt has been redeemed. Fifth, exits follow endogenous default on the debts. These ingredients help characterize firms' optimal decisions (capital, prices, the decision to call, and the refinancing).

Economy. The economy lasts three (3) periods, $t = 0, 1, 2$. Risk-neutral shareholders own the firms. I consider a firm characterized by its productivity, which produces with only capital and is exposed to idiosyncratic capital shocks. Each firm is financed through debt (bond) and equity payout. Equity is initially costless issued in the first period $t = 0$, e_0 and second period $t = 1$, e_1 . Equity is positive in this setup ($e > 0$), representing shareholder payout. The firm raises its debt by issuing two-period defaultable non-callable and callable bonds. For simplicity, I suppose that to solve the firm's problem, each firm does not have an initial asset.

Technology. The production process occurs during the periods $t = 1, 2$. The firm produces

goods y with the production function:

$$y = zk^\alpha, \text{ with } \alpha \in (0, 1)$$

where z is a persistent total factor productivity shock realized in periods $t = 0, 1, 2$. The initial productivity z_0 defines the type of the firm. The productivity shock evolves following a log-AR(1) process $\log z_t = \rho \log z_{t-1} + \epsilon_{zt}$, where $\epsilon_{zt} \sim i.i.d. \mathcal{N}(0, \sigma_z^2)$.

Capital Quality Shocks. The firm receives, after production, an idiosyncratic capital quality shock ε i.i.d. across time and firms. It is drawn from a normal distribution $\mathcal{N}(0, \sigma_\varepsilon^2)$. It influences the un-depreciated capital of the firm after production. The capital quality shocks allow the model to generate the default risk and match the default rates observed in the data. It can be viewed as an unforeseen force in the efficiency of capital, such as an unmodeled technological decline that reduces the value of the firm's capital.

3.1 Setup

A non-callable debt issued in the period $t = 0$ is a promise to pay the fixed coupon c_1 in period $t = 1$ and to repay the principal of the debt to the bondholder together with the fixed coupon c_1 , in period $t = 2$. The market price of such a non-callable bond is set at p_0^{nc} in period $t = 0$.

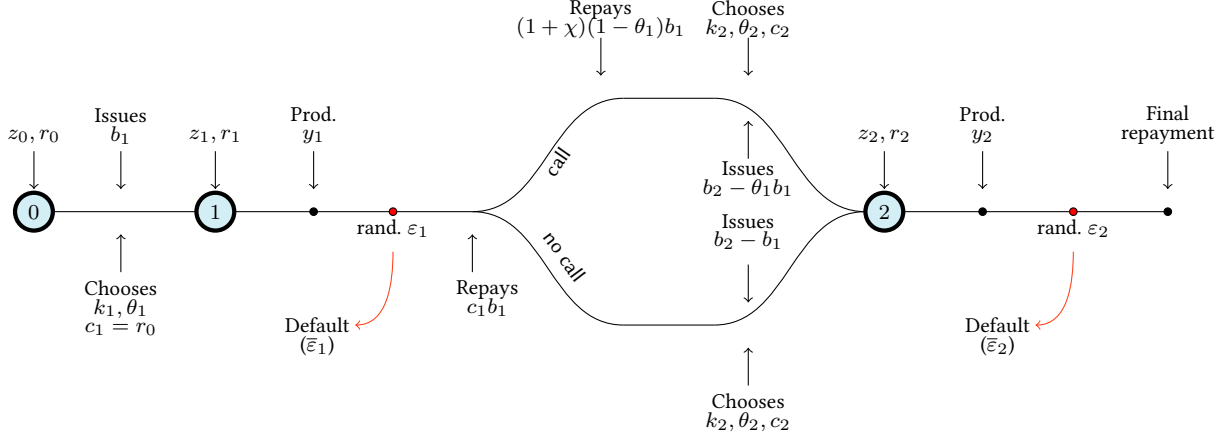
A callable debt issued in the period $t = 0$ with initial maturity in period $t = 2$. It has embedded in it, a *call option*, which gives the right to the issuer to call back or redeem the bond early to its initial maturity (here, in period $t = 1$). The callable bond carries two promises. If the issuer decides to exercise the call option, it repays the debt to the bondholder with a premium set at the coupon c_1 . The other promise is to pay the fixed coupon of c_1 and continue as a non-callable bond if it is not called. The market price of the callable bond is denoted p_0^c and set in period $t = 0$. The protection period is at half of the maturity in this three-period model.¹⁴

The timing of the three-period model is depicted in Figure 7 and explained in the following lines. At the beginning of the period $t = 0$, the firm gets the productivity shock z_0 and the information on the interest rate r_0 prevailing. It has an initial stock of assets q_0 (which is null for this model) and issues an equity e_0 . It chooses its capital k_1 by issuing a fixed-coupon (c_1) bond b_1 , composed of non-callable bond in share θ_1 and callable bond in share $1 - \theta_1$. Both bonds are supposed to mature initially in period $t = 2$. I allow the exercise of the call option to happen in the period $t = 1$.¹⁵ For simplicity, I also assume that issuance is costly for each type of bond, and

¹⁴The protection period is the period during which the issuer of the bond cannot call or redeem the bond before its stated call date. This period is specified in the bond indenture or prospectus and is typically several years following the issuance of the debt. Data suggest that callable bonds, when called, are called around 58% of their initial lifetime.

¹⁵This is inspired by the empirical resulting A2 about the timing of calling relative to the initial maturity, which

Figure 7: TIMING FOR THE THREE-PERIOD SETUP



I adopt the specification of a quadratic form as is done in the literature.

$$\eta_b = (\eta_{nc}\theta_1^2 + \eta_c(1 - \theta_1)^2) \cdot [\max(0, b)]^2$$

The capital constituted for production in the period $t = 1$ is:

$$k_1 = e_0 + p_0 b_1 \quad (2)$$

At the beginning of period $t = 1$, the firm has a new productivity z_1 conditional on its previous one and observes the latest interest rate r_1 . The firm produces and receives the capital quality shock ε_1 . For simplicity, there is no depreciation of the capital. The firm decides to default when its value is less than zero or to repay its debt obligations otherwise. It depends on the realization of capital quality, and when it is under a certain threshold $\bar{\varepsilon}_1$, the firm defaults with a null value. Conditional on surviving, the firm decides whether to call the bond $(1 - \theta_1)b_1$ or not. Whether the firm calls or not, it has to pay the interest on debts. The stock of assets of the firm in the period $t = 1$, when:

- it does not call the bond is:

$$q_1^{nocall} = k_1 + z_1(k_1)^\alpha + \varepsilon_1 k_1 - c_1 b_1 \quad (3)$$

is around 50%. Different call options are embedded in callable bonds: European Call Option, American Call Option, Contingent Call Option, and Soft Call Provision. In my model, a callable bond coincides with the European Call Option and the American Call Option since we know that the issuer can call its bond at any time between the issuance date $t = 0$ and the maturity $t = 2$; thus, the redemption date is $t = 1$.

- it calls the bond is:

$$q_1^{call} = k_1 + z_1(k_1)^\alpha + \varepsilon_1 k_1 - c_1 b_1 - (1 + \chi)(1 - \theta_1)b_1 \quad (4)$$

The term $-c_1 b_1$ is the periodic payment of coupon on the outstanding debt b_1 when the firm chooses not to call during the period $t = 1$, while χ represents the call premium paid additionally to the principal of the callable bond $(1 - \theta_1)b_1$. χ is an amount over the bond's face value paid to the bondholder if the bond is called early. This call premium compensates for the bondholder's loss of future income.

The firm considers all the state variables and new capital quality shock to decide whether to call or not $(1 - \theta_1)b_1$. Following this decision, it chooses its new level of capital k_2 by choosing the amount of debt b_2 for the next period $t = 2$.¹⁶ The firm chooses the non-callable composition (θ_2) of the next period debt by choosing the non-callable share θ_2^{new} in the debt newly issued. I consider interest payment as set in a weighted average coupon c_2 to be paid on the next period debt b_2 .¹⁷

- when the firm does not call the bond, the non-callable fraction, and the periodic coupon c_2 evolve as follows:

$$\begin{cases} \theta_2 &= \theta_1 \frac{b_1}{b_2} + \theta_2^{new} \frac{b_2 - b_1}{b_2} \\ c_2 &= c_1 \frac{b_1}{b_2} + r_1 \frac{b_2 - b_1}{b_2} \end{cases} \quad (5)$$

where $\theta_2^{new} = 1$, as the new debt is a non-callable one-period bond.

- when the firm calls its callable bond, the non-callable fraction, and the periodic coupon c_2 evolve as follows:

$$\begin{cases} \theta_2 &= \theta_1 \frac{b_1}{b_2} + \theta_2^{new} \frac{b_2 - \theta_1 b_1}{b_2} \\ c_2 &= c_1 \frac{\theta_1 b_1}{b_2} + r_1 \frac{b_2 - \theta_1 b_1}{b_2} \end{cases} \quad (6)$$

¹⁶Based on the empirical result of refinancing, the firm is allowed to issue new (one-period) bond at price p_1 when it decides to call $(1 - \theta_1)b_1$. This is a modified motive of refinancing as in the findings in [Chen et al. \(2010\)](#), and my empirical facts suggest that firms need capital to pursue operations once they exercise their call option.

¹⁷This weighted average coupon is essential for accurately tracking the firm's debt servicing costs over time. [Ippolito et al. \(2018\)](#) use a similar formulation to model the share of hedged floating-rate debt. The coupon rate determines the periodic interest payments the firm needs to make on its debt. The firm's decision to call or not call its bonds impacts future coupon payments. By incorporating the coupon rate formulas, the model captures the cost of debt accurately, influencing the firm's investment and financing decisions. The formulas also reflect changes in market interest rates over time and ensure that when the firm issues new debt, the coupon rate aligns with prevailing market conditions.

In period $t = 2$, the firm gets its new productivity z_2 and observes the interest rate r_2 .¹⁸ The stock of assets of the firm in the period $t = 2$, after producing, would be:

$$q_2 = k_2 + z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2)b_2 \quad (7)$$

where $(1 + c_2)b_2$ is the final debt reimbursement.

3.2 Firm Problem

In period $t = 0$, the firm chooses the capital k_1 , the debt b_1 , the non-callable fraction of debt θ_1 , and the coupon rate c_1 . In the period $t = 1$, the firm produces with k_1 , and there is a threshold of capital quality shock $\bar{\varepsilon}_1$ under which the firm's value is null. I denote by $s_1 = \{z_1, r_1\}$ the state variables vector at the beginning of the period $t = 1$ and by $x_1 = \{k_1, b_1, \theta_1^{new}, c_1\}$ the policy vector.¹⁹ The firm maximizes its present value, which is the value of the shareholders, by solving the following:

$$V_0(s_0) = \max_{x_1 = \{k_1, b_1, \theta_1, c_1\}} \left\{ -k_1 + p_0 b_1 - \eta_b + \beta_0 \mathbb{E}_{s_1|s_0} \int_{\bar{\varepsilon}_1}^{\infty} \bar{q}_1 + \bar{V}_1(s_1, x_1) \varphi(\varepsilon_1) d\varepsilon_1 \right\} \quad (8)$$

$$\text{subject to: } \bar{q}_1 = k_1 + z_1(k_1)^\alpha + \varepsilon_1 k_1$$

$$\bar{V}_1(s_1, x_1) = \max_{\mathbb{1}_{\text{call}}(s_1, x_1)} \left\{ \bar{V}_1^{\text{call}}(s_1, x_1), \bar{V}_1^{\text{nocall}}(s_1, x_1) \right\}$$

$$\bar{\varepsilon}_1 : 0 = \bar{q}_1 + \bar{V}_1(s_1, x_1)$$

$$p_0 = \theta_1 p_0^{nc} + (1 - \theta_1) p_0^c$$

$$\eta_b = (\eta_{nc} \theta_1^2 + \eta_c (1 - \theta_1)^2) \cdot [\max(0, b)]^2$$

The following indicator states the decision to call $(1 - \theta_1)b_1$ in $t = 1$:

$$\mathbb{1}_{\text{call}}(s_1, x_1) = \begin{cases} 1, & \text{if } \bar{V}_1^{\text{call}}(s_1, x_1) > \bar{V}_1^{\text{nocall}}(s_1, x_1) \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

Where \bar{V}_1^{call} ($\bar{V}_1^{\text{nocall}}$) is the firm's value when $(1 - \theta_1)b_1$ is called (not called) in $t = 1$.

When the firm does not call its callable debt in period $t = 1$, it adjusts its capital by choosing k_2 and decides the new period debt by issuing $b_2 - b_1$. In period $t = 2$, there exists a threshold $\bar{\varepsilon}_2^{\text{nocall}}$ such that under this value, $V_2^{\text{nocall}} = 0$. The firm obtains its maximal value by solving the

¹⁸The interest rate r_2 is irrelevant since the period $t = 2$ is the final period, and there is no new bond issuance.

¹⁹In the period $t = 0$, the non-callable fraction $\theta_1 = \theta_1^{new}$, because there is no outstanding bond at the beginning of this period.

following:

$$\bar{V}_1^{nocall}(s_1, x_1) = \max_{k_2, b_2, \theta_2^{new}, c_2} \left\{ -e + \beta_1 \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{nocall}}^{\infty} V_2^{nocall}(s_2, x_2) \varphi(\varepsilon_2) d\varepsilon_2 \right\} \quad (10)$$

$$\text{subject to: } k_2 = e - c_1 b_1 + p_1(b_2 - b_1)$$

$$V_2^{nocall}(s_2, x_2) = \max_{\bar{\varepsilon}_2^{nocall}} \{0, k_2 + z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2)b_2\}$$

$$\bar{\varepsilon}_2^{nocall} : 0 = V_2^{nocall}(s_2, x_2)$$

$$\theta_2 = \theta_1 \frac{b_1}{b_2} + \theta_2^{new} \frac{b_2 - b_1}{b_2}$$

$$c_2 = c_1 \frac{b_1}{b_2} + r_1 \frac{b_2 - b_1}{b_2}$$

$$p_1 = \theta_2^{new} p_1^{nc} + (1 - \theta_2^{new}) p_1^c$$

Where p_1 is the weighted average price of the new debt, composed by the price of the non-callable (callable) new bond $p_1^{nc}(p_1^c)$. These prices are given by:

$$p_1^{nc} = p_1^c = \frac{1}{(1 + r_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{nocall}}^{\infty} (1 + r_1) \varphi(\varepsilon_2) d\varepsilon_2 = \mathbb{E}_{z_2|z_1} (1 - \Phi(\bar{\varepsilon}_2^{nocall})) \quad (11)$$

$(1 - \Phi(\bar{\varepsilon}_2^{nocall}))$ is the probability of not defaulting in the period $t = 2$ after the capital quality shock when the firm does not call in period $t = 1$.

In period $t = 1$, when the firm calls its callable debt $(1 - \theta_1)b_1$, it returns the debt and issues a new one $b_2 - \theta_1 b_1$. In period $t = 2$, there exists a threshold $\bar{\varepsilon}_2^{call}$ such that under this value, $V_2^{call} = 0$. The firm then solves:

$$\bar{V}_1^{call}(s_1, x_1) = \max_{k_2, b_2, \theta_2^{new}, c_2} \left\{ -e + \beta_1 \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{call}}^{\infty} V_2^{call}(s_2, x_2) \varphi(\varepsilon_2) d\varepsilon_2 \right\} \quad (12)$$

$$\text{subject to: } k_2 = e - c_1 b_1 - (1 + \chi)(1 - \theta_1)b_1 + p_1(b_2 - \theta_1 b_1)$$

$$V_2^{call}(s_2, x_2) = \max_{\bar{\varepsilon}_2^{call}} \{0, k_2 + z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2)b_2\}$$

$$\bar{\varepsilon}_2^{call} : 0 = V_2^{call}(s_2, x_2)$$

$$\theta_2 = \theta_1 \frac{b_1}{b_2} + \theta_2^{new} \frac{b_2 - \theta_1 b_1}{b_2}$$

$$c_2 = c_1 \frac{\theta_1 b_1}{b_2} + r_1 \frac{b_2 - \theta_1 b_1}{b_2}$$

$$p_1 = \theta_2^{new} p_1^{nc} + (1 - \theta_2^{new}) p_1^c$$

where χ is the call premium, the additional amount over the face value the issuer pays to redeem the callable bond $(1 - \theta_1)b_1$ early. p_1 is the weighted average price of the new debt, composed by the price of the non-callable (callable) new bond $p_1^{nc}(p_1^c)$. These prices are given by:

$$p_1^{nc} = p_1^c = \frac{1}{(1 + r_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{call}}^{\infty} (1 + r_1) \varphi(\varepsilon_2) d\varepsilon_2 = \mathbb{E}_{z_2|z_1} (1 - \Phi(\bar{\varepsilon}_2^{call})) \quad (13)$$

$(1 - \Phi(\bar{\varepsilon}_2^{call}))$ is the probability of not defaulting in the period $t = 2$ after the capital quality shock when the firm calls in period $t = 1$.

3.3 Creditors' Problem

Creditors are risk neutral and discount the future at the same rate $\beta_0 = 1/(1 + r_0)$ in period $t = 0$ and $\beta_1 = 1/(1 + r_1)$ in period $t = 1$ as shareholders. This part follows the structure of creditors' problem in [Jungherr and Schott \(2021, 2022\)](#).

