Sampling

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Content

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from here let's assume CNF

Inducing PCFGs with EM

Let us define the parameters $\boldsymbol{\theta} \in [0, 1]^{|R|}$ where[1]
$p(r) = \theta_r$ where $r = A \to \alpha$ is a rule in R

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E-step conditions on current model $\boldsymbol{\theta}$
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Expectations: Inside-Outside dynamic program $O(|V|^3 |\mathbf{w}|^3)$

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  sampling

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If we could solve 1

then $\hat{\Phi} \equiv \frac{1}{N} \sum_{i=1}^{N} \phi(x^{(i)})$

Robert and Casella [2004]

Monte Carlo estimates

Accuracy of an MC estimate is independent of dimensionality

$$\hat{\Phi} \equiv \frac{1}{N} \sum_{i=1}^{N} \phi(x^{(i)})$$

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However, it is **very hard** to sample from high dimensional spaces!

Sampling from chart

Given a string $\mathbf{w}$, assume we can build the chart $\mathcal{T}(\mathbf{w})$

- $\langle i, A, j \rangle$ where $A \in N$ and $0 \leq i < j \leq |\mathbf{w}|$ represents a chart cell

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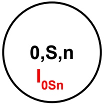
then,

expectations trivial Inside-Outside run

sampling trivial random tree traversal from start symbol

Top-down sampling illustration



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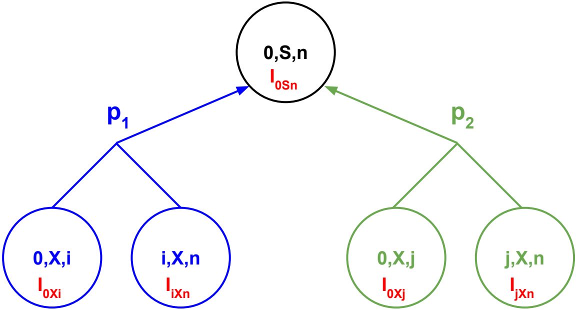

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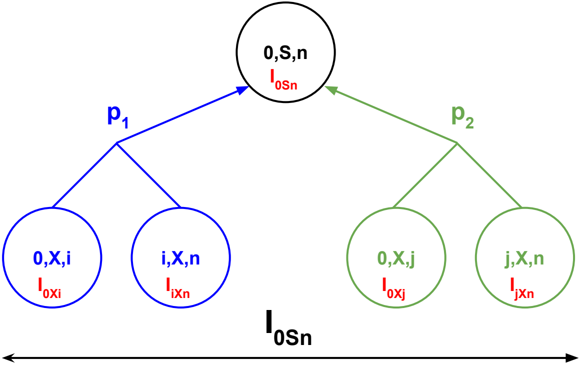

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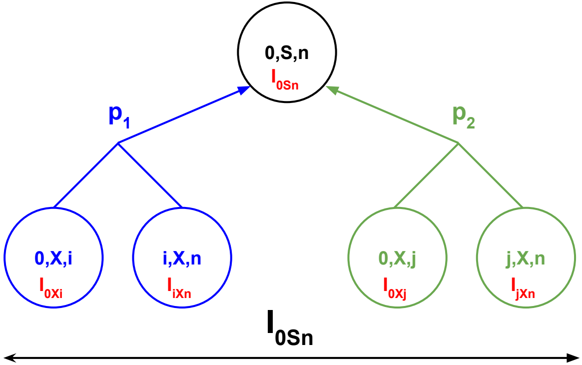

$p_1 \times I_{0xi} \times I_{iXn}$

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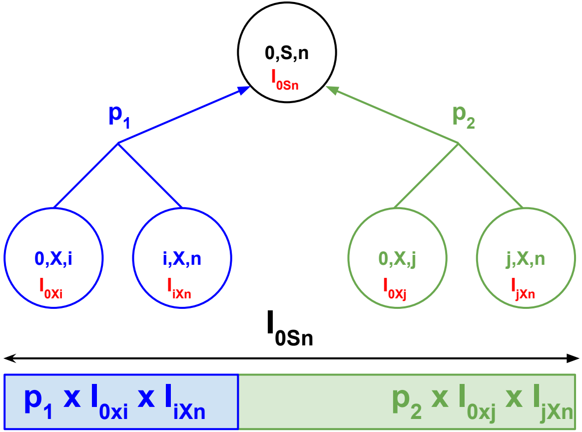

