13.1 Ballon Design, Introduction

Balloons are one of the earliest aeronautical devices, and yet understanding their deceptively simple behavior can be a challenge. Fortunately their mysterious ways can be understood by a 1st principles approach. The balloon technology addressed in this chapter deals mainly with large plastic high-altitude scientific gas balloons that fly in the stratosphere. These giant balloons have a launch, ascent, float, and termination phase. To give the reader an idea of size, the stratospheric scientific balloons commonly used ranges from 11 to 40 *million cubic feet* (MCF), with typical altitudes of 120,000 ft (36.5 km) and gross inflations up to 14500 lbs. Serious research in developing these types of stratospheric balloons have at their origins the US air force (for cosmic ray effects on pilots), and for cold war high-altitude spying platforms. Much UFO sightings can be traced back to these large balloons (but that's another story). A spin-off of this research was to apply these vehicles for acquiring data from science payloads, now conducted through the Columbia Scientific Balloon Facility supported by NASA.

What makes a lighter-than-air vehicle possible is the principle of buoyancy or Archimedes principle where the buoyancy force is equal to the weight of the displaced fluid, as in a boat displacing water. In the case of a balloon, the buoyancy is complicated by the presence of a lighter compressible lifting gas, and so the principle is more accurately described as: *the net force of buoyancy is equal to the difference in weight between the displaced air and the lift gas*. The buoyancy force for the whole volume is known as the gross inflation, and the lift gas has a lifting ability per unit volume known as the specific buoyancy b (N/m^3), which changes with temperature, pressure, and lift gas species. When a balloon along with its payload is floating in perfect equilibrium, the system density is exactly equal to the density of the ambient air. That is the system density is all the mass of the balloon material, the payload, the lift gas mass, divided by the total displaced volume.

A lifting gas such as helium is always required because it provides reacting pressure such that thin-film membrane structures can be used in the construction. If one wanted to reinvent the so-called "vacuum balloon" then the additional weight of a lift gas will seem trivial compared to the huge weight penalty from a structure that has to take the full compressive stress of atmospheric pressure (and not buckle).

There are many types of balloons and hybrids between the groups as well. Latex weather balloons are characteristic in that they have a skin that keeps stretching until they burst (usually at a calibrated diameter) while having no fixed float altitude in particular. Hot air balloons have constant pressure and volume but variable temperature differentials between inside and outside the gas envelope, also with no particular fixed float altitude. Gas balloons can have variable or fixed volume with an uncontrolled temperature differential (some control with optical properties of the skin). They can be designed to be pressurized as a constant volume / variable pressure device or unpressurized as a constant pressure / variable volume device. With several parameters to play with (volume,

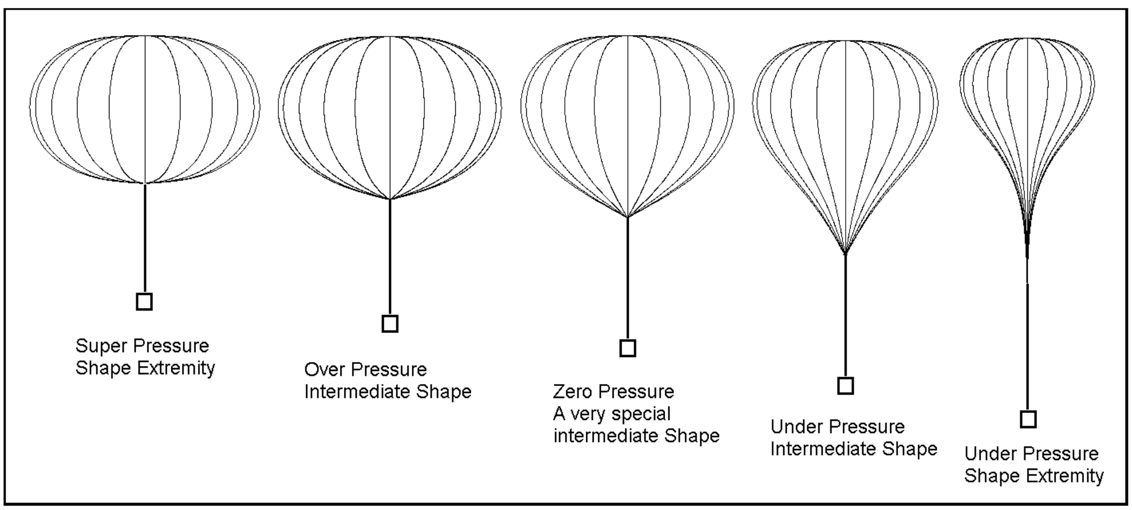


Figure 13.1   Spectrum of axisymmetric natural balloon shapes with horizontal tops

pressure, temperature) one can come up with combinations and hybrids to fulfill any requirements, including a fixed design altitude. Balloons can be made to have many different shapes, but the most common flavor of shape is known as the "natural shape", whereby the skin stress in the circumferential (transverse) direction is zero. Meridionally-lobed membrane structures such as circular parachutes and balloons, where cords or tapes run in a vertical plane, concentrate the loads in the "longitude direction", or meridional as it is called. The stresses in the transverse direction (circumferential, hoop) are very small compared to the stresses in the meridional direction which concentrate in the load tapes. This leads to the natural shape, and depending on the internal pressurization and skin weight, will lead to the variation of natural shapes as illustrated in figure 13.1.

The super pressure shape extremity on the far left is fondly called a pumpkin balloon and has the property that once the shape has been established then more pressure will not change the basic shape (with bulges it is a pumpkin, without it is an *isotensoid*). On the far right is the under pressure shape extremity, so noted because the cone angle of the skin at the bottom is vertical. Reducing the pressure further will shrink the balloon but it will still have the same scale shape, only more "roped" section of uninflated balloon below the vertical point. There is an infinite spectrum of intermediate shapes in between with one special case in the middle, the zero pressure shape. An engineered balloon only has the intended amount of skin material about the design point, and so the intermediate shapes actually are more complex then the simplifications shown above. They will have wrinkles and bulges resembling scoops of ice cream on a cone.

When we speak of pressure in a balloon, we are more accurately describing the "differential pressure", the difference between internal lift gas pressure and the external atmospheric pressure. The lift gas will have a measurable gradient of differential pressure from bottom to top (base to apex, in balloon jargon). The zero pressure shape is notable because at the very bottom, the differential pressure is zero. In other words, at the base point, the lift gas pressure is equal to ambient atmospheric pressure. As one sounds upward through the balloon, the differential pressure increases until a maximum at the apex is reached. This is a manifestation of the lifting force created by a less dense lift gas. A hot-air balloon (Montgolfiere) is also an example of a zero pressure balloon, as the bottom is open such that the differential pressure at the base is zero. Gas balloons as well as hot-air balloons have been made in shapes other than the "natural" shape, such as cylinders, cones, and tetrahedral shapes known as "tetroons".

