

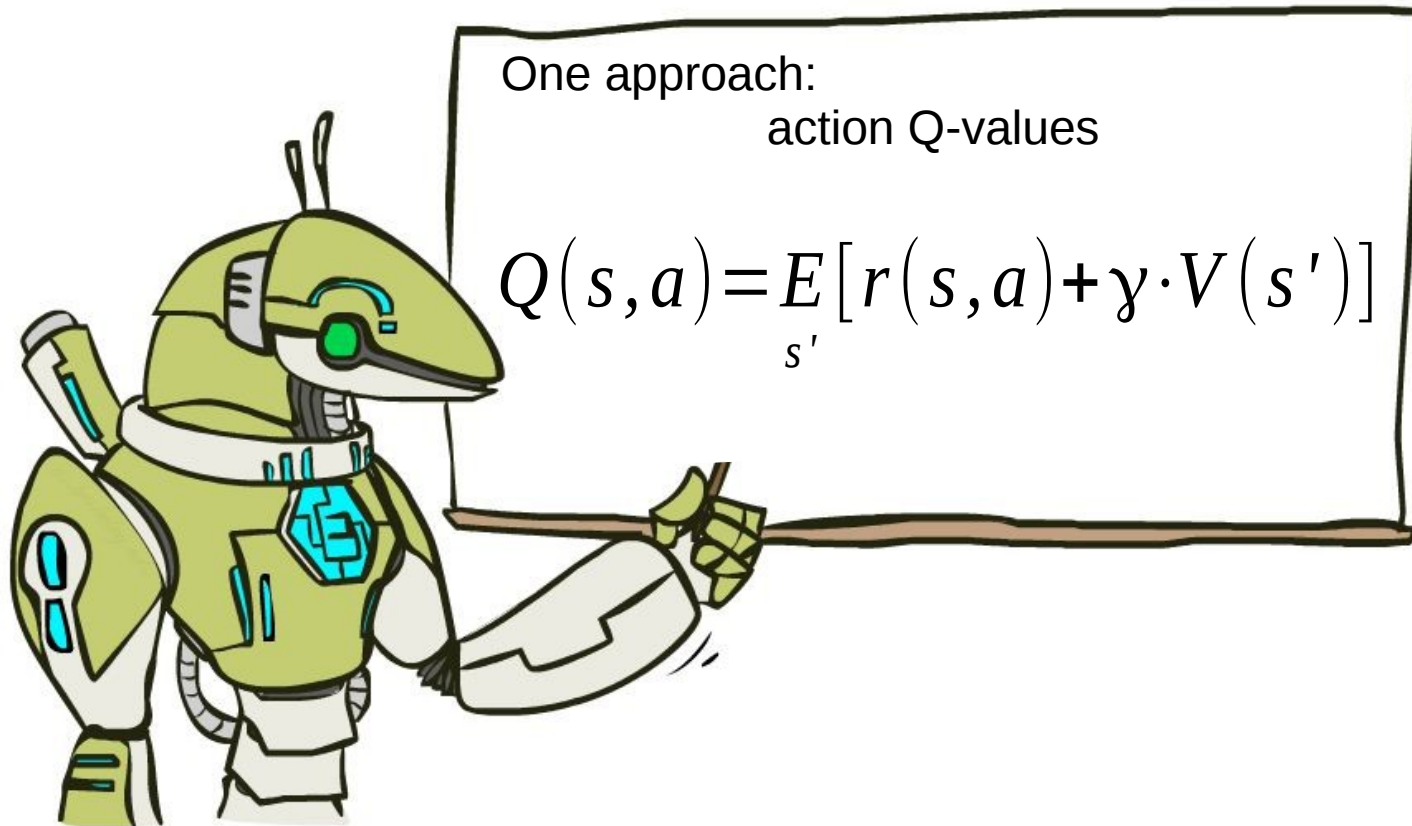
Reinforcement learning

Episode 4

Approximate reinforcement learning



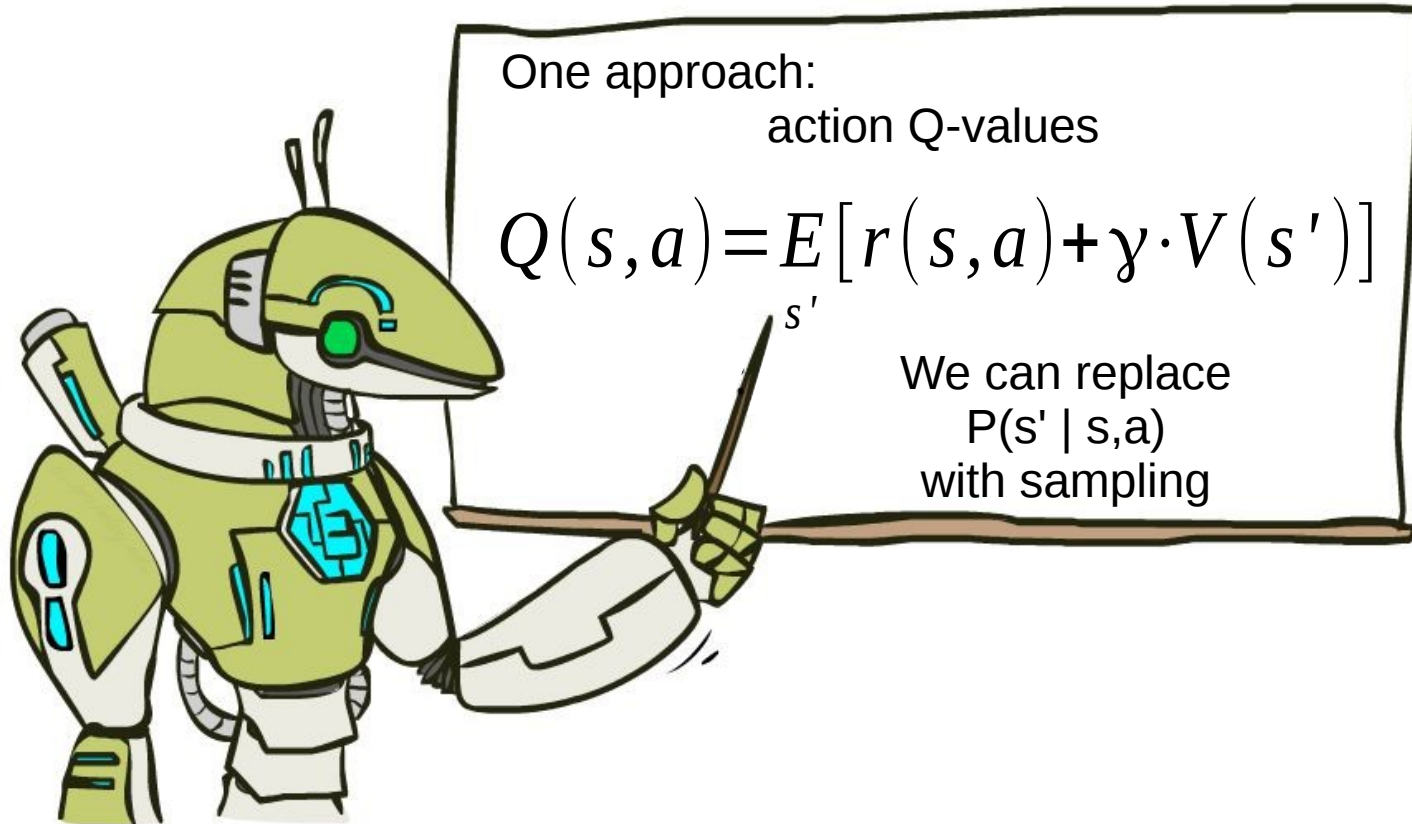
Recap: Q-learning



Action value $Q(s, a)$ is the expected total reward **R** agent gets from state **s** by taking action **a** and following policy **π** from next state.

$$\pi(s) : \operatorname{argmax}_a Q(s, a)$$

Recap: Q-learning



$$Q(s_t, a_t) \leftarrow \alpha \cdot (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')) + (1 - \alpha) Q(s_t, a_t)$$

$$\pi(s) : \operatorname{argmax}_a Q(s, a)$$

Q-learning as MSE minimization

Given $\langle \mathbf{s}, \mathbf{a}, \mathbf{r}, \mathbf{s}' \rangle$ minimize

$$L = [Q(s_t, a_t) - Q^{true}(s_t, a_t)]^2$$

$$L \approx [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]^2$$

How to optimize?

Q-learning as MSE minimization

Given $\langle \mathbf{s}, \mathbf{a}, \mathbf{r}, \mathbf{s}' \rangle$ minimize

$$L = [Q(s_t, a_t) - Q^{true}(s_t, a_t)]^2$$

$$L \approx [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]^2$$

For tabular $Q(s, a)$

$$\nabla L = 2 \cdot [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]$$

Q-learning as MSE minimization

Given $\langle \mathbf{s}, \mathbf{a}, \mathbf{r}, \mathbf{s}' \rangle$ minimize

$$L = [Q(s_t, a_t) - Q^{true}(s_t, a_t)]^2$$

$$L \approx [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]^2$$

For tabular $Q(s, a)$

$$\nabla L \approx 2 \cdot [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]$$

Something's sooo wrong!

Q-learning as MSE minimization

Given $\langle \mathbf{s}, \mathbf{a}, \mathbf{r}, \mathbf{s}' \rangle$ minimize

$$L = [Q(s_t, a_t) - \underline{Q^{true}(s_t, a_t)}]^2 \quad \text{const}$$

$$L \approx [Q(s_t, a_t) - \underline{(r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))}]^2 \quad \text{const}$$

For tabular $Q(\mathbf{s}, \mathbf{a})$

$$\nabla L \approx 2 \cdot [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]$$

Q-learning as MSE minimization

For tabular $Q(s,a)$

$$L \approx [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]^2$$

$$\nabla L \approx 2 \cdot [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]$$

Gradient descent step:

$$Q(s, a) := Q(s, a) - \alpha \cdot 2 [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]$$

Q-learning as MSE minimization

For tabular $Q(s,a)$

$$L \approx [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]^2$$

$$\nabla L \approx 2 \cdot [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]$$

Gradient descent step:

$$Q(s, a) := Q(s, a)(1 - 2\alpha) + 2\alpha(r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))$$

Q-learning as MSE minimization

For tabular $Q(s,a)$

$$L \approx [Q(s_t, a_t) - (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))]^2$$

