SLE via QIO

Andrew Yaremenko

Description

Solving linear equation systems using quantum inspired optimization (QIO) for solving quadratic unconstrained binary optimizatio problem (QUBO)

Formulas

$$Ax = b \iff \|Ax - b\|^2 \longrightarrow \min \iff x^T A^T Ax - 2b^T Ax \longrightarrow \min$$

$$x = A^T (AA^T)^{-1} b$$

$$\|x\|^2 = b^T (AA^T)^{-1} AA^T (AA^T)^{-1} b = b^T (AA^T)^{-1} b$$

$$\|x\|^2 \leq \|b\|^2 \ \|(AA^T)^{-1}\| \leq \frac{\|b\|^2}{\text{smallest singular value of } AA^T}$$

Algorithm 1

1. Formulate SLE as quadratic minimization problem
2. Pick initial bounds for each coordinate
3. Substitute $x_i = lb_i + \frac{ub_i - lb_i}{2^p}\left(\frac{1}{2} + q_{i1} + 2q_{i2} + 4q_{i3} + ... + 2^{p-1} q_{p-1i}\right)$
4. Solve QUBO for q_{ij}
5. Update bounds with the neighbourhood of found solution
6. Repeat untill bounds are small enough

Improvement 1

After finding a solution x_0, substitute $y = x + x_0$ and repeat the algorithm.

Improvement 2

Add perturbation to step 3. Choose the constant and the coefficients of q_{ij} with the following procedure:

```
rnd = ss.truncnorm(-1 / 2 / sigma, 1 / 2 / sigma, 1 / 2, sigma).rvs()
mn, mx = rnd, rnd
const = rnd * lengths[i] + lb[i]
coefs = []
for k in range(prec):
    rnd = ss.truncnorm(-mn / sigma, (1 - mx) / sigma, 0, sigma).rvs()
    coefs += [(rnd + 2 ** k) * lengths[i]]
    mn = min(mn, mn + rnd)
    mx = max(mx, mx + rnd)
```