3.3.1 Non-callable bond pricing

After producing, when the firm decides to default, in $t = 1$, the creditors of the non-callable bonds recover the remaining fraction $(1 - \xi)$ of the firm's liquidation value \underline{q}_1 , with the fraction ξ being lost and:

$$\underline{q}_1 \equiv k_1 + y_1 + \varepsilon_1 k_1 \quad (14)$$

In this three-period model, for simplicity, I consider that the whole value of the firm is lost when it defaults on its debt, so $\xi = 1$. The coupon received by the debt holders in period $t = 1$ is r_0 , then the price of the non-callable debt $\theta_1 b_1$ is:

$$p_0^{nc} = \frac{1}{(1 + r_0)} \mathbb{E}_{s_1|s_0} \int_{\bar{\varepsilon}_1}^{\infty} (r_0 + \bar{p}_1^{nc}) \varphi(\varepsilon_1) d\varepsilon_1 \quad (15)$$

with \bar{p}_1^{nc} the average expected continuation price of $\theta_1 b_1$ in $t = 1$:

$$\bar{p}_1^{nc} = \mathbb{1}_{\text{call}} \times \bar{p}_1^{nc_call}(s_1, x_1) + (1 - \mathbb{1}_{\text{call}}) \times \bar{p}_1^{nc_nocall}(s_1, x_1)$$

where $\mathbb{1}_{\text{call}}$ is the dummy that indicates the firm decision to call or not, $\bar{p}_1^{nc_call}$ ($\bar{p}_1^{nc_nocall}$) is the continuation price of $\theta_1 b_1$ when $(1 - \theta_1)b_1$ is called (not called) in $t = 1$:

$$\bar{p}_1^{nc_call}(s_1, x_1) = \frac{1}{(1 + r_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{call}}^{\infty} (1 + c_1) \varphi(\varepsilon_2) d\varepsilon_2 = \beta_1 (1 + c_1) \mathbb{E}_{z_2|z_1} (1 - \Phi(\bar{\varepsilon}_2^{call}))$$

and

$$\bar{p}_1^{nc_nocall}(s_1, x_1) = \frac{1}{(1+r_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{nocall}}^{\infty} (1+c_1) \varphi(\varepsilon_2) d\varepsilon_2 = \beta_1(1+c_1) \mathbb{E}_{z_2|z_1} (1 - \Phi(\bar{\varepsilon}_2^{nocall}))$$

3.3.2 Callable bond pricing

The price of the callable debt $(1 - \theta_1)b_1$ is generally set relative to a similar non-callable debt, such that the option is not exercised in the period $t = 1$. The issuer has to consider the minimum between the non-callable bond price and the exercise price (here at par), using a callable bond premium.²⁰ I use the following formula to model the callable bond pricing at the issuance in the period $t = 0$:

$$p_0^c = \frac{1}{(1+r_0)} \mathbb{E}_{s_1|s_0} \int_{\bar{\varepsilon}_1}^{\infty} [r_0 + (1 - \mathbb{1}_{call}) \bar{p}_1^{c_nocall} + \mathbb{1}_{call} (1 + \chi)] \varphi(\varepsilon_1) d\varepsilon_1 \quad (16)$$

The term $\bar{p}_1^{c_call}$ is the market price of the callable bond in the call scenario. It reflects the bond's value in the secondary market when the call decision is made. The term $(1 + \chi)$ represents the bondholder's potential payoff, ensuring that they do not receive more than the fixed call price even if the market price $\bar{p}_1^{c_call}$ is higher at the time of the call, this aligns with the fact that the call scenario occurs when interest rates drop, increasing the continuation value of the bond. This cap protects the issuer and defines the financial outcome of the early redemption. The call premium $\chi \in (0, 1)$ is typically a small percentage of the face value to compensate bondholders for the early redemption.²¹

$\bar{p}_1^{c_call} (\bar{p}_1^{c_nocall})$ is the market price of $(1 - \theta_1)b_1$ when $(1 - \theta_1)b_1$ is called (not called) in $t = 1$. Because of the issuance of both bonds in the same period by the same issuer facing the same conditions, and due to the fact there is only one period left at this stage of the timing, a callable bond that is not called is priced as a similar one-period non-callable one: $\bar{p}_1^{c_call} = \bar{p}_1^{nc_call}$ and $\bar{p}_1^{c_nocall} = \bar{p}_1^{nc_nocall}$.

²⁰The pricing of callable bonds is similar to the extension of defaultable callable bonds valuation' model in [Jarrow et al. \(2010\)](#) which extended the model of defaultable callable bond in [Duffie and Singleton \(1999\)](#). But I consider endogenous decisions to default and to call the callable bonds. In [Duffie and Singleton \(1999\)](#), the exercise price at the call period is set at par. Ultimately, the firm internalizes the minimal payment in the call scenario.

²¹Setting the call premium as a percentage of the face value aligns with the idea that the premium compensates for the risk relative to the prevailing interest rate environment. This ensures that the premium is proportionate to market conditions during issuance. Callable bonds usually offer a higher coupon rate than non-callable bonds to offset call risk. However, the call premium is generally lower than this higher coupon rate, as it is designed to offer a buffer rather than full compensation.

3.4 Characterization of firm optimal policies

According to this framework within a partial equilibrium setting, the model is solved by backward induction, considering the problems in (10), (12), and (8), alongside the creditors' problems through (15), (16). The subsection discusses the first-order conditions regarding capital choices for both initial and intermediate periods ($t = 0, 1$), the central decision to call a callable bond in period $t = 1$, and the dynamics of debt issuance across these time frames. The feature of calling the callable bond $(1 - \theta_1)b_1$ at $t = 1$ serves to differentiate those optimal choices. I don't use superscripts "nocall" and "call" when there is no ambiguity in the notation.

3.4.1 Refinancing decision in the intermediate period $t = 1$

Now, I analyze the optimal decisions on issuing new debt after not calling and after calling $(1 - \theta_1)b_1$ in this period $t = 1$. The first-order conditions with respect to $b_2 - b_1$ ($b_2 - \theta_1 b_1$) when the firm does not call (calls) $(1 - \theta_1)b_1$ are the following:

nocall

$$[b_2 - b_1] : -(1 + c_2) \frac{b_2 - b_1}{k_2} \mathbb{E}_{z_2|z_1} (\varphi(\bar{\varepsilon}_2)) - \beta_1(c_1 - r_1) \frac{b_1}{b_2} \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2)] = 0 \quad (17)$$

call

$$[b_2 - \theta_1 b_1] : -(1 + c_2) \frac{b_2 - \theta_1 b_1}{k_2} \mathbb{E}_{z_2|z_1} (\varphi(\bar{\varepsilon}_2)) - \beta_1(c_1 - r_1) \frac{\theta_1 b_1}{b_2} \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2)] = 0 \quad (18)$$

The proof of these optimal decisions can be found in Appendix C.1. These conditions present the net cost of refinancing and indicate that the optimal refinancing after the call decision should account for the new interest rate (r_1), the level of the periodic coupon (c_1), and the expected effect on default risk ($\bar{\varepsilon}_2$).

High level b_2 (or $b_2 - b_1$) increases the default probability, as $\frac{\partial \bar{\varepsilon}_2}{\partial (b_2 - b_1)} = \frac{1 + c_2}{k_2} > 0$. It reduces the revenue $p_1(b_2 - b_1)$ from selling the new bond. In period $t = 1$, the optimal refinancing b_2 increases in the stock of existing debt b_1 .²² In the second term, the denominator is the next period stock of non-callable debt in both cases. However, for the no-call scenario, the uncalled bond in the same proportions at both the numerator and the denominator levels cancels out the effects of θ_1 . The implication is that θ_1 does not influence this optimal refinancing choice when the firm does not call the fraction $(1 - \theta_1)$ of its debt b_1 . When the firm calls the

²²This result is consistent with the literature. See the slow debt property in [Jungherr and Schott \(2022\)](#).

share $(1 - \theta_1)$ of its debt, the optimal new debt $b_2 - \theta_1 b_1$ also increases the default probability $\frac{\partial \bar{\varepsilon}_2}{\partial (b_2 - \theta_1 b_1)} = \frac{1 + c_2}{k_2} > 0$, since:

$$c_2 = \begin{cases} r_1 + (c_1 - r_1) \frac{b_1}{b_2} & \text{when no call} \\ r_1 + (c_1 - r_1) \frac{\theta_1 b_1}{b_2} & \text{when call} \end{cases} \quad (19)$$

Then, θ_1 emphasizes the role of the outstanding debt b_1 at the beginning of the period $t = 1$. It amplifies the positive impact of the stock of debt on the choice of optimal refinancing b_2 when the firm calls. Considering firms with default threshold at the right of the distribution of capital quality shock ($\bar{\varepsilon}_1 < \bar{\varepsilon}$), φ increases in $\bar{\varepsilon}_1$, so firms with high default risk face the more severe cost of refinancing (roll-over risk). I derive the following properties of debt callability in refinancing.

Proposition 1. *In period $t = 1$,*

- θ_1 plays an (asymmetric) role in refinancing only when the callable debt is called.
- If the interest rate does not change, i.e., is set s.t. $r_1 = c_1$, θ_1 (or the share of callable bond) has no effect in refinancing, whether the firm decides to call or not.

The proof of Proposition 1 can be found in Appendix C.4. Proposition 1 shows that the callable bonds are important in the amplitude of refinancing when the firm decides to call it. It also shows that the interest rate prevailing at the moment of call decision matters for this role. The asymmetry means that the share of callable bonds significantly impacts refinancing decisions when interest rates are falling, allowing firms to take advantage of lower borrowing costs. In contrast, when interest rates rise, the share of callable bonds has a less significant impact, as firms are less likely to refinance at higher rates.

3.4.2 Optimal capital choice in the intermediate period $t = 1$

I consider the optimal choices of capital under two scenarios: when the firm doesn't call the bond (k_2^{nocall}) and when it does (k_2^{call}). The first-order conditions to k_2 for these cases are respectively the following:

nocall

$$[k_2] : -1 - \frac{\partial \bar{\varepsilon}_2}{\partial k_2} \left[\beta_1 k_2 [1 - \Phi(\bar{\varepsilon}_2)] - (b_2 - b_1) \frac{\partial p_1}{\partial \bar{\varepsilon}_2} \right] + \beta_1 \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} (\varepsilon_2 - \bar{\varepsilon}_2) \varphi(\varepsilon_2) d\varepsilon_2 = 0 \quad (20)$$

call

$$[k_2] : -1 - \frac{\partial \bar{\varepsilon}_2}{\partial k_2} \left[\beta_1 k_2 [1 - \Phi(\bar{\varepsilon}_2)] - (b_2 - \theta_1 b_1) \frac{\partial p_1}{\partial \bar{\varepsilon}_2} \right] + \beta_1 \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} (\varepsilon_2 - \bar{\varepsilon}_2) \varphi(\varepsilon_2) d\varepsilon_2 = 0 \quad (21)$$

Where the capital effect on default risk is:

$$\frac{\partial \bar{\varepsilon}_2}{\partial k_2} = - \frac{1 + \alpha z_2 k_2^{\alpha-1} + \bar{\varepsilon}_2}{k_2} = \frac{(1 - \alpha) z_2 k_2^{\alpha} - (1 + c_2) b_1}{(k_2)^2} \quad (22)$$

Despite the positive impact of additional capital on production, the diminishing return to capital (due to $\alpha < 1$) reduces the firm's ability to absorb shocks, contributing to a high risk of default. I then consider that capital reduces the default risk ($\partial \bar{\varepsilon}_2 / \partial k_2 < 0$). In this circumstance, a high debt stock b_1 decreases the optimal choice k_2 . θ_1 does not influence the optimal investment when the firm does not call $(1 - \theta_1) b_1$ in $t = 1$.

As c_2 increases, the default risk induced by capital is supposed to be more important. But this effect depends on the composition of c_2 , which depends in turn on θ_1 and the relative level of the interest rate r_1 to the average coupon c_1 .

Corollary 3.0.1. θ_1 (or callable share) reduces (or amplifies) the firm's optimal investment when it exercises the option to call its callable debt.

1. The callable fraction of bonds does not influence the optimal investment decision when the firm does not call its debt.
2. Following an easing of monetary policy (i.e., a decrease in r_1), firms with a high share of callable bonds (i.e., low θ_1) increase their investment more when they call these bonds compared to firms with a low share of callable bonds.
3. Following a tightening of monetary policy (i.e., an increase in r_1), firms with a high share of callable bonds (low θ_1) decrease their investment more when they call these bonds compared to firms with a low share of callable bonds.

The proof of Corollary 3.0.1 can be found in Appendix C.5. Corollary 3.0.1 aligns with Becker et al. (2024) results by highlighting how callable bonds enhance a firm's flexibility in managing debt and investment, particularly under varying monetary policy conditions and for firms with high-yield ratings.

3.4.3 The decision to call (in the intermediate period $t = 1$)

The firm decides to call its callable bond $(1 - \theta_1)b_1$ **iff**:

$$\bar{V}_1^{nocall}(s_1, x_1) < \bar{V}_1^{call}(s_1, x_1) \quad (23)$$

Proposition 2. *In period $t = 1$, the firm's decision to call its callable bond $(1 - \theta_1)b_1$ is determined by the following inequality:*

$$\mathbb{E}_{z_2|z_1} \left\{ \left(-b_1 + \frac{(1 - \alpha)y_2^{nocall}}{1 + c_2} \right) [1 - \Phi(\bar{\varepsilon}_2^{nocall})] \right\} < -(1 + \chi)(1 - \theta_1)b_1 + \mathbb{E}_{z_2|z_1} \left\{ \left(-\theta_1 b_1 + \frac{(1 - \alpha)y_2^{call}}{1 + c_2} \right) [1 - \Phi(\bar{\varepsilon}_2^{call})] \right\} \quad (24)$$

The proof of Proposition 2 can be found in Appendix C.6. Proposition 2 presents, in its simplest form, the firm's decision to call the bond based on comparing the expected net returns under the no-call and call scenarios. The firm calls its callable bond if the expected net return from calling the bond (RHS of (23)) exceeds the expected net return from not calling the bond (LHS of (23)).

The term $-(1 + \chi)(1 - \theta_1)b_1$ represents the immediate cost of calling the bond for the firm in period $t = 1$. Conditional to not default, the term in the expectation consists of the minimal value of the capital return of the firm after refinancing through $b_2 - b_1$ ($b_2 - \theta_1 b_1$ after calling).

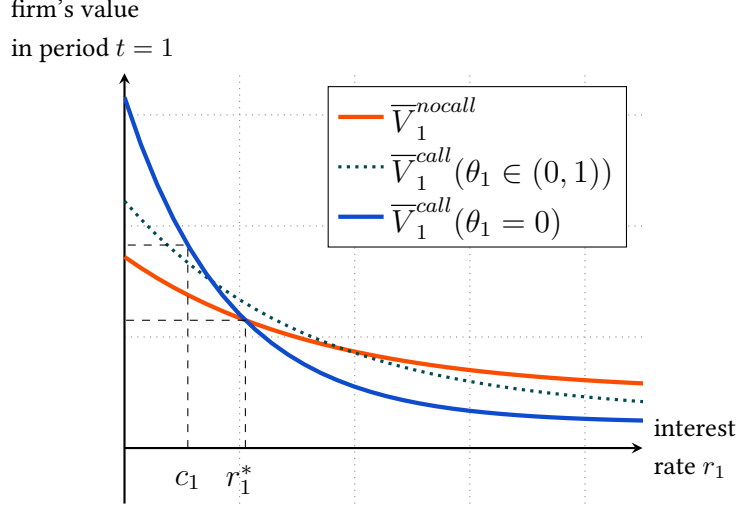
Proposition 3. *For a given state s_1 , an average coupon c_1 on existing debt b_1 which has a callable bond share $(1 - \theta_1 \in [0, 1])$, $\exists! r_1^* > c_1$, such that: $\forall r_1 < r_1^*$, $\bar{V}_1^{nocall}(s_1, x_1) < \bar{V}_1^{call}(s_1, x_1)$.*

The proof of Proposition 3 can be found in Appendix C.7. The main object of interest in Proposition 3 is the level of the interest rate set for the period $t = 1$. This rate influences the value of the firm's repayment in both periods $t = 1, 2$ in the call and non-call scenarios. Therefore, depending on the level of the share of non-callable bonds θ_1 , these value can change their relative position along interest rates. I derive the following Corollary from the Proposition. Both Proposition 3 and Corollary 3.0.2 are illustrated in Figure 8.

Corollary 3.0.2. *A decrease in the interest rate in the period $t = 1$:*

1. *improves the credit quality of the firm;*
2. *increases the probability of calling the callable debt in this period;*
3. *these effects are stronger for firms with a smaller share of callable bonds (i.e., high θ_1) and weaker for firms with a high share of callable bonds.*

Figure 8: ILLUSTRATION OF CALL DECISION IN PERIOD $t = 1$



Corollary 3.0.2 explores the conditions behind the decision to call for firms. The first point indicates that low interest rates improve the firm's credit quality by reducing its future default probability. Therefore, a decline in interest rates can be associated with favorable opportunities that encourage firms to call.²³ This is because the default threshold is positively sensitive to changes in the periodic coupon. So, a decrease in interest rate reduces this periodic coupon, then minimizes the cutoff capital quality in the future (See Appendix C.8 for the proof.). Second, when the original coupon rate c_1 is sufficiently high (due to a high interest rate r_0 in period $t = 0$), the cost of continuing to pay these high-interest rates (reflected in the left-hand side of the (23)) may exceed the cost of facing new debt payments through a potentially lower rate r_1 . Then, a decrease in the interest rate in $t = 1$ raises this benefit of calling. This demonstrates what is theoretically behind the purpose of callable bonds. Third, a higher share of callable bonds indicates a higher debt burden, which leads firms to internalize and endogenize the fact they will face difficulties refinancing their debt once they call.

3.4.4 The optimal choice of callable share $(1 - \theta_1)$ in period $t = 0$

The FOCs for the share of non-callable bond in the period $t = 0$ is given by:

$$[\theta_1] : b_1 \frac{\partial p_0}{\partial \theta_1} + p_0 \frac{\partial b_1}{\partial \theta_1} - \frac{\partial \eta_b}{\partial \theta_1} \leq -\beta_0 \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \theta_1} \quad (25)$$

²³Becker et al. (2024) found the same results empirically and emphasized the role of credit quality improvement.

where I denote V_1 the firm's continuation value in the period $t = 1$, given the realization of new shocks and when default is avoided:

$$V_1(s_1, x_1) = \int_{\bar{\varepsilon}_1}^{\infty} \left[\bar{q}_1 + \mathbb{1}_{\text{call}} \times \bar{V}_1^{\text{call}}(s_1, x_1) + (1 - \mathbb{1}_{\text{call}}) \times \bar{V}_1^{\text{nocall}} \right] \varphi(\varepsilon_1) d\varepsilon_1 \quad (26)$$

The optimal share of non-callable bonds (θ_1) is defined by the following expression:

$$\begin{aligned} \beta_0 \mathbb{E}_{s_1|s_0} \left\{ \mathbb{1}_{\text{call}} \times \frac{\partial \bar{V}_1^{\text{call}}}{\partial \theta_1} \times \left[\underbrace{\left[r_0 + \theta_1 \bar{p}_1^{\text{nc-call}} + (1 - \theta_1)(1 + \chi) \right]}_{\text{effective value of firm's leverage in } t=1 \text{ when the callable part } (1 - \theta_1) \text{ of the bond is called}} \frac{b_1}{k_1} \varphi(\bar{\varepsilon}_1) + \underbrace{\left[1 - \Phi(\bar{\varepsilon}_1) \right]}_{\text{repayment probability in } t=1} \right] \right\} \\ - 2[(\eta_{mc} + \eta_c)\theta_1 - \eta_c] b_1^2 = 0 \end{aligned} \quad (27)$$

Firstly, this condition shows that the presence of callable bonds in the firm decomposition justifies the possibility of a call depending on the future change in the firm's and aggregate conditions. In the first term, the first two factors indicate the call effect of the non-callable share (callable share). According to the proposition (2), the value of $\mathbb{1}_{\text{call}}$ as well as the sign of $\partial \bar{V}_1^{\text{call}}(s_1, x_1) / \partial \theta_1$ turns respectively from 1 to 0 and from negative to positive when the optimal interest rate r_1 in $t = 1$ changes from $< r_1^*$ to $> r_1^*$. The last term is also a key element in explaining the optimal share θ_1 . Therefore, I make the following distinction.

When the initial coupon c_0 is too low, such that future rates are more likely to be higher, the probability of calling the callable debt is almost null, and this absorbs the callable bond effect entirely. Based on this, the call scenario is less likely to happen in the intermediate period $t = 1$. However, the optimal share of callable bonds depends uniquely on the relative issuance cost (η_c / η_{mc}).

- $\eta_c \ll \eta_{mc}$: all firms tend to issue more callable bonds because of their low cost.
- $\eta_c \gg \eta_{mc}$: all firms tend to issue less callable bonds because of their low cost.
- $\eta_c \sim \eta_{mc}$: the issuance costs do not affect the optimal share of callable, which is 50 % in this case.

