Решение СЛАУ при помощи QIO

Яременко Андрей

Description

Solving linear equation systems using quantum inspired optimization (QIO) for solving quadratic unconstrained binary optimization (QUBO)

Formulas

$$Ax = b \Longleftrightarrow \|Ax - b\|^2 \longrightarrow \min \Longleftrightarrow x^T A^T Ax - 2b^T Ax \longrightarrow \min$$

$$x = A^T \left(AA^T\right)^{-1} b \ - \text{ is one solution}$$

$$A^T A \text{ has n real non-negative eigenvalues}$$

$$\|x\|^2 = b^T \left(AA^T\right)^{-1} AA^T \left(AA^T\right)^{-1} b = b^T \left(AA^T\right)^{-1} b$$

$$\|x\|^2 \leq \|b\|^2 \ \|\left(AA^T\right)^{-1}\| \leq \frac{\|b\|^2}{\text{smallest singular value of } AA^T} \ - \text{ bound on the solution}$$

Generally there is no upper bound on the solution because A can be arbitrarily close to singular. Also we don't want to compute SVD or the inverse matrix.

Algorithms

Initial algorithm

1. Find bounds on x_i
2. Folmulate initial problem in terms of quadratic optimization
3. Split the hypercube of possible solutions into 2^n parts and find their middle points ($x_i = x_i' + \Delta_i q_i,\ \ q_i \in \{0, 1\}$)
4. Solve QUBO problem (substitute new variables and use $0^2 = 0,\ \ 1^2 = 1$)
5. Update bounds and repeat.

Cons: Doesn't necessarily converge to an exact solution. Also has issues with errors in quantum computations.

Potential improvements:

1. Choose points randomly
2. Run multiple times, subtract previous solution and scale up system to increase precision and reliability.

Generalized algorithm from the paper

1. Find bounds on x_i
2. Folmulate initial problem in terms of quadratic optimization

3. Represent variables with finite precision ($x_i = (-2^p + 2^r)q_p + \sum_{i=r}^{p-1} 2^i q_i$)
4. Solve QUBO problem with new variables

Cons: Number of variables grows quadratically with precision.