Sparsely Activated Layers for Text Classifiers

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SEA @ IvI

Text classifiers

Let's consider a general text classifier these days

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$$Y|x \sim \mathrm{Cat}(f(x;\theta))$$

- x is some (high-dimensional) input text
  e.g. a sentence, short paragraph, pair of texts
- y is a K-valued label
  e.g. sentiment, logical entailment
- $f(\cdot;\theta)$ maps from text to a K-dimensional probability vector
  e.g. a NN encoder and a softmax output layer

We call this an *observation model*

Parameter estimation

Given N i.i.d. observations, a step in the direction

$$\boldsymbol{\nabla}_\theta \log \mathrm{Cat}(y^{(s)}|f(x^{(s)};\theta))$$

takes us closer to a local optimum of the log-likelihood function.

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As long as we keep everything about f fully differentiable
it can be as fancy as we like!

Fancy f

In sentiment classification
f is usually a bidirectional recurrent encoder

In natural language inference (aka textual entailment)
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But, make f too fancy and

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3. we can never tell what the classifier is doing

In this talk I will focus on (3)
collaboration with Joost Bastings and Ivan Titov

Outline

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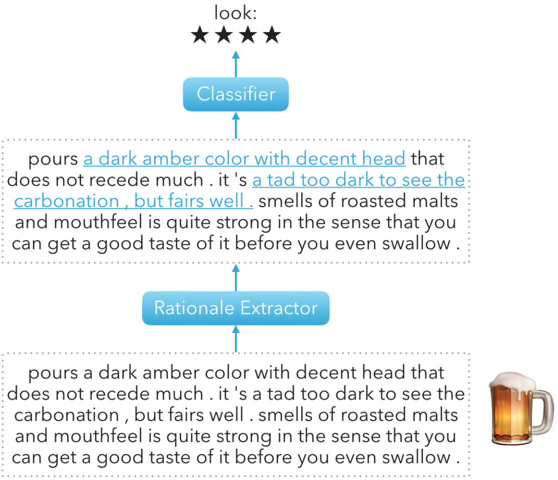
A step towards transparency

We give a NN lots of data to crunch and it makes decisions for us

- why certain decisions take place?
- based on what evidence?
- can we take a peek at what correlations a NN is likely exploiting?

Rationale

What if we classified based on a compact view of the input?



Lei et al. (2016) called this view a *rationale*

Inducing latent rationales

I will denote this view by $x \odot z$

- think of $z = \langle z_1, \ldots, z_n \rangle$ as an elementwise mask
  it selects what parts of the input $x = \langle x_1, \ldots, x_n \rangle$ are available for classification

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We want to *learn* what to select, thus we introduce a *latent model*

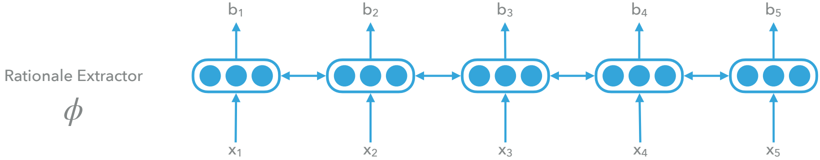
$$Z_i | x \sim \mathrm{Bern}(g_i(x; \phi))$$
$$Y | x, z \sim \mathrm{Cat}(f(x \odot z; \theta))$$

and have a NN $g(x; \phi)$ parameterise n Bernoulli selectors

Latent rationales with Bernoulli selectors

x_1 x_2 x_3 x_4 x_5

Latent rationales with Bernoulli selectors



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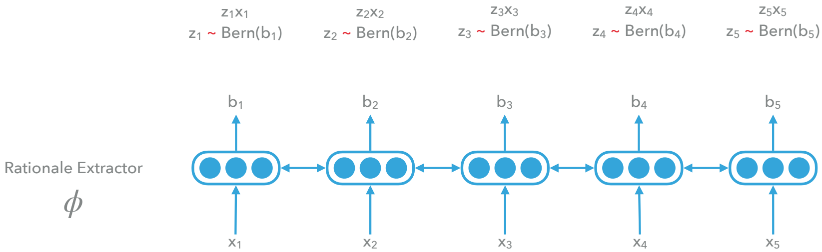