When the initial coupon c_0 is sufficiently high to increase the chance to call the callable bonds in the future (through small interest rate $r_1 < r_1^*$), the expected probability of calling is then one and $\partial \bar{V}_1^{\text{call}}(s_1, x_1) / \partial \theta_1 < 0$. So we can discuss the competing effects of the two components of

the second factor under the expectation, in (27). Since φ follows a normal distribution, I define $\bar{\varepsilon}$ as the average of the capital shock, and I consider the firm's risk profile when analyzing the optimal share of callable bonds (θ_1) in different interest rate environments. We have the following distinctions:

With high r_0 , the callable bonds are more likely to be called next period $\mathbb{1}_{\text{call}} \rightarrow 1$. Therefore, the benefit of having callable debts is effectively positive. Additionally, we show in Proposition (3) that the advantages to call decreases as θ_1 increases. This means that surviving to default in the next period will reduce this advantage to call. This is likely to happen if the firm is in the less risky part of the firm's default distribution ($\bar{\varepsilon}_1 < \bar{\varepsilon}$). We can deduce that firms in this area have less benefit to issue callable bonds.

With low r_0 , the likelihood of calling in the future is very low $\mathbb{1}_{\text{call}} \rightarrow 0$. I recall the result from Proposition (3) saying that the benefit from calling increases (decreases) in θ_1 (callable share), See $\partial \bar{V}_1^{\text{call}} / \partial \theta_1 > 0$ in the right part of Figure (8). Then, the benefit from having a callable bond in its debts is stronger when the firm is in a situation close to default (less likely to survive default) in the next period. This is because the realization default threshold $\varphi(\bar{\varepsilon}_1)$ is less important as the firm the probability to survive $1 - \Phi(\bar{\varepsilon}_1)$ decreases. This corresponds to the right of firm default distribution ($\bar{\varepsilon}_1 > \bar{\varepsilon}$). I derive, then, the same result that firms with high default risk tend to choose low θ_1 , i.e., a high share of callable.

4 Numerical analysis

Now, I focus on a quantitative exercise involving using fixed values for parameters. I also calibrate the three-period model for some in solving the partial equilibrium. Given these values, I obtain and analyze the firm's optimal capital (k_1 in the initial period $t = 0$ and k_2 the intermediate period $t = 1$), the firm's optimal share of callable bonds (in the initial period $t = 0$), the optimal debt b_1 (in the initial period $t = 0$), and call decision policies. I use the solutions to illustrate the mechanism explained in the previous section and give a sense of firms' decisions on debt callability and its implications.

5 Full Model

I develop a heterogeneous firm general equilibrium model with risky debt and debt callability. The model has the following ingredients: (1) firms use labor and capital as factors of production; (2) to finance their capital investments, they combine equity and debt issuance decisions; (3) debts are sold as long-term defaultable bonds; (4) debt consists of non-callable bond and callable bond

following a composition law of the share of non-callable; (5) upon non-default, firms can call their callable debt before the new optimal decisions.²⁴ Callable bond offers higher yields to investors in compensation for the risk of early redemption during the bond's duration. Still, it is supposed to save costs of interest payments (when it is called) and provide greater flexibility to its issuer. The model economy has four types of agents: firms, creditors, government, and households. Firms with different persistent productivity first hire labor, produce, and then receive capital quality shock upon which they decide to default or not. They decide whether to call their callable bond and then invest in the capital by dividend and debt issuance. There is a continuum of risk-neutral creditors who price the bonds; a government decides the tax and interest rates level; a representative household completes the model. Time is discrete, and as we set in a period, I use the prime symbol (') to denote the future values.

5.1 Firms

5.1.1 Technology and Productivity

Firms are perfectly competitive and produce a single, unique, homogeneous final good. Each firm produces by combining capital k and labor l in a decreasing returns-to-scale technology and using a Cobb-Douglas production function:

$$y = z (k^\psi l^{1-\psi})^\nu$$

where $\psi, \nu \in (0, 1)$. z is the total factor productivity following a persistent shock learned from the previous period. The idiosyncratic productivity z follows an AR(1) process:

$$\log z' = \rho_z \log z + \epsilon'_z, \quad \epsilon'_z \sim^{i.i.d.} \mathcal{N}(0, \sigma_z^2), \quad \rho_z \in (0, 1)$$

The firms pays every period a fixed cost of operation f . The firm receives, after production, an idiosyncratic capital quality shock ε i.i.d. across time and firms. It is drawn from a normal distribution $\mathcal{N}(0, \sigma_\varepsilon^2)$. This shock defines the threshold which influences the firm's decision to default after production.

5.1.2 Financing

Now, I define the types of bonds and present how the firm issues and combines them in financing.

Non-callable Debt. A long-term debt issued in the period t promises to pay a fixed coupon c each period. I assume that a fraction γ of the outstanding principal matures each period, following

²⁴I assume that only public firms can access defaultable debts with a call option. They issue only long-term debts (See [Karabarounis and Macnamara \(2021\)](#).)

Hatchondo and Martinez (2009); Chatterjee and Eyigungor (2012); Gomes et al. (2016). It means that the firm pays back γ additionally to the coupon, and the debt is of finite maturity. The market price of such a non-callable bond is set at p^{nc} at the issuance period.

Callable Debt. A long-term debt issued in the period t promises to pay a fixed coupon c each period. It has embedded in it, a *call option*, which gives the right to the issuer to call back or redeem the bond early to its initial maturity. The callable bond carries two promises. If the issuer decides to exercise the call option, it repays the debt to the bondholder with a premium set at the coupon c . The other promise is to pay the fixed coupon of $c + \gamma$ on the outstanding debt and continue as a non-callable bond if it is not called. The market price of the callable bond is denoted p^c and set at the issuance period.

Share of bond not to call. Considering the quantity of the outstanding debt b at the beginning of a period, I denote by θ the bond share not be called in this period. Then, if the firm exercises its call option, it will call $(1 - \theta)b$. Also, if the firm issues new debt, the share of bond not to call is denoted $\tilde{\theta}$.

The firm can only issue new debt if $b' \geq (1 - \gamma)b$ when there is no call and $b' \geq (1 - \gamma)\theta b$ when there is a call. I denote by \tilde{b} the new debt in each scenario when there is no ambiguity about the call decision of the firm.

Bond Issuance Cost. I assume that retiring all the outstanding debt ($\tilde{b} \geq 0$) is costless for the firm. I adopt the specification of a quadratic form as is done in the literature (see Jungherr and Schott (2021)). The cost of issuing new debt is:

$$\eta_b = \left(\eta_{nc}\tilde{\theta}^2 + \eta_c(1 - \tilde{\theta})^2 \right) \cdot \left[\max(0, \tilde{b}) \right]^2$$

The firm can also issue equity but with a lower bound $e \geq -\underline{e}$ where $\underline{e} > 0$, avoiding the possibility of financing constantly through equity Ponzi games.

Equity Issuance Cost. I assume that the firm incurs a cost when it issues an external equity ($e \geq 0$), and no cost when it distributes dividends ($e < 0$). I adopt again the specification of a quadratic form inspired from Jungherr and Schott (2021). The cost of issuing new external equity is:

$$\eta_e = \eta_e \cdot \left[\max(0, e) \right]^2$$

5.1.3 Callability structure and Call decision

At the call decision, the amount of debt the firm can call is $(1 - \theta)b$. Whatever its decision, it pays the fraction γ on the non-callable share of its debt. Then, depending on its decision, the fraction of

debt not to call in the next period in the remaining debt is different. The direct implication is that the composition of the new debt issued, denoted by \tilde{b} , and the periodic coupon payment differs. This justifies the introduction of the firm idiosyncratic variables θ , and c , and also the index for the call decision, $j \in \{\text{nocall}, \text{call}\}$. As the debt chosen for the next period is denoted by b' , the new debt issued \tilde{b} is described as follows:

$$\forall j \in \{\text{nocall}, \text{call}\}, \tilde{b}_j = \begin{cases} b'_j - (1 - \gamma)b, & \text{if } b'_j > (1 - \gamma)b, & \text{when no call,} \\ b'_j - (1 - \gamma)\theta b, & \text{if } b'_j > (1 - \gamma)\theta b, & \text{when call.} \end{cases} \quad (28)$$

When deciding on the new debt, the non-callable composition (θ') of the next period debt is the weighted average of the remaining non-callable share (which is not to be called) θb in the outstanding debt $(1 - \gamma)b$, and the non-callable share $\tilde{\theta}$ in the new debt issued \tilde{b} . It has the following law of motion: $\forall j \in \{\text{nocall}, \text{call}\}$,

$$\theta'_j = \begin{cases} \theta \frac{(1 - \gamma)b}{b'} + \tilde{\theta}_j \frac{\tilde{b}_j}{b'}, & \text{if } b'_j > (1 - \gamma)b, \text{ when no call } \left[b'_j > (1 - \gamma)\theta b, \text{ when call} \right] \\ \theta, & \text{otherwise.} \end{cases} \quad (29)$$

The periodic coupon c' of the firm is set as the weighted average of the current coupon c on the outstanding debt and the current interest rate r which is the coupon on the new debt issued. The new interest rate r is the coupon attached to the new bond \tilde{b} , while the not-matured-yet and uncalled bond left in the next period bond b' will have to pay the same coupon c . This formulation allows me to track the coupon associated with the not-to-call part of the bond in the next period.

For the nocall scenario, the remaining debt after repaying the fraction γ is $(1 - \gamma)b$, so we have:

$$c'_{\text{nocall}} = \begin{cases} c \frac{(1 - \gamma)b}{b'} + r \frac{\tilde{b}_j}{b'}, & \text{if } b'_j > (1 - \gamma)b, \\ c, & \text{otherwise.} \end{cases} \quad (30)$$

For the call scenario, the remaining debt after repaying the fraction γ is $(1 - \gamma)\theta b$, since the callable debt $(1 - \theta)b$ is called, so we have:

$$c'_{\text{call}} = \begin{cases} c \frac{(1 - \gamma)\theta b}{b'} + r \frac{\tilde{b}_j}{b'}, & \text{if } b'_j > (1 - \gamma)\theta b, \\ c, & \text{otherwise.} \end{cases} \quad (31)$$

Now, I define the firm's internal funds to be the net worth after production, tax, depreciation, and

interest payment:

$$n_j(s, x) = \begin{cases} k - \gamma b + (1 - \tau) [y + \varepsilon k - \delta k - wl - cb - f], & \text{when no call} \\ k - [\gamma\theta + (1 + \chi)(1 - \theta)] b + (1 - \tau) [y + \varepsilon k - \delta k - wl - cb - f], & \text{when call.} \end{cases}$$

The call premium $\chi \in (0, 1)$, over the face value, is provided as compensation for the potential loss of future interest payments due to the early redemption of the bond. The premium might be predetermined or calculated as a percentage of the face value of the outstanding callable bond. It is a penalty during the call protection period and gradually declines as the maturity date approaches. This formulation ensures that the call premium decreases as the bond matures, aligning with the intuition that the cost of calling the bond should reduce over time.²⁵ Although endogenously fixed, it allows traceability in the model.²⁶

5.1.4 Value Functions

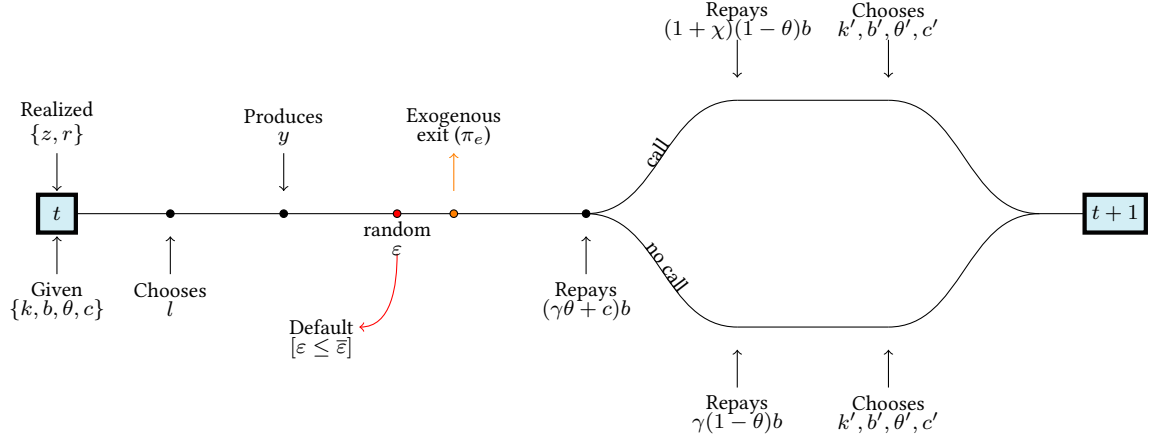
Each period, the firm has its new productivity z , it gets the information on the latest interest rate r , and based on its vector of state variable $x = \{k, b, \theta, c\}$ decided in the previous period, it chooses its labor to produce. I denote the vector of shock variables by $s = \{z, r\}$. The firm maximizes the shareholder value and discounts its future cash flows at the current interest rate. Without any ambiguity, the set of state variables is (s, x) to simplify the notations. Let $V(s, x)$ be the value of the solvent firm making the finance-investment decision.

The timing within a period t is illustrated in Figure 9 and is as follows. At the beginning of a period t , the economy is characterized by the interest rate r . A firm carries capital k , debt b , the share of the not-to-call bond θ , and its weighted average coupon rate c . Given its realized productivity z , it chooses the labor l to produce y —the decision to call interacts with the firm's optimal decisions. Knowing the distribution of the capital quality shock, the firm decides on the cutoff $\bar{\varepsilon}$. Upon non-default, the firm chooses to call or not the callable share $1 - \theta$ of its outstanding

²⁵The call premium increases with the prevailing interest rate, reflecting the higher cost of calling the bond early when interest rates are high. It is proportional to the remaining principal of the callable bond. The call premium also decreases as the bond principal decreases over time due to repayments. This relationship is consistent with the model's incorporation of the call decision indicator and default condition, which adjust the bond's value based on whether the bond is called or not. The call premium also shows an inverse relationship with the bond's original maturity. A longer original maturity typically results in a lower call premium for a given remaining principal and interest rate, as the longer duration allows more opportunity for rates to change, affecting the likelihood of calling. Note that with each period, repayment of the bond at a fraction γ leads to a geometric decrease in the outstanding principal. This geometric decay ensures that the remaining unmatured and uncalled principal of callable bonds decreases over time, which aligns with the model's treatment of the call premium and its adjustment based on the bond's callability and default conditions.

²⁶The call premium adjusts dynamically with changes in the prevailing interest rate, ensuring that the model remains responsive to economic conditions. The formulation aligns with important papers in macro-finance and term structure models (refer to Joslin et al. (2014); Cochrane (2017)).

Figure 9: TIMING FOR THE FIRM'S PROBLEM



Notes: The first blue indicates the beginning of period t and the last marks the beginning of the next period $t + 1$. The firm decides to continue operations or to default endogenously (red arrow) after the capital quality shock ε . The exogenous exit is illustrated in orange and happens with a probability π_e .

bond b . Thus, it can choose k', b', θ', c' . Then, at the beginning of the period, the value of the firm is given by:

$$V(s, x) = \max \left\{ \underbrace{V^r(s, x)}_{\text{continue}}, \underbrace{0}_{\text{default}} \right\}. \quad (32)$$

I assume that in default, the firm exists and receives 0, which implies the following definition of the default cutoff $\bar{\varepsilon}$:

$$\bar{\varepsilon} : 0 = V^r(s, x) \quad (33)$$

Upon non-default, the value received by the firm is $V^r(s, x)$, defined by:

$$V^r(s, x) = (1 - \pi_e)\bar{V}(s, x) + \pi_e V^{ex}(s, x). \quad (34)$$

V^r represents the repayment value of the firm when it decides to continue. In this case, a probability π_e still exists to exit exogenously; then, with its earnings after production, the firm has to liquidate its capital and debt. It pays back its outstanding debt and pays out the dividend to shareholders. Its value in exogenous exit is:

$$V^{ex}(s, x) = k - \gamma b + (1 - \tau)[y + \varepsilon k - \delta k - wl - cb - f] - (1 - \gamma)bp. \quad (35)$$

Then, the firm continues operations when it survives default and exogenous exit by choosing

whether to call or not the callable fraction $1 - \theta$ of its outstanding bond b . Based on its choice, it chooses equity to complement the internal fund from its asset n . I now define the firm's value of continuation.

$$\bar{V}(s, x) = \max_{\mathbb{1}_{\text{call}}(s, x)} \{ \bar{V}_{\text{nocall}}(s, x), \bar{V}_{\text{call}}(s, x) \} \quad (36)$$

The indicator function $\mathbb{1}_{\text{call}}(s, x)$ is one if and only the firm decides to exercise the option to call. I define the continuation price of each type of bond as their expected continuation prices:

$$\begin{cases} \bar{p}^{nc} &= (1 - \mathbb{1}_{\text{call}}(s, x)) \cdot \bar{p}_{\text{nocall}}^{nc}(s, x) + \mathbb{1}_{\text{call}}(s, x) \cdot \bar{p}_{\text{call}}^{nc}(s, x) \\ \bar{p}^c &= (1 - \mathbb{1}_{\text{call}}(s, x)) \cdot \bar{p}_{\text{nocall}}^c(s, x) + \mathbb{1}_{\text{call}}(s, x) \cdot \bar{p}_{\text{call}}^c(s, x) \end{cases} \quad (37)$$

When the firm takes the decision $j \in \{\text{nocall}, \text{call}\}$ on the callable share $1 - \theta$ of its bond b , it receives:

$$\bar{V}_j(s, x) = \max_{e, k'_j, b'_j, \theta'_j, c'_j} \left\{ -e - \eta_e(e) + \frac{1}{1+r} \mathbb{E}_{s'|s} \int_{\varepsilon'} V(s', x') d\Phi(\varepsilon') \right\} \quad (38)$$

subject to:

$$\begin{aligned} k'_j &= e + n_j + p\tilde{b}_j - \eta_b(\tilde{b}_j), \\ V(s', x') &= \max \{ V^r(s', x'), 0 \}, \\ V^r(s', x') &= (1 - \pi_e) \bar{V}(s', x') + \pi_e V^{ex}(s', x'), \\ \bar{V}(s', x') &= \max_{\mathbb{1}_{\text{call}}(s', x')} \{ \bar{V}_{\text{nocall}}(s', x'), \bar{V}_{\text{call}}(s', x') \}, \\ p &= \tilde{\theta}_j p^{nc} + (1 - \tilde{\theta}_j) p^c. \end{aligned}$$

Where p is the weighted average price of the new debt, composed by the price of the non-callable (callable) new bond $p^{nc}(p^c)$.

Firms Entry

In each period, there is free entry. I follow [Hopenhayn \(1992\)](#), I assume that a potential entrant draws an initial level of productivity z^e from an invariant distribution after paying the entry cost. A constant mass M of potential entrants enters the economy without any initial capital $k = 0$, any initial debt $b = 0$. The free entry condition is the following:

$$\int V(z_e, r, 0, 0, \theta, c) = c_e.$$

5.1.5 Bond Pricing

When the firm decides to default on its debts, to liquidate, and to exit, the creditors recover a fraction $1 - \xi$ of the firm's liquidation \underline{n} defined by:

$$\underline{n}(\varepsilon) = \max \left\{ 0, k + (1 - \tau) \left[y + \varepsilon k - \delta k - wl - f \right] \right\} \quad (39)$$

ξ is the parameter that indicates the fraction loosed when the firm is liquidated, and $(1 - \xi)$ is, then, the rate of recovery of capital for the creditor. I assume that all creditors have the same seniority over the firm's liquidation value claim.