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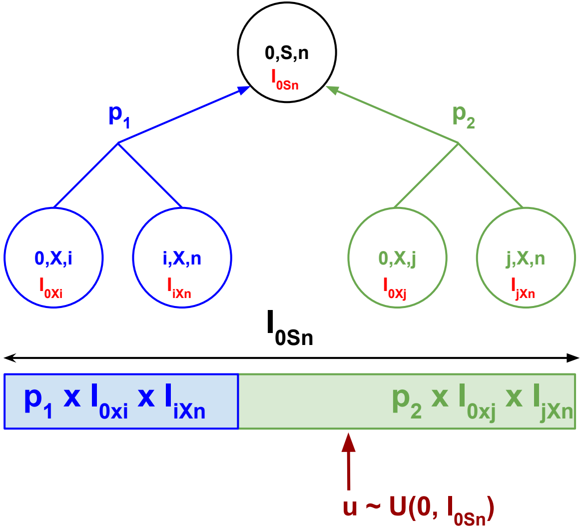

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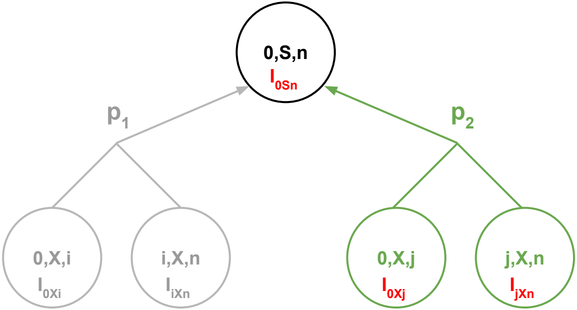

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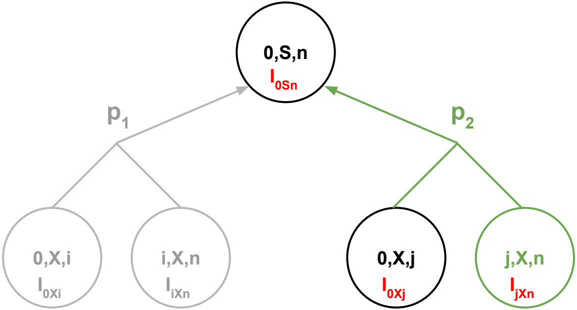

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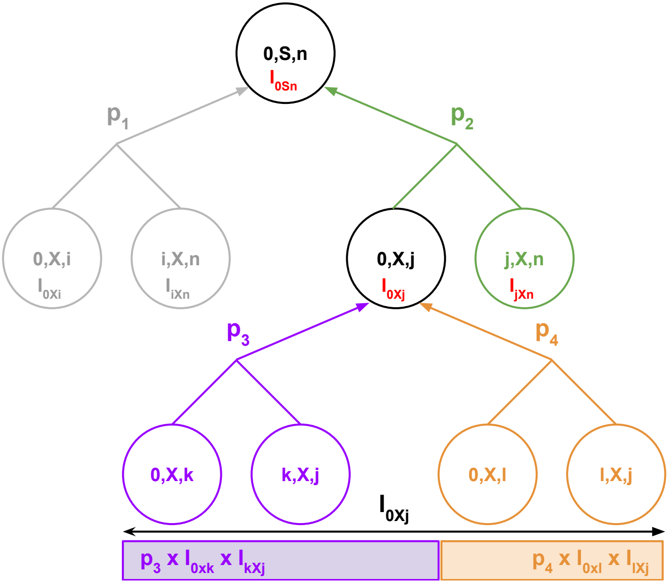