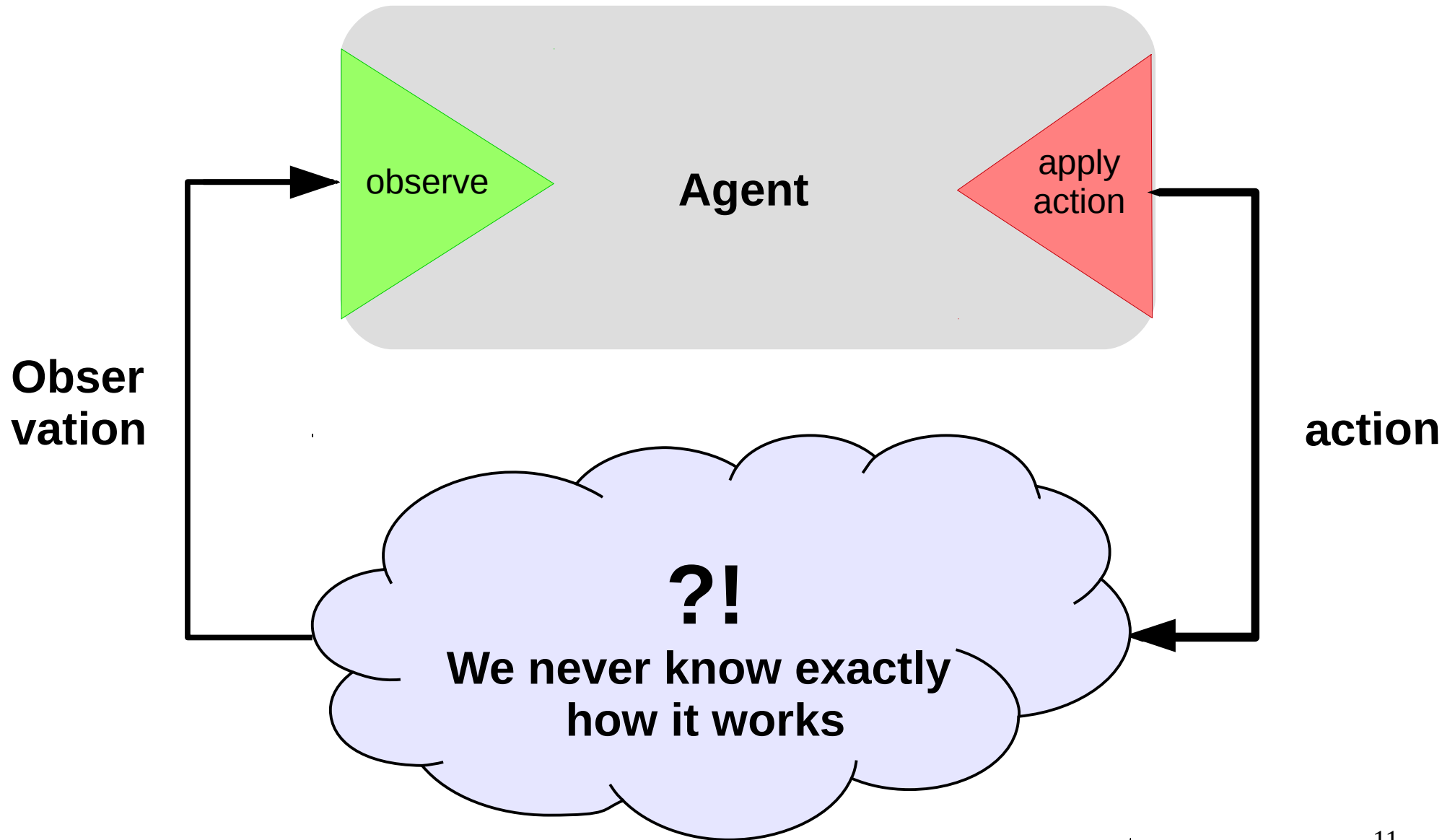
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Gradient descent step:

$$Q(s, a) := Q(s, a)(1 - 2\alpha) + 2\alpha(r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a'))$$

= moving average formula
(define $\alpha' = 2 \cdot \alpha$)

Real world



Problem:

State space is usually large,
sometimes continuous.

And so is action space;

However, states do have a structure, similar
states have similar action outcomes.

Problem:

State space is usually large,
sometimes continuous.

And so is action space;

Two solutions:

- Binarize state space
- Approximate agent with a function

Which one would you prefer for atari?

Problem:

State space is usually large,
sometimes continuous.

And so is action space;

Two solutions:

- Binarize state space  Too many bins or handcrafted features
- Approximate agent with a function  Let's pick this one

From tables to approximations

- Before:
 - For all states, for all actions, remember $Q(s,a)$
- Now:
 - Approximate $Q(s,a)$ with some function
 - e.g. linear model over state features

$$\operatorname{argmin}_{w,b} \left(Q(s_t, a_t) - [r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')] \right)^2$$

Trivia: should we use **classification** or **regression** model?
(e.g. logistic regression Vs linear regression)

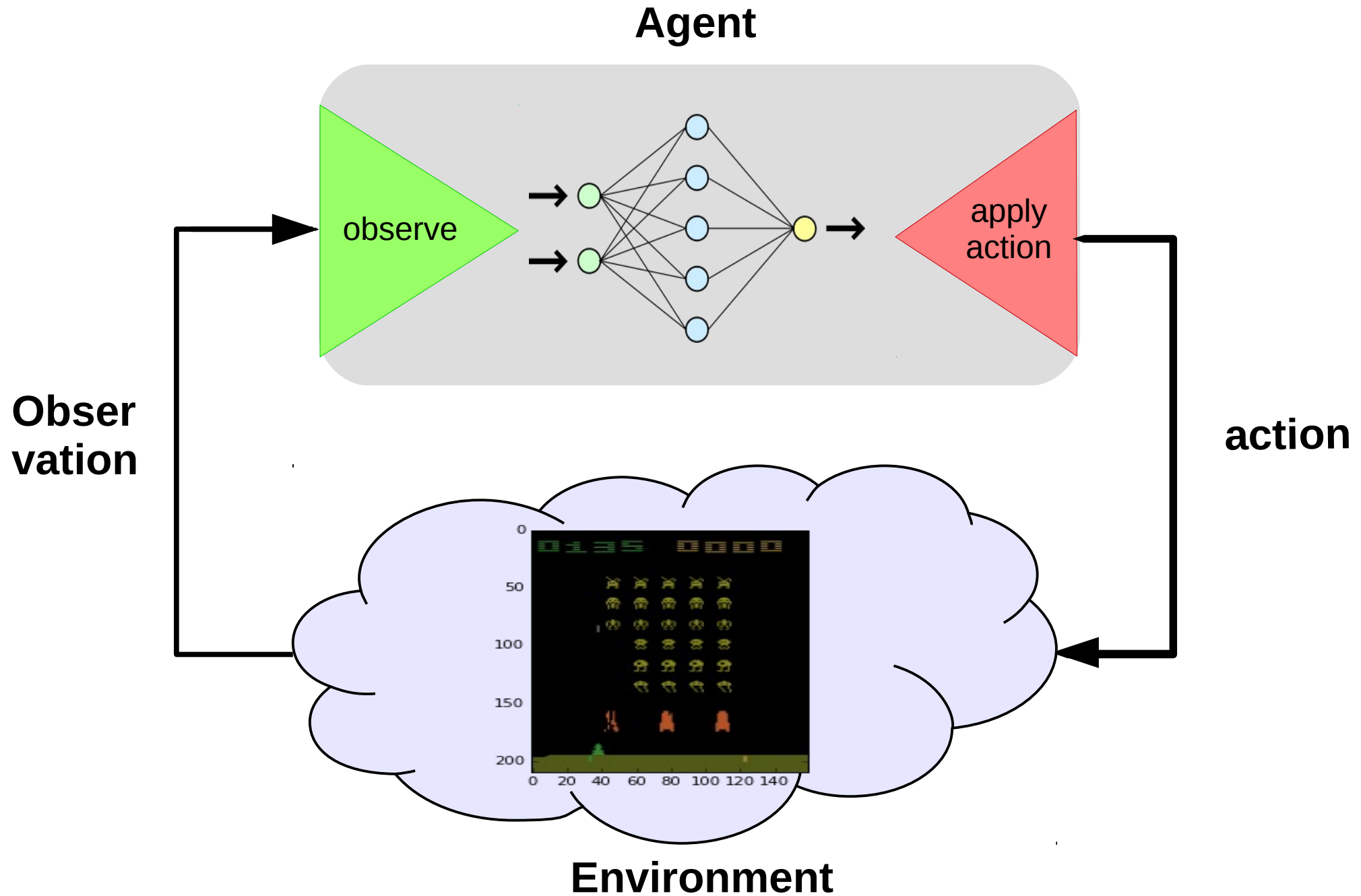
From tables to approximations

- Before:
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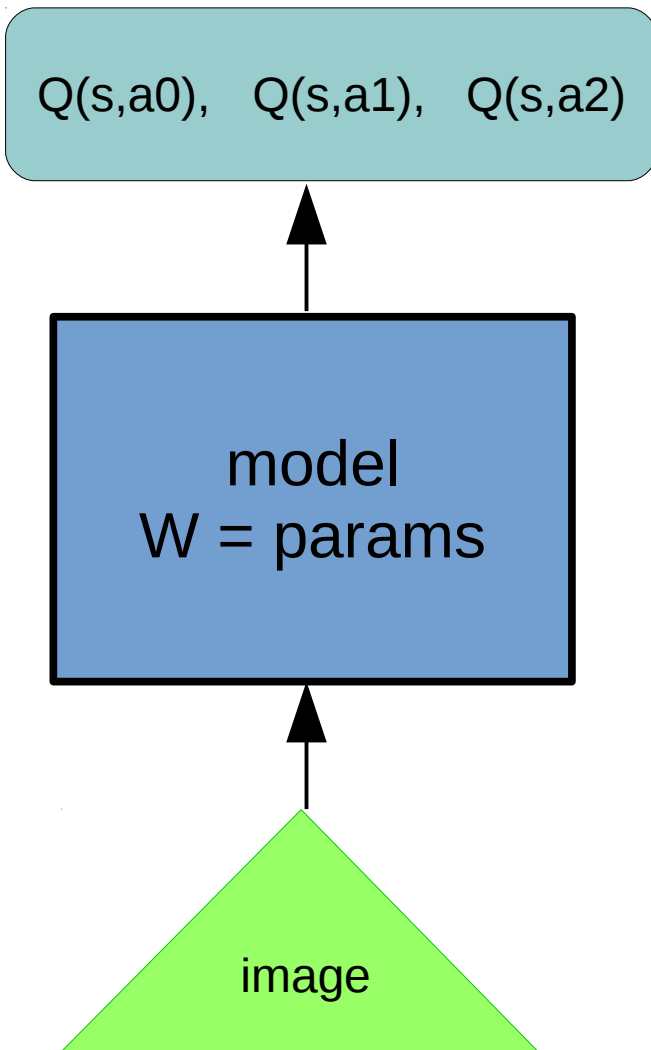
$$\operatorname{argmin}_{w,b} \left(Q(s_t, a_t) - [r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')] \right)^2$$

- Solve it as a **regression** problem!

MDP again



Approximate Q-learning



Q-values:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot \max_{a'} \hat{Q}(s_{t+1}, a')$$

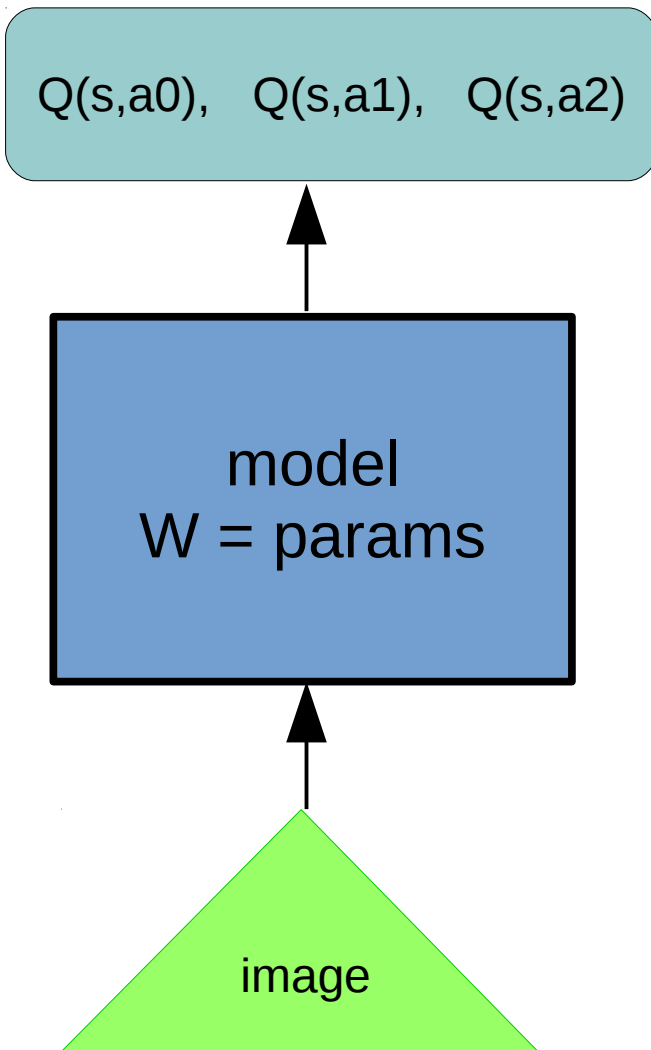
Objective:

$$L = \left(Q(s_t, a_t) - [r + \gamma \cdot \max_{a'} Q(s_{t+1}, a')] \right)^2$$