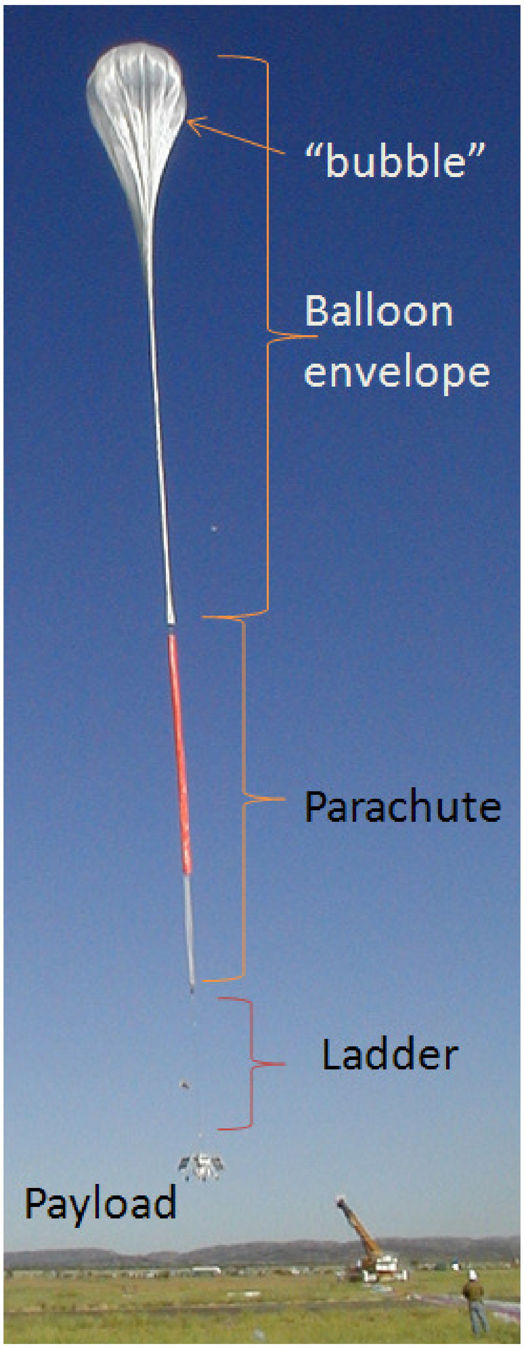


Figure 13.2. Launch in Australia (CSBF)

Mental Experiment - a day in the life of a scientific balloon

A visceral appreciation of modern high-altitude balloon flight can be attained if we for a moment imagine ourselves to be a balloon, in other words we shall *think* like a balloon. This experiment begins just after release from the launch site (fig 13.2), where most of our clear plastic loose polyethylene skin is stretched out vertically, with a small bubble of helium gas at the top. It's quite stressful, having all that lifting force concentrated in a small bubble of skin, which is why we are designed with a *cap*, an extra 1,2,or 3 layers of film just on our top.

We resemble a skinny multi-scoop ice cream cone, with a flight train and gondola hanging from the bottom point of the cone. As we rise with more buoyancy lift than we weigh, the balloon accelerates to a speed of approximately 4-5 m/s where the aerodynamic drag force settles into equilibrium with the 'excess' buoyancy, known as *free lift*.

As we rise the atmospheric pressure is decreasing with altitude expanding the helium gas, and so the flaccid portion of balloon skin begins to fill out catching the sun. The helium gas cools down in temperature due to the expansion, but the sun and Earth are warming our plastic skin which in turn warms up the helium via internal convection. The gas expansion would truly chill the helium to the point where our upward momentum would stop, but radiant energy from the sun (and some slight help from atmospheric convection) keeps us going. Our ascent slows down considerably at around 4.5 km due to our rapidly expanding volume and resulting adiabatic cooling, but we still have the sun and the ambient air to add to our warmth. Somewhere past 10 km the atmosphere starts to rob us of warmth so that we are more reliant now than ever of the sun to replace the heat energy lost to expansion. It's just as well since with the thinner air the sun is stronger and helps us through the coldest portion of the troposphere, gateway to the stratosphere (at the tropopause). High winds in contrary directions hit us here, and our ground path direction changes. We are also being sustained from below as our balloon skin absorbs the reflected sunlight known as albedo and the warm Earth bathes us with infrared energy. Our volume and surface area is ever expanding, catching more radiant energy as we rise higher. We pass through the *trope* into the stratosphere where the thin air temperature begins to get warmer instead of colder due to the concentration of ozone which absorbs ultra violet. As a result the warmer air rapidly decreases in density, slowing our ascent at about 18 km altitude, since after all we need something to float in. The less dense the air the less we are able to float. The air is so thin at this point that the temperature of our plastic skin is dominated by radiant energy balance. That is, our temperature is mostly the result from the balance of direct sun, albedo, and infrared energy absorbed vs. infrared energy lost, emitted from our plastic skin. The heat from our skin warms up the helium via internal convection so as to be lagging behind in temperature, but for the most part the helium is at the same temperature as our skin when we are at float conditions. Even with cold space above our top gets slightly warmer than the bottom due to the extra layers of cap material absorbing energy.

At a float altitude that we were designed for, generally with only 1% of the atmosphere left overhead, our skin is expanded completely to the full volume that it is capable of, a hundred-fold or so increase from when we first launched. The extra helium gas that was used to give us free lift to ascend to this height is now by its own impetus venting through "pony-tail" ducts to the atmosphere whose lower openings are at the same level as the bottom of the balloon. The ducts are attached about 1/3 of the way up from the bottom so as to prevent siphoning of air into the balloon. As we vent helium, the gas pressure decreases slightly, expanding the helium slightly which lowers our temperature momentarily. The sun warms us and we float into an equilibrium temperature. Depending whether we are in Antarctica or at mid latitude, the gas may be slightly colder or warmer than the surrounding air.

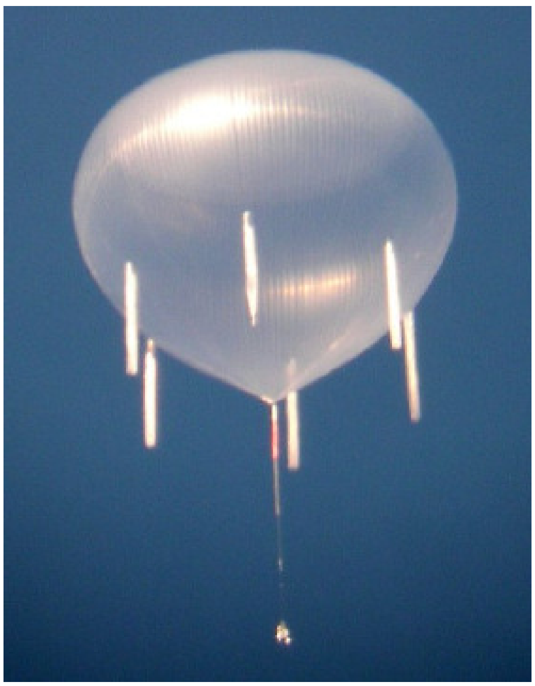


Figure 13.3. 40 million cubic foot zero pressure balloon at float (Mike Smith, Aerostar)

We are a typical high altitude balloon known as a zero pressure balloon because our bottom pressure is equal to atmospheric pressure but our top has a slightly higher pressure than ambient, which is where we derive our buoyant lift. You see, the lift gas is less dense than air and if we are 120m tall that height of helium gas weighs less than the equivalent height of air, and so there is more helium pressure remaining at the top that was not burdened with the job of reacting gas weight. That extra pressure is usefully employed lifting the top of our skin which in turn is able to lift the payload below us. The sun feels 1.5 times as strong up here and the Earth's infrared is about 2/3 of its maximum strength down at the ground since now it has to pass through the atmosphere where some is absorbed in a complex exchange with clouds and sky. The sun gets higher, the ground gets warmer, and more skin area becomes illuminated due to a characteristic of our geometry which in turn heats up the helium. The helium gas expands venting some gas out of the bottom ducts. At solar noon we have the minimum amount of gas (normally the warmest period). This sets us up for the maximum gas we'll have for tomorrow, so some ballast drops will be coming soon to maintain altitude.

We float over a large cold lake, which fills up half of our view of the earth below us, which delivers to us less albedo energy and less infrared than the ground did, and our skin cools down somewhat. The gas contracts, our volume decreases, and we are not as buoyant as before resulting in our sinking. A command is given and some ballast is released which means we weigh less now. Our smaller buoyancy now matches our smaller weight and we stop sinking. We pass by the cold lake, warming back up again expanding to our former volume. Things are different now because we weigh less, and so we float slightly higher than before the lake and the ballast drop. Going higher means a less dense atmosphere and less pressure, so now the correct amount of gas we had for the first float altitude is too much for this slightly higher altitude and so we vent out more of our precious helium.