I first present the pricing schedule for the non-callable and callable components of the new bond \tilde{b} , and second, I define the continuation price of the two types of bonds in cases of no-call and call.

Non-callable bond pricing

For the non-callable $\tilde{\theta}$ in the new debt \tilde{b} , the coupon rate attached to it is the current interest rate r , so next period, when the firm does not default, it has to pay the fraction γ and the interest r . After this payment, the remaining fraction of the debt will be $1 - \gamma$, valued by creditors at \bar{p}^{nc} . But the current interest rate of r has the weight \tilde{b}/b' in the composition of the new average periodic coupon for the next period c' , defined in (30) and (31), and the continuation price \bar{p}^{nc} contains information on this future average coupon c' instead of direct value of the current rate r .

$$p^{nc}(z, r, k', b', \tilde{\theta}, r) = \frac{1}{1 + r} \mathbb{E}_{s'|s} \left\{ \int_{\tilde{\varepsilon}'}^{\infty} \left[\gamma + r + (1 - \gamma) \bar{p}^{nc}(z', r', \hat{x}^k, \hat{x}^b, \tilde{\theta}, r) \right] d\Phi(\varepsilon') \right. \\ \left. + \int_{-\infty}^{\tilde{\varepsilon}'} \frac{1 - \xi}{b'} \underline{n}' d\Phi(\varepsilon') \right\} \quad , \quad (40)$$

where $\bar{p}^{nc}(z', r', \hat{x}^k, \hat{x}^b, \tilde{\theta}, r)$ is the continuation price of this non-callable bond ($\tilde{\theta}\tilde{b}$) in the next period.²⁷ I define it as the continuation price of the previous non-callable debt θb , so \bar{p}^{nc} instead as shown in (37). Its formulation does not change depending on the call decision in the current period. It is important to notice that the prevailing non-callable bond θb has to pay the periodic

²⁷Notice that \bar{p}^{nc} is a function, and by keeping the arguments $k', b', \tilde{\theta}, r$ in the price, I am sure to keep the current interest rate r on the non-callable bond $\tilde{\theta}\tilde{b}$. See detailed explanations in Appendix (D.1.1).

coupon c set at its issuance. Then, it is set as:

$$\begin{aligned} \bar{p}_j^{nc}(z, r, k', b', \theta, c) = \frac{1}{1+r} \mathbb{E}_{s'|s} \left\{ \int_{\bar{\varepsilon}'}^{\infty} [\gamma + c + (1-\gamma) \bar{p}^{nc}(z', r', \hat{x}_j(s', x'))] d\Phi(\varepsilon') \right. \\ \left. + \int_{-\infty}^{\bar{\varepsilon}'} \frac{1-\xi}{b'} \underline{n}' d\Phi(\varepsilon') \right\}, \quad \forall j \in \{\text{nocall}, \text{call}\} \end{aligned} \quad (41)$$

where $\hat{x}_j(s, x) = [\hat{x}_j^k(s, x), \hat{x}_j^b(s, x), \hat{x}_j^\theta(s, x), \hat{x}_j^c(s, x)]$ is the vector of policy functions for k', b', θ' , and c' in the scenario j .

Callable bond pricing

The price of a callable bond follows three steps: (i) a similar non-callable bond is priced; (ii) the endogenous probability to call the callable bond in the next period; (iii) the potential loss of return for the bondholder in case of call. The callable bond is in fraction $(1 - \tilde{\theta})$ in the new debt \tilde{b} issued in the current period. I provide a pricing model that incorporates the expected average loss of the creditor in case of the call option exercise. Based on the pricing of the non-callable bond above, in (40), the callable bond is priced in $t = 0$, at:

$$\begin{aligned} p^c(z, r, k', b', \tilde{\theta}, r) = \frac{1}{1+r} \mathbb{E}_{s'|s} \left\{ \int_{\bar{\varepsilon}'}^{\infty} \left[r + (1 - \mathbb{1}'_{\text{call}}) (\gamma + (1-\gamma) \bar{p}_{\text{nocall}}^c) \right. \right. \\ \left. \left. + \mathbb{1}'_{\text{call}} (1 + \chi) \right] d\Phi(\varepsilon') + \int_{-\infty}^{\bar{\varepsilon}'} \frac{1-\xi}{b'} \underline{n}' d\Phi(\varepsilon') \right\} \end{aligned} \quad (42)$$

where $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}(s', \hat{x}(s, x))$ is the call decision in the next period based on $(s', \hat{x}(s, x))$, $\bar{p}_j^c(z, r, k', b', \tilde{\theta}, r)$ is the continuation price of the callable bond $(1 - \tilde{\theta})\tilde{b}$ in the next period in the scenario $j \in \{\text{nocall}, \text{call}\}$. The formulation in (42) considers the potential loss to the bondholder due to the issuer's option to call the bond. For simplicity, instead of \bar{p}^c , I present the continuation price of the current callable bond $(1 - \theta)b$.

$$\bar{p}^c(z, r, k', b', \theta, c) = (1 - \mathbb{1}_{\text{call}}(s, x)) \cdot \bar{p}_{\text{nocall}}^c(s, x) + \mathbb{1}_{\text{call}}(s, x) \cdot \bar{p}_{\text{call}}^c(s, x) \quad (43)$$

Contrarily to non-callable bonds, the pricing of the continuation value of callable bonds should differ depending on the call option exercise. While for the no-call scenario, an outstanding callable bond is priced similarly to a non-callable bond, in (41), in the case of calling, its pricing should stop at the final repayment, which is the principal and the call premium. This difference is then

reconsidered in the continuation prices as it follows.

$$\begin{aligned} \bar{p}_j^c = \frac{1}{1+r} \mathbb{E}_{s'|s} \left\{ \int_{\bar{\varepsilon}'}^{\infty} \left[c + (1 - \mathbb{1}'_{\text{call}}) (\gamma + (1 - \gamma) \bar{p}_{\text{nocall}}^{c'}) \right. \right. \\ \left. \left. + \mathbb{1}'_{\text{call}} (1 + \chi) \right] d\Phi(\varepsilon') + \int_{-\infty}^{\bar{\varepsilon}'} \frac{1 - \xi}{b'} \underline{n}' d\Phi(\varepsilon') \right\} \end{aligned} \quad (44)$$

where $\bar{p}_j^{c'} = \bar{p}_j^c(z', r', \hat{x}(s', x'))$, $\forall j \in \{\text{nocall}, \text{call}\}$. The price above is the callable bond's market value in scenario j . The bond is priced like it is newly issued with the current market conditions and the firm's characteristics. I provide further discussion of the pricing of callable bonds in Appendix (D.1.2).

5.2 Households

I consider a representative infinitely-lived household that owns all firms and receives the income in the economy. The household consumes, works, and invests its savings in equity and debt. Government revenue from taxation is returned to the household as a lump-sum transfer. The household has preferences over consumption C_t and labor supply L_t . The utility function is:

$$U(C_t, L_t) = \ln(C_t) - \frac{L_t^{1+\vartheta}}{1+\vartheta}$$

The household discounts the future by the discount factor β (which corresponds to the average interest rate) and makes optimal choices through the marginal rate of substitution and inter-temporal substitution, i.e.

$$\begin{aligned} 1 &= \mathbb{E} \left[\beta \frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)} \right] \\ w &= - \frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} \end{aligned} \quad (45)$$

5.3 Policy Authority

The government sets a fixed corporate tax τ on corporate periodic earnings. Concerning the monetary policy, I consider a simple framework where the authority decides on the real interest rate exogenous path. As in some works using New Keynesian Models (Jeenas (2018), Ottonello and Winberry (2020), Jungherr et al. (2024)) or using a simple exogenous interest rate setting (e.g. Ippolito et al. (2018), Deng and Fang (2022)), I assume an AR(1) process:

$$\log r' = \mu_r + \rho_r \log r + \epsilon'_r, \quad \epsilon'_r \sim^{i.i.d.} \mathcal{N}(0, \sigma_r^2), \quad \mu_r, \rho_r \in (0, 1) \quad (46)$$

5.4 Equilibrium

Now, I define the competitive recursive equilibrium of the model, considering the economy's steady state. First, I describe the law of motion of the firms' distribution, and I define the stationary equilibrium.

Definition 5.1 (Law of motion of the firms' distribution). *Let Γ be the distribution of incumbent firms at the beginning of the period, and Ω be the distribution of entrants firms. The distribution of firms that will produce in the next period is determined as follows:*

$$\Gamma'(z', k', b') = \int_0^\infty \int_0^\infty \Gamma(z, k, b) \pi_e(z'|z) \left[1 - \Phi(\bar{\varepsilon}'(z, r, k, b, \theta, c)) \right] \cdot \mathbb{1}_{B_{inc}} dz db + \Omega'(z', k', b') \quad (47)$$

where $B_{inc} = \left\{ (z, k, b, k', b') \mid b' = (1 - \gamma) \cdot b(z, r, k, b, \theta, c) \right\}$; and the distribution function of future entrants $\Omega'(z', b'_{nc}, b'_c)$ is defined as:

$$\Omega'(z', k', b') = M \cdot \mathbb{1}_{B_{entr}}$$

where $B_{entr} = \left\{ (z', k', b') \mid k' = 0, b' = 0 \right\}$.

Definition 5.2 (Stationary Equilibrium). *A stationary equilibrium in this economy consists of a set of: (i) value functions $V(z, r, k, b, \theta, c)$, $V^r(z, r, k, b, \theta, c)$, $V^x(z, r, k, b, \theta, c)$, $\bar{V}(z, r, k, b, \theta, c)$, $\bar{V}_{nocall}(z, r, k, b, \theta, c)$, and $\bar{V}_{call}(z, r, k, b, \theta, c)$; (ii) a vector of policy functions $\hat{x}(z, r, k, b, \theta, c) = \{k', b', \theta', c'\}$; (iii) bond pricing functions $p^{nc}(z, r, k, b, \theta, c)$ and $p^c(z, r, k, b, \theta, c)$ given by (40) and (42); (iv) a mass of entrants M^* and a stationary distribution Γ^* ; (v) household consumption C^* and aggregate labor supply L^* ; and (vi) a wage w^* and an interest rate r^* , such that:*

1. *Given the bond price functions $p^{nc}(z, r, k, b, \theta, c)$ and $p^c(z, r, k, b, \theta, c)$, the policy vector $\hat{x}(z, r, k, b, \theta, c)$, the value function $V(z, r, k, b, \theta, c)$, and the default decision $\bar{\varepsilon}$ solve the firm's optimization problem (38).*
2. *The free entry condition holds: $V(z_e, 0, 0, \theta, c) = 0$.*
3. *The bond price functions $p^{nc}(z, r, k, b, \theta, c)$ and $p^c(z, r, k, b, \theta, c)$ are consistent with the zero expected profit condition for the investors and the default probabilities and expected recovery rates satisfy the repayment policy;*
4. *The representative household chooses optimally C^* and L^* , consistent with (45).*
5. *The goods market clears:*

$$\begin{aligned}
Y &\equiv \int_0^\infty \int_0^\infty \left[y - f - H \left(\tilde{b}(z, r, k, b, \theta, c), b, \theta, c \right) \right. \\
&\quad \left. - \xi \int_{-\infty}^{\tilde{\varepsilon}(z, r, k, b, \theta, c)} \underline{n} d\Phi(\varepsilon) \right] \Gamma^*(z, b) dz db \\
&= C^* + I^*
\end{aligned} \tag{48}$$

with C^* , the household optimal consumption and I^* , the aggregate investment, defined as:

$$I^* \equiv \delta \int_0^\infty \int_0^\infty k(z, b_{nc}, b_c) \Gamma^*(z, b_{nc}, b_c) dz db \tag{49}$$

5. The labor market clears

$$L^* \equiv \int_0^\infty \int_0^\infty l(z, b_{nc}, b_c) \Gamma^*(z, b_{nc}, b_c) dz db \tag{50}$$

I follow the solution methods in [Hatchondo et al. \(2016\)](#), and [Jungherr and Schott \(2021\)](#) to solve the model equilibrium, see details in [Appendix E.2](#).

6 Quantitative Analysis

This section presents the quantitative analysis of the model, including the calibration strategy, model validation, dynamic analysis, and counterfactual experiments. After calibrating the model to match key empirical moments, I assess its ability to replicate observed firm behavior and macroeconomic dynamics, focusing on the role of callable debt in firm financing decisions and its implications for the broader economy.

6.1 Calibration

The calibration process is divided into two parts: externally fixed parameters, which are chosen based on values found in the literature or empirically observed data, and internally calibrated parameters, which are estimated by fitting the model to match key empirical moments.

6.1.1 Externally fixed parameters

The externally fixed parameters are drawn from existing studies or empirical data and represent well-established or relatively stable aspects of the economy across different models. These parameters are not estimated within the model but are held constant throughout the calibration.

These externally fixed parameters are the model's foundation, ensuring the baseline environment reflects the broader macroeconomic and firms' average-level context. By grounding these

Table 3: EXTERNALLY FIXED PARAMETERS

Params.	Description	Value	Source/Target
ϑ	Inverted Frisch elasticity	0.5	Arellano et al. (2019)
τ	Corporate tax rate	0.4	Gomes et al. (2016)
ψ	Capital share	0.33	Bloom et al. (2018)
ζ	Decreasing returns-to-scale	0.75	Bloom et al. (2018)
δ	Quarterly depreciation rate	0.025	Standard (BEA)
π_e	Exogenous exit rate	0.01	Ottonello and Winberry (2020)
γ	Debt repayment rate	0.05	As in Jungherr and Schott (2021)
ρ_z	Productivity shock persistence	0.9	Ottonello and Winberry (2020)

parameters in empirical evidence and the literature, I ensure that the model’s dynamics are consistent with well-established economic relationships. For the parameters of the production function, I take the capital share ζ and the decreasing returns-to-scale ψ from [Bloom et al. \(2018\)](#). The capital quarterly depreciation $\delta = 0.025$ is set to fit estimates from BEA. I use the persistence of the productivity process, $\sigma_z = 0.03$, estimated by [Ottonello and Winberry \(2020\)](#). Following [Gomes et al. \(2016\)](#), I fix the corporate tax rate τ to 0.4.

6.1.2 Internally fitted parameters

The internally fitted parameters are those that are calibrated by matching the model to a set of essential empirical moments. These parameters are adjusted to capture firm-specific financial behaviors, particularly related to debt issuance and investment. The calibration process is based on minimizing the distance between the model-implied moments and their empirical counterparts. Below, I detail the nine internally calibrated parameters, their roles within the model, the empirical moments they target, and the data sources used accordingly for calibration.

Table 4: INTERNALLY FITTED PARAMETERS

Params.	Description	Value	Target	Data	Model
η_{mc}	Non-callable issuance cost	0.0026	Leverage ratio	33 %	31 %
η_c	Callable issuance cost	0.0042	Share of callable debt	62 %	64 %
ξ	Default cost	0.6892	Avg credit spread (all debt)	3.1 %	3.3 %
χ	Call premium	0.0201	Avg credit spread (callable debt)	3.2 %	3.4 %
f	Fixed operation cost	0.3191	Investment rate	22 %	21 %
σ_ε	Capital quality shock volatility	0.7286	Average call rate	47 %	46 %
σ_z	Productivity shock volatility	0.0210	Average exit rate	2.2 %	2.1 %
ρ_r	Interest rate persistence	0.9367	Average callable bond lifespan	48.6 %	48.1 %
σ_r	Interest rate volatility	0.0908	Average long-run interest rate	2.94 %	2.91 %

The issuance cost parameters (η_{mc} , η_c) are primarily identified through their impact on the leverage ratio and the share of callable debt. Higher issuance costs for callable debt (η_c) decrease

its prevalence, thereby directly affecting the share of callable debt in the capital structure. The importance of these targets are emphasized with observations and intuitions on investment dynamics, and refinancing flexibility, documented by [Covas and Haan \(2011\)](#), [Crouzet \(2018\)](#), [Begenau and Salomao \(2019\)](#), and [Becker et al. \(2024\)](#). The default cost parameter (ξ) is identified through its influence on credit spreads. Higher default costs lead to wider credit spreads, aligning with the observed average credit spreads across debt types. [Acharya and Carpenter \(2002\)](#), who emphasize the role of default costs in shaping corporate debt dynamics.

The call premium is central for matching the average credit spread on callable debt. By adjusting χ , we ensure that the model accurately reflects the market's additional risk assessment associated with callable features. A higher call premium discourages the issuance of callable debt, thereby influencing its relative pricing compared to non-callable debt. This approach is consistent with [Duffie and Singleton \(1999\)](#), who explore the pricing implications of defaultable bonds with callable features.

Fixed operating costs influence the profitability and the investment rate. By calibrating f to match the average investment rate, we ensure that firms' investment behaviors in the model reflect empirical observations. High fixed operating costs constrain firms' ability to undertake growth-enhancing investments, highlighting the role of operating expenses as an essential determinant of firm-level decisions, as discussed in the framework of [Hopenhayn \(1992\)](#).

Capital quality shock volatility measures the uncertainty in firms' capital effectiveness, directly influencing default risk. Higher volatility implies increased earnings uncertainty and a greater likelihood of default, leading to wider credit spreads. The calibration of σ_ε targets the average call rate, capturing the overall risk premium demanded by financial markets. It allows the model to be in the sens of important works in the literature. (e.g. [Gilchrist and Zakrajšek \(2012\)](#) on credit spreads as a reflection of firm-specific risks).

Productivity shock volatility represents the unpredictability in firms' productivity levels, affecting their operational efficiency and survival prospects. This parameter is calibrated to match the average exit rate, ensuring that the model accurately reflects the impact of productivity fluctuations on firm dynamics and macroeconomic outcomes. By capturing the variability in productivity, σ_z plays a key role in determining firms' investment and exit decisions. On the interest rate process, its persistence ρ_r is identified through its effect on the lifespan of callable bonds. Its effect, combined with the volatility of the process σ_r , also determines the intensity of the call decision. Higher persistence leads to longer-lived callable bonds, aligning the model with empirical observations of bond lifespans.

My calibration relies on firm-level data sources, primarily drawn from Compustat and the Fi-

financial Information System Dataset (FISD). I use FRED data for the long-run interest rate. The leverage ratios, share of callable debt, and investment rates are, for instance, sourced from Compustat. Moreover, secondary, some moments are taken coherently from important works in the literature (e.g. [Arellano et al. \(2019\)](#), [Ottonello and Winberry \(2020\)](#)).