Gradient step:

$$w_{t+1} = w_t - \alpha \cdot \frac{\delta L}{\delta w}$$

Approximate Q-learning



Q-values:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot \max_{a'} \hat{Q}(s_{t+1}, a')$$

Objective:

$$L = \left(Q(s_t, a_t) - \underbrace{\left[r + \gamma \cdot \max_{a'} Q(s_{t+1}, a') \right]}_{\text{consider const}} \right)^2$$

consider const

Gradient step:

$$w_{t+1} = w_t - \alpha \cdot \frac{\partial L}{\partial w_t}$$

Approximate SARSA

Objective:

$$L = \left(Q(s_t, a_t) - \underbrace{\hat{Q}(s_t, a_t)}_{\text{consider const}} \right)^2$$

Q-learning:

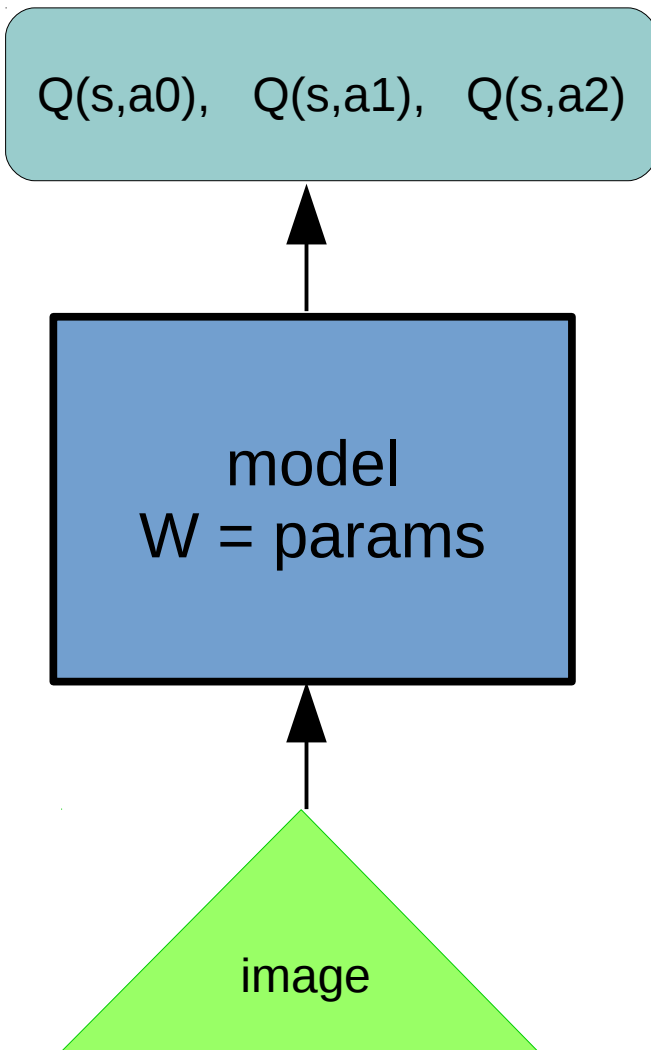
$$\hat{Q}(s_t, a_t) = r + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$

SARSA:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot Q(s_{t+1}, a_{t+1})$$

Expected Value SARSA:

$$\hat{Q}(s_t, a_t) = ???$$



Approximate SARSA

Objective:

$$L = \left(Q(s_t, a_t) - \underbrace{\hat{Q}(s_t, a_t)}_{\text{consider const}} \right)^2$$

Q-learning:

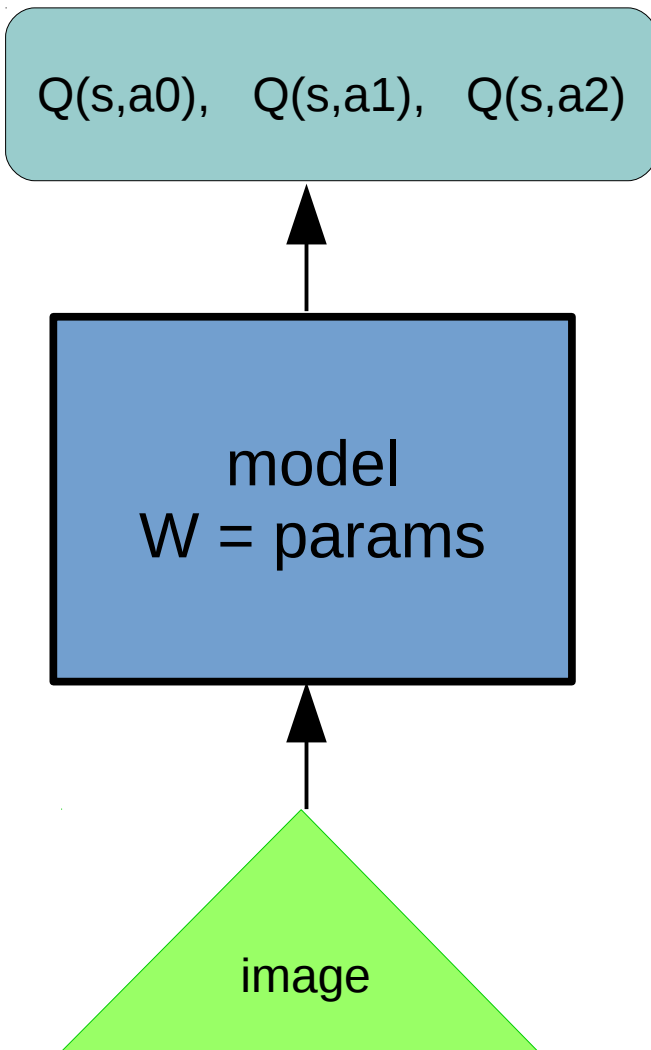
$$\hat{Q}(s_t, a_t) = r + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$

SARSA:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot Q(s_{t+1}, a_{t+1})$$

Expected Value SARSA:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot E_{a' \sim \pi(a|s)} Q(s_{t+1}, a')$$



Approximate n-step methods

Objective:

$$L = \left(Q(s_t, a_t) - \underbrace{\hat{Q}(s_t, a_t)}_{\text{consider const}} \right)^2$$

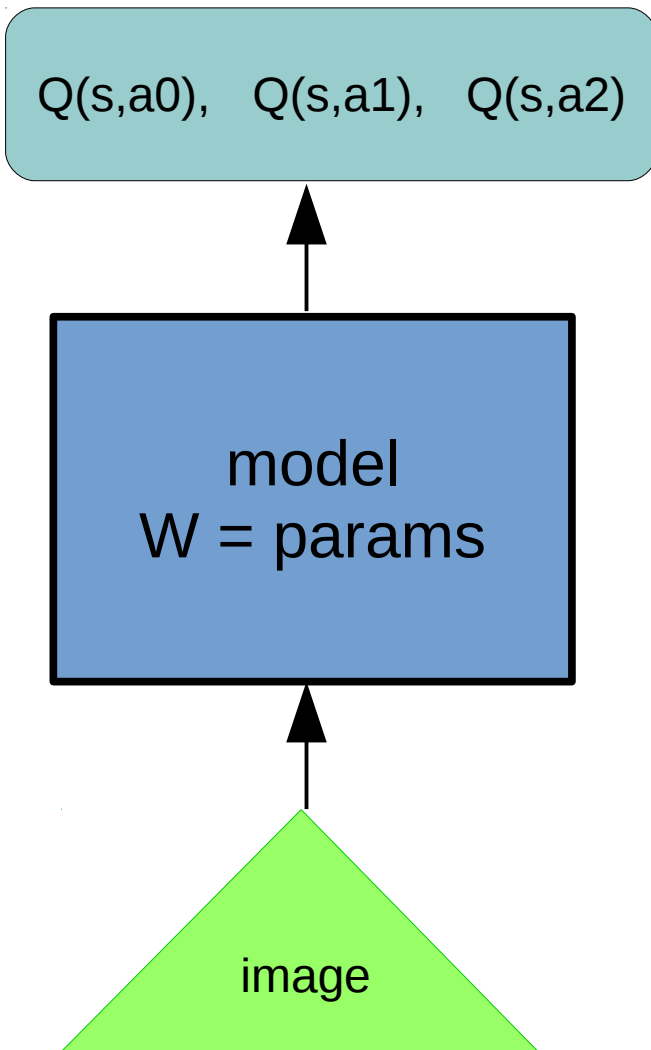
Q-learning:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$

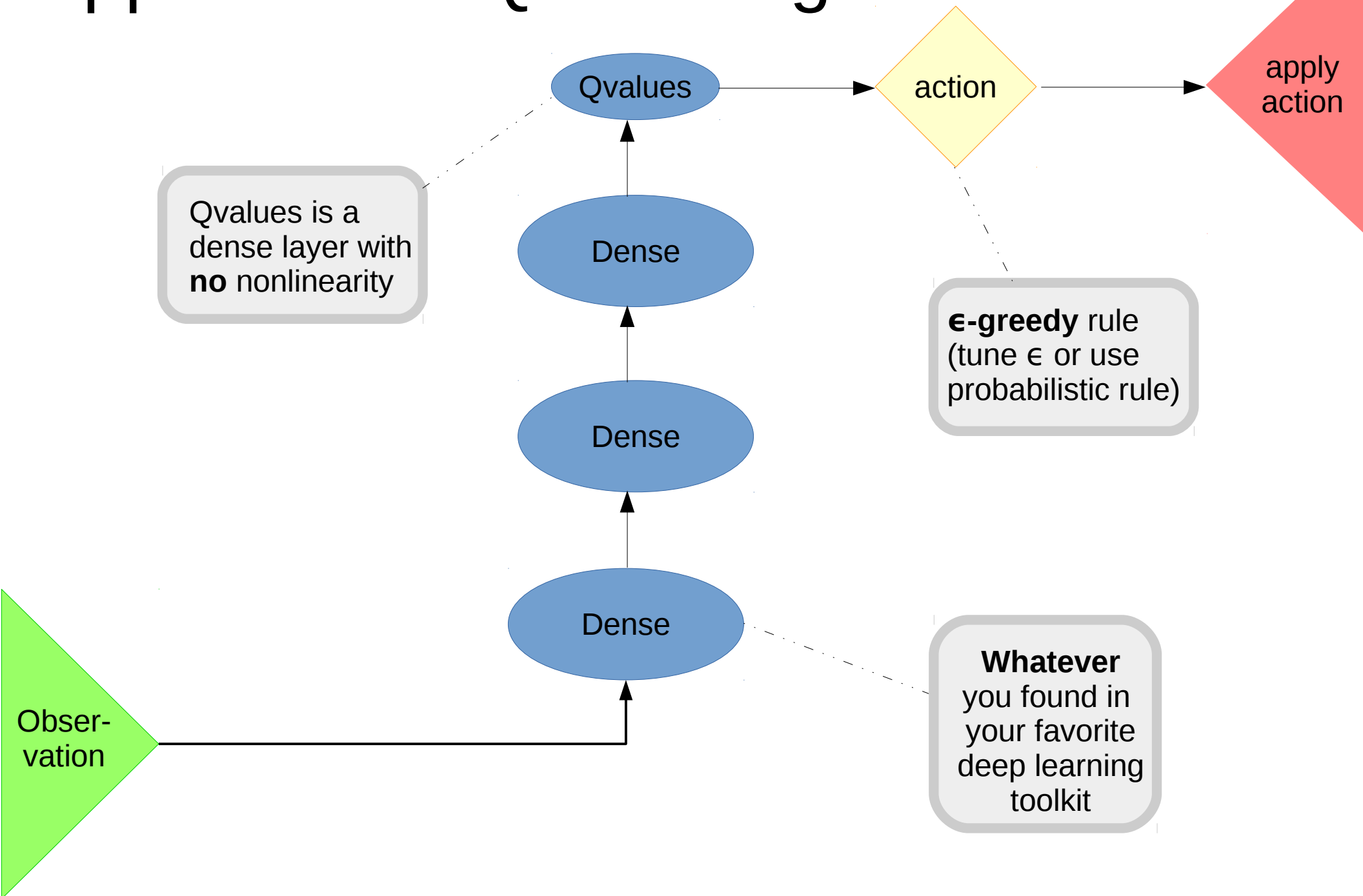
Q-learning n-step:

$$\hat{Q}(s_t, a_t) = r(s_t, a_t) + \gamma r(s_{t+1}, a_{t+1}) + \gamma^2 Q(s_{t+2}, a_{t+2})$$