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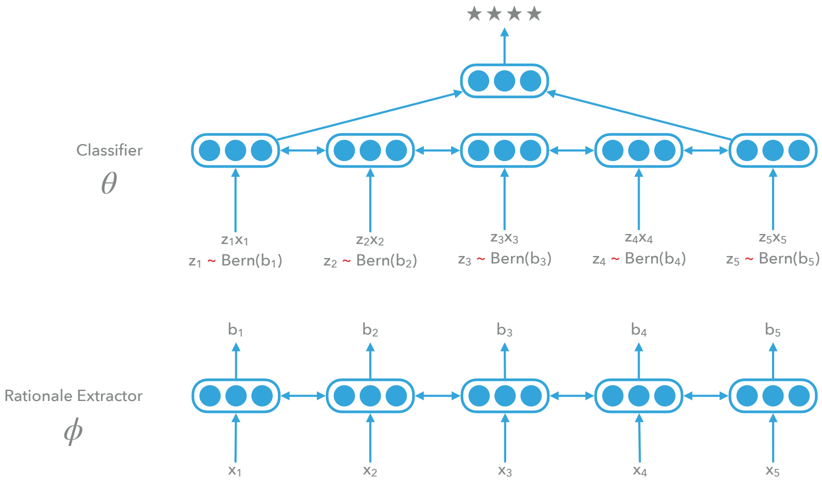
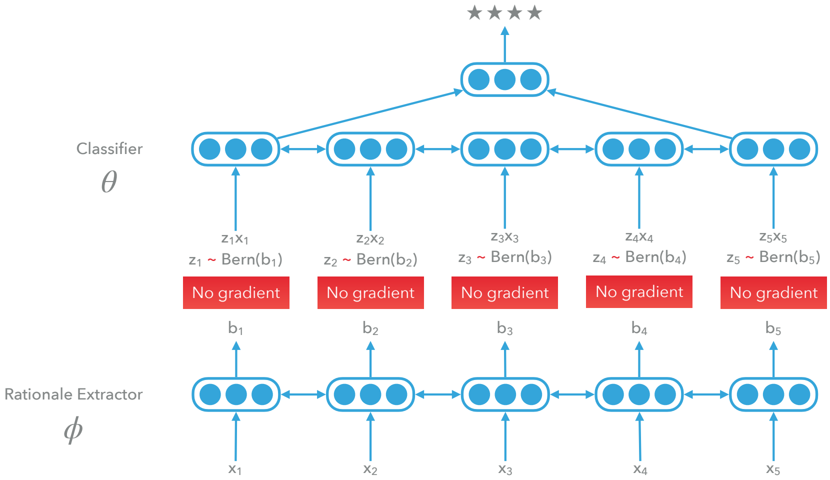


Latent rationales with Bernoulli selectors

Requires gradient estimation via REINFORCE!



Outline

A rectified distribution

Consider a Gaussian variable $\epsilon \sim \mathcal{N}(0, 1)$

Rectified Gaussians in machine learning (Socci et al. 1998, Winn and Bishop 2005)

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- What is the probability of sampling **exactly** 0? 0!
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- What's the probability of sampling ϵ exactly 0? 0!
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- Where is the $\max$ non-differentiable?

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- Will we ever sample $\epsilon = 0$?

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- Where is the $\max$ non-differentiable? At $\epsilon = 0$
- Will we ever sample $\epsilon = 0$? No :D

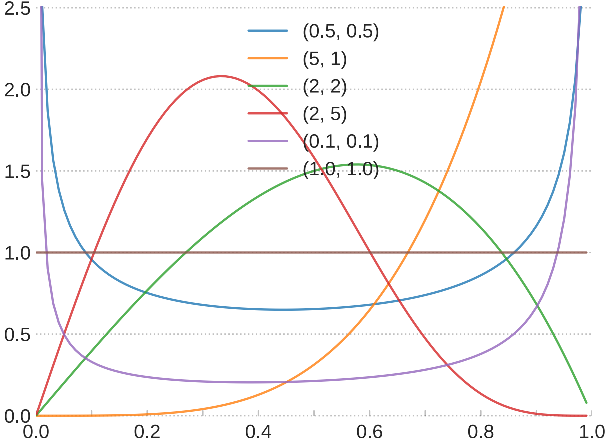
Rectified Gaussians in machine learning (Socci et al. 1998, Winn and Bishop 2005)