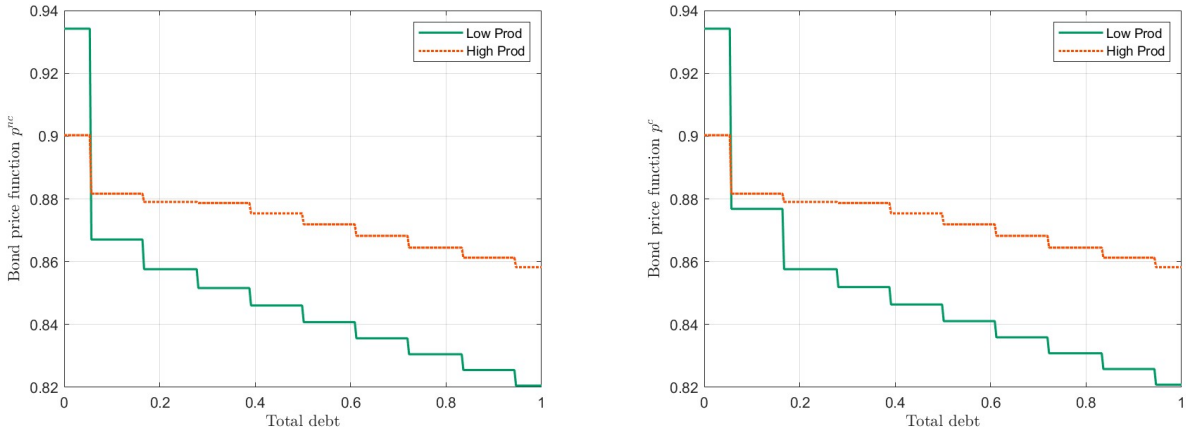
6.2 Dynamic effects of callability

In this section, we explore the dynamics of the calibrated model, focusing on the implications of callable debt on firm behavior and macroeconomic outcomes. We analyze bond prices, capital and debt decisions, and the response of key variables to shocks.

6.2.1 Bond Prices

This subsection investigates how bond prices vary with the firm's debt levels, capital stock, and productivity. Understanding the relationship between these variables and bond prices is crucial for analyzing the cost of financing and the risk premium investors require. I examine the price of non-callable and callable bonds as functions of the firm's state variables: total debt b , capital k , and productivity z . The bond price reflects the present value of future coupon payments, adjusted for the probability of default and the probability that the bond will be called in the case of callable bonds.

Figure 10: BONDS PRICES AS FUNCTION OF DEBT



(a) Average non-callable bond prices

(b) Average callable bond prices

Notes: This figure plots the prices of non-callable and callable bonds across different levels of total debt for two distinct productivity levels: low ($z = z_{low}$), and high ($z = z_{high}$).

Higher Debt Levels: As debt increases, bond prices decline, reflecting the increased default risk associated with higher leverage. This effect is more pronounced for callable bonds, where the call option introduces additional pricing considerations.

Productivity Influence: Higher productivity levels z are associated with higher bond prices, as firms are less likely to default when productivity is strong. This relationship is consistent across both callable and non-callable bonds.

Capital Stock: A larger capital stock k generally leads to higher bond prices, which signals a stronger balance sheet and lower default risk. These findings highlight the link between a firm's financial position and the cost of debt financing, particularly when callable debt is involved.

6.2.2 Heterogeneous decision rules

6.2.3 Heterogeneous responses due to debt callability

7 Conclusion

In this paper, I investigated the relationship between callable debt and firm decisions, grounded in three key stylized facts regarding the callability structure of corporate bonds among U.S. non-financial firms from 1990 to 2018. My analysis revealed that the use of callable bonds has risen significantly over the past decades, with 62 % of corporate bonds now issued with a call option. Additionally, I found that firms holding a larger share of callable bonds tend to have more assets and higher investment levels, though significant heterogeneity exists across firms.

Empirical evidence indicates that larger firms with higher creditworthiness are more likely to call their callable bonds, underscoring the importance of firm size and credit quality in callability decisions. These firms benefit from more favorable borrowing conditions, enabling them to strategically call their bonds to reduce borrowing costs. In contrast, smaller, higher-risk firms are more constrained in exercising their call options due to the rollover risks associated with refinancing, which incentivizes them to retain callable bonds as a way to secure long-term financing.

To further explore the dynamics of callable debt, I developed a quantitative model that incorporates firm decisions on bond issuance, call options, and default risk, with particular attention to the role of credit quality. The results suggest that firms face a trade-off when choosing callable over non-callable debt: the potential to refinance under better terms versus the exposure to greater debt obligations and default risk. Callable bonds offer firms flexibility to adjust their debt burden in response to productivity shocks and changes in interest rates, thereby mitigating default risk. However, the effectiveness of this flexibility is heavily contingent on both market conditions and the firm's credit quality. Firms with higher credit ratings are better equipped to leverage callable debt as a refinancing tool, whereas lower-rated firms may struggle to do so due to heightened refinancing risks and limited access to favorable terms.

Consistent with the findings of [Gilchrist and Zakrajšek \(2012\)](#), my model shows that callable

bonds generally reduce default risk for issuers by allowing them to refinance at more favorable terms during periods of declining interest rates. However, the impact of callability on default risk is asymmetric: smaller, higher-risk firms tend to call their bonds less frequently than larger, more creditworthy firms, reflecting the former's greater exposure to refinancing risk and limited flexibility.

The model also highlights significant asymmetries in callable bond issuance across different interest rate environments and credit quality distributions. In a low-interest-rate environment, where firms expect future rates to rise, high-risk firms with lower credit ratings are more likely to favor non-callable bonds, as callable bonds become less advantageous in the context of rising rates. Conversely, in a high-interest-rate environment, where firms anticipate stable or declining future rates, callable bonds become more attractive—especially for smaller firms—since they offer the potential for refinancing at lower rates, despite the increased default risk. In this scenario, firms with stronger credit quality are better positioned to manage the trade-offs between flexibility and debt servicing costs.

Large, low-risk firms approach callable debt issuance more strategically, carefully balancing the benefits of future refinancing flexibility against the higher interest rates associated with callable bonds. These firms are generally better able to manage the trade-offs between the debt revenue effect (from future refinancing) and the repayment effect (reflecting current debt obligations), optimizing their debt structure in response to economic conditions.

In conclusion, this study underscores the important role of callable debt in managing firm debt obligations and in responding to productivity and interest rate shocks. Credit quality plays a pivotal role in determining how firms utilize callable bonds, influencing their ability to refinance and mitigate default risk. Firms optimize their debt structure based on productivity levels, creditworthiness, volatility in productivity shocks, and prevailing economic conditions such as interest rates. The findings suggest significant heterogeneity in how firms of different sizes and credit qualities utilize callable debt, impacting their investment and financing decisions.

Moreover, the framework developed in this paper provides an insightful tool for future research on the macroeconomic implications of debt callability. It offers a structured approach for examining how callable bonds influence firm behavior in different economic contexts, and how this behavior interacts with broader macroeconomic forces such as monetary policy and aggregate demand fluctuations. Future studies could build on this model to explore the aggregate effects of callable debt across business cycles and assess the financial stability implications of widespread callability structures in corporate debt markets.

Looking forward, further research could explore the aggregate implications of debt callability,

particularly its role in the transmission of monetary policy shocks. The rise of callable debt in corporate financing raises important questions about how the structure of debt callability and the distribution of credit quality influence firms' responsiveness to monetary policy and macroeconomic fluctuations. These questions present promising avenues for further investigation into the broader macroeconomic impact of callable bonds on financial stability and economic growth.

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APPENDIX TO "DEBT CALLABILITY AND FIRM DYNAMICS"

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A	Data	1
A.1	Sample Selection	1
A.2	Variable Construction	1
B	Additional Empirical Results	3
B.1	Callable Bond prevalence across Industry	3
B.2	Bonds Characteristics	3
B.3	Time of calling	3
C	Three-period Model Appendix	5
C.1	Proof of the refinancing decision	5
C.2	Optimal values when $(1 - \theta_1)b_1$ is not called in $t = 1$	6
C.3	Optimal values when $(1 - \theta_1)b_1$ is called in $t = 1$	7
C.4	Proof of Proposition 1	8
C.5	Proof of Corollary 3.0.1	9
C.6	Proof of Proposition 2	10
C.7	Proof of Proposition 3	10
C.8	Proof of Corollary 3.0.2	12
C.9	Proof of the optimal callability in the condition (27)	13
D	Full Model Appendix	15
D.1	Discussion of bonds pricing	15
D.2	First Order Conditions	17
E	Quantitative Appendix	21
E.1	Model Moments	21
E.2	Solving the Model	24

A Data

In this paper, I use two datasets: bond-level data on bonds' characteristics from Mergent FISD, and firm-level data on investment and financial information from CRSP-Compustat. In this section, I describe the sample selection and the variables used in the empirical analysis.

A.1 Sample Selection

The sample used in this study consists of non-financial publicly listed firms in the United States from January 1990 to May 2018. The primary data sources include:

Mergent Fixed Income Securities Database (Mergent FISD): This database provides detailed information on corporate bonds, including issuance date, original issuance amount, callability, convertibility, covenants, and other bond-specific attributes. The sample is filtered to include only U.S. corporate bonds issued by non-financial firms, excluding utilities (SIC codes 4900–4999) and financial firms (SIC codes 6000–6999).

Compustat: Compustat offers quarterly data on firm-specific variables, including financial positions and balance sheet information. Firms in the final sample are required to have total debt representing at least 5% of their assets. All variables are winsorized at the 1% and 99% levels to mitigate the influence of extreme values.

Center for Research in Security Prices (CRSP): Stock price data and other stock-related variables are obtained from CRSP.

These data sources are merged using the CUSIP identifier, which uniquely identifies each firm or issuer, a la [Jungherr and Schott \(2021\)](#).

A.2 Variable Construction

The variables below provide insights into the characteristics of firms that issue callable bonds and their impact on firm behavior and financing decisions.

Callable Bonds Indicator:

A bond is classified as callable if the issuer has the option to redeem the bond before its maturity. The proportion of callable bonds issued by a firm is calculated as the share of callable bonds relative to the total bonds issued.

Firm Size (Log of Real Total Assets):

Firm size is measured as the logarithm of the firm's total assets, using Compustat. I also define

firm size as the size of its average stock of debt. In the callable structure analysis, I use the credit rating as a proxy for the firm size.

Investment Rate (%):

The investment rate is computed as the ratio of capital expenditures to total assets.

Leverage (% Debt/Assets):

Leverage is calculated as the ratio of total debt to total assets.

Return on Assets (ROA, %):

ROA is measured as the ratio of net income to total assets.

Market-to-Book Ratio:

This ratio is calculated as the market value of equity divided by the book value of equity.

Credit Rating:

Firms are sorted into credit rating categories based on their long-term credit ratings, with lower scores indicating higher credit risk.

Interest Coverage Ratio:

This ratio is computed as earnings before interest and taxes (EBIT) divided by interest expenses.

Cash Holdings (% Assets):

Cash holdings are measured as the ratio of cash and cash equivalents to total assets.

Debt Maturity:

Debt maturity is calculated as the proportion of long-term debt (debt maturing in more than one year) relative to total debt. This variable captures the firm's debt structure and potential refinancing risks. In Mergent FISD, I use the "action" variable to ensure that the bond is still unmatured and have its status of callability.

Call Date and Call Dummy:

The call date is recorded as the date when a callable bond is effectively called by the issuer. It excludes then the unmatured bonds which disappear for other reasons. Based on this definition, the call dummy indicates 1 if the bond is callable and called, and 0 if the bond is callable, still alive, and not matured yet. I use the "action" variable in Mergent FISD and the redemption file to identify properly the firm's decision of call.

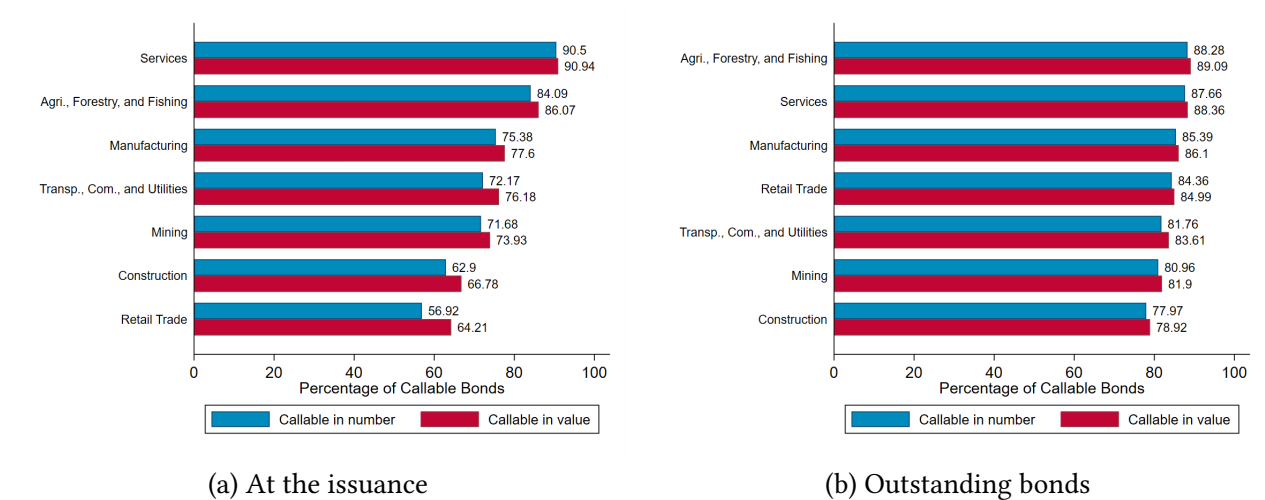
B Additional Empirical Results

This subsection includes additional empirical exercises that support differences brought by callable bond issuance.

B.1 Callable Bond prevalence across Industry

Using SIC codes, I categorized bonds into different industries and calculated the number of callable and total bonds issued by each industry. I also computed the percentage of callable bonds in terms of both the number of bonds issued and the total value of bonds issued for each industry.

Figure A1: CALLABLE BONDS BY INDUSTRY



Notes: I compute the share of callable bonds for the case of the issuance and the case of outstanding bonds. Data are from Mergent FISD & CRSP-Compustat.

B.2 Bonds Characteristics

Callable bonds are generally issued at lower prices compared to non-callable bonds. Their coupon rate is on average higher because it makes them more attractive and justifies the risk of reinvesting for the holder of the bonds.

B.3 Time of calling

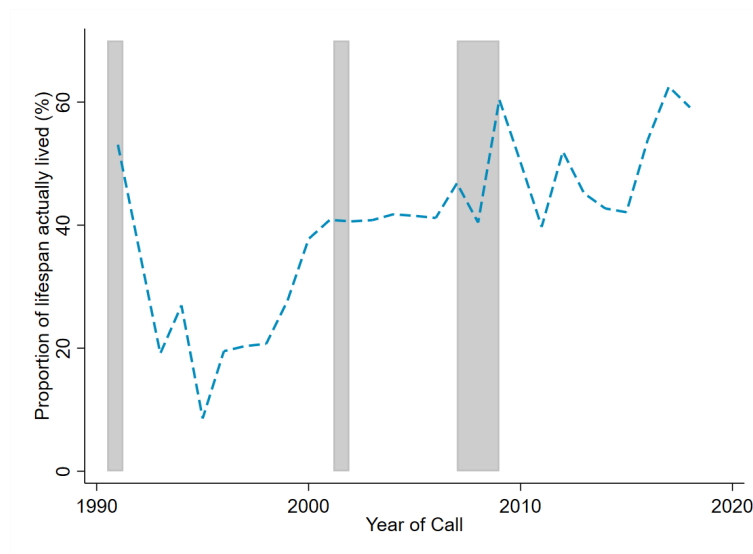
This section adds empirical evidence about the average timing of the call option exercise. The figure shows that callable bonds are typically called on average at the middle of their initial maturity.

Table A1: CALLABLE BONDS VS NON CALLABLE BONDS

	Callable bonds	N Callable bonds
Maturity	12.9657 years	12.4684 years
Issue Size (\$ mn)	49.1148	24.7659
Coupon Rate (%)	6.8351	6.61763
Offering Price	98.2012	99.5552

Notes: The differences in the two columns are significant at 5%. Data are from Mergent FISD.

Figure A2: AFTER WHICH PERCENTAGE OF THEIR LIVES, BONDS ARE CALLED?



Notes: I identify what percentage of the initial maturity of the debt has elapsed between the issuance and the redemption of the debt. This percentage is computed as the annual average over our sample, with the call option exercise happening from 1991 to 2018, in our data. Data are from Mergent FISD.

C Three-period Model Appendix

This subsection provides details on theoretical derivations and proofs of propositions, corollaries, and other optimal characteristics of the three-period model presented in the section (3).

C.1 Proof of the refinancing decision

In this proof of (17) and (18), I provide the details of the optimal decision on refinancing in period $t = 1$ after respectively, not calling and calling the callable debt $(1 - \theta_1)b_1$. The new debt chosen is denoted by b_2 . The first order condition with respect to the debt issued, respectively $b_2 - b_1$ and $b_2 - \theta_1 b_1$, is described in the following lines.

For the no-call scenario, we have:

$$[b_2 - b_1] : p_1 + \frac{\partial p_1}{\partial(b_2 - b_1)}(b_2 - b_1) + \beta_1 \frac{\partial}{\partial(b_2 - b_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{nocall}}^{\infty} V_2^{nocall}(s_2, x_2) \varphi(\varepsilon_2) d\varepsilon_2 \leq 0 \quad (A1)$$

Using the definition of the price of the new debt in (10) and its components in (11), I have the derivatives of the price and the expected future value of the firm with respect to the default risk:

$$\frac{\partial p_1}{\partial \bar{\varepsilon}_2} = \theta_2^{new} \frac{\partial p_1^{nc}}{\partial \bar{\varepsilon}_2} + (1 - \theta_2^{new}) \frac{\partial p_1^c}{\partial \bar{\varepsilon}_2} = -\mathbb{E}_{z_2|z_1} [\varphi(\bar{\varepsilon}_2)] \quad (A2)$$

$$\frac{\partial}{\partial \bar{\varepsilon}_2} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} V_2(s_2, x_2) \varphi(\varepsilon_2) d\varepsilon_2 = -k_2 \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2)] \quad (A3)$$

A high level of of next period debt b_2 increases the default risk, shown by the following:

$$\frac{\partial \bar{\varepsilon}_2}{\partial(b_2 - b_1)} = \frac{1 + c_2}{k_2} \quad (A4)$$

When combining (A2), (A3), and (A4), I have the variations in the new debt, of the price of the new debt and the expected future value of the firm:

$$\frac{\partial p_1}{\partial(b_2 - b_1)} = -\frac{1 + c_2}{k_2} \mathbb{E}_{z_2|z_1} [\varphi(\bar{\varepsilon}_2)] \quad (A5)$$

$$\frac{\partial}{\partial(b_2 - b_1)} \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} V_2(s_2, x_2) \varphi(\varepsilon_2) d\varepsilon_2 = -(1 + c_2) \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2)] \quad (A6)$$

I rewrite the FOC of the new debt $b_2 - b_1$:

$$[b_2 - b_1] : -(1 + c_2) \frac{b_2 - b_1}{k_2} \mathbb{E}_{z_2|z_1} (\varphi(\bar{\varepsilon}_2)) + (1 - \beta_1(1 + c_2)) \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2)] = 0 \quad (\text{A7})$$

I can deduce the form shown in (17) by using the composition of the average coupon defined in (19).

