CS 535 Homework 6

Due: 6pm Dec. 2, 2022.

1. The following questions highlight the differences betwen MSA and its undirected counterpart MST.

   (a) Give an example of a digraph in which the lightest edge does not belong to any MSA rooted at r, even though it does belong to some SA rooted at r.

   (b) Give an example of a digraph in which the heaviest edge belongs to each MSA rooted at r, even though there are some SA rooted at r that avoids it.

   (c) Give an example of a digraph in which all edge weights are distinct but the MSA is not unique.

2. Consider the following instance of the **Load Balancing** problem: There are m machines and $n = 2m+1$ jobs: 3 jobs are of length m, and 2 jobs are of length $m+i$ for each $1 \leq i \leq m-1$. Show that on this instance the list-scheduling algorithm with the LPT rule achieves the approximation ratio $\frac{4}{3} - \frac{1}{3m}$.

3. Consider the following *preemptive* variant of the **Load Balancing** problem: There are m identical machines, and n jobs with processing times $t_1, t_2, \cdots, t_n$ respectively. Each machine can process at most one job at a time. Each job can run on more than one machines but must run on at most one machine at any time. Let $T = \max\left\{\max_{1\leq j\leq n} t_j, \frac{1}{m}\sum_{1\leq j\leq n} t_j\right\}$.

   (a) Prove that the optimal makespan is at least T.

   (b) Give a linear-time algorithm which produces a preemptive schedule with makespan exactly T.

4. Consider the following instance of the **Bin Packing** problem: Suppose $0 < \varepsilon < 1/8$ and $n > 1$. There are $30n$ items: $6n$ items of size $1/2 + \varepsilon$, $6n$ items of size $1/4 + 2\varepsilon$, $6n$ items of size $1/4 + \varepsilon$, and $12n$ items of size $1/4 - 2\varepsilon$. All bins have unit capacity. Show that on this instance the FFD algorithm achieves the approximation ratio 11/9.

5. Give a 4-appoximation algorithm for finding a maximum cut in a directed graph $D = (V, A)$.

6. **[PhD Session only]** Prove that the list-scheduling algorithm with the LPT rule for the **Load Balancing** problem has approximation ratio $\frac{4}{3} - \frac{1}{3m}$.