HardKumaraswamy

We propose a distribution that

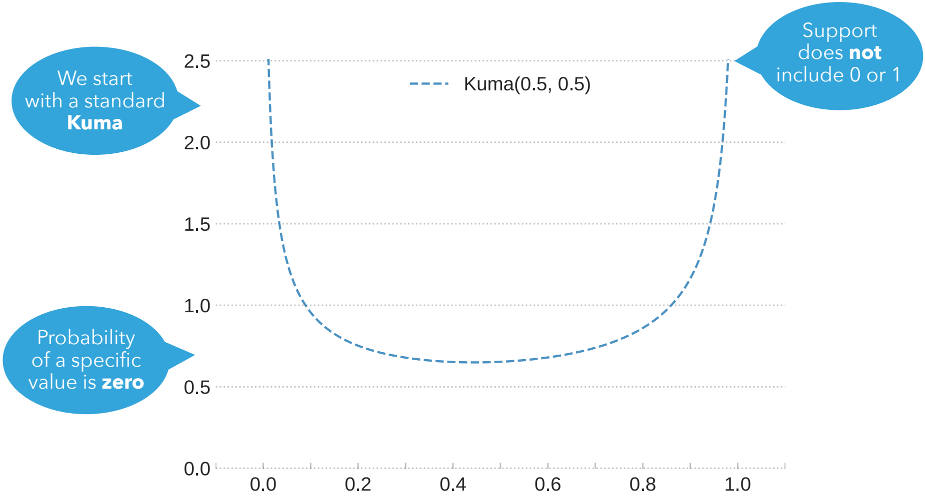
- gives support to the **closed** interval $[0, 1]$
- and assign non-zero probability to outcomes 0 and 1
  $\mathbb{P}(z \in \{0\}) > 0$ and $\mathbb{P}(z \in \{1\}) > 1$

HardKumaraswamy



Kumaraswamy distribution (Kumaraswamy 1980) in machine learning (Nalisnick and Smyth 2016)

HardKumaraswamy



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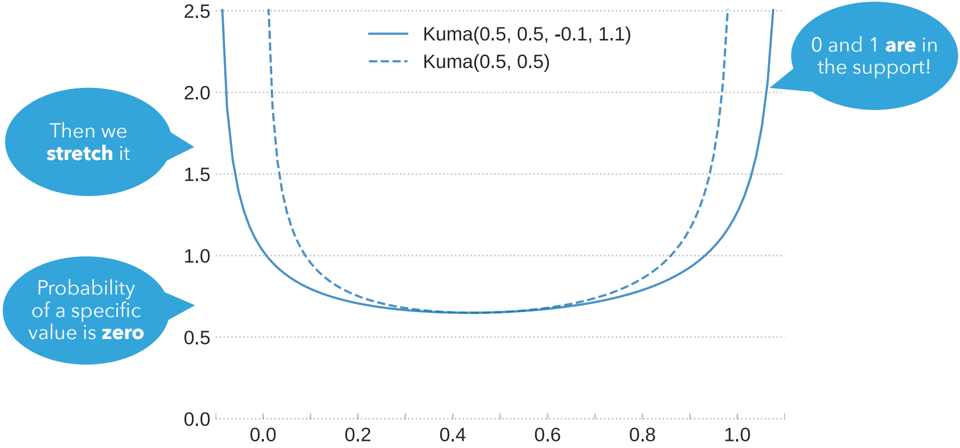

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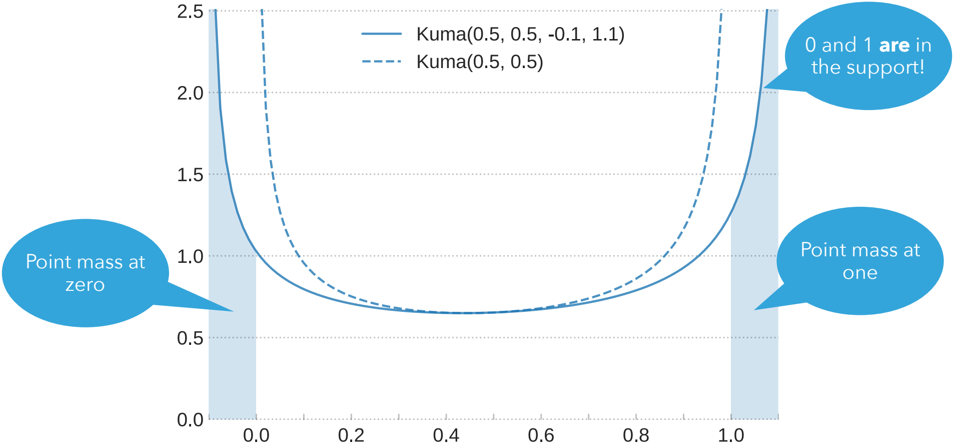

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Sampling a HardKumaraswamy variable

If $Z \sim \mathrm{HKuma}(a, b, l, r)$

Louizos et al. (2017) proposed this *stretch-and-rectify* technique using Binary Concrete variables (Maddison et al. 2017, Jang et al. 2017) in the context of Bayesian NNs.