Concerning the call scenario, the method is the same, except for the debt b_1 which is replaced by $\theta_1 b_1$ in the outstanding debt. I write then the FOC of the new debt $b_2 - \theta_1 b_1$:

$$[b_2 - \theta_1 b_1] : -(1 + c_2) \frac{b_2 - \theta_1 b_1}{k_2} \mathbb{E}_{z_2|z_1} (\varphi(\bar{\varepsilon}_2)) + (1 - \beta_1(1 + c_2)) \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2)] = 0 \quad (\text{A8})$$

In turn, I consider the composition of the average coupon when the firm exercises its call option, to obtain finally the optimal refinancing in the call scenario in (18). ■

C.2 Optimal values when $(1 - \theta_1)b_1$ is not called in $t = 1$

The default threshold $\bar{\varepsilon}_2^{nocall}$ in $t = 2$, when the callable bond is not called in $t = 1$, is defined as follows:

$$\begin{aligned} \bar{\varepsilon}_2^{nocall} : 0 &= V_2^{nocall}(s_2, x_2) \\ &: 0 = z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2)b_2 \end{aligned}$$

I derive the threshold $\bar{\varepsilon}_2^{nocall}$:

$$\bar{\varepsilon}_2^{nocall} = - \frac{k_2 + y_2 - (1 + c_2)b_2}{k_2} \quad (\text{A9})$$

I reformulate the optimal value of the firm (in (10)) when it doesn't call its callable bond:

$$V_1^{nocall}(s_1, x_1) = -c_1 b_1 - k_2 + p_1(b_2 - b_1) + \beta_1 \cdot k_2 \cdot \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} (\varepsilon_2 - \bar{\varepsilon}_2) \varphi(\varepsilon_2) d\varepsilon_2 \quad (\text{A10})$$

The variation in the value function above in (A10), induced by changes in the default threshold $\bar{\varepsilon}_2^{nocall}$ is defined as follows:

$$\Delta \bar{\varepsilon}_2^{nocall} = -(b_2 - b_1)(1 + c_2) \mathbb{E}_{z_2|z_1} [\varphi(\bar{\varepsilon}_2)] - \beta_1 \cdot k_2 \cdot \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2)] \quad (\text{A11})$$

The first-order condition with respect to k_2 in the period $t = 1$ is explained in:

$$-1 + \frac{\partial \bar{\varepsilon}_2}{\partial k_2} \cdot \Delta \bar{\varepsilon}_2^{nocall} + \beta_1 \cdot \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} (\varepsilon_2 - \bar{\varepsilon}_2) \varphi(\varepsilon_2) d\varepsilon_2 = 0 \quad (\text{A12})$$

with

$$\frac{\partial \bar{\varepsilon}_2}{\partial k_2} = - \frac{1 + mpk_2 + \bar{\varepsilon}_2}{k_2} = \frac{(1 - \alpha)y_2 - (1 + c_2)b_2}{k_2^2} \quad (\text{A13})$$

Where $mpk_2 = \alpha z_2(k_2)^{\alpha-1}$ is the marginal productivity of the capital k_2 in the period $t = 2$. I use the optimal condition in (A12) multiplied by k_2 to write the optimal value of the function in (A10):

$$V_1^{nocall}(s_1, x_1) = -c_1 b_1 + p_1(b_2 - b_1) - \frac{\partial \bar{\varepsilon}}{\partial k_2} \cdot \Delta \bar{\varepsilon}_2^{nocall} \cdot k_2 \quad (\text{A14})$$

Now, I replace $\frac{\partial \bar{\varepsilon}_2}{\partial k_2}$ and $\Delta \bar{\varepsilon}_2^{nocall}$ by their respective expressions in (A13) and (A11), and I use the optimal refinancing formula in (17). I obtain the expression of the optimal value of the firm used in the proposition (2):

$$V_1^{nocall}(s_1, x_1) = -c_1 b_1 + \mathbb{E}_{z_2|z_1} \left\{ \left[-b_1 + \frac{(1 - \alpha)y_2}{1 + c_2} \right] \times [1 - \Phi(\bar{\varepsilon}_2^{nocall})] \right\} \quad (\text{A15})$$

■

C.3 Optimal values when $(1 - \theta_1)b_1$ is called in $t = 1$

When the firm calls its the callable bond in $t = 1$, the default threshold $\bar{\varepsilon}_2^{call}$ in $t = 2$, is obtained as follows:

$$\begin{aligned} \bar{\varepsilon}_2^{call} : 0 &= V_2^{call}(s_2, x_2) \\ &: 0 = z_2(k_2)^\alpha + \varepsilon_2 k_2 - (1 + c_2)b_2 \end{aligned}$$

I derive the threshold $\bar{\varepsilon}_2^{call}$:

$$\bar{\varepsilon}_2^{call} = - \frac{k_2 + y_2 - (1 + c_2)b_2}{k_2} \quad (\text{A16})$$

I reformulate the optimal value of the firm (in (12)) when it doesn't call its callable bond:

$$\begin{aligned} V_1^{call}(s_1, x_1) &= -(1 + \chi)(1 - \theta_1)b_1 - c_1 b_1 - k_2^{call} + p_1(b_2 - \theta_1 b_1) \\ &\quad + \beta_1 \cdot k_2^{call} \cdot \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2^{call}}^{\infty} (\varepsilon_2 - \bar{\varepsilon}_2^{call}) \varphi(\varepsilon_2) d\varepsilon_2 \end{aligned} \quad (\text{A17})$$

I derive here the variation in the value function above in (A17), induced by changes in the default threshold $\bar{\varepsilon}_2^{call}$ defined as:

$$\Delta \bar{\varepsilon}_2^{call} = -(b_2 - \theta_1 b_1)(1 + c_2) \mathbb{E}_{z_2|z_1} [\varphi(\bar{\varepsilon}_2^{call})] - \beta_1 \cdot k_2^{call} \cdot \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2^{call})] \quad (\text{A18})$$

The first-order condition with respect to k_2 in the period $t = 1$ is shown in:

$$-1 + \frac{\partial \bar{\varepsilon}_2}{\partial k_2} \cdot \Delta \bar{\varepsilon}_2^{call} + \beta_1 \cdot \mathbb{E}_{z_2|z_1} \int_{\bar{\varepsilon}_2}^{\infty} (\varepsilon_2 - \bar{\varepsilon}_2) \varphi(\varepsilon_2) d\varepsilon_2 = 0 \quad (\text{A19})$$

with

$$\frac{\partial \bar{\varepsilon}_2}{\partial k_2} = -\frac{1 + mpk_2 + \bar{\varepsilon}_2}{k_2} = \frac{(1 - \alpha)y_2 - (1 + c_2)b_2}{k_2^2} \quad (\text{A20})$$

I multiply by k_2 the optimal condition in (A19) to write the optimal value of the function in (A17):

$$V_1^{call}(s_1, x_1) = -(1 + \chi)(1 - \theta_1)b_1 - c_1 b_1 + p_1(b_2 - \theta_1 b_1) - \frac{\partial \bar{\varepsilon}_2}{\partial k_2} \cdot \Delta \bar{\varepsilon}_2^{call} \cdot k_2 \quad (\text{A21})$$

Now, I replace $\frac{\partial \bar{\varepsilon}_2}{\partial k_2}$ and $\Delta \bar{\varepsilon}_2^{call}$ by their respective expressions in (A20) and (A18), and I consider the final expression of the optimal refinancing in (18). I get the expression of the optimal value of the firm used in the proposition 2:

$$V_1^{call}(s_1, x_1) = -(1 + \chi)(1 - \theta_1)b_1 - c_1 b_1 + \mathbb{E}_{z_2|z_1} \left\{ \left[-\theta_1 b_1 + \frac{(1 - \alpha)y_2}{1 + c_2} \right] \times [1 - \Phi(\bar{\varepsilon}_2^{call})] \right\} \quad (\text{A22})$$

■

C.4 Proof of Proposition 1

Proof. First, let recall the expression of the optimal refinancing in the no-call scenario.

$$[b_2 - b_1] : -(1 + c_2) \frac{b_2 - b_1}{k_2} \mathbb{E}_{z_2|z_1} (\varphi(\bar{\varepsilon}_2)) - \beta_1 (c_1 - r_1) \frac{b_1}{b_2} \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2)] = 0 \quad (\text{A23})$$

θ_1 has no effect on the optimal refinancing when the callable part is not called. This is due to the same impact of θ_1 in the second fraction in front of the expectation term.

Now, to complete the proof, I discuss the relative levels of the coupon rate c_1 on existing debt and the new interest rate r_1 .

If $c_1 < r_1$, refinancing is not so beneficial for the firm whatever its level of debt (same for

calling). A high interest rate of r_1 is not an ideal moment for refinancing in the no-call scenario. But if the firm calls its debt despite the level of interest rate, the share θ_1 of non-callable will serve to reduce the cost of debt since it carries smaller interest expenses. So firms with high θ_1 are more likely to refinance after calling when $c_1 < r_1$ through the reduction in the cost of their new total debt. θ_1 is then important for cheaper refinancing when a bond is called in a period of $c_1 < r_1$.

If $c_1 > r_1$, it is the right time to refinance (eventually to call), and the refinancing cost can be reduced. Moreover, firms with small θ_1 will be more encouraged to refinance, because more weight is put on the interest rate (which is smaller) for future debt costs. Even if refinancing is more supportable in the future when the new interest rate is smaller, firms with high θ_1 are stuck with the relatively high-interest payment discouraging the refinancing. θ_1 is then detrimental for lower-cost refinancing in the call scenario in a period of $c_1 > r_1$. ■

C.5 Proof of Corollary 3.0.1

Proof. I recall the definition in (19) of the firm-specific periodic coupon after the call decision. The change in this coupon with respect to the change in the interest rate is given by:

$$\frac{\partial c_2}{\partial r_1} = \begin{cases} 1 - \frac{\theta_1 b_1}{b_2} & \text{when no call} \\ 1 - \frac{\theta_1 b_1}{b_2} & \text{when call} \end{cases} \quad (\text{A24})$$

If $c_1 < r_1$, when the firm does not call, having existing debt b_1 will attenuate the rise in the cost of capital, whatever the stock of (non)callable bond. When the firm calls, having high θ_1 will serve to drag down the rising cost of capital c_2 . So firms with a high θ_1 face a relatively low new cost of capital in c_2 than firms with low θ_1 . Then, their investment decreases less when they call. This is because they have a high share of existing callable bonds, so they will face more fully the new rate r_1 after calling it.

If $c_1 > r_1$, when the firm decides not to call its callable debt, having a small stock of debt b_1 contributes to the reduction of the new cost of capital c_2 . This is independent of the share of callable in this existing debt. In the case of calling, the part that is not called $\theta_1 b_1$ has to pay the differential interest $c_1 - r_1$. So firms with a high share of callable bonds (which means low θ_1) increase the optimal capital more when they call than firms with lower callable shares.

This completes the proof of the corollary. ■

C.6 Proof of Proposition 2

Proof. To prove Proposition 2, I first recall the price formulas of new bonds in period $t = 1$:

$$p_1 = \begin{cases} \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2^{nocall})] & \text{when no call} \\ \mathbb{E}_{z_2|z_1} [1 - \Phi(\bar{\varepsilon}_2^{call})] & \text{when call} \end{cases} \quad (\text{A25})$$

I consider the inequality with the transformed formulations of the firm's optimal value in the cases of not calling ((A15)) and calling((A22)), to establish the inequality:

$$\begin{aligned} & -e_1 b_1 + \mathbb{E}_{z_2|z_1} \left\{ \left(-b_1 + \frac{(1-\alpha)y_2^{nocall}}{1+c_2} \right) [1 - \Phi(\bar{\varepsilon}_2^{nocall})] \right\} \\ & < -(1+\chi)(1-\theta_1)b_1 - e_1 b_1 + \mathbb{E}_{z_2|z_1} \left\{ \left(-\theta_1 b_1 + \frac{(1-\alpha)y_2^{call}}{1+c_2} \right) [1 - \Phi(\bar{\varepsilon}_2^{call})] \right\} \end{aligned} \quad (\text{A26})$$

To complete the proof, we just to make the term $-c_1 b_1$ disappear at both sides of the inequality. ■

C.7 Proof of Proposition 3

Proof. The proposition concerns the possibility of crossing for the firm's value in both cases of not calling and calling. I prove the proposition through four points. It is important to note that the signs of both values do not matter because they are monotone and continue functions.

1. Based on the form of the firm's value in the no-call scenario, I have:

$$\frac{\partial \bar{V}_1^{nocall}(s_1, x_1)}{\partial \theta_1} = 0. \quad (\text{A27})$$

2. Now, I derive the change in the firm's value in both scenarios, following a change in interest rate r_1 .

$$\begin{aligned} \frac{\partial \bar{V}_1^{nocall}(s_1, x_1)}{\partial r_1} &= \frac{\partial}{\partial r_1} \mathbb{E}_{z_2|z_1} \left\{ \left(-b_1 + \frac{(1-\alpha)y_2^{nocall}}{1+c_2} \right) [1 - \Phi(\bar{\varepsilon}_2^{nocall})] \right\} \\ &= \mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{nocall})] (1-\alpha)y_2^{nocall} \frac{\partial}{\partial r_1} \left(\frac{1}{1+c_2} \right) \right\} \\ &= -\mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{nocall})] \frac{(1-\alpha)y_2^{nocall}}{(1+c_2)^2} \frac{b_2 - b_1}{b_2} \right\} < 0 \end{aligned} \quad (\text{A28})$$

$$\begin{aligned}
\frac{\partial \bar{V}_1^{call}(s_1, x_1)}{\partial r_1} &= \frac{\partial}{\partial r_1} \mathbb{E}_{z_2|z_1} \left\{ \left(-\theta_1 b_1 + \frac{(1-\alpha)y_2^{call}}{1+c_2} \right) [1 - \Phi(\bar{\varepsilon}_2^{call})] \right\} \\
&= \mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{call})] (1-\alpha)y_2^{call} \frac{\partial}{\partial r_1} \left(\frac{1}{1+c_2} \right) \right\} \\
&= -\mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{call})] \frac{(1-\alpha)y_2^{call}}{(1+c_2)^2} \frac{b_2 - \theta_1 b_1}{b_2} \right\} < 0
\end{aligned}$$

3. At this step, I need to study the (the sign of) second derivative of \bar{V}_1^{nocall} and \bar{V}_1^{call} with respect to the interest rate r_1 :

$$\begin{aligned}
\frac{\partial^2 \bar{V}_1^{nocall}(s_1, x_1)}{\partial r_1^2} &= -\frac{\partial}{\partial r_1} \left(\mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{nocall})] \frac{(1-\alpha)y_2^{nocall}}{(1+c_2)^2} \frac{b_2 - b_1}{b_2} \right\} \right) \\
&= -\mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{nocall})] (1-\alpha)y_2^{nocall} \frac{b_2 - b_1}{b_2} \frac{\partial}{\partial r_1} \left(\frac{1}{(1+c_2)^2} \right) \right\} \\
&= 2 \cdot \mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{nocall})] \frac{(1-\alpha)y_2^{nocall}}{(1+c_2)^3} \left(\frac{b_2 - b_1}{b_2} \right)^2 \right\} > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \bar{V}_1^{call}(s_1, x_1)}{\partial r_1^2} &= -\frac{\partial}{\partial r_1} \left(\mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{call})] \frac{(1-\alpha)y_2^{call}}{(1+c_2)^2} \frac{b_2 - \theta_1 b_1}{b_2} \right\} \right) \\
&= -\mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{call})] (1-\alpha)y_2^{call} \frac{b_2 - \theta_1 b_1}{b_2} \frac{\partial}{\partial r_1} \left(\frac{1}{(1+c_2)^2} \right) \right\} \\
&= 2 \cdot \mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{call})] \frac{(1-\alpha)y_2^{call}}{(1+c_2)^3} \left(\frac{b_2 - \theta_1 b_1}{b_2} \right)^2 \right\} > 0
\end{aligned}$$

Therefore, considering the two above inequalities, I arrive at both functions \bar{V}_1^{nocall} and \bar{V}_1^{call} are convex in r_1 , with other four other parameters fix.

4. Lastly, to know the relative positions of the slope of both curves, I derive their asymptotic behavior with respect to θ_1 , as follows:

$$\begin{aligned}
\frac{\partial^2 \bar{V}_1^{call}(s_1, x_1)}{\partial \theta_1 \partial r_1} &= -\frac{\partial}{\partial \theta_1} \left(\mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{call})] \frac{(1-\alpha)y_2^{call}}{(1+c_2)^2} \frac{b_2 - \theta_1 b_1}{b_2} \right\} \right) \\
&= -\mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{call})] (1-\alpha)y_2^{call} \frac{\partial}{\partial \theta_1} \left(\frac{b_2 - \theta_1 b_1}{b_2} \frac{1}{(1+c_2)^2} \right) \right\} \\
&= 2 \cdot \mathbb{E}_{z_2|z_1} \left\{ [1 - \Phi(\bar{\varepsilon}_2^{call})] \frac{(1-\alpha)y_2^{call}}{(1+c_2)^2} \frac{b_1}{b_2} \left[1 + 2 \cdot \frac{b_2 - \theta_1 b_1}{b_2} \frac{c_1 - r_1}{1+c_2} \right] \right\}
\end{aligned}$$

The sign of the derivative in the previous line depends on the sign and the amplitude of $c_1 - r_1$. Considering the case of $r_1 \rightarrow 0$, the y-intercept of $\bar{V}_1^{call}(s_1, x_1)|_{r_1=0} > \bar{V}_1^{nocall}(s_1, x_1)|_{r_1=0}, \forall \theta_1 <$

1.

$$\left. \frac{\partial^2 \bar{V}_1^{call}(s_1, x_1)}{\partial \theta_1 \partial r_1} \right|_{r_1 \rightarrow 0} > 0 \quad (\text{A29})$$

$$\left. \frac{\partial^2 \bar{V}_1^{call}(s_1, x_1)}{\partial \theta_1 \partial r_1} \right|_{r_1 \rightarrow +\infty} < 0 \quad (\text{A30})$$

5. \bar{V}_1^{nocall} coincides with \bar{V}_1^{call} when $\theta_1 = 1$: $\bar{V}_1^{call}(s_1, x_1)|_{\theta_1=1} = \bar{V}_1^{nocall}(s_1, x_1), \forall c_1$ and $\forall r_1$.

When I combine the five points above, I can now deduce the proof of the existence of the interest rate r_1^* , such that \bar{V}_1^{call} and \bar{V}_1^{nocall} cross at r_1^* , based on the intermediate value theorem.

The existence of r_1^* is then proved, therefore through the signs described in (A29) and (A30), I state that when $r_1 < r_1^*$, the exercise of the call option is the unique optimal decision: $\bar{V}_1^{nocall}(s_1, x_1) < \bar{V}_1^{call}(s_1, x_1)$. This completes the proof of Proposition 3. ■

C.8 Proof of Corollary 3.0.2

Proof. To prove Corollary 3.0.2, I need two ingredients:

1. The last terms at both sides of the inequality in (24) concern the probability of not defaulting. This term decreases in the capital quality cutoff $\bar{\varepsilon}_2$. This threshold value, in turn, increases in the periodic coupon c_2 :

$$\frac{\partial \bar{\varepsilon}_2}{\partial c_2} = \frac{b_2}{k_2} > 0 \quad (\text{A31})$$

Within the decomposition of c_2 , we showed through (A24) that θ_1 amplifies the sensitivity of the default threshold.