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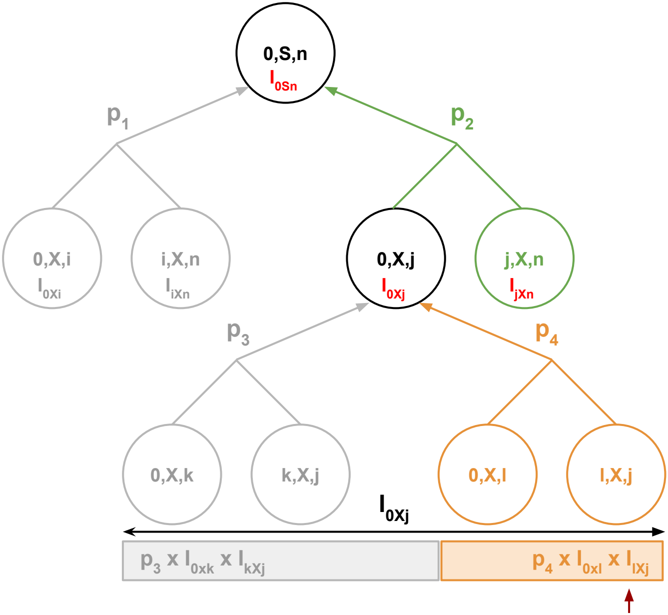

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~~Given a string $\mathbf{w}$, assume we can build the chart $\mathcal{T}(\mathbf{w})$~~

We care about the cases in which we cannot instantiate the chart!

Why is it hard to sample from high dimensional spaces?

Let's rewrite the density

$$p(x) = \frac{p^*(x)}{Z_p} = \frac{p^*(x)}{\int_{\mathcal{X}} p^*(x)\mathrm{d}x}$$

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In parsing

it is the inside at the root of the chart

but we cannot afford building the chart!

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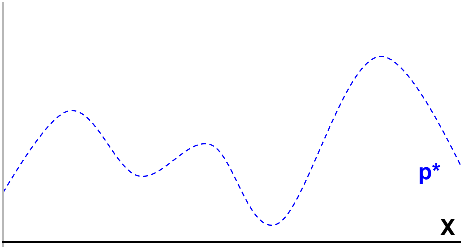
Can we get by without expressing Z_p?

We could sample uniformly directly from the support $\mathcal{X}$ approximating Z_p by how much of it we have seen

$$Z_N = \sum_{i=1}^{N} p^*(x^{(i)})$$

Uniform sampling



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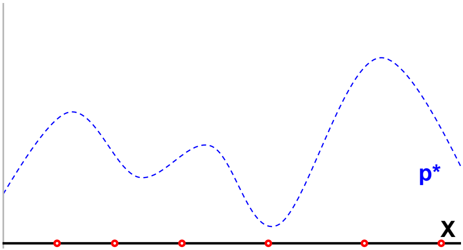

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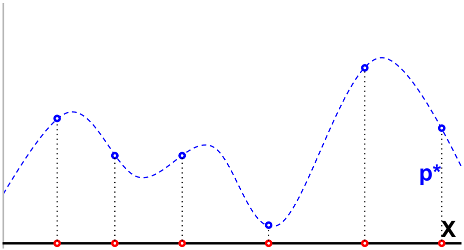

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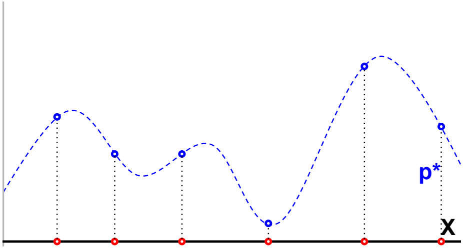

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- there are 2^d states — think of it as derivation trees
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Suppose, 10^3 bits

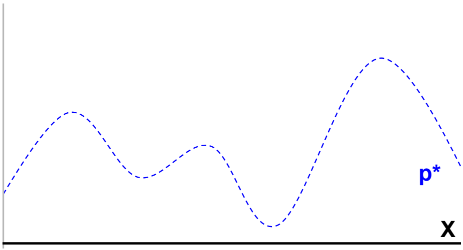
think of it as rules in a chart for $|\mathbf{w}| = 10$

- $2^{500} \approx 10^{150}$ trials
  square of the number of particles in the universe
  [MacKay, 1998]

Lessons

1. assessing Z_p in high dimensional spaces is hard
2. sampling is hard even when $p^*(x)$ is easy to evaluate
   (and direct access to Z_p is not required)

Importance sampling



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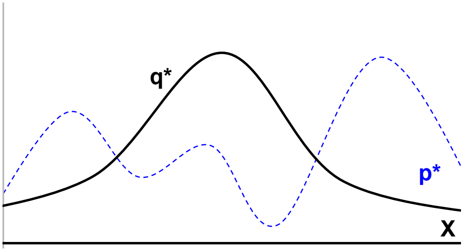