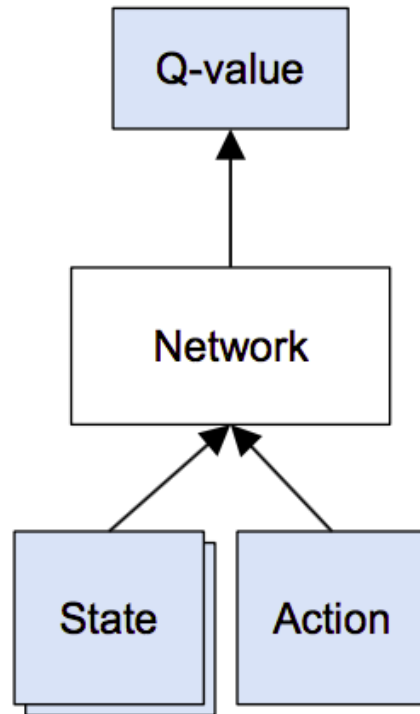
$$\hat{Q}(s_t, a_t) = \left[\sum_{\tau=t}^{\tau=t+n} \gamma^\tau r(s_{t+\tau}, a_{t+\tau}) \right] + \gamma^n \cdot \max_a Q(s_{t+n}, a)$$



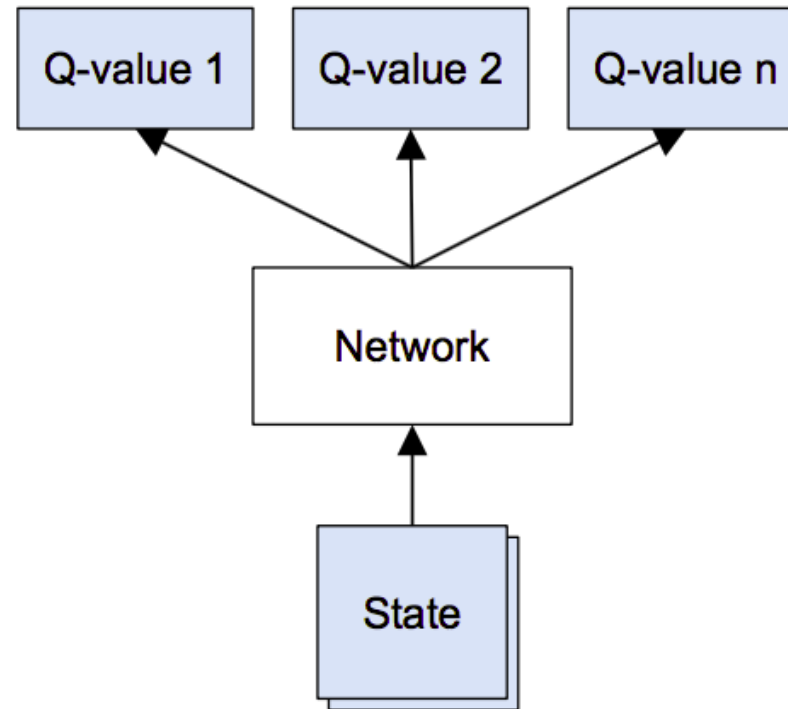
Approximate Q-learning



Architectures

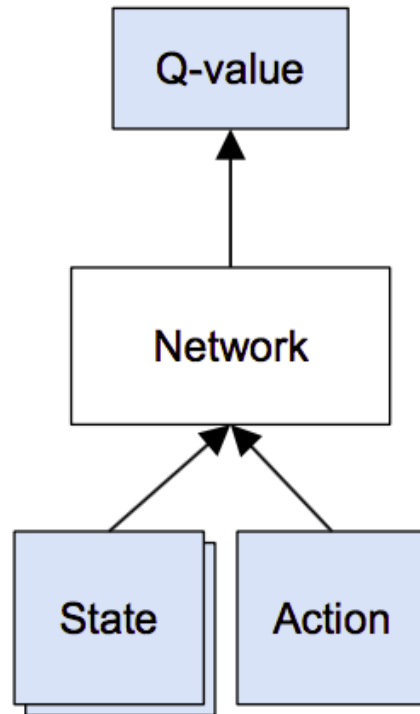


Given (\mathbf{s}, \mathbf{a})
Predict $Q(\mathbf{s}, \mathbf{a})$

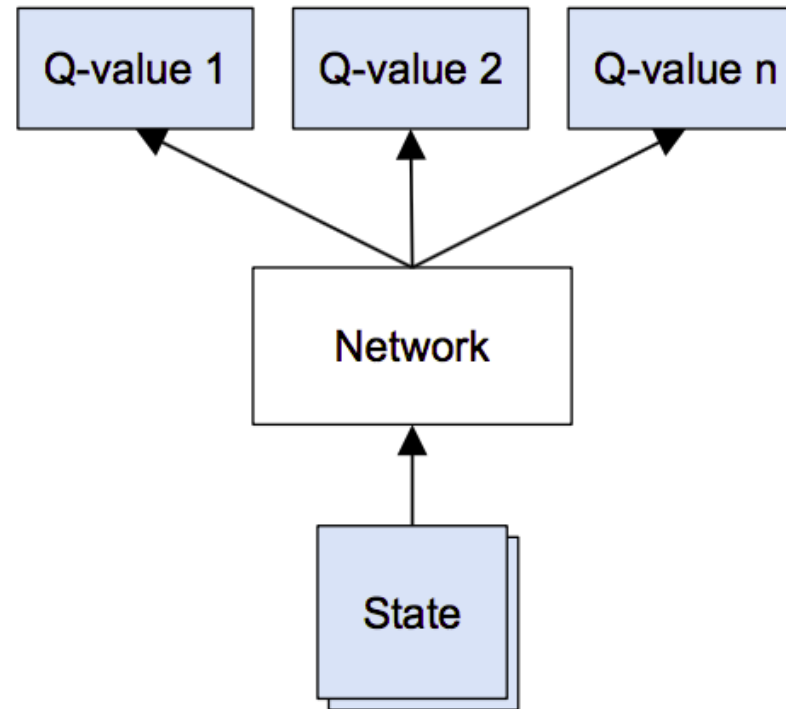


Given \mathbf{s} predict all q-values
 $Q(\mathbf{s}, \mathbf{a}_0)$, $Q(\mathbf{s}, \mathbf{a}_1)$, $Q(\mathbf{s}, \mathbf{a}_2)$

Architectures



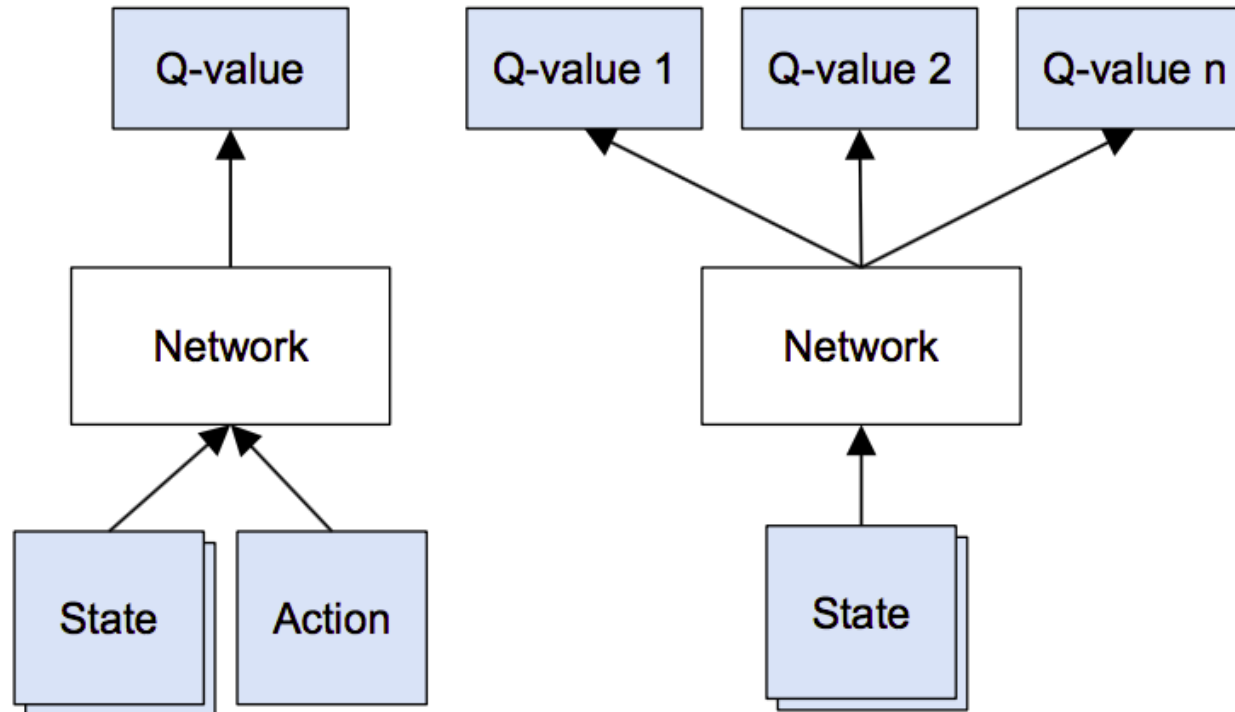
Given (\mathbf{s}, \mathbf{a})
Predict $Q(\mathbf{s}, \mathbf{a})$



Given \mathbf{s} predict all q-values
 $Q(\mathbf{s}, \mathbf{a}_0), Q(\mathbf{s}, \mathbf{a}_1), Q(\mathbf{s}, \mathbf{a}_2)$

Trivia: in which situation does **left** model work better?
And right?

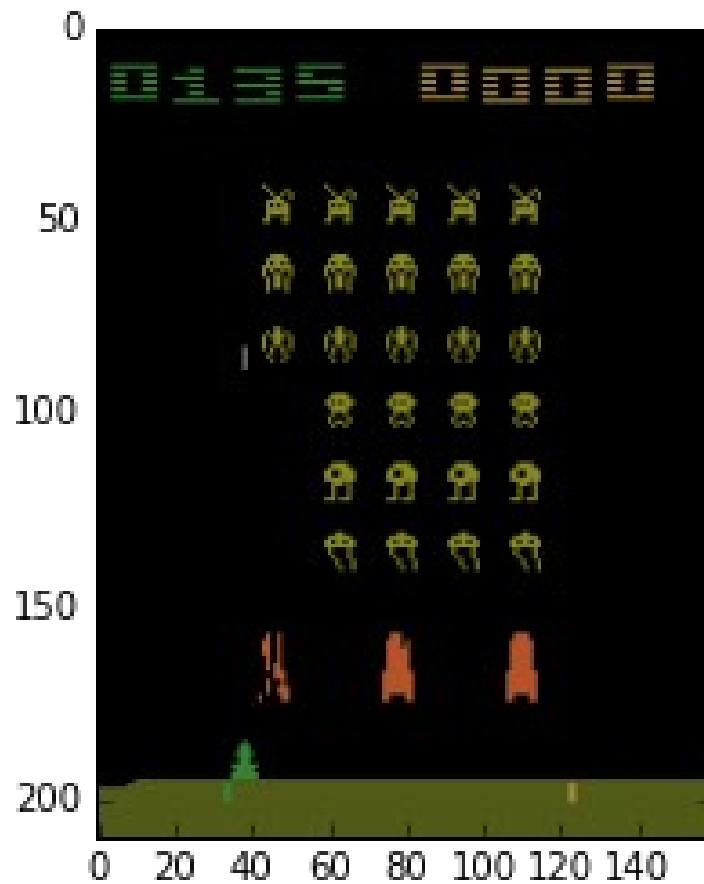
Architectures



Needs one forward pass
for **each action**

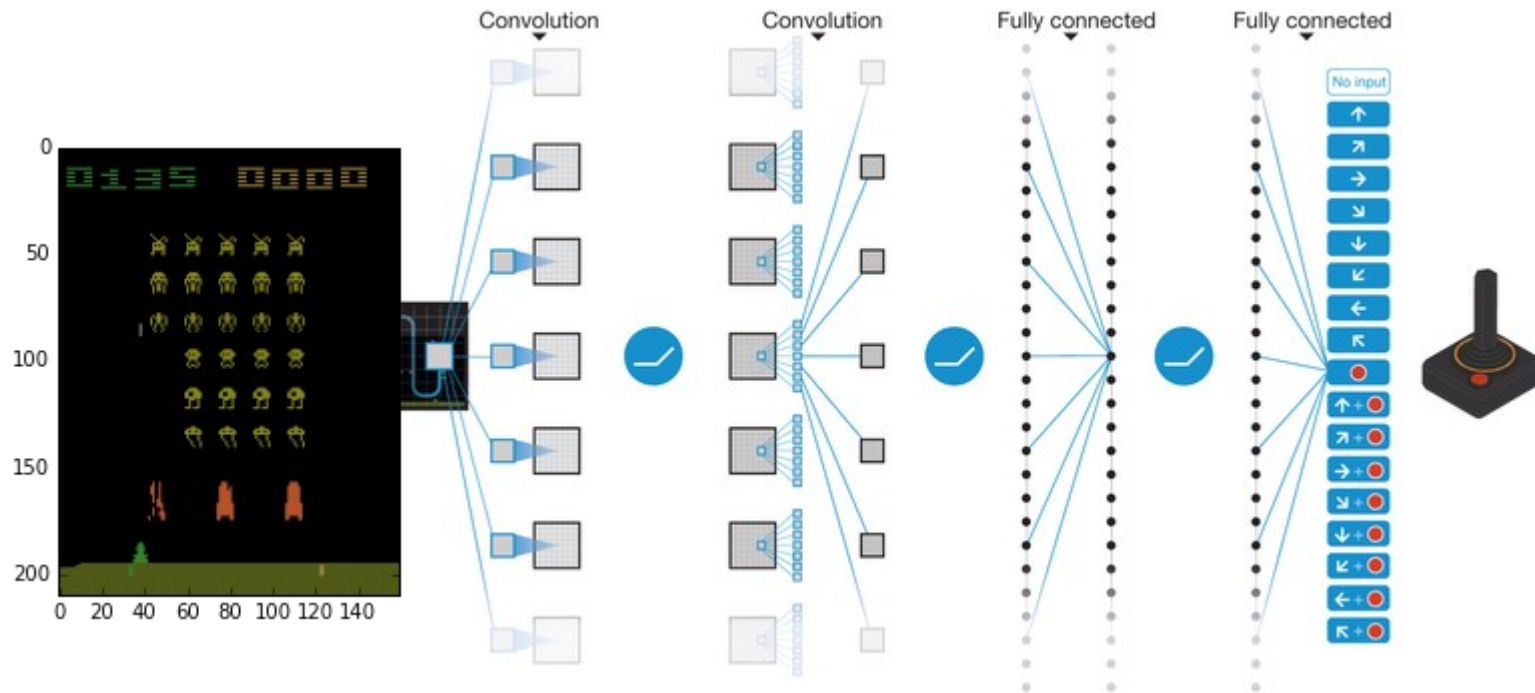
Works if action space is
large / continuous

Needs one forward pass
for **all actions**
(faster)

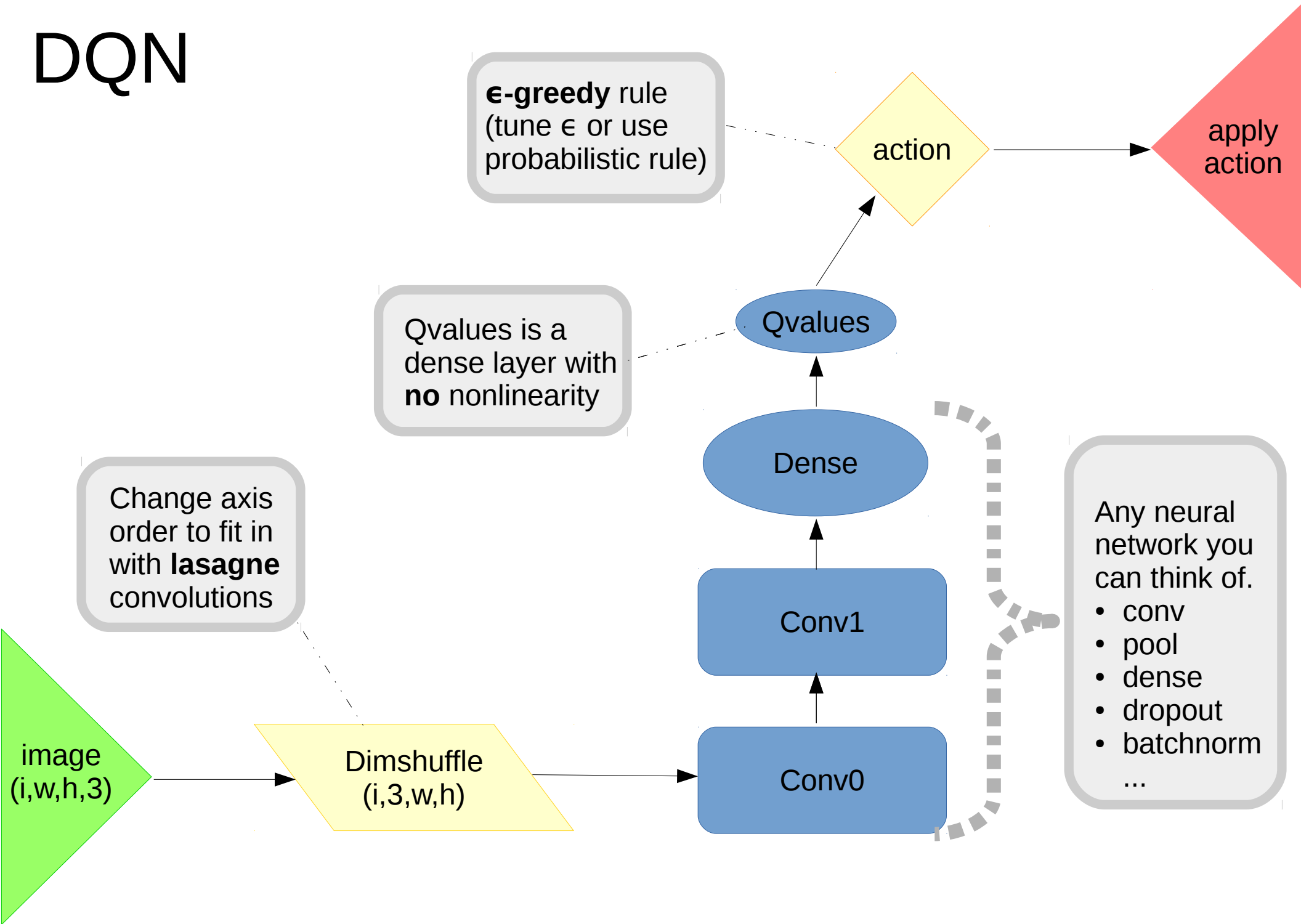


What kind of network digests images well?

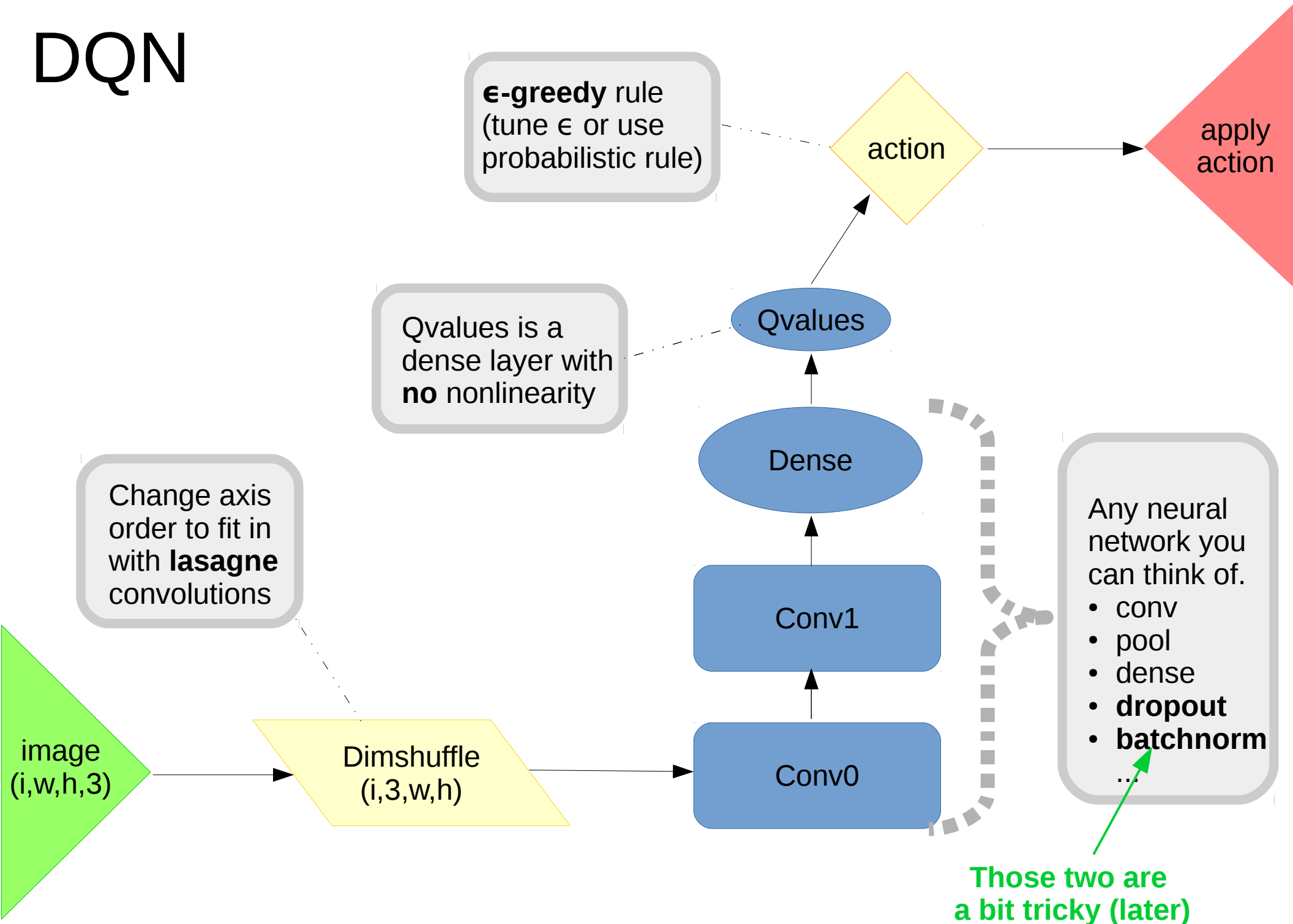
Deep learning approach: DQN



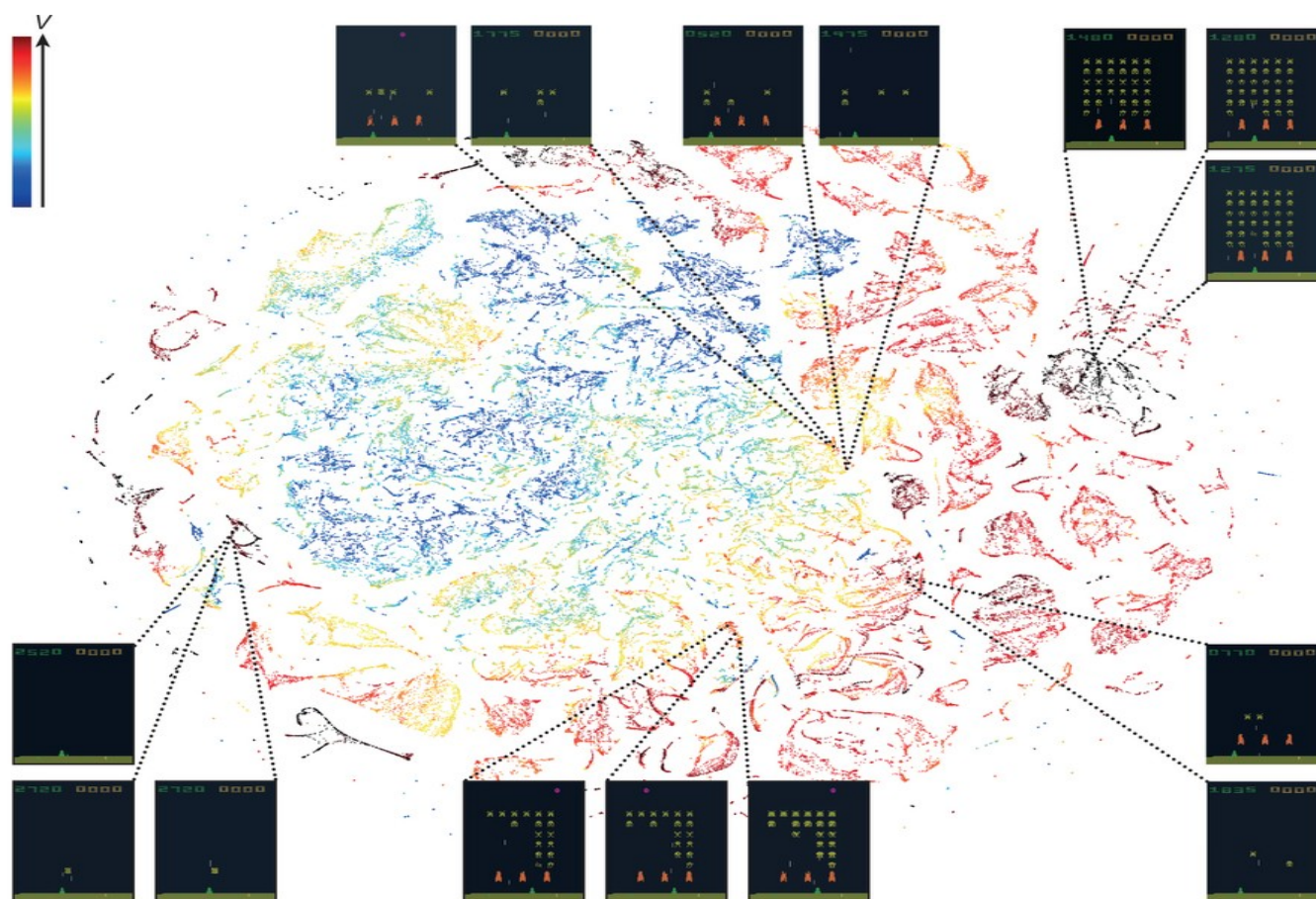
DQN



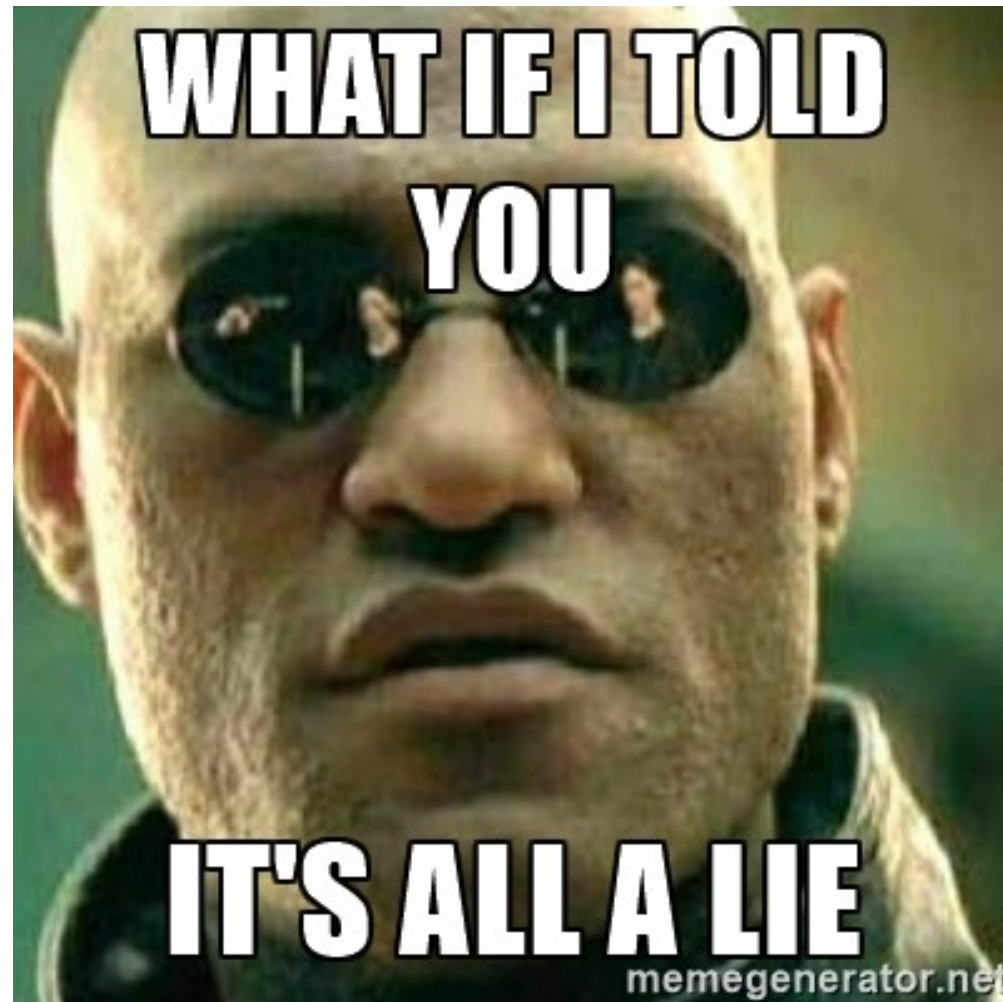
DQN

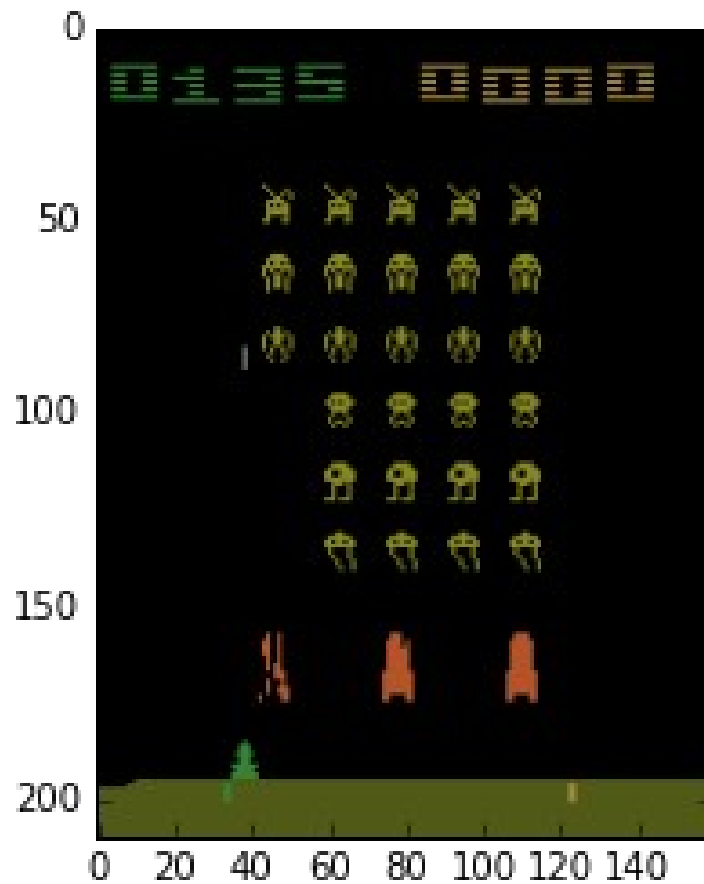


TSNE makes every slide 40% better



- Embedding of pre-last layer activations
- Color = $V(s) = \max_a Q(s,a)$





How bad it is if agent spends
next 1000 ticks under the left rock?
(while training)

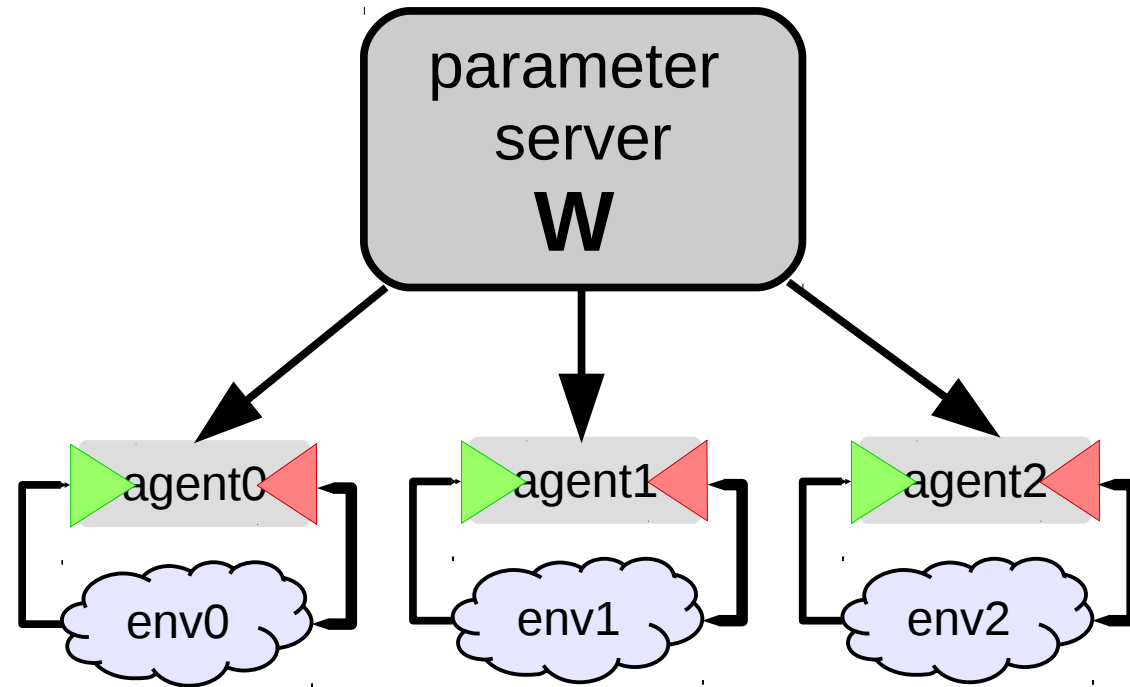
Problem

- Training samples are **not** “i.i.d”,
- Model forgets parts of environment it hasn't visited for some time
- Drops on learning curve
- **Any ideas?**



Multiple agent trick

Idea: Throw in several agents with shared \mathbf{W} .