2. I recall the sign of $\frac{\partial^2 \bar{V}_1^{call}(s_1, x_1)}{\partial \theta_1 \partial r_1}$ in the last line (4) and use what the signs in (A29) and (A30) say. This sign shows how strong the effect of the share of non-callable θ_1 (so that the share of callable) is in amplifying the decision to call regarding the variation in market interest rates.

$$\frac{\partial^2 \bar{V}_1^{call}(s_1, x_1)}{\partial \theta_1 \partial r_1} \begin{cases} > 0 & \text{if } r_1 < r_1^* \text{ (when call)} \\ < 0 & \text{if } r_1 > r_1^* \text{ (when no call)} \end{cases} \quad (\text{A32})$$

With these inequalities, I have all elements illustrated in Figure 8. ■

C.9 Proof of the optimal callability in the condition (27)

Proof. To establish the expression of the optimal condition for the choice of θ_1 in (27), let's rewrite the F.O.C. expressed in (25) as an equality.

$$\begin{aligned} [\theta_1] : b_1 \frac{\partial p_0}{\partial \theta_1} + p_0 \frac{\partial b_1}{\partial \theta_1} - \frac{\partial \eta_b}{\partial \theta_1} + \beta_0 \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \theta_1} &= 0 \\ : \frac{\partial \bar{\varepsilon}_1}{\partial \theta_1} \left[b_1 \frac{\partial p_0}{\partial \bar{\varepsilon}_1} + p_0 \frac{\partial b_1}{\partial \bar{\varepsilon}_1} + \beta_0 \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \bar{\varepsilon}_1} \right] - \frac{\partial \eta_b}{\partial \theta_1} &= 0 \end{aligned} \quad (\text{A33})$$

First, I derive the default cutoff $\bar{\varepsilon}_1$:

$$\bar{\varepsilon}_1 = - \frac{k_1 + y_1 + \mathbb{1}_{\text{call}} \times \bar{V}_1^{\text{call}}(s_1, x_1) + (1 - \mathbb{1}_{\text{call}}) \times \bar{V}_1^{\text{nocall}}(s_1, x_1)}{k_1} \quad (\text{A34})$$

Now, I define the components of the above equation. I get the default risk derivative relative to the non-callable bond's share.

$$\frac{\partial \bar{\varepsilon}_1}{\partial \theta_1} = - \frac{1}{k_1} \times \mathbb{1}_{\text{call}} \times \frac{\partial \bar{V}_1^{\text{call}}}{\partial \theta_1} \quad (\text{A35})$$

I then consider the effects of the default risk on the price of the initial bond p_0 within both types of bond, and the expected firm's value in the intermediate period $\mathbb{E}_{s_1|s_0} V_1$.

$$\begin{aligned} \frac{\partial p_0^{nc}}{\partial \bar{\varepsilon}_1} &= -\beta_0 \mathbb{E}_{s_1|s_0} \left\{ (r_0 + \bar{p}_1^{nc}) \varphi(\bar{\varepsilon}_1) \right\} \\ \frac{\partial p_0^c}{\partial \bar{\varepsilon}_1} &= -\beta_0 \mathbb{E}_{s_1|s_0} \left\{ (r_0 + 1 + \chi) \varphi(\bar{\varepsilon}_1) \right\} \\ \frac{\partial \mathbb{E}_{s_1|s_0} V_1}{\partial \bar{\varepsilon}_1} &= -k_1 \mathbb{E}_{s_1|s_0} [1 - \Phi(\bar{\varepsilon}_1)] \end{aligned}$$

Concerning the issuance cost, its variation on the share of non-callable is described by:

$$\frac{\partial \eta_b}{\partial \theta_1} = 2 [(\eta_{nc} + \eta_c) \theta_1 - \eta_c] b_1^2 \quad (\text{A36})$$

By replacing the five above derivatives in the expression (A33), I obtain the four following components of the optimal choice of non-callable share:

1. for the non-callable share of bond, I obtain: $\beta_0 \mathbb{E}_{s_1|s_0} \left\{ \mathbb{1}_{\text{call}} \times \frac{\partial \bar{V}_1^{\text{call}}}{\partial \theta_1} \times \left[(r_0 + \bar{p}_1^{nc}) \frac{b_1}{k_1} \varphi(\bar{\varepsilon}_1) \right] \right\}$
2. for the callable share of bond, I obtain:

$$\beta_0 \mathbb{E}_{s_1|s_0} \left\{ \mathbb{1}_{\text{call}} \times \frac{\partial \bar{V}_1^{\text{call}}}{\partial \theta_1} \times \left[\mathbb{1}_{\text{call}} (r_0 + 1 + \chi) \frac{b_1}{k_1} \varphi(\bar{\varepsilon}_1) \right] \right\}$$

3. for the continuation value in next period $t = 1$, the component is:

$$\beta_0 \mathbb{E}_{s_1|s_0} \left\{ \mathbb{1}_{\text{call}} \times \frac{\partial \bar{V}_1^{\text{call}}}{\partial \theta_1} \times [1 - \Phi(\bar{\varepsilon}_1)] \right\}$$

4. for the issuance costs, the derivation is: $-2[(\eta_{nc} + \eta_c)\theta_1 - \eta_c] b_1^2$

I recall the key property of any indicator variable which is idempotent, then I use the following for the rest of the proof:

$$\mathbb{1}_{\text{call}} \times \mathbb{1}_{\text{call}} = \mathbb{1}_{\text{call}} \quad (\text{A37})$$

I use also the following simplification:

$$\mathbb{1}_{\text{call}} \times \bar{p}_1^{nc} = \bar{p}_1^{nc_call} \quad (\text{A38})$$

Back to the optimal equation on θ_1 , we have:

$$\begin{aligned} \beta_0 \mathbb{E}_{s_1|s_0} \left\{ \mathbb{1}_{\text{call}} \times \frac{\partial \bar{V}_1^{\text{call}}}{\partial \theta_1} \times \left[\theta_1 (r_0 + \bar{p}_1^{nc_call}) + (1 - \theta_1) (r_0 + 1 + \chi) \frac{b_1}{k_1} \varphi(\bar{\varepsilon}_1) + [1 - \Phi(\bar{\varepsilon}_1)] \right] \right\} \\ - 2[(\eta_{nc} + \eta_c)\theta_1 - \eta_c] b_1^2 = 0 \end{aligned}$$

Last, I bring the interest rate r_0 out of the continuation returns of the bond to derive the final expression:

$$\begin{aligned} \beta_0 \mathbb{E}_{s_1|s_0} \left\{ \mathbb{1}_{\text{call}} \times \frac{\partial \bar{V}_1^{\text{call}}}{\partial \theta_1} \times \left[[r_0 + \theta_1 \bar{p}_1^{nc_call} + (1 - \theta_1) (1 + \chi)] \frac{b_1}{k_1} \varphi(\bar{\varepsilon}_1) + [1 - \Phi(\bar{\varepsilon}_1)] \right] \right\} \\ - 2[(\eta_{nc} + \eta_c)\theta_1 - \eta_c] b_1^2 = 0 \end{aligned}$$

(A39)

■

D Full Model Appendix

D.1 Discussion of bonds pricing

D.1.1 Importance of distinct continuation prices for callable and non-callable bonds

The continuation price, denoted as \bar{p}^{nc} (or \bar{p}^c), reflects the valuation of *outstanding debt* in the next period based on the initial terms established at issuance. Distinguishing between continuation prices for non-callable and callable bonds is essential due to differences in contractual terms and payment expectations between these bonds. Specifically, the continuation price for a non-callable bond, \bar{p}^{nc} , and for a callable bond, \bar{p}^c , serve distinct functions by reflecting either fixed or adjustable payment obligations.

- **Non-callable bond and fixed coupon rate:** The continuation price \bar{p}^{nc} for a non-callable bond incorporates the original coupon rate c determined at issuance. This fixed coupon ensures that creditors holding non-callable debt are paid at a stable rate, unaffected by shifts in the current market interest rate r . Thus, \bar{p}^{nc} accurately represents the creditor's expectation of receiving consistent payments as per the bond's initial terms, upholding the contractual integrity of non-callable debt.
- **Callable bond and call decision flexibility:** The continuation price \bar{p}^c for a callable bond varies according to the firm's call decision. If the firm refrains from calling, \bar{p}^c aligns with \bar{p}^{nc} , continuing under the original coupon payments. However, if the call option is exercised, the continuation price reflects only the principal repayment plus the call premium χ , omitting future coupon payments. This adjustment is crucial for callable bondholders, who face the potential for early repayment, including a compensatory premium for their investment. Therefore, \bar{p}^c accounts for both no-call scenarios, where future coupons are expected, and call scenarios, where payment is limited to principal plus premium.

Can a single continuation price be used? Although it might seem appealing to simplify the model by using a single continuation price based on a weighted average of periodic coupon payments (as defined in (30) and (31)), this approach would introduce inaccuracies:

1. *Creditor composition differences:* Newly issued debt and outstanding debt are held by different creditor groups with distinct expectations. New creditors enter under current market conditions, while existing creditors rely on previously set coupon terms. A unified continuation price could obscure these differences, misrepresenting the expectations of each creditor class.²⁸

²⁸See e.g. [Ippolito et al. \(2018\)](#) for the weighted average periodic coupon, in the context of fixed vs floating rate debts.

2. *Contractual integrity of fixed coupons:* Outstanding debt is typically structured with fixed coupons that do not vary with interest rate changes. Averaging coupons across different debt types would imply contractual adjustments to these fixed payments, contradicting the standard fixed-rate structure in financial contracts.

Thus, maintaining distinct continuation prices—one based on the fixed coupon for outstanding debt and another on the current interest rate for newly issued debt—provides an accurate depiction of the firm’s obligations and aligns with the varying expectations of each creditor group. For non-callable bonds, this distinction ensures stability through a fixed coupon rate, while for callable bonds, it allows for flexible adjustment based on the firm’s optimal call decision. This dual pricing approach is particularly valuable for accurately modeling scenarios with mixed debt instruments, as it reflects the financial strategies of firms in fluctuating market conditions and their impact on bondholder returns.

D.1.2 Callable bond pricing

The callable bond pricing model must incorporate both the firm’s endogenous call decision and the potential loss for bondholders when the call option is exercised. Equation (42) reflects this structure, providing a dynamic framework that captures how the firm optimizes its value under the possibility of calling the bond.

1. Endogenous Call Decision:

In this framework, the firm decides whether to call the bond by comparing the **value of calling** with the **value of holding the bond until maturity**. The decision to call is endogenous, depending on the firm’s state variables s , and policy functions $\hat{x}(s, x)$. This decision is modeled through the call indicator $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}(s', \hat{x}(s, x))$, which determines whether the callable bond will be called or not. This allows for a more accurate representation of the firm’s incentives and behavior, as opposed to assuming a fixed probability or exogenously determined call timing. The literature, such as [Duffie and Singleton \(1999\)](#); [Jarrow et al. \(2010\)](#), often relies on exogenous factors to model the call decision, which may not fully capture the issuer’s strategic behavior in response to market conditions (See also [Chen et al. \(2010\)](#)).

2. Min Function and Redundancy:

In traditional models, the **min function** is used to ensure that the bondholder’s payoff does not exceed the call price, even if the market price of the bond exceeds it. However, in our extended model, this comparison is **internalized** in the firm’s value-maximizing decision to call or not. Therefore, the **min function** becomes redundant since the firm already

optimizes between calling the bond or not based on the comparison of its continuation value. The decision rule internalizes the market price dynamics and the call price, making the explicit use of **min** unnecessary (See [Brennan and Schwartz \(1977\)](#) for classical discussions on callable bond pricing.).

3. The model incorporates a dynamic continuation price for both callable and non-callable bonds, taking into account future state variables and policy functions, as is done in the dynamic debt model literature in macroeconomics (e.g. [Crouzet \(2017\)](#); [Karabarbounis and Macnamara \(2021\)](#); [Jungherr and Schott \(2021\)](#)). This contrasts with simpler models where the continuation value is often static or based on less granular assumptions. By considering the future coupon payments and the possibility of a call, the model provides a more comprehensive valuation that aligns with real market dynamics.
4. The call premium is intrinsically set as a function of the prevailing interest rate r , making the model sensitive to market conditions at issuance. This allows for the call premium to decrease as the bond matures without being called, reflecting the decreasing call risk. This feature provides a more accurate representation of the bond's pricing over its life, something not always addressed in the literature where call premiums may be assumed constant or independent of market rates.

D.2 First Order Conditions

In this subsection, I present the first-order conditions for the decision variables: the capital k' , the debt b' , and the share of non-callable θ' . Without any specificity, I consider the rewritten value function defined in (38), so for a given state variables $s = \{z, r\}$, $x = \{k, b, \theta, c\}$, for the case $j \in \{\text{nocall}, \text{call}\}$ we have:

$$\bar{V}_j(s, x) = \max_{e, k'_j, b'_j, \theta'_j, c'_j} \left\{ -k'_j + n_j + p\tilde{b}_j - \eta_b(\tilde{b}_j) - \eta_e(e) + \frac{1}{1+r} \mathbb{E}_{s'|s} \int_{\bar{\varepsilon}'}^{\infty} V^r(s', x') d\Phi(\varepsilon') \right\} \quad (\text{A1})$$

The weighted average price of the new debt is: $p = \tilde{\theta}_j p^{nc} + (1 - \tilde{\theta}_j) p^c$. I define a new pseudo-value function of the firm which consider the part of its value function concerned by its optimal choices.

$$W_j(s', x') = -k'_j + p\tilde{b}_j - \eta_b(\tilde{b}_j) - \eta_e(e) + \frac{1}{1+r} \mathbb{E}_{s'|s} \int_{\bar{\varepsilon}'}^{\infty} V^r(s', x') d\Phi(\varepsilon') \quad (\text{A2})$$

D.2.1 Optimal capital k'

The firm's first-order condition for capital k'_j follows:

$$\left(1 + \frac{\partial \eta_e(e)}{\partial e}\right) \left\{ -1 + b' \left[\tilde{\theta}_j \frac{\partial p^{nc}}{\partial k'_j} + (1 - \tilde{\theta}_j) \frac{\partial p^c}{\partial k'_j} \right] \right\} + \frac{1}{1+r} \mathbb{E}_{s'|s} \int_{\bar{\varepsilon}'}^{\infty} \frac{\partial V^r(s', x')}{\partial k'_j} d\Phi(\varepsilon') = 0 \quad (\text{A3})$$

Where the repayment value evolves depending on the capital as follows:

$$\begin{aligned} \frac{\partial V^r(s', x')}{\partial k'_j} = & \left[1 - \mathbb{1}'_{\text{call}} \right] \left\{ \frac{\partial n'_{\text{nocall}}}{\partial k'_j} \cdot \left[(1 - \pi_e) \mathbb{E}_{s''|s'} \left(1 + \frac{\partial \eta_e(e')}{\partial e'} \right) + \pi_e \left(1 - (1 - \gamma) b' \mathbb{E}_{s''|s'} \frac{\partial p'_{\text{nocall}}}{\partial n'_{\text{nocall}}} \right) \right] \right\} \\ & + \mathbb{1}'_{\text{call}} \left\{ \frac{\partial n'_{\text{call}}}{\partial k'_j} \cdot \left[(1 - \pi_e) \mathbb{E}_{s''|s'} \left(1 + \frac{\partial \eta_e(e')}{\partial e'} \right) + \pi_e \left(1 - (1 - \gamma) \theta' b' \mathbb{E}_{s''|s'} \frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}} \right) \right] \right\} \end{aligned}$$

where $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}(s', x')$ indicates whether the firm will choose to call the callable debt $(1 - \theta')$ in the next period; $n'_{\text{nocall}} = n_{\text{nocall}}(s', x')$, and $n'_{\text{call}} = n_{\text{call}}(s', x')$ are respectively the internal fund of the firm after producing for both future choices. Then, we have:

$$\forall j \in \{\text{nocall}, \text{call}\}, \frac{\partial n'_j}{\partial k'_j} = \frac{\partial n_j(s', x')}{\partial k'_j} = 1 + (1 - \tau) [z' \alpha k_j'^{\alpha-1} + \epsilon' - \delta] \quad (\text{A4})$$

To obtain the derivatives of the prices relative to the next period capital, I need the effects on the default threshold, $\bar{\varepsilon}'$, which is defined implicitly in (33).

$$\frac{\partial \bar{\varepsilon}}{\partial k'_j} = - \frac{1 + (1 - \tau) (z' \alpha k_j'^{\alpha-1} + \epsilon' - \delta)}{(1 - \tau) k'_j} \quad (\text{A5})$$

I can then compute the non-callable bond price derivative with respect to k'_j :

$$\begin{aligned} \frac{\partial p^{nc}}{\partial k'_j} = & \beta \mathbb{E}_{s'|s} \left\{ \int_{\bar{\varepsilon}'}^{\infty} (1 - \gamma) \mathbb{E}_{s''|s'} \left[(1 - \mathbb{1}'_{\text{call}}) \frac{\partial p'^{nc}_{\text{nocall}}}{\partial n'_{\text{nocall}}} \frac{\partial n'_{\text{nocall}}}{\partial k'_j} + \mathbb{1}'_{\text{call}} \frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}} \frac{\partial n'_{\text{call}}}{\partial k'_j} \right] d\Phi(\varepsilon') \right. \\ & + \int_{-\infty}^{\bar{\varepsilon}'} \frac{1 - \xi}{b'} [1 + (1 - \tau) (z' \alpha k_j'^{\alpha-1} + \epsilon' - \delta)] d\Phi(\varepsilon') \\ & \left. + \varphi(\bar{\varepsilon}') \left[-(\gamma + r + (1 - \gamma) \mathbb{E}_{s''|s'} p'^{nc}) + \frac{1 - \xi}{b'} \underline{n}(\bar{\varepsilon}') \right] \frac{\partial \bar{\varepsilon}'}{\partial k'_j} \right\} \end{aligned}$$

I derive the variation of the callable bond price with respect to k'_j :

$$\begin{aligned} \frac{\partial p^c}{\partial k'_j} = & \beta \mathbb{E}_{s'|s} \left\{ \int_{\bar{\varepsilon}'}^{\infty} (1 - \gamma) \mathbb{E}_{s''|s'} \left[(1 - \mathbb{1}'_{\text{call}}) \frac{\partial p^c}{\partial n'_{\text{nocall}}} \frac{\partial n'_{\text{nocall}}}{\partial k'_j} \right] d\Phi(\varepsilon') \right. \\ & + \int_{-\infty}^{\bar{\varepsilon}'} \frac{1 - \xi}{b'} [1 + (1 - \tau) (z' \alpha k_j'^{\alpha-1} + \epsilon' - \delta)] d\Phi(\varepsilon') \\ & \left. + \varphi(\bar{\varepsilon}') \left[-[r + (1 - \mathbb{1}'_{\text{call}}) [\gamma + (1 - \gamma) \mathbb{E}_{s''|s'} p_{\text{call}}^c] + \mathbb{1}'_{\text{call}} (1 + \chi)] + \frac{1 - \xi}{b'} \underline{n}(\bar{\varepsilon}') \right] \frac{\partial \bar{\varepsilon}'}{\partial k'_j} \right\} \end{aligned}$$

D.2.2 Optimal debt b'

Now, I derive the optimal condition for the choice of the next period level of debt b' .

$$\left(1 + \frac{\partial \eta_e(e)}{\partial e} \right) \left\{ p + b' \left[\tilde{\theta}_j \frac{\partial p^{nc}}{\partial b'} + (1 - \tilde{\theta}_j) \frac{\partial p^c}{\partial b'} \right] - \frac{\partial \eta_b(b')}{\partial b'} \right\} + \frac{1}{1 + r} \mathbb{E}_{s'|s} \int_{\bar{\varepsilon}'}^{\infty} \frac{\partial V^r(s', x')}{\partial b'} d\Phi(\varepsilon') = 0 \quad (\text{A6})$$

Where the derivative of the repayment value with respect to b' is:

$$\begin{aligned} \frac{\partial V^r(s', x')}{\partial b'} = & [1 - \mathbb{1}'_{\text{call}}] \mathbb{E}_{s''|s'} \left\{ \frac{\partial n'_{\text{nocall}}}{\partial b'} \cdot \left[(1 - \pi_e) \left(1 + \frac{\partial \eta_e(e')}{\partial e'} \right) + \pi_e \left(1 - (1 - \gamma) b' \frac{\partial p'_{\text{nocall}}}{\partial n'_{\text{nocall}}} \right) \right] \right. \\ & \left. + \frac{\partial \tilde{b}'_{\text{nocall}}}{\partial b'} \cdot \left[(1 - \pi_e) \frac{\partial W(s'', x''_{\text{nocall}})}{\partial \tilde{b}'_{\text{nocall}}} - \pi_e \left(p'_{\text{nocall}} + b' \frac{\partial p'_{\text{nocall}}}{\partial n'_{\text{nocall}}} \right) \right] \right\} \\ & + \mathbb{1}'_{\text{call}} \mathbb{E}_{s''|s'} \left\{ \frac{\partial n'_{\text{call}}}{\partial b'} \cdot \left[(1 - \pi_e) \left(1 + \frac{\partial \eta_e(e')}{\partial e'} \right) + \pi_e \left(1 - (1 - \gamma) \theta' b' \frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}} \right) \right] \right. \\ & \left. + \frac{\partial \tilde{b}'_{\text{call}}}{\partial b'} \cdot \left[(1 - \pi_e) \frac{\partial W(s'', x''_{\text{call}})}{\partial \tilde{b}'_{\text{call}}} - \pi_e \left(p'^{nc}_{\text{call}} + b' \frac{\partial p'^{nc}_{\text{call}}}{\partial n'_{\text{call}}} \right) \right] \right\} \end{aligned}$$

where the function $W(s'', x''_j)$ is the function defined in (A2) when the decision j will be taken the next period. $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}(s', x')$ indicates whether the firm will choose to call the callable debt $(1 - \theta')$ in the next period; $n'_{\text{nocall}} = n_{\text{nocall}}(s', x')$, and $n'_{\text{call}} = n_{\text{call}}(s', x')$ are respectively the internal fund of the firm after producing for both future choices. Additionally, I define the following components of the above derivatives.