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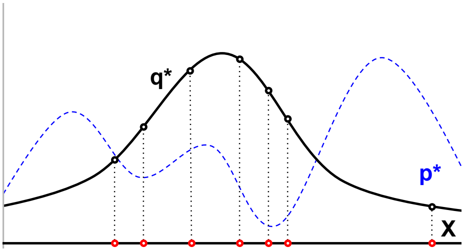

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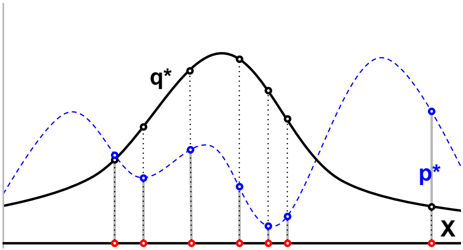

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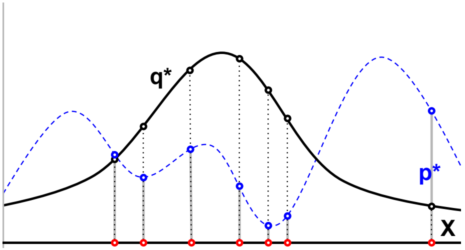

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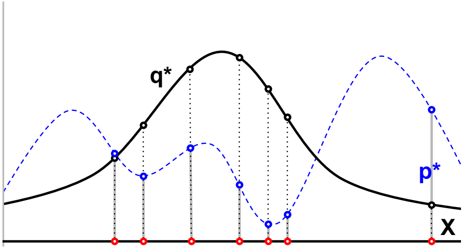

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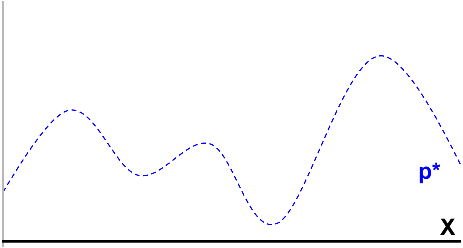
Importance sampling

Introduces an instrumental distribution $q(x)$

- a better guess than sampling uniformly from the state space
- $q(x)$ is such that sampling from it is trivial
- the variance of the estimate becomes a $q(x)$

Rejection sampling



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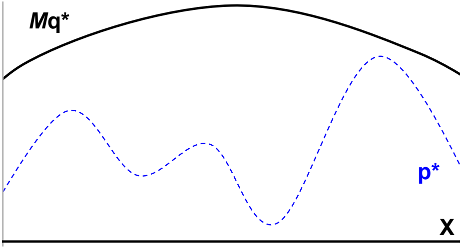

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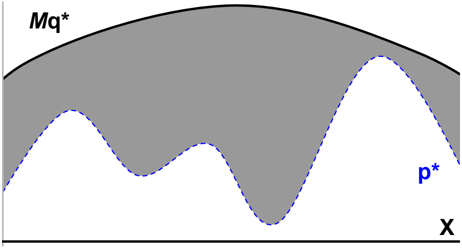

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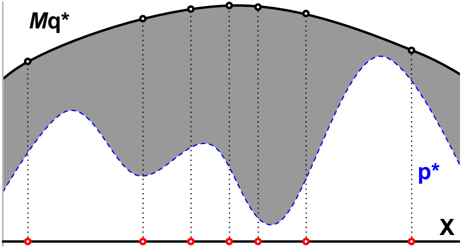

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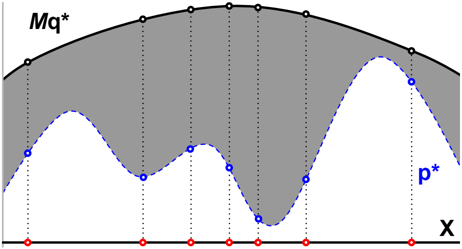

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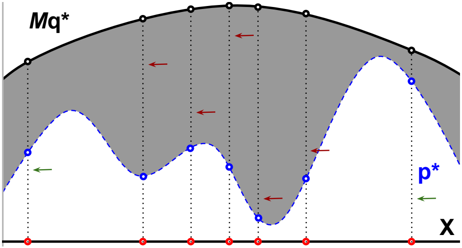