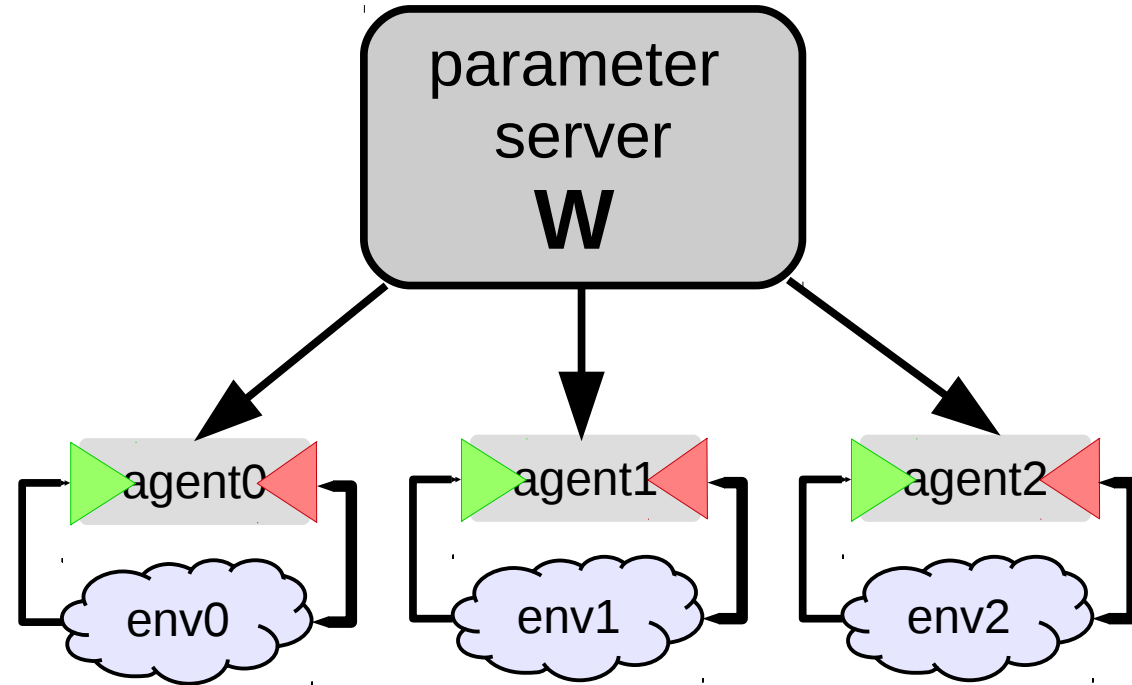


Multiple agent trick

Idea: Throw in several agents with shared W .

- Chances are, they will be exploring different parts of the environment,
- More stable training,
- Requires a lot of interaction

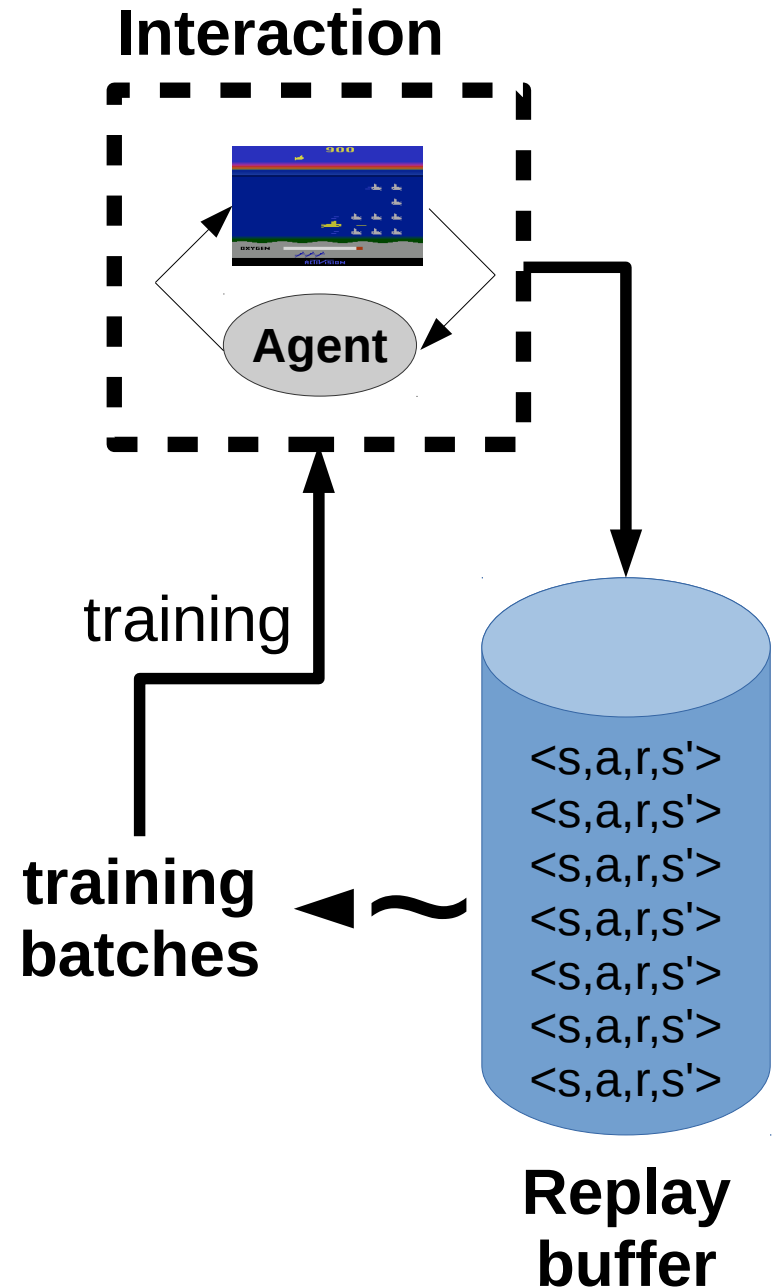
Trivia: your agent is a real robot car. Any problems?



Experience replay

Idea: store several past interactions
 $\langle s, a, r, s' \rangle$
Train on random subsamples

Any +/- ?



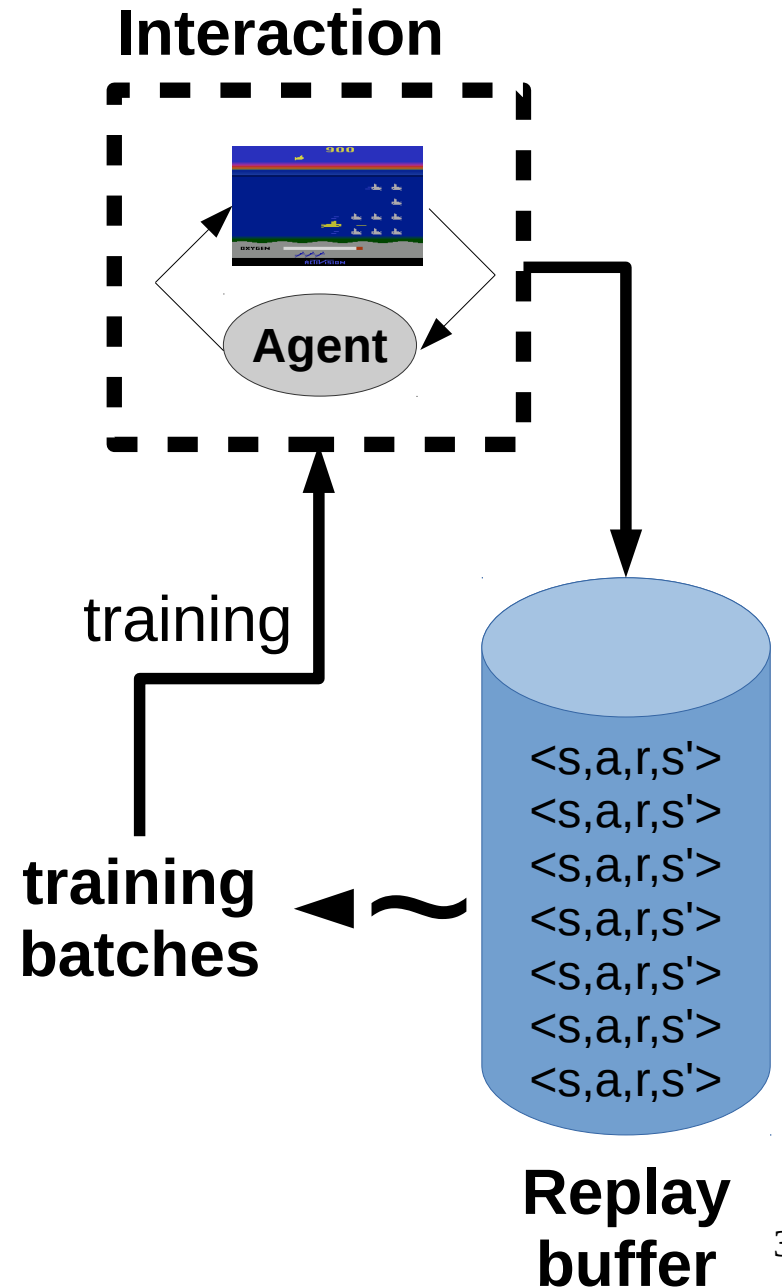
Experience replay

Idea: store several past interactions
 $\langle s, a, r, s' \rangle$

Train on random subsamples

- Atari DQN: $>10^5$ interactions
- Closer to i.i.d
pool contains several sessions
- Older interactions were obtained
under weaker policy

Better versions coming next week



Summary so far

to make data closer to i.i.d.

Use one or several of

- **experience replay**
- **multiple agents**
- Infinitely small learning rate :)

advanced stuff coming next lecture

An important question

- You approximate $Q(s,a)$ with a neural network
- You use **experience replay** when training

Trivia: which of those algorithms will fail?

- Q-learning
- SARSA
- 15-step q-learning
- Expected Value SARSA

An important question

- You approximate $Q(s,a)$ with a neural network
- You use **experience replay** when training

Agent trains off-policy on an older version of him

Trivia: which of those algorithms will fail?

Off-policy methods work, On-policy is super-slow (fail)

- | | |
|--------------|------------------------|
| – Q-learning | – 15-step q-learning |
| – SARSA | – Expected Value SARSA |

When training with on-policy methods,

- use no (or small) experience replay
- compensate with parallel game sessions

Deep learning meets MDP

- Dropout, noize
 - **Used in experience replay only:** like the usual dropout
 - **Used when interacting:** a special kind of exploration
 - You may want to decrease p over time.
- Batchnorm
 - Faster training but may break moving average
 - **Experience replay:** may break down if buffer is too small
 - **Parallel agents:** may break down under too few agents
<same problem of being non i.i.d.>

Final problem



Left or right?

Problem:

Most practical cases are partially observable:

Agent observation does not hold all information about process state
(e.g. human field of view).

Any ideas?