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  Yes, reparameterised gradients are available!

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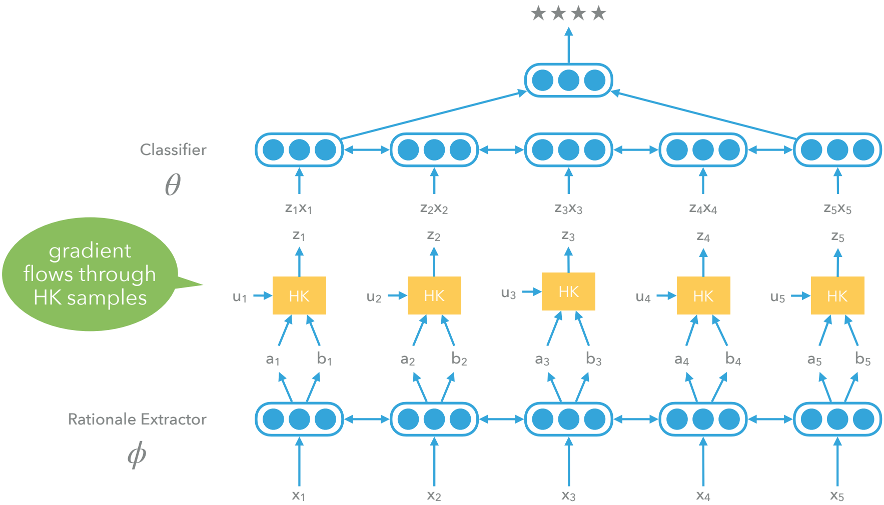
Latent rationales with HardKuma selectors



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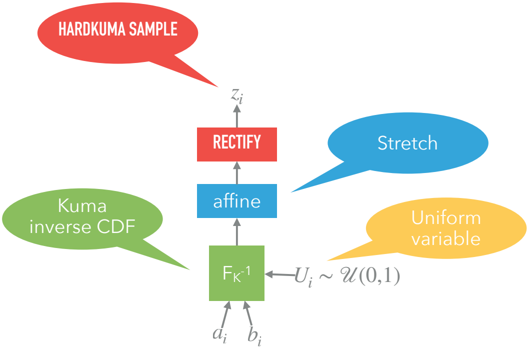

Strictly positive shape parameters $a_i, b_i = g_i(x; \phi)$

Promoting sparsity

Short selections: penalise expected number of non-zero selectors

$$\boxed{\mathbb{E}_{P(z|x,\phi)}\left[\sum_{i=1}^{n}\mathbb{I}[z_i \neq 0]\right]}$$

Relaxed L_0 due to Louizos et al. (2017)

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Coherent groups: penalise expected number of zero-to-nonzero and nonzero-to-zero changes

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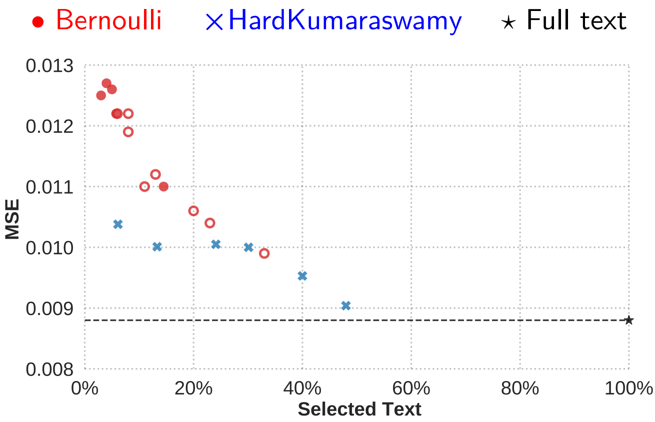
Tractable and differentiable function of ϕ