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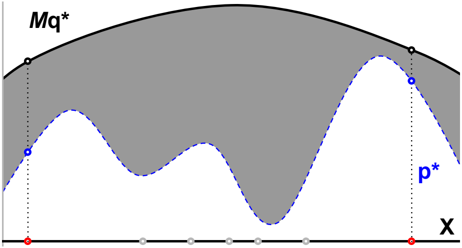

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Accepted x's make an exact sample from $p(x)$

$$\hat{\Phi} = \sum_{i=1}^{N} \phi(x^{(i)})$$

Rejection sampling

Introduces an upperbound $Mq^*(x) \geq p^*(x)$

1. sample (x, u) uniformly distributed over the $(d+1)$-dimensional surface under $Mq^*(x)$
2. retain only points uniformly distributed under $p^*(x)$

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Problem

- low acceptance rate
- in high dimensional spaces, M is typically huge
  the ratio $\frac{Z_p}{MZ_q} \to 0$

MC for parsing

Consider the integration of a parser and a 2nd order HMM tagger

$$p(\mathbf{t}) = p_G(\mathbf{t}) p_{H_2}(h(\mathbf{t}))$$

where $h(\mathbf{t})$ is the sequence of tags

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Rejection sampling

replace p_{H_2} by a lower-order upperbound (e.g. 0-order HMM)

$$q(\mathbf{t}) = p_G(\mathbf{t}) q_{H_0}(h(\mathbf{t}))$$

Markov chain Monte Carlo

A Markov chain that leaves the desired distribution **invariant**

- unlike MC, samples are not independent
- in the limit of an infinite chain, the state of the chain converges to the target distribution
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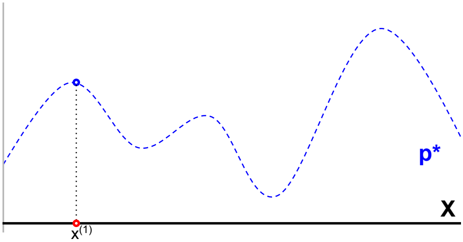
Samples and expectation

1. $\{x^{(i)}\}_{i=k}^{N}$
2. $\hat{\Phi} = \sum_{i=k}^{N} \phi(x^{(i)})$

Slice sampling



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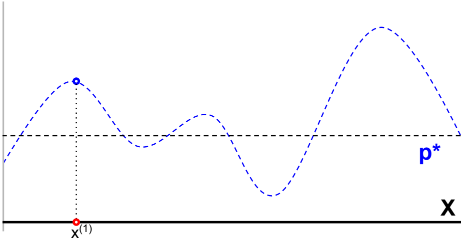

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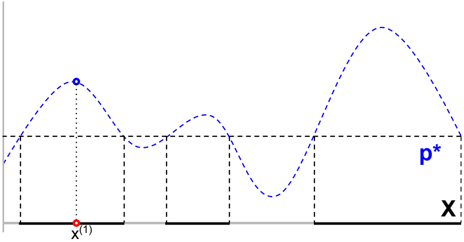

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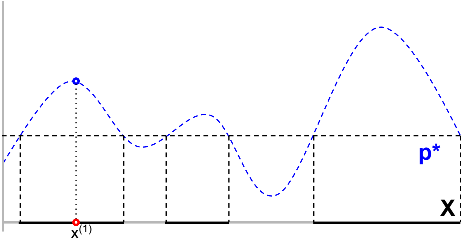

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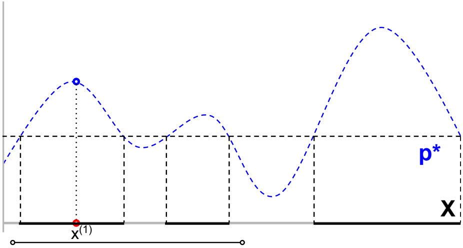

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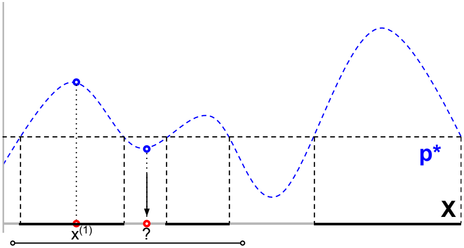

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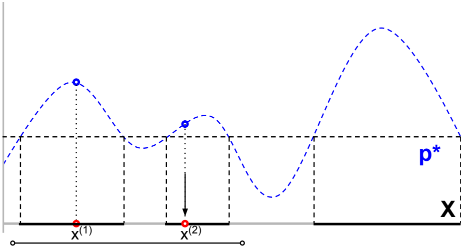