The sun starts to set cooling down our skin and the helium, and we sink as before. The sun sets, the ground cools, and we only have Earth's infrared energy to keep our skin warm now. The helium cools and contracts, we sink down until another command is given to release a good amount of ballast. The sinking stops and we wait out the night.

A glorious morning sun warms us back up again, but as we are even lighter than before we sail up to an even higher altitude. This of course means we vent out more helium when we attain this higher altitude, warmed up by the sun as before. Several more diurnal cycles like the previously described and we will run out of ballast. It is now time to return to Earth. The destruct command is given; the payload is pyro-separated and begins to fall. As it falls the destruct line yanks out a banana-peel slice out from one of the gores, ripping the gore the entire length. The balloon bursts and the payload waits for the parachute to inflate, waiting for the neck-snap. As the payload parachutes down the balloon folds itself into a giant plastic-wrap football and impacts the ground in a safe area. The payload settles down in a safe area as well, its impact lessened by the cardboard crush pads on the bottom of the gondola's legs.

13.2 Modern Zero Pressure Balloons

There are many kinds of balloons, but one that is frequently spoken of is the "natural shape". Natural-shape refers to designing the balloon such that the skin has zero or near-zero circumferential (transverse) stress, leaving all the major stress in the meridional (vertical plane) direction as in a parachute. Natural shapes with different loading ratios (suspended weight/gross inflation) and skin density ratios are known as "Sigma shapes" from Justin Smalley's reports in the 1950's and '60's. Different Sigma numbers will result in different shapes being derived due to the chosen areal weight density of the membrane film. A heavier film will of course result in a larger required volume for the same payload. Justin Smalley in the 1960's developed the famous Sigma tables by which one could design natural-shaped zero pressure balloons with zero circumferential stress [1]. The tables would give the shape, film tension to payload ratio T/L, and balloon weight to payload ratio W/L parameterized according to the Smalley Sigma number Σ.

If the differential pressure between the ambient air and the lifting gas at the base position (bottom of the balloon) is zero, then it is a 'zero pressure' design and has the familiar upside-down onion shape. Hot-air balloons are zero pressure designs as the bottom is not sealed, and so must be in equilibrium with the ambient pressure at the bottom opening. In their practical construction these balloons are made with vertical 'banana-peel slices' known as gores. The polyethylene gores are heat sealed together along their long edges and reinforced at the seams with embedded cord known as load tapes. With internal buoyancy pressure the gores can bulge out between the loaded tendons to give the lobed appearance although they do not have to. Hot air balloons do this to gain a margin of volume, where as a zero pressure gas balloon has its skin in a stress equilibrium with the buoyancy pressure using the meridional radius of curvature (without any help from the transverse direction), resulting in a smooth skin. Zero pressure designs have relatively low skin and tendon (or load tape) loads. Being unsealed a helium zero-pressure balloon will vent overboard helium that gets heated if the expansion is above the volume capacity of the balloon. But at night it will drop in altitude due to the cooling of the gas. This is why zero pressure balloons only last several diurnal cycles, as ballast runs out in an attempt to maintain altitude to make up for gas being chuffed out by flying higher each day (ballast drops makes it lighter), and more gas venting during hot portions of the day. Ballast and gas can easily be wasted if the corrections are too severe which over shoots the float altitude and vents precious lift gas.

These high-altitude balloons are constructed with polyethylene film in the 0.3 to 0.8 mil range (0.0003" – 0.0008"). The film is special in that it is co-extruded from several layers that melt into one layer. The purpose is to minimize the chance of ever having pin-holes that line up leading to undesirable porosity above and beyond the normal amount of helium permeability. Although strictly speaking helium permeability is a function of film temperature, film thickness, and differential pressure, the differential pressures would have to be orders of magnitude higher than what is seen in large balloons to make differential pressure of any concern (temperature and film thickness having the greatest effect).

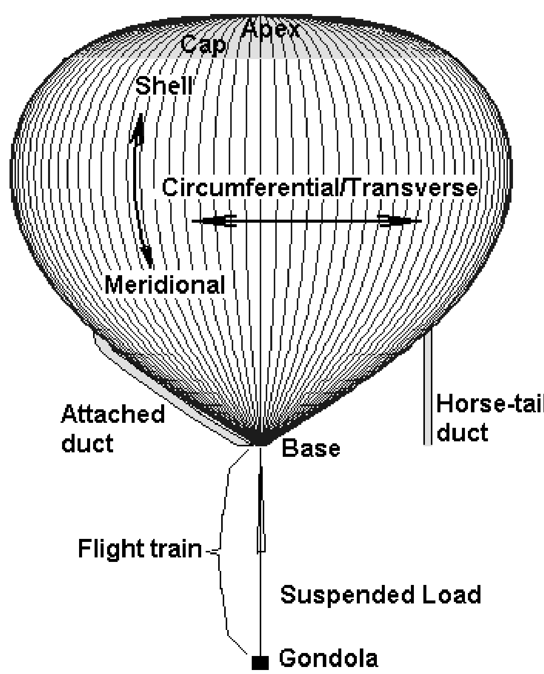


Figure 13.4. ZP definitions.

Balloons can bob at float altitudes due to their inherent mass-on-a-spring behavior. Adiabatic expansion/compression is complicit in that if there is a perturbation in altitude, say upwards, there is an expansion and cooling of the gas which in turn contracts to reduce buoyancy. The balloon sinks, over travels the equilibrium point, and then compresses with an increase in temperature, increasing buoyancy. The atmosphere as a body behaves similarly with so-called Vaisala-Brunt gravity waves, and when the wind blows over mountains the gravity waves can set up the perturbations necessary to disturb the balloon in the stratosphere. When the super temperatures are just right (near zero), vertical bobbing resonances can occur.

Zero pressure balloons need ducts to prevent over-pressurization of the envelope which would result in a structural failure. These ducts can hang down, or be pulled up to the shell, known as *attached*. This is done for the sake of some payloads which might have a telescope that can be occluded by a hanging duct. One can by use of the rate of climb and pipe flow equations determine the film stress in a duct. Typically an equation based on years of experience is used instead.

Figure 13.5. Dynamic launch [CSBF]

There are numerous ways to launch large balloons, but the one preferred at CSBF (Columbia Scientific Balloon Facility) is the "dynamic" launch method. Launch dates are set according to the upper level winds, such as during turn-around at the equinoxes, but launch times are planned for when surface winds are minimal. Flights are meticulously planned using reliability probabilities leading to a casualty expectation (CE) analysis. If CEs are above a given threshold, then there is no flying that day.

13.3 Modern Super Pressure Balloons

If the envelope is sealed and pressurized substantially above having a zero differential base pressure, then these designs are known as super-pressurized so as to survive diurnal temperature effects without losing altitude. Super pressure designs generally come in two categories, spherical and pumpkin shaped. In a spherical design all the stress is carried by the skin, where as a pumpkin design mostly separates the roles of structural and gas containment. A super-pressurized balloon in effect has reserve pressure such that a drop in temperature should not drop the pressure below the point where a large change in shape volume occurs. Since the mass and volume of the gas is a constant, the average density is also constant which preserves the float altitude. While zero pressure balloons can have substantial altitude variation as the gas envelope expands and contracts over the course of a day, a super pressure balloon will have a rock solid altitude profile so long as pressure is maintained in the envelope. That is where the magic lies, in determining the expected temperature variation for the flight environment and seeing that the structure can take the stress for the required super pressure. If one examines the ideal gas law and takes the derivative dP/dT, it will equal the gas density times the gas constant. This implies that the lower the density, the smaller the ΔP will be for a given ΔT, which implies that the higher it flies then the lower the ΔP that one has to deal with over a diurnal cycle (because the gas density will be lower at higher altitudes). Higher is easier for a super pressure balloon (> 33.5km, 110000 ft for a large pumpkin).