Relaxed L_0 due to Louizos et al. (2017)

Outline

BeerAdvocate



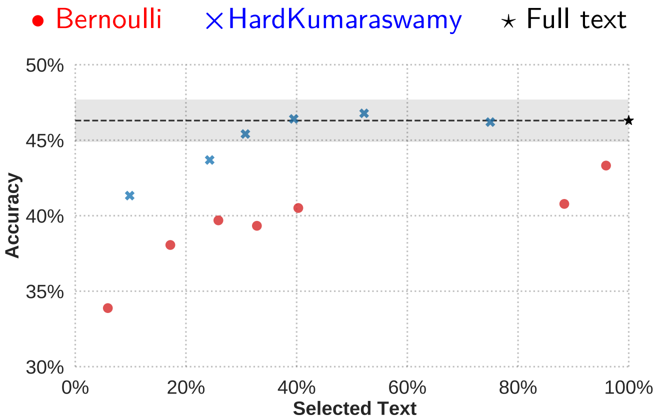
Regression to sentiment score $[0, 1]$

BeerAdvocate

Method	Target rate	Look		Smell		Taste	
		% Precision	% Selected	% Precision	% Selected	% Precision	% Selected
Attention (Lei et al.)	Threshold	80.6	13	88.4	7	65.3	7
Bernoulli (Lei et al.)	Grid	96.3	14	95.1	7	80.2	7
Bernoulli *(reimpl.)*	Grid	94.8	13	95.1	7	80.5	7
HardKuma	Lagrange	**98.1**	13	**96.8**	7	**89.8**	7

Regression to sentiment score $[0, 1]$

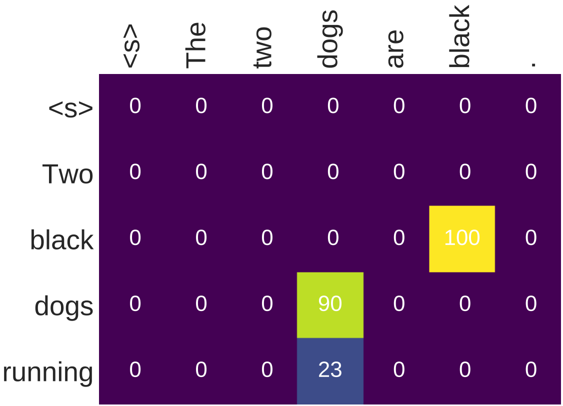
Stanford sentiment classification



Classification from very negative to very positive

Stanford natural language inference

Entailment



Stanford natural language inference

Contradiction

	<s>	Three	cats	race	on	a	track	.
<s>	0	0	0	0	0	0	0	0
Three	0	84	0	0	0	0	0	0
dogs	0	0	100	0	0	0	18	0
racing	0	0	0	87	0	0	43	0
on	0	0	0	0	0	0	0	0
racetrack	0	0	33	48	0	0	73	0

Stanford natural language inference

	Accuracy	
	Dev	Test
LSTM (Bowman et al. 2016)	–	80.6
DA (Parikh et al. 2016)	–	86.3
DA *(reimplementation)*	86.9	86.5
DA with HardKuma attention	86.0	85.5

1% drop with 8.6% of non-zero attention cells

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Remarks

Distributions that mix discrete and continuous behaviour are typically used to sparsify models (i.e. parameters)

We show how to use them to construct differentiable sparse layers

- for sentiment classification (sparse rationale)
- and natural language inference (sparse attention)

Other applications we are looking into include

- adjacency in a graph
- keys/values in memory networks

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Thanks!

References I

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John Winn and Christopher M Bishop. Variational message passing. *Journal of Machine Learning Research*, 6(Apr):661–694, 2005.