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- find an interval I that contains $x^{(i)}$ and as much of S as feasible
- draw x' uniformly from I
- make $x^{(i+1)} = x'$ if $x' \in S$, that is, $p^*(x') > y$

Slice sampling

An attempt to get a "black box" sampler

- form of auxiliary variable sampling
- no need for proxy distributions
- requires assessing p for a given sample and for the boundaries of an interval I
- finding I can be hard

Gibbs sampling

Task

sample from the joint $p(\mathbf{x} = x_1, \ldots, x_n)$

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- can be done when we know how to sample from all the required conditional distributions
- running the sampler for a sufficiently long time produces a samples of values for x from close to the target distribution

MCMC pros and cons

Cons

1. slow mixture (particularly Gibbs)
2. hard to diagnose convergence

Pros

1. enable inference when $p(x)$ is just too complex for dynamic programming
2. estimates can always be improved by increasing the number of samples

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MLE via EM requires Inside-Outside $O(|\mathbf{x}|^3 |\mathbf{y}|^3)$ prohibitive!

Slice sampling for synchronous parsing

We introduce an auxiliary variable per chart cell

- chart
  $S = \{\langle A, i, j, k, l\rangle : 0 \leq i < j \leq |\mathbf{x}|, 0 \leq k < l \leq |\mathbf{y}|, A \in V\}$
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- rule applications r_s with $\theta_{r_s} \leq u_s$ are pruned from the dynamic program

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Sampling $p(\mathbf{u}|\mathbf{t})$

- u_s are conditionally independent

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The hyperparameter a controls the degree of pruning

Blunsom and Cohn [2010]

Bayesian inference

Bayesian MAP inference

$$p(\boldsymbol{\theta}|\mathcal{W}) \propto p_G(\mathcal{W}|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

- $\mathcal{W}$ data (set of strings)
- $p_G(\mathcal{W}|\boldsymbol{\theta})$ likelihood of data given model $\boldsymbol{\theta}$
- $p(\boldsymbol{\theta})$ prior (if uniform we get MLE)

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Posterior is also a Dirichlet

- $p_D(\boldsymbol{\theta}|\mathcal{T}; \boldsymbol{\alpha}) = p_D(\boldsymbol{\theta}|\mathbf{f}(\mathcal{T}) + \boldsymbol{\alpha})$

"updates the prior conditioning on evidence"

Gibbs sampling

Let's rewrite the posterior in terms of a joint distribution

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Gibbs sampling

Let's rewrite the posterior in terms of a joint distribution

$$p(\boldsymbol{\theta}|\mathcal{W};\boldsymbol{\alpha}) = \sum_{\mathcal{T}\in G(\mathcal{W})} p(\mathcal{T},\boldsymbol{\theta}|\mathcal{W};\alpha)$$

In Gibbs sampling, we can sample from the joint $p(\mathcal{T},\boldsymbol{\theta})$ by sampling from the conditionals

- $p(\mathcal{T}|\boldsymbol{\theta},\mathcal{W};\boldsymbol{\alpha}) = \prod_{\mathbf{w}\in\mathcal{W}} p(\mathbf{t}|\mathbf{w},\boldsymbol{\theta};\boldsymbol{\alpha})$

$$\mathbf{t} \sim p(\mathbf{t}|\mathbf{w},\boldsymbol{\theta};\boldsymbol{\alpha})$$

requires Inside weights

- $p(\boldsymbol{\theta}|\mathcal{T},\mathcal{W};\boldsymbol{\alpha}) = p_D(\boldsymbol{\theta}|\mathbf{f}(\mathcal{T})+\boldsymbol{\alpha}) = \prod_{A\in N} p_D(\boldsymbol{\theta}_A|\mathbf{f}_A(\mathcal{T})+\boldsymbol{\alpha})$

$$\boldsymbol{\theta}_A \sim p_D(\boldsymbol{\theta}_A|\mathbf{f}_A(\mathcal{T})+\boldsymbol{\alpha})$$

there exists efficient techniques to sample from a Dirichlet

Johnson et al. [2007]

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One can sample in order to

- compute expectations (e.g. when performing MLE)
- marginalise latent variables
- approximate distributions in general

Questions?

References I

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