When the firm does not call in the next period, $\mathbb{1}'_{\text{call}} = 0$, $\frac{\partial n'_{\text{nocall}}}{\partial b'} = -(\gamma + (1 - \tau)c')$, and $\frac{\partial \tilde{b}'_{\text{nocall}}}{\partial b'} = 1 - \gamma$, while when the firm decides to call in the next period, $\mathbb{1}'_{\text{call}} = 1$, we have $\frac{\partial n'_{\text{call}}}{\partial b'} = -[\gamma\theta' + (1 + \chi)(1 - \theta') + (1 - \tau)c']$, and $\frac{\partial \tilde{b}'_{\text{call}}}{\partial b'} = \theta'(1 - \gamma)$.

I derive the variation of the pseudo-value functions. In the no-call scenario in the next period, we have:

$$\frac{\partial W(s'', x''_{\text{nocall}})}{\partial \tilde{b}'_{\text{nocall}}} = -p'_{\text{nocall}} \left(1 + \frac{\partial \eta_e(e')}{\partial e'} \right) - \frac{\partial \eta_b(\tilde{b}'_{\text{nocall}})}{\partial \tilde{b}'_{\text{nocall}}} \quad (\text{A7})$$

and for the call scenario, we have:

$$\frac{\partial W(s'', x''_{\text{call}})}{\partial \tilde{b}'_{\text{call}}} = -p'^{\text{nc}}_{\text{call}} \left(1 + \frac{\partial \eta_e(e')}{\partial e'} \right) - \frac{\partial \eta_b(\tilde{b}'_{\text{call}})}{\partial \tilde{b}'_{\text{call}}} \quad (\text{A8})$$

Lastly, to complete the optimal conditions components, I derive the prices sensitivities to the next period debt level.

I compute the non-callable bond price derivative with respect to b' :

$$\begin{aligned} \frac{\partial p^{\text{nc}}}{\partial b'} = & \beta \mathbb{E}_{s'|s} \left\{ \int_{\bar{\varepsilon}'}^{\infty} (1 - \gamma) \mathbb{E}_{s''|s'} \left[(1 - \mathbb{1}'_{\text{call}}) \left(\frac{\partial p'^{\text{nc}}_{\text{nocall}}}{\partial n'_{\text{nocall}}} \frac{\partial n'_{\text{nocall}}}{\partial b'} + \frac{\partial p'^{\text{nc}}_{\text{nocall}}}{\partial \tilde{b}'_{\text{nocall}}} \frac{\partial \tilde{b}'_{\text{nocall}}}{\partial b'} \right) \right. \right. \\ & \left. \left. + \mathbb{1}'_{\text{call}} \left(\frac{\partial p'^{\text{nc}}_{\text{call}}}{\partial n'_{\text{call}}} \frac{\partial n'_{\text{call}}}{\partial b'} + \frac{\partial p'^{\text{nc}}_{\text{call}}}{\partial \tilde{b}'_{\text{call}}} \frac{\partial \tilde{b}'_{\text{call}}}{\partial b'} \right) \right] d\Phi(\varepsilon') \right. \\ & - \int_{-\infty}^{\bar{\varepsilon}'} \frac{1 - \xi}{(b')^2} [k' + (1 - \tau)(z'k'^{\alpha} + \epsilon'k' - \delta k' - f)] d\Phi(\varepsilon') \\ & \left. + \varphi(\bar{\varepsilon}') \left[-(\gamma + r + (1 - \gamma) \mathbb{E}_{s''|s'} p'^{\text{nc}}) + \frac{1 - \xi}{b'} \underline{n}(\bar{\varepsilon}') \right] \frac{\partial \bar{\varepsilon}'}{\partial b'} \right\} \quad (\text{A9}) \end{aligned}$$

For the callable bond, the difference stands from the call price in the call scenario. I compute the non-callable bond price derivative with respect to b' :

$$\begin{aligned} \frac{\partial p^c}{\partial b'} = & \beta \mathbb{E}_{s'|s} \left\{ \int_{\bar{\varepsilon}'}^{\infty} (1 - \gamma) \mathbb{E}_{s''|s'} \left[(1 - \mathbb{1}'_{\text{call}}) \left(\frac{\partial p'^c_{\text{nocall}}}{\partial n'_{\text{nocall}}} \frac{\partial n'_{\text{nocall}}}{\partial b'} + \frac{\partial p'^c_{\text{nocall}}}{\partial \tilde{b}'_{\text{nocall}}} \frac{\partial \tilde{b}'_{\text{nocall}}}{\partial b'} \right) \right. \right. \\ & - \int_{-\infty}^{\bar{\varepsilon}'} \frac{1 - \xi}{(b')^2} [k' + (1 - \tau)(z'k'^{\alpha} + \epsilon'k' - \delta k' - f)] d\Phi(\varepsilon') \\ & \left. + \varphi(\bar{\varepsilon}') \left[-[r + (1 - \mathbb{1}'_{\text{call}}) [\gamma + (1 - \gamma) \mathbb{E}_{s''|s'} p'^c_{\text{call}}] + \mathbb{1}'_{\text{call}}(1 + \chi)] + \frac{1 - \xi}{b'} \underline{n}(\bar{\varepsilon}') \right] \frac{\partial \bar{\varepsilon}'}{\partial b'} \right\} \quad (\text{A10}) \end{aligned}$$

E Quantitative Appendix

In this section, I describe the model counterparts of key empirical moments for the full model, present the solution method, and discuss further quantitative results.

E.1 Model Moments

This subsection guides the connection between the theoretical framework and the empirical evidence presented earlier. The empirical moments we focus on include bond pricing, firm default, debt issuance, and investment dynamics, which are all essential for understanding the impact of debt callability on firm behavior and aggregate outcomes.

At the long-term steady-state equilibrium, several simplifying assumptions hold to facilitate the calculation of model moments. Specifically, the coupon rate (c^*) equals the risk-free rate (r^*), implying that the prices of riskless non-callable and callable debts are normalized to 1: $c^* = r^*$.

E.1.1 Bond Pricing and Credit Spreads

The pricing of callable and non-callable debt within the model is fundamental for understanding corporate financing strategies. The bond price is determined by evaluating the firm's decision to either call or not call the debt. This decision hinges on comparing the present value of callable debt, which includes the call premium, against the value of non-callable debt.

The price of callable debt in the model is determined by comparing the call and no-call decisions. The bond price depends on whether the firm exercises the call option and the associated future expected value of the callable and non-callable debt. The price computation is done for each state, accounting for firm productivity, interest rates, debt levels, and capital.

$$\begin{aligned} \text{average non-callable price} &= \hat{p}^{nc} \\ &= \mathbb{E}_{z,r} \int_K \int_B \int_{\Theta} \int_{\bar{C}} (1 - \mathbb{1}_{\text{call}}) \cdot \bar{p}_{\text{nocall}}^{nc} + \mathbb{1}_{\text{call}} \cdot \bar{p}_{\text{call}}^{nc} \quad (\text{A1}) \end{aligned}$$

where $\bar{p}_j^{nc} = \bar{p}_j^{nc}(z, r, k', b', \theta, c)$ for $j \in \{\text{nocall}, \text{call}\}$ are defined in Equation (41). K , B , Θ , and \bar{C} are the state spaces for respectively the state variables k , b , θ , and c , they are defined in the subsection below. The indicator $\mathbb{1}_{\text{call}}$ denotes whether the firm opts to call the debt in the current period.

Similarly, the price of callable debt is computed as:

$$\begin{aligned} \text{average callable price} &= \hat{p}^c \\ &= \mathbb{E}_{z,r} \int_K \int_B \int_{\Theta} \int_{\bar{C}} (1 - \mathbb{1}_{\text{call}}) \cdot \bar{p}_{\text{nocall}}^c + \mathbb{1}_{\text{call}} \cdot \bar{p}_{\text{call}}^c \end{aligned} \quad (\text{A2})$$

where $\bar{p}_j^c = \bar{p}_j^c(z, r, k', b', \theta, c)$ for $j \in \{\text{nocall}, \text{call}\}$ are defined in Equation (44). These prices are essential for calculating the average credit spreads, which reflect the compensation investors require for bearing credit risk.

Now, I derive the credit spreads. The credit spread measures the additional yield that investors demand to hold corporate debt over risk-free government bonds. It is the key indicator of the perceived riskiness of the firm's debt. At the steady state equilibrium, the prices have characteristics with fixed constant average call rate, and constant rate, and the price of riskless non-callable and callable debts is 1.

The credit spread on non-callable debt is calculated as:

$$\text{spread}^{nc} = \left[\frac{\gamma + c^*}{\hat{p}^{nc}} + (1 - \gamma) \right]^4 - (1 + r^*)^4 \quad (\text{A3})$$

Here, γ represents the quarterly rate of debt repayment, and c^* is the steady-state coupon rate on the firm's debts. The exponent 4 annualizes the quarterly returns, aligning with the typical frequency of debt repayments.

For callable debt, the spread is given by:

$$\text{spread}^c = \left\{ \frac{1}{\hat{p}^c} \left[c^* + (1 - q^*) \left(\gamma + (1 - \gamma) \hat{p}^c \right) + q^* (1 + \chi) \right] \right\}^4 - (1 + r^*)^4 \quad (\text{A4})$$

In this equation, q^* denotes the average call rate, and χ is the call premium. The spread on callable debt accounts for the additional costs associated with the option to call, reflecting the increased risk for investors.

The overall average credit spread is a weighted average of the spreads on non-callable and callable debt:

$$\text{spread} = \theta^* \cdot \text{spread}^{nc} + (1 - \theta^*) \cdot \text{spread}^c \quad (\text{A5})$$

where θ^* is the steady-state share of non-callable debt. This aggregation ensures that the model captures the combined effect of both debt types on the overall credit risk premium.

E.1.2 Rates of exit

First, the default rate reveals the probability that a firm will fail to meet its debt obligations, influenced by productivity shocks, debt levels, and most importantly capital quality shock. It is determined by the threshold $\bar{\varepsilon}$, below which a firm defaults.

$$\text{default rate} = \mathbb{E}_{z,r} \int_K \int_B \int_{\Theta} \int_{\bar{C}} \mathbb{P}[\varepsilon \leq \bar{\varepsilon}] = \mathbb{E}_{z,r} \int_K \int_B \int_{\Theta} \int_{\bar{C}} \Phi[\bar{\varepsilon}] = \mathbb{E}_{z,r} \int_K \int_B \int_{\Theta} \int_{\bar{C}} \int_{\bar{\varepsilon}}^{\infty} d\Phi(\varepsilon) \quad (\text{A6})$$

where Φ is the cumulative distribution function of capital quality shocks, and $\bar{\varepsilon}$ is the default threshold in the current period. This expectation integrates over all possible states, providing an average default probability across the economy.

The exit rate accounts for the probability that a firm exits the market, either through default or exogenously. It is a combination of the default rate and the probability of an exogenous exit, π_e . As described on the timing of the model (9), the firm decides to default or not before the exogenous exit shock:

$$\text{exit rate} = \text{default rate} + (1 - \text{default rate}) \cdot \pi_e \quad (\text{A7})$$

E.1.3 Callable bond lifespan

The average callable bond lifespan measures the expected duration that callable bonds remain outstanding before being redeemed by the firm. This moment captures the temporal aspect of debt callability, reflecting how often and under what conditions firms choose to exercise their call options.

$$\text{average callable bond lifespan} = \mathbb{E}_{z,r} \int_K \int_B \int_{\Theta} \int_{\bar{C}} \hat{T}(z, r, k, b, \theta, c) \cdot \mathbb{1}_{\text{callable}}(z, r, k, b, \theta, c) \quad (\text{A8})$$

where $\hat{T}(z, r, k, b, \theta, c)$ represents the remaining lifespan of a callable bond given the current state. $(1 - \theta(z, r, k, b, \theta, c))$ denotes the share of callable debt in the current state, ensuring that only callable bonds contribute to the average lifespan. This moment constitutes one of this paper's contributions to the literature of macroeconomic implications of firms' debt heterogeneity. It helps for understanding the persistence of callable debt in the firm's capital structure and its implications for refinancing and financial flexibility.

E.1.4 Callability

The callable share rate measures the proportion of a firm's debt that is callable, reflecting the firm's reliance on callable debt instruments for financing flexibility. I consider the average value of the optimal share of callable within the solution of the model.

$$\text{callable share rate} = (1 - \theta^*) = \mathbb{E}_{z,r} \int_K \int_B \int_{\Theta} \int_{\bar{C}} (1 - \theta'(z, r, k, b, \theta, c)) \quad (\text{A9})$$

where K , B , Θ , and \bar{C} are the state spaces for respectively the state variables k , b , θ , and c .

The decision to call the bond is based on comparing the value of calling vs. not calling, considering the cost of paying the call premium and the potential gains from refinancing. Therefore, the average call rate quantifies the frequency with which firms exercise their option to call debt before maturity, influenced by factors such as interest rate movements, debt levels, and firm productivity.

$$\text{average call rate} = q^* = \mathbb{E}_{z,r} \int_K \int_B \int_{\Theta} \int_{\bar{C}} \mathbb{1}'_{\text{call}}(s', \hat{x}(s, x)) \cdot (1 - \theta'(z, r, k, b, \theta, c)) \quad (\text{A10})$$

where $\mathbb{1}'_{\text{call}} = \mathbb{1}_{\text{call}}(s', \hat{x}(s, x))$ is the indicator of the call decision the next period, and $\hat{x}(s, x)$ is the vector of policy functions. This formula ensures that the call rate reflects only those bonds eligible for calling.

E.1.5 Investment

The investment rate reflects the proportion of a firm's earnings allocated to capital investments, influenced by factors such as profitability, fixed operating costs, and available financing.

$$\text{Investment Rate} = \mathbb{E}_{z,r} \int_K \int_B \int_{\Theta} \int_{\bar{C}} I(z, r, k, b, \theta, c) \quad (\text{A11})$$

Where $I(z, r, k, b, \theta, c)$ denotes the firm's investment decision function. This moment serves to control the aggregate investment behavior of firms, and the capital accumulation.

E.2 Solving the Model

The components of the model solution are the value functions, the bond price schedules, and the firm-level policies on k' , b' , θ' , c' . To solve it, I use dynamic programming and numerical algorithms. The procedure iterates on the value functions and the price functions until the convergence. I start by discretizing the state space. Using the method in [Tauchen \(1986\)](#), I transform the AR(1) process of the productivity z into a 25-point grid, and the interest rate turns into a 10-point grid. For other state variables, I set a 25-point grid for bonds b , a 10-point grid for non-callable

share θ , and a 10-point grid for the periodic coupon c .

At every step of the algorithm, I use interpolation on b, θ, c grids to approximate the policy functions. The main algorithm to solve the model works through the following steps.

1. I start with the finite-horizon (T periods) version of the model. I set the final value and price to be null, $V_{T+1} = 0, p = 0$.
2. I set the initial guesses for value and price functions, by solving the period $T - 1$. In this case, I set the policy on new debt issued such as only non-callable bonds are issued ($\tilde{\theta}_T = 1$), and the parameter of debt repayment at $\gamma = 1$, i.e. full repayment at the final period of non-callable bonds and unmatured and uncalled callable bonds. The following lines detail this step:
 - (a) Guess of capital k_T , deduction of the labor supply, and using labor market clearing, get the wage. Guess future non-callable bonds share $\tilde{\theta}$ (for the period T , $\tilde{\theta}_T = 1$), and compute the periodic coupon for the next period, given by (30) and (31).
 - (b) Using these values, for each point on the space state grid, solve the two optimization problems in (38), subsequently the call decision problem in (36), then in (34), and the optimal value in (32). To do it, first, given the guessed k_T and continuation value of decision variables, I compute the default $\bar{\varepsilon}$ given by (33) and the default probability. Second, I compute the prices p^{nc} , \bar{p}_j^{nc} , p^c , and $\bar{p}_j^c, j \in \{\text{nocall}, \text{call}\}$, using (40)-(44) through inner loop-outer loop procedure.
 - (c) Based on these objects, I optimize the firm objective function to obtain the optimal firm's decision k, b, θ, c and the prices' schedules.
 - (d) The continuation values and prices obtained here serve as the new guesses for the rest of the algorithm.
3. For each period $t < T - 1$, I use these guesses as the continuation values \bar{V}_j and prices p^{nc} , \bar{p}_j^{nc} , p^c , and \bar{p}_j^c for the rest of the algorithm.
4. The solution is found if the distance between the new and previous continuation values \bar{V}_j , and the distance between the new and previous prices p^{nc} , p^c are below the tolerance level (set at 10^{-6}). If this criterion is not satisfied, I get back to step (2) using these new \bar{V}_j and prices p^{nc} , \bar{p}_j^{nc} , p^c , and \bar{p}_j^c as the new continuation values and prices.