Figure 13.6. Julian Nott's super pressure pumpkin balloon at float

Super pressure spheres (fig 13.7) are optimal for small balloons, but for large balloons the skin weight becomes too much a liability. As a pressure vessel the skin stress is proportional to 1/the radius of curvature, so a large sphere will have huge skin stresses while a

Figure 13.7. 1960's polyester super pressure sphere GHOST

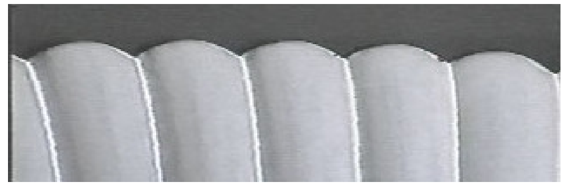


Figure 13.8. Super pressure pumpkin lobed bulges

pumpkin reduces the hoop stress by having a small bulge radius on each lobed gore (small compared to the global radius of the balloon).

The cords that look like parachute lines in figure 13.8 are called tendons; they are made with a very stiff fiber material called PBO. PBO however has degradation issues from ultraviolet and moisture. Without protection measures one must assume that at least 50% of the strength will diminish over the course of the flight. And in any case the failure mode is a creep-rupture type which further de-rates the tensile ability. Still, the stiffness and lightness of this material is phenomenal.

The major problem with super pressure pumpkins is a global shape instability that can occur when there is an excess of circumferential material, forming what is known as an s-cleft. Analyzing for such instability in the intended shape is extremely complex and has occupied years of effort by mathematicians and other researchers. Fortunately their efforts as well as empirical scaling from flight and laboratory observations have resulted in a much greater understanding of this phenomenon. For instance, we now know that this is indeed a low potential energy state where if the balloon has an inclination to do so it will to go there. It is not an artifact of friction, and in fact modeling shows that the volume slightly increases from the design shape when in an s-cleft shape. The majority of the potential energy is in compressing the gas, so having a slightly larger volume option for the balloon (to lower the pressure) becomes irresistible when the conditions are ripe for an s-cleft. Empirical

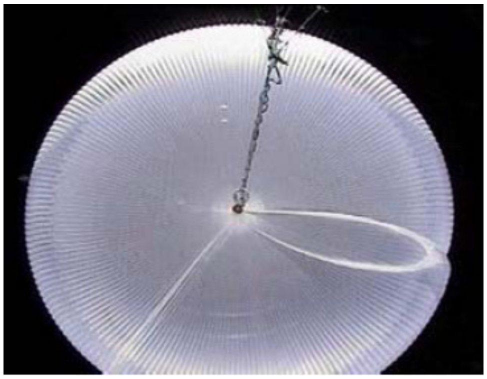


Figure 13.9. Flight 555 Sweden, up-looking camera viewing an s-cleft (NASA, CSBF)

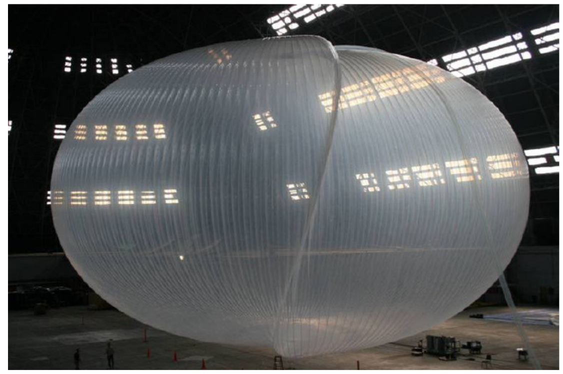


Figure 13.10. 27 meter diameter test pumpkin designed to show an s-cleft simulating Flight 555

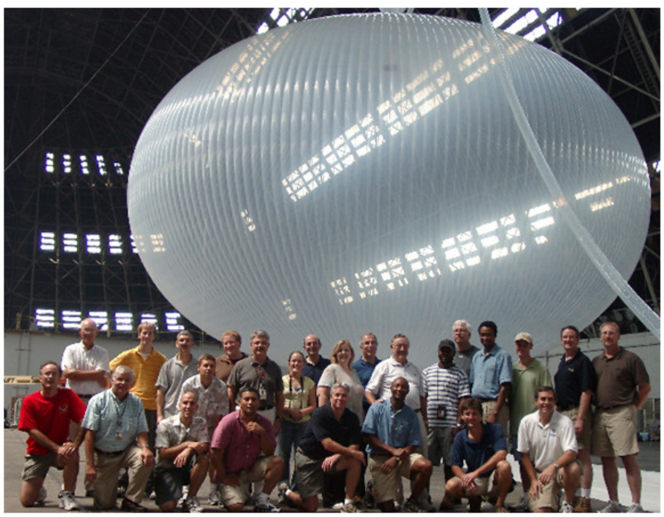


Figure 13.11. 27 meter diameter test pumpkin designed to be cleft free

rules-of thumb known as cleft factors (CF) are considered when designing super pressure pumpkins, to be later confirmed with the new analytical methods (under the auspices of NASA's Balloon Program Office). A 7 million cubic foot pumpkin with 200 gores (Flight 591) now holds the endurance record for a balloon of that size with a 54 day flight circling Antarctica 3 times (2008, NASA and CSBF).

With no circumferential stress and no lobing this shape would be an isotensoid, following Euler's Elastica formulation. A sphere is famous for its maximum volume to surface area$^{3/2}$ ratio, but a fascinating fact is that this shape has a volume to gore length^3 ratio greater than a sphere, which is why filament-wound spacecraft propulsion tanks use this shape.

Recommended Super Pressure at Design Float Altitudes

This is the super pressure required for any pressurized balloon to stay afloat:

$$\Delta P_{Required} = \frac{P_{air}}{T_{air}} \cdot \Delta T_D + \Delta P_{Reserve}$$

Which is approximated by:

$$\Delta P_{Required} \approx \left[475 \cdot e^{-0.155 \cdot (Altitude_{km} - 1)} \right] \cdot \Delta T_D + \Delta P_{Reserve} \qquad (13.1)$$

$\Delta P_{Reserve}$ is the minimum differential pressure you wish to have in reserve, Pascals.

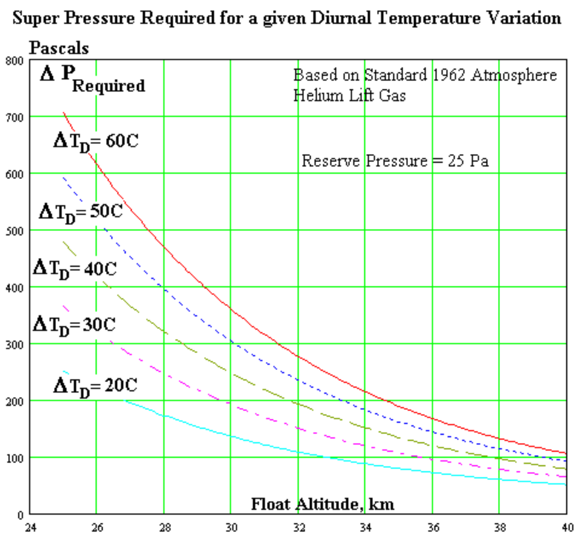


Figure 13.12. Required super pressure as function of float altitude

$Altitude_{km}$ is the altitude in kilometers

ΔT_D = maximum gas day temperature – minimum gas night temperature, deg C

Note: Recommendations for balloons having a solar absorptivity to IR emissivity ratio of 0.2 (the lower the a/e ratio, the lower the required differential pressure): Desert regions can have up to $\Delta T_D \sim$ 70C, whereas $\Delta T_D \sim$ 30 to 50C for most other places when at high altitudes.