Problem:

Most practical cases are partially observable:

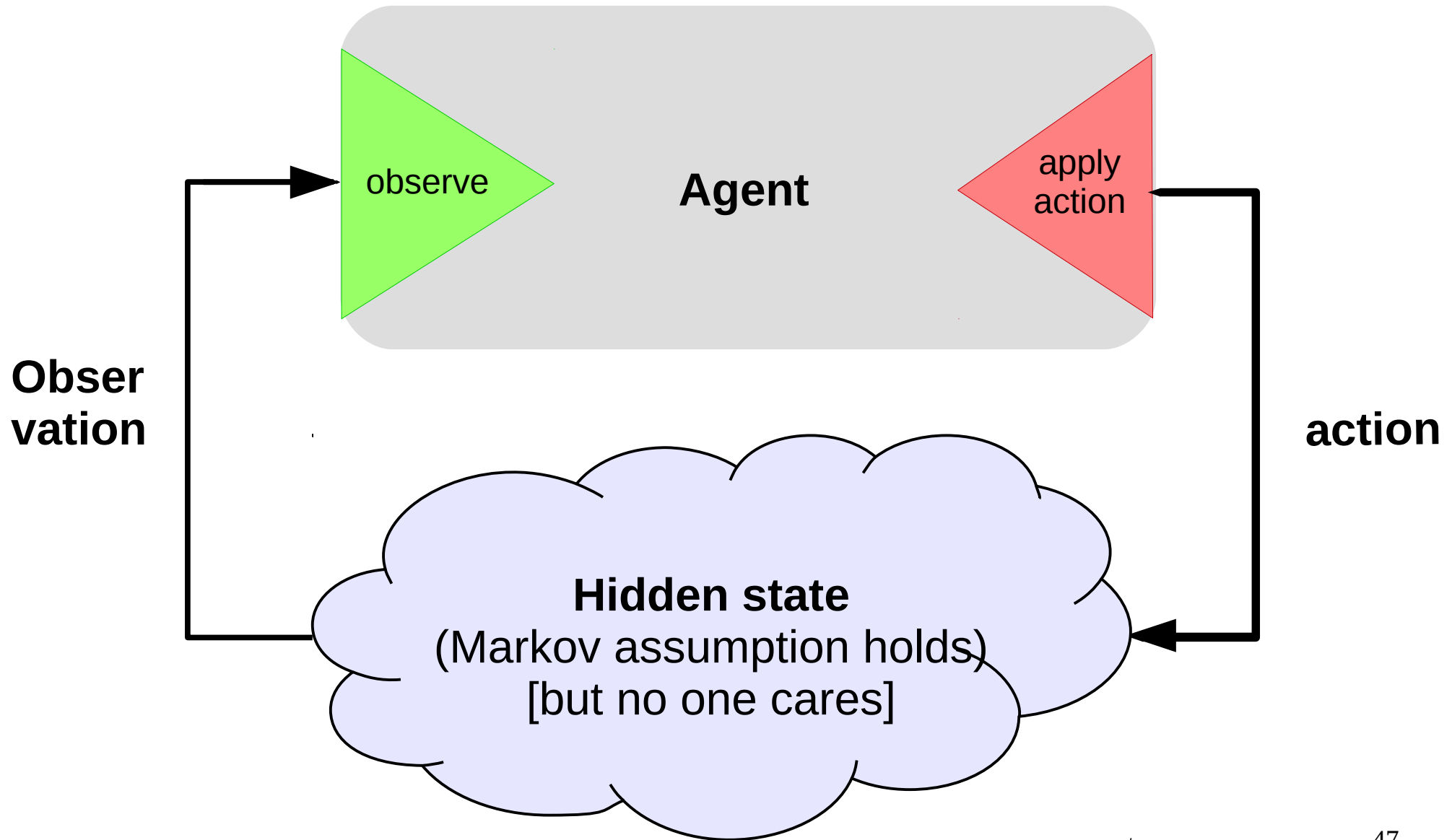
Agent observation does not hold all information about process state
(e.g. human field of view).

- However, we can try to infer hidden states from sequences of observations.

$$s_t \simeq m_t : P(m_t | o_t, m_{t-1})$$

- Intuitively that's agent memory state.

Partially observable MDP



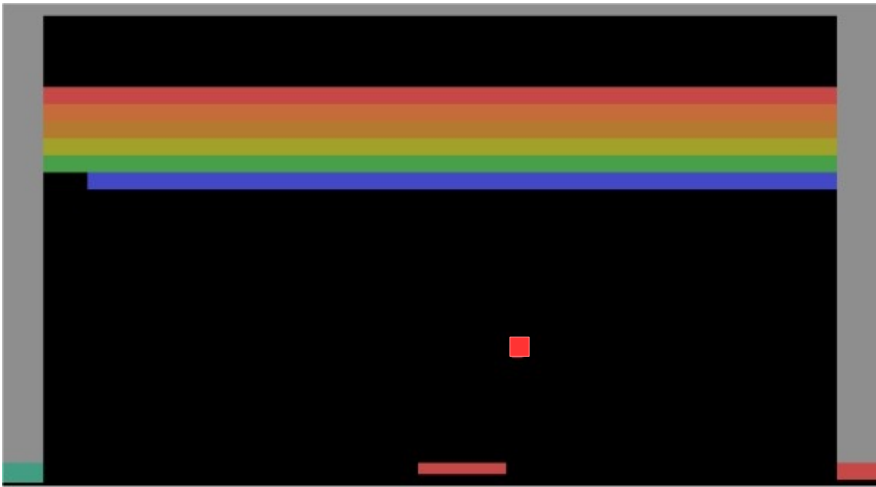
N-gram heuristic

Idea:

$$s_t \neq o(s_t)$$

$$s_t \approx (o(s_{t-n}), a_{t-n}, \dots, o(s_{t-1}), a_{t-1}, o(s_t))$$

e.g. ball movement in breakout

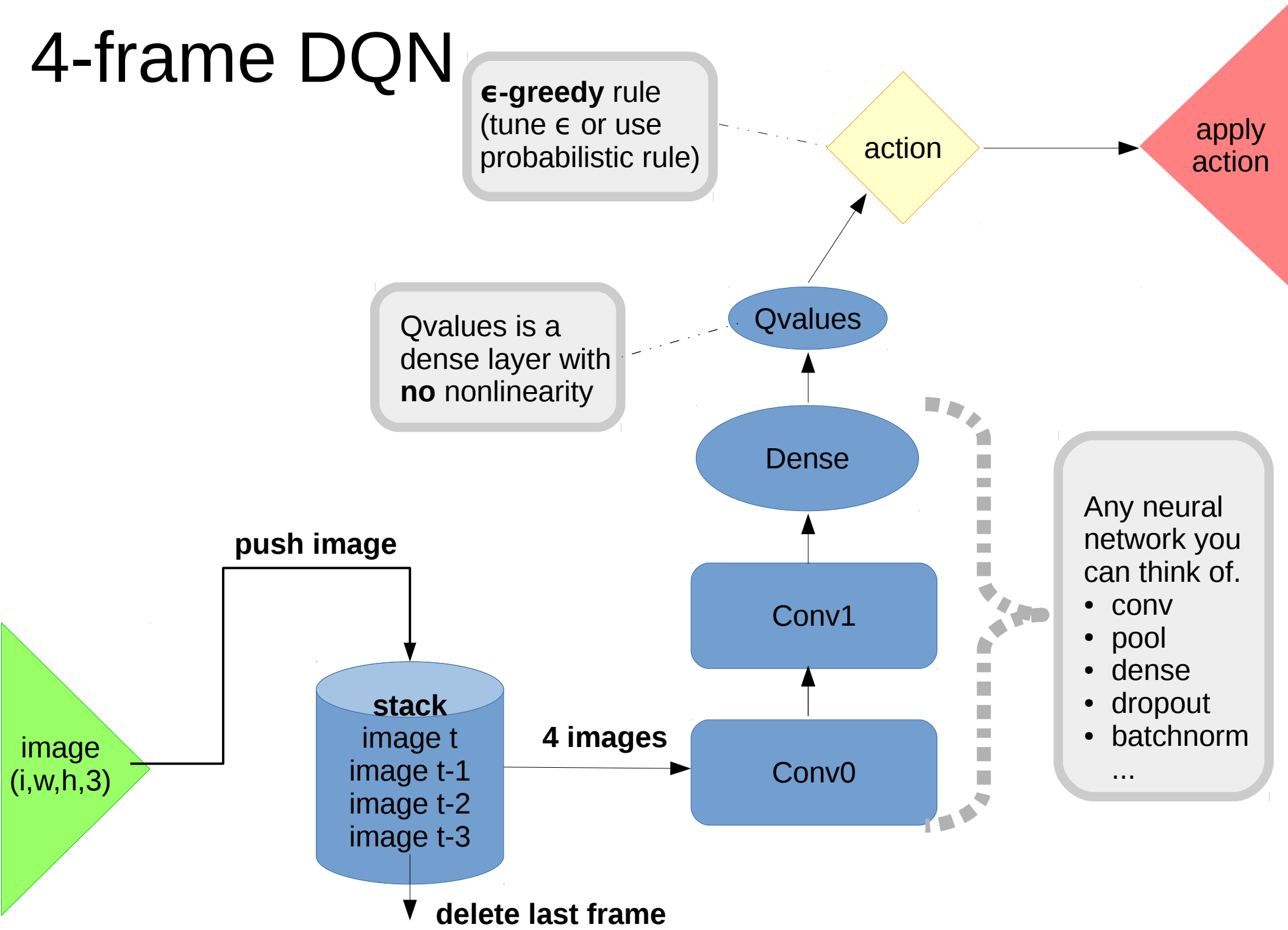


• One frame



• Several frames

4-frame DQN



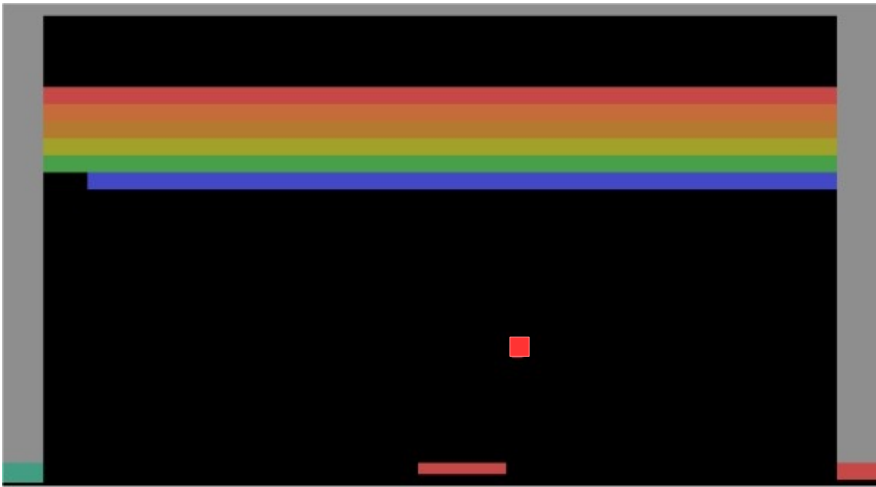
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e.g. ball movement in breakout



• One frame



• Several frames

Alternatives

Ngrams:

- Nth-order markov assumption
- Works for velocity/timers
- Fails for anything longer than N frames
- Impractical for large N

Alternative approach:

- Infer hidden variables given observation sequence
- Kalman Filters, Recurrent Neural Networks
- More on that in a few lectures

Seminar



Autocorrelation

- Reference is based on predictions

$$r + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$

- Any error in Q approximation is propagated to neighbors
- If some $Q(s,a)$ is mistakenly over-exaggerated, neighboring qvalues will also be increased in a cascade
- Worst case: divergence
- **Any ideas?**

Target networks

Idea: use older network snapshot
to compute reference

$$L = \left(Q(s_t, a_t) - \left[r + \gamma \cdot \max_{a'} Q^{old}(s_{t+1}, a') \right] \right)^2$$

- Update Q old periodically
 - Slows down training

Target networks

Idea: use older network snapshot
to compute reference

$$L = \left(Q(s_t, a_t) - [r + \gamma \cdot \max_{a'} Q^{old}(s_{t+1}, a')] \right)^2$$

- Update Q old periodically
 - Slows down training
- Smooth version:
 - use moving average

$$\theta^{old} := (1 - \alpha) \cdot \theta^{old} + \alpha \cdot \theta^{new}$$

- Θ = weights