Controlled sparsity

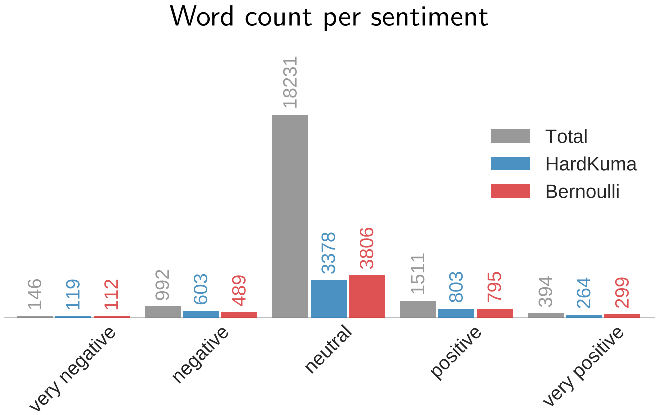
We specify target values t for the sparsity-inducing penalties $R(\phi)$ and employ Langrangian relaxation

$$\min_{\lambda} \max_{\phi,\theta} \mathcal{L}(\phi, \theta) - \lambda^\top (R(\phi) - t)$$

where $\mathcal{L}(\theta, \phi)$ is a lowerbound on the log-likelihood function

A simple form of variational inference (Jordan et al. 1999), an instance of a VAE (Kingma and Welling 2014).

Sentiment words



Reparameterised gradients

$$\frac{\partial \mathcal{L}}{\partial u} = \frac{\partial \mathcal{L}}{\partial h} \times \frac{\partial h}{\partial t} \times \frac{\partial t}{\partial k} \times \frac{\partial k}{\partial u}$$

$$\begin{aligned} k &= F_K^{-1}(u; a, b) \\ t &= l + (r - l)k \\ h &= \min(1, \max(0, t)) \end{aligned}$$

ELBO

We need to marginalise all possible latent assignments:

$$\log P(y|x,\theta,\phi) = \log \sum_z P(z|x,\phi) P(y|x \odot z, \theta)$$

but there 2^n of those!

Let's derive a lowerbound

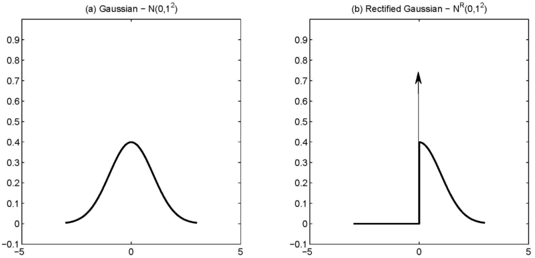
$$\log P(y|x,\theta,\phi) \overset{\text{JI}}{\geq} \underbrace{\sum_z P(z|x,\phi) \log P(y|x \odot z, \theta)}_{\mathcal{L}(\theta,\phi|x,y)}$$

and work with gradient estimates instead

$$\begin{aligned}
\boldsymbol{\nabla}_\theta \mathcal{L}(\theta,\phi|x,y) &= \mathbb{E}_{P(z|x,\phi)}[\boldsymbol{\nabla}_\theta \log P(y|x \odot z, \theta)] \\
\boldsymbol{\nabla}_\phi \mathcal{L}(\theta,\phi|x,y) &= \mathbb{E}_{P(z|x,\phi)}[\log P(y|x \odot z, \theta) \boldsymbol{\nabla}_\phi \log P(z|x,\phi)]
\end{aligned}$$

Rectified Gaussian

As we know the cdf of a Gaussian variable
we can collapse some of the probability mass to a single point



This variable mixes discrete and continuous behaviour.

Images from Wikipedia
Rectified Gaussians in machine learning (Socci et al. 1998, Winn and Bishop 2005)

Distribution function

For the rectified Gaussian

$$f_H(h) = F_\epsilon(0|\mu,\sigma)\delta(h) + (1 - F_\epsilon(0|\mu,\sigma))\mathcal{N}(h|\mu,\sigma^2)\mathbf{1}_{\mathbb{R}_{>0}}(h)$$

For the Hard Kumaraswamy

$$\begin{aligned}
f_H(h;a,b,l,r) &= \mathbb{P}(h=0)\delta(h) + \mathbb{P}(h=1)\delta(h-1) \\
&\quad + \mathbb{P}(0<h<1) f_T(h;a,b,l,r)\mathbf{1}_{(0,1)}(h) \\
f_T(t;a,b,l,r) &= f_K\left(\tfrac{t-l}{r-l};a,b\right)\frac{1}{(r-l)} \\
F_T(t;a,b,l,r) &= f_K\left(\tfrac{t-l}{r-l};a,b\right) \\
f_K(k;a,b) &= abk^{a-1}(1-k^a)^{b-1} \\
F_K(k;a,b) &= 1-(1-k^a)^b \\
F_K^{-1}(u;a,b) &= \left(1-(1-u)^{1/b}\right)^{1/a}
\end{aligned}$$