As can be seen in the graph, the higher the balloon flies the much easier it gets as far as the super pressure requirement.

13.4 Balloon Physics Fundamentals

As was stated in the introduction, balloons are easily investigated using a 1^{st} principles approach. The 1^{st} principles used are:

Ideal gas law
Aerostatics
1^{st} law of thermodynamics
Heat transfer by radiation
Heat transfer by convection
and of course Newton's laws

13.4.1 Ideal gas law

A given number of gas molecules at a given absolute temperature T in a given volume V will produce the same pressure P no matter the molecular species, that is, for an 'ideal gas' (Eq. 13.2a).

$$P = n \cdot R \cdot T/V \qquad (N/m^2 = Pa) \qquad (13.2a)$$

Where R is the ideal gas constant of 0.83141 Joules/mole/K, n is the number of moles of molecules (6.022×10^{23} molecules per mole). A customized version of the ideal gas law using the mass density and a customized ideal gas constant R_{gas} has the form shown in Eq. 13.2b:

$$P = \rho \cdot R_{gas} \cdot T \qquad (Pa) \qquad (13.2b)$$

where ρ is the mass density (kg/m^3) of the gas.

To customize the equation for any particular gas:

$$R_{gas} = 8314.1 \,/\, \text{molecular weight} \qquad (m^2/s^2/K)$$

The ideal gas constant for helium	$R_{he} = 2077.2$
The ideal gas constant for diatomic hydrogen	$R_{h2} = 4148.7$
Air (20% oxygen and 80% nitrogen)	$R_{air} = 287.1$

These numbers are slightly off from the direct calculation due to the fact that these gases are not ideal, but close. Non ideal gases have additional degrees of freedom (rotation and internal vibration) and sometimes other sources for creating a potential field such as weak electrical forces. These elements combine to add inaccuracies to the simple statistical bouncing-particle based ideal gas law. This non-linear behavior largely becomes important when compressing the gasses into tanks, but not for typical balloon flight.

13.4.2 Buoyancy and Aerostatics

A difference in densities between the ambient air and the lifting gas produce an aerostatic pressure differential within the envelope volume to produce buoyancy lift. In terms of the Archimedes principle, the difference in weight of the displaced heavier fluid and the weight of the lighter fluid is equal to the buoyant force, or gross lift.

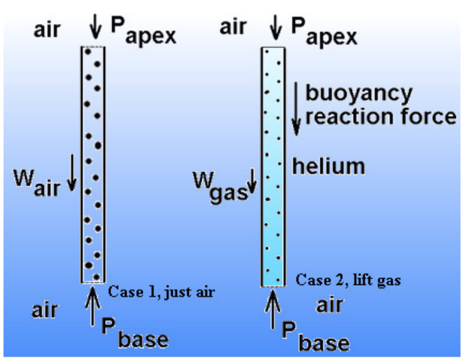


Figure 13.13. Air or helium in a column

Looking at these two cases (fig 13.13) we can set up two equilibrium equations for a cylinder of air of short length L, and one which has a lighter gas substituting the air:

$$P_{base} \cdot Area = W_{air} + P_{apex} \cdot Area$$
$$P_{base} \cdot Area = W_{gas} + P_{apex} \cdot Area + buoyancy_{reaction} \quad (13.3a, 13.3b)$$

where W_{air} and W_{gas} are the weights of air and gas respectively in the cylinders.

The buoyancy reaction force is holding the case for equilibrium. If we divide by the volume (Area · L), and diminish the length to a differential dz (z going up), we arrive at the aerostatic principle (such that $P_{apex} = P_{base} + dP$):

$$\frac{dP_{air}}{dz} = -g \cdot Density_{air}$$
$$\frac{dP_{gas}}{dz} = -g \cdot Density_{gas} \quad (13.4a, 13.4b)$$

If we subtract the two equilibrium equations and substitute buoyancy = - buoyancy reaction, we arrive at Archimedes principle:

$$buoyancy = W_{air} - W_{gas} \quad (13.5a)$$

If we further divide this by the volume, it leads us to the specific buoyancy parameter b:

$$b = g \cdot \left(Density_{air} - Density_{gas} \right) \quad (13.5b)$$

Newtons of lift per cubic meter of lift gas.

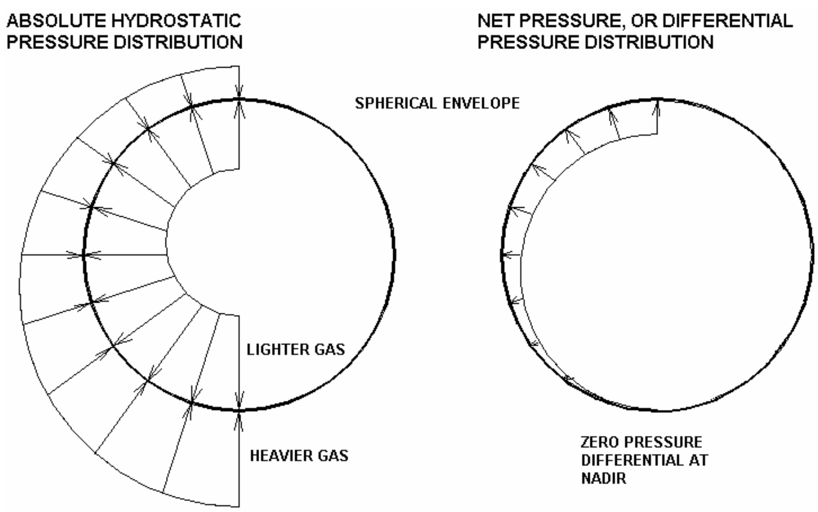


Figure 13.14. Nature of buoyancy in a zero pressure condition

Figure 13.14 illustrates the pressure distribution in a spherical envelope with the pressures being equal and opposite only at the bottom. The resultant pressure differential is characteristic of a 'zero-pressure' design by subtracting the absolute gas pressure from the absolute air pressure. The pressure differential ΔP distribution from apex to base is simply:

$$\Delta P = \Delta P_{apex} - b \cdot z \qquad (13.6)$$

where ΔP_{apex} is the maximum differential pressure at the apex and z is the vertical coordinate starting at the apex and pointing in the base direction.

The total lifting force is defined as the gross inflation, and follows the relation:

$$GI = b \cdot \text{Volume} \qquad (13.7)$$

where Volume is the volume of the gas envelope. Under equilibrium conditions the gross inflation = gross mass (which equals suspended mass + balloon envelope structure).

Combining the aerostatic differential equation 13.4a with the ideal gas law (Eq. 13.2b) as applied to air, integrating vertically upwards in z with a linear temperature profile (known as the temperature lapse rate) arrives at equation 13.8, useful for modeling atmospheric pressure at altitude:

$$P_2 = P_1 \cdot \left(\frac{T_2}{T_1}\right)^{\left(\frac{-g}{L \cdot R_{air}}\right)} \quad \text{where} \quad L = \frac{T_2 - T_1}{z_2 - z_1} \qquad (13.8)$$

13.4.3 Force Balance and Free Lift

Any additional lift above and beyond what it takes to lift the gross weight is known as the free lift force, F. During equilibrium ascent the instantaneous free lift will be balanced by aerodynamic drag. Lets us take a look at the simple relationships:

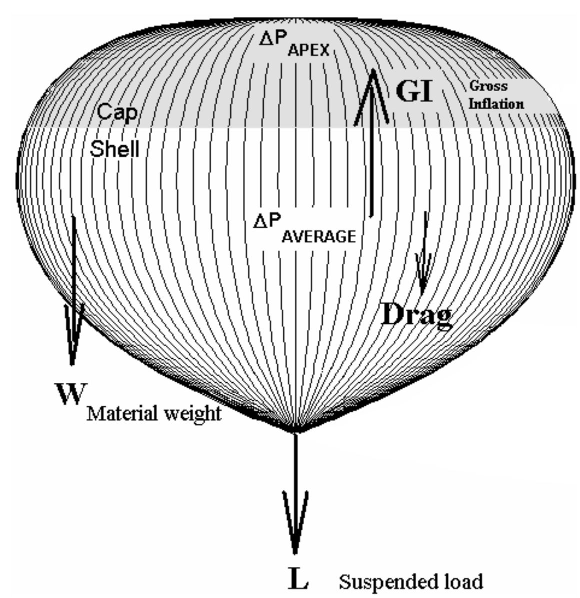


Figure 13.15. Force balance

Gross Inflation GI = b · Volume

Gross Weight = Balloon material weight + Suspended Load
Gross Inflation = Gross Weight + Freelift
Gross Inflation = Gross Weight + Drag @ equilibrium ascent
Drag = Freelift @ equilibrium ascent

GI Gross Inflation, N
G Gross weight = W + L
W Balloon carcass weight (do not include lift gas)
L Suspended load
F Free lift (excess buoyancy) Free lift ratio = 1+F/G = GI/G

For the isothermal and zero pressure case the specific buoyancy b (with g = acceleration of gravity = 9.807 m/s^2 and density in kg/m^3):

$$b = g \cdot Density_{air} \cdot \left(1 - \frac{R_{air}}{R_{gas}}\right) \quad \text{N/m}^3 \qquad (13.9)$$

Here we define the bulk super temperature and super pressure:

$$\Delta T = T_{gas} - T_{air}$$
$$\Delta P = P_{gas} - P_{air} \qquad (13.10a, 13.10b)$$

This next equation combines buoyancy and the ideal gas law to produce the free lift ratio ($FreeLift_{ratio}$ = 1+F/G) at any condition where T and P are the ambient air temperature and pressure, Δ T and Δ P are the super temperature and pressure of the lifting gas.

$$FreeLift_{ratio} = \frac{M_{gas}}{M_{gross}} \cdot \left[\frac{\left(1 + \frac{\Delta T}{T_{air}}\right)}{\left(1 + \frac{\Delta P}{P_{air}}\right)} \cdot \frac{R_{gas}}{R_{air}} - 1 \right] \qquad (13.11)$$

M_{gas} is the gas mass, and M_{gross} is the gross mass (G/g). Easily rearrange the equation to determine the required gas mass if the gross mass is known and a launch free lift ratio established (it usually is). Equilibrium is when the free lift ratio = 1.0. Isothermal freelift is when ΔT = 0. For zero pressure balloons ΔP = 0. Normal day launch free lift ratios are in the 10% range (= 1.1), while launching at night could use free lift ratios in the 25% range (= 1.25).

Montgolfier Hot Air Balloon

With this type of balloon, the super pressure = 0, and if we assume perfect equilibrium (freelift ratio = 1.0), then for a Montgolfiere type of balloon we can simplify:

Gross mass (suspended mass + envelope mass, kg) for a zero-pressure Montgolfier hot-air balloon with super temperature ΔT °C as shown in Eq. 13.12:

$$GrossMass = Volume \cdot \frac{P_{air}}{R_{air}} \cdot \left[\frac{1}{T_{air}} - \frac{1}{(T_{air} + \Delta T)} \right]$$

where,
Volume is the gas envelope volume in cubic meters
P_{air} is the atmospheric pressure in Pascals (Pascal = millibars × 100=N/m^2)
T_{air} is the atmospheric temperature in °K (°K = °C + 273.2)

Or, in terms of required super temperature (FreeLiftForce in Newtons) the super temperature required for the given volume, gross mass, and free lift force is shown in Eq. 13.13 (degrees C):

$$\Delta T = \left[\frac{1}{T_{air}} - \frac{GrossMass + \dfrac{FreeLiftForce}{g}}{Volume} \cdot \frac{R_{air}}{P_{air}} \right]^{-1} - T_{air}$$

13.4.4 Balloon Thermal Fundamentals

Temperatures play a key part in the performance of a balloon. The optical properties of the skin and the energy flux of the environment (the watts per unit area of radiant energy) will determine this temperature. High altitude balloons fly in an environment dominated by radiant thermal energy such as direct solar, reflected solar (albedo) from the ground, clouds, and sky, and infrared energy (up-welling and down-welling). In the case of an ascending balloon the gas is expanding and the energy flux impinging the skin is constantly changing. To determine the temperature of the balloon skin in this case one must employ the 1st law of thermodynamics which is a statement of conservation of energy, also known as the energy balance equation (leaving out the macroscopic kinetic and potential energies).

$$\Delta \text{NetHeatEnergy} = \Delta \text{InternalEnergy} + \Delta \text{Work done}$$

Or in classical form: $dQ = dU + PdV$.

Internal convection is rather important as the transport mechanism of heat energy from the skin to the lift gas. Conduction however does not play much of a role in such thin film membrane structures.

For both static and dynamic conditions, heat transfer by radiation is an important element to understand. Starting with optical properties, this refers to a surface's solar absorptivity α, transmissivity τ, reflectivity r, and infrared emissivity ε and transmissivity τ_{IR}. The parameters α, ε, τ, and r are expressed as fractions in that:

$$1 = r + \alpha + \tau$$

$$1 = r_{IR} + \alpha_{IR} + \tau_{IR}$$

where the 1st implies solar wavelengths and 2nd IR wavelengths.

From Kirchhoff's law of radiation heat transfer, at any specific wavelength the absorptivity is equal to the emissivity while in thermal equilibrium. This is important because absorptivity is generally given for solar short-wave frequencies, but emissivity is given for IR wavelengths. So if we are interested in the absorption of IR radiant energy, we use the given IR emissivity as equal to the IR absorptivity ($\alpha_{IR} = \varepsilon$).

A property of any surface with an absolute temperature T; it emits radiant energy according to the Stefan-Boltzmann relationship:

$$q = \varepsilon \cdot \sigma \cdot T^4 \qquad \text{IR flux, Watts/m}^2$$

Where σ = Stefan-Boltzmann constant (5.67×10^{-8} W/m²K⁴), ε is the surface emissivity, and T is in degrees Kelvin.

The energy absorbed Q on a surface from a radiant heat source with flux q depends on the absorptivity α of the receiving surface and how much of the receiving surface has a view of the source:

$$Q = \alpha \cdot SurfaceArea \cdot ViewFactor \cdot q \qquad \text{Watts}$$

Where ViewFactor is the factor to take into account geometric viewing effects, q is the source flux (watts/m²), and SurfaceArea is the exposed surface area of the receiving object.

ViewFactor for a balloon skin surface is the diffuse-radiant view factor of the earth, also called Fbe. It is the ratio of the balloon surface area that "sees" the earth divided by the total exposed balloon surface area. In the view factor equation below for a sphere, Z is the balloon altitude in meters:

$$HalfConeAngle = \arcsin\left(\frac{R_{earth}}{R_{earth}+Z}\right) \qquad R_{earth} = 6.371 \cdot 10^6$$

$$ViewFactor = \frac{1-\cos(HalfConeAngle)}{2} \qquad (13.14, 13.15)$$

At high altitudes the ViewFactor is approximately 0.45. As one ventures higher in altitude, the Earth's horizon "dips" away to be below horizontal due to curvature effects. Dip = $\pi/2$ – $HalfCone_{angle}$, and the view radius on the surface is simply = 6371·Dip (km).

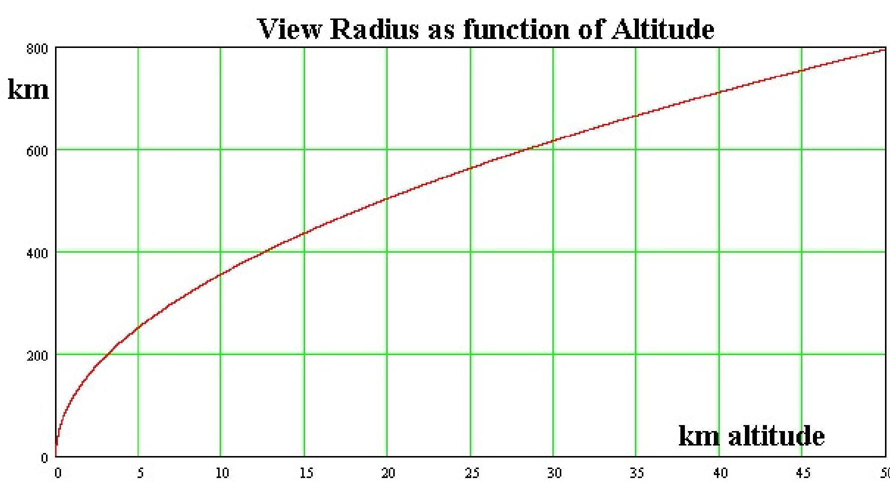


 Figure 13.16. Ground view radius of balloon at altitude

Convection effects can be expressed with heat transfer factors that can be calculated for a sphere. Internally there is a natural free convection that is probably much more complicated than what has been researched for spheres, but it is useful enough for balloons. Internal convection heat transfer from film to gas happens faster than one might suppose, taking only 15-20 minutes to get the gas temperature nearly equal to the film temperature for the large balloons at float (from simulations matching flights). Here is the general form of the heat transfer from convection:

$$Q_{ConvectionInternal} = HC_{int} \cdot (T_{gas} - T_{film}) \cdot A_{reference}$$

$$Q_{ConvectionExternal} = HC_{ext} \cdot (T_{air} - T_{film}) \cdot A_{reference}$$

where $A_{reference}$ is the reference surface area and HC is the heat transfer coefficient (watts/m^2 per deg K). Equation 13.16 is an approximate HC for external natural convection as a function of altitude and super temperature for a sphere which is good enough for preliminary design (Alt is altitude in meters, ΔT in degrees K).

$$HC_{sphere}(Alt, \Delta T) = e^{-0.00007 \cdot Alt} \cdot \left[0.38 + 0.7 \cdot \left(1 - e^{-0.06 \cdot |\Delta T|}\right)\right] \tag{13.16}$$

There are reflections of solar and infrared energy that reflect inside the balloon giving more opportunities for energy absorption. Looking at the diagram one can add up the effective energy absorbed by looking at the yellow dots (Fig 13.17) and seeing that the total energy absorbed is proportional to:

$\alpha + \alpha \cdot \tau\,(1 + r + r^2 + r^3 + r^4 + r^5 + \ldots)$

where we can define an effective reflectivity:

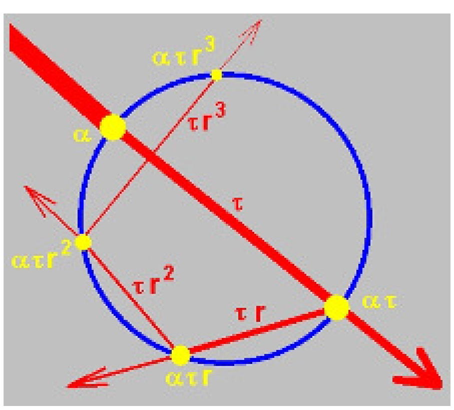


Figure 13.17. Internal reflections

$$r_{effective} = r + r^2 + r^3 + r^4 + r^5 + \ldots = r/(1-r)$$

13.4.5 Balloon Shape Fundamentals

The natural shape equations are a specialization of the membrane-shell equations. So let's first examine the membrane equations for a differential area element.

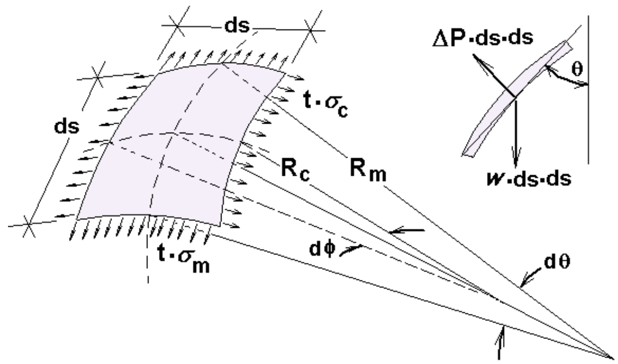


Figure 13.18. Membrane differential element in equilibrium

This differential element that is ds x ds has a compound curvature with radius of curvature Rm and Rc. It has a differential pressure ΔP acting on it as well as a weight per unit area *w* acting in the direction of g. Film stresses σ_m and σ_c multiplied by the thickness t produce a line load N/m along the orthogonal edges. For simplicity there are differential changes not shown on this area element so as to derive the basic equilibrium equation in the surface normal direction.

Geometric relationships:

$$ds = R_m \cdot d\theta = R_c \cdot d\phi$$

$$dA = ds \cdot ds = R_m \cdot d\theta \cdot R_c \cdot d\phi$$

Setting up equilibrium in the ΔP direction with sin(dθ) ~ dθ:

$$\Delta P \cdot dA = 2 \cdot \sigma_m \cdot t \cdot \frac{d\theta}{2} \cdot ds + 2 \cdot \sigma_c \cdot t \cdot \frac{d\phi}{2} \cdot ds + w \cdot dA \cdot \sin(\theta)$$

Finally substituting the geometric equations and simplifying:

$$\frac{\Delta P}{t} = \frac{\sigma_m}{R_m} + \frac{\sigma_c}{R_c} + \frac{w \cdot \sin(\theta)}{t}$$

One can see that membrane equilibrium has "presure vessel" components from both the meridional and circumfrential directions. It is in this manner that a zero pressure balloon can have vertical wrinkles indicating no circumferential stress even while having buoyancy differential pressure inside it. The meridional curvalure is supplying all the stress needed to satisfy equilibrium.

Non-dimensionalization of balloon parameters

Balloon researchers in the 1950's and 1960's were able to discover the commonality of natural shapes by viewing them in non-dimensional form. The purpose to nondimensionalize a system is to remove scale effects, such that large or small, a system can be seen as a ratio of forces and geometries. If the ratios between different balloons are the same, then we can expect similar shape regardless of relative size. The first job is to identify the relevant dimensional parameters, which for a simple balloon has the following:

Parameter	*Variable name*	*Dimension*	*Note*
Volume	V	m^3	volume of the gas
Specific buoyancy	b	N/m^3	lift ability of the gas
Gross Inflation	GI	N	total lift capacity
Free Lift	F	N	excess lift
Gross Weight	G = GI - F	N	equilibrium, G = GI
Suspended load	L	N	
Differential pressure	ΔP	N/m^2	
Total Meridional Tension	T	N	
Skin areal weight density	w	N/m^2	
Radial coordinate of gore	r or x	m	
Vertical coordinate of gore	z	m	
Gore length	s	m	

The load becomes nondimensionalized as Lbar = L/GI.

The Smalley "natural length" is defined as $\lambda = \left(\frac{L}{b}\right)^{\frac{1}{3}}$ which is still dimensional (meters).

Parameter	*Nondimensional variable*	*Equation*
Suspended load ratio	Lbar	L/GI
Skin areal weight Sigma	Σ	$w/(b \cdot \lambda \cdot \kappa)$
Differential pressure ratio	Abar	$\Delta P/(b \cdot \lambda)$
Tension ratio	Tbar	$T/(b \cdot \lambda^3)$

Where $\kappa = \left[\frac{1}{2\pi}\right]^{\frac{1}{3}}$ = 0.541926 (vestige of early research on spheres)

The early researchers used the suspended load L to non-dimensionalize forces which have some advantages. Or, one can use the gross inflation to nondimensionalize forces which leads to a very interesting relationship; load ratio Lbar and Sigma become linear, making calculations simple. With this new definition:

The new "natural length" is defined as $\lambda_F = \left(\frac{GI}{b}\right)^{\frac{1}{3}} = V^{\frac{1}{3}}$ (13.17a)

Skin areal weight $\Sigma_F = \frac{w}{b \cdot \lambda_F \cdot \kappa} = \Sigma \cdot Lbar^{\frac{1}{3}}$ (13.17b)

Generally speaking the relationship between load ratio Lbar and Σ_F for *any shape* is:

$$\text{Lbar} = 1 - (\kappa \cdot \text{SurfaceArea}/\text{Volume}^{2/3}) \cdot \Sigma_F$$

Every shape has a characteristic (SurfaceArea/ Volume$^{2/3}$) ratio that is a constant for rigid shapes regardless of absolute size, and even though natural zero pressure shapes change with Sigma number, the relationship is still surprisingly linear.

For a sphere:

$$Lbar = 1 - 2.62074 \cdot \Sigma_F$$

For a zero pressure natural-shape balloon:

$$Lbar = 1 - 2.682 \cdot \Sigma_F \tag{13.17c}$$

It is interesting to note in figure 13.19 that when the load ratio is zero (it can only pick itself up, no payload), that the shape becomes the same as a super pressure pumpkin shape (next section), and the classic Sigma number goes to infinity. In the classic definition of Sigma, there are no Sigma numbers that won't produce a viable balloon. Using the SigmaF definition allows balloons to be designed that cannot fly but can be used as ground test models for study (if Σ_F is greater than 0.37). Most modern scientific zero pressure balloons (ZPB) have Sigma numbers between 0.1 and 0.2 in any case.

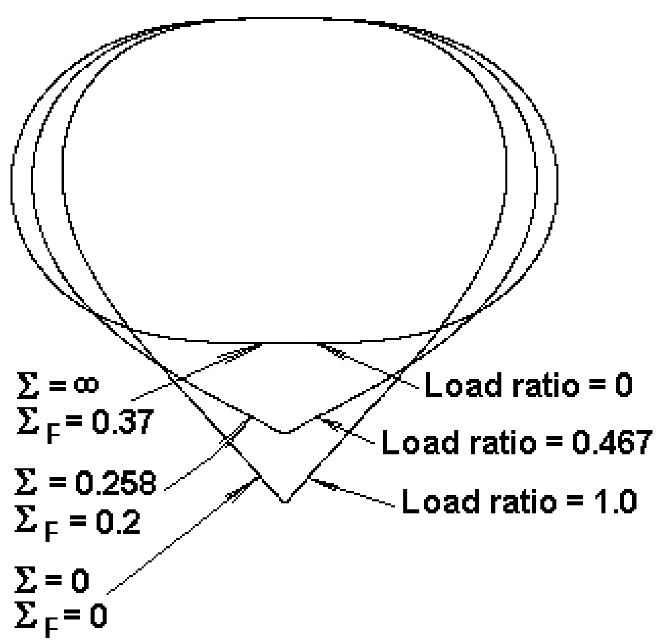


Figure 13.19. Natural zero pressure shapes with different load ratios

13.4.6 Balloon Film Fundamentals

Linear low density polyethylene co-extruded film is used for both zero pressure and super pressure balloons. Smaller balloons have used Mylar (polyester), but the large scientific balloons are manufactured with polyethylene. The film structural properties are a function of temperature, applied stress, and the length of time the stress is applied. Quite unlike designing with a fabric or a metal, there is no one-size-fits-all approach when dealing with this material. The glass transition temperature is of critical importance as these balloons cross the tropopause (coldest portion of the atmosphere). The expected film temperatures must remain above this threshold of approximately -98C (175K). It becomes critical for a super pressure design to know the limits for yield point and tertiary creep which are changing with the environment. Generally speaking, it is a good idea to keep the non-thermal portion of strain to below 2-3%. Strains are divided up into the initial elastic strain, the thermal strain, and the creep strain. Complex visco-elastic constitutive materials models have been developed [2] which guide the usage for super pressure designs. What has been theorized and experimentally verified is that these polyethylene films behave best when in a biaxial stress state. There is a term called “effective stress” where it is minimized when a nearly 1:1 biaxial stress state exists.

In a super pressure pumpkin, additional effects occur with the film material that benefits the stress state over time. As the lobed-bulge creeps in the hoop direction, the radius of curvature reduces, thus reducing the hoop stress. As the material creeps in the meridional direction, more shared load is passed to the tendons reducing the meridional stress. This general beneficial effect has been called “strain arrest”.

13.5 Balloon Environments

The high-altitude polyethylene scientific balloon is a thermal vehicle. It lives in a mostly radiant thermal balance from the many heat sources that influence it (Fig. 13.20). When warm, the gas expands and either pressurizes the gas envelope or is vented. If vented, then nighttime cooling makes ballast drops necessary.

These are the basic heat sources that influence balloon flight:

a) Direct sunshine on the skin membrane
b) Reflected diffuse sunshine in the form of ground albedo
c) Reflected diffuse sunshine in the form of cloud albedo
d) Diffuse infrared from the ground/atmosphere (up-welling)
e) Diffuse infrared from the clouds
f) Atmospheric convection
g) Diffuse infrared from the sky (down-welling)

At 33.5 kilometers altitude on an average sunny day in Ft. Sumner, the fraction of heat loads absorbed in the skin of a large scientific balloon (10 to 40 million cubic foot volume) is approximately:

Direct Solar	37%
Indirect Solar (albedo)	14%
Earth IR	49%
Atmospheric Convection	-1%

At night convection starts to have more effect, but not much.

It should be noted that direct sun impingement is a collimated source and affects a projected area of the balloon, while the other environments have a diffuse source nature to them requiring the use of the ViewFactor term applied to the full surface area.

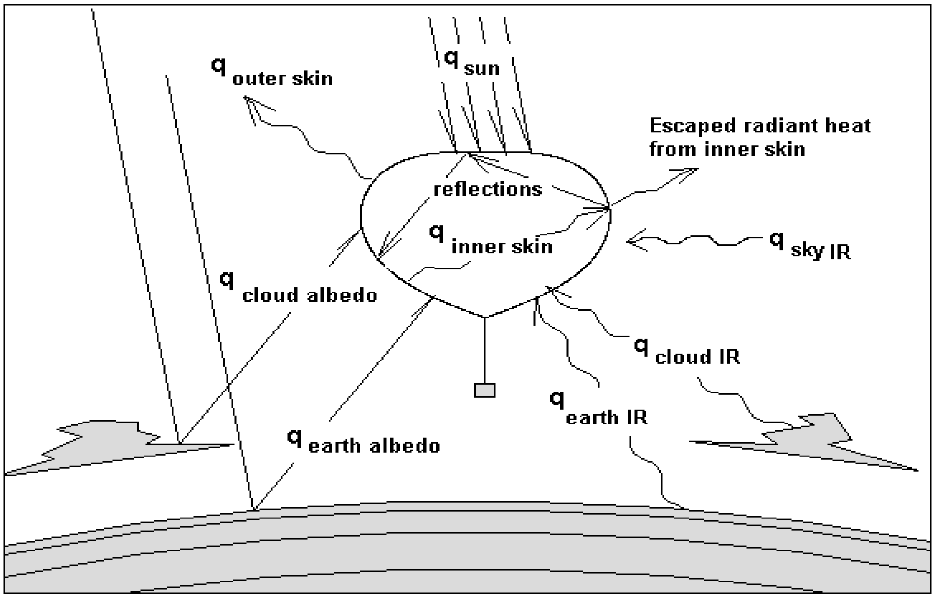


Figure 13.20. Radiant environment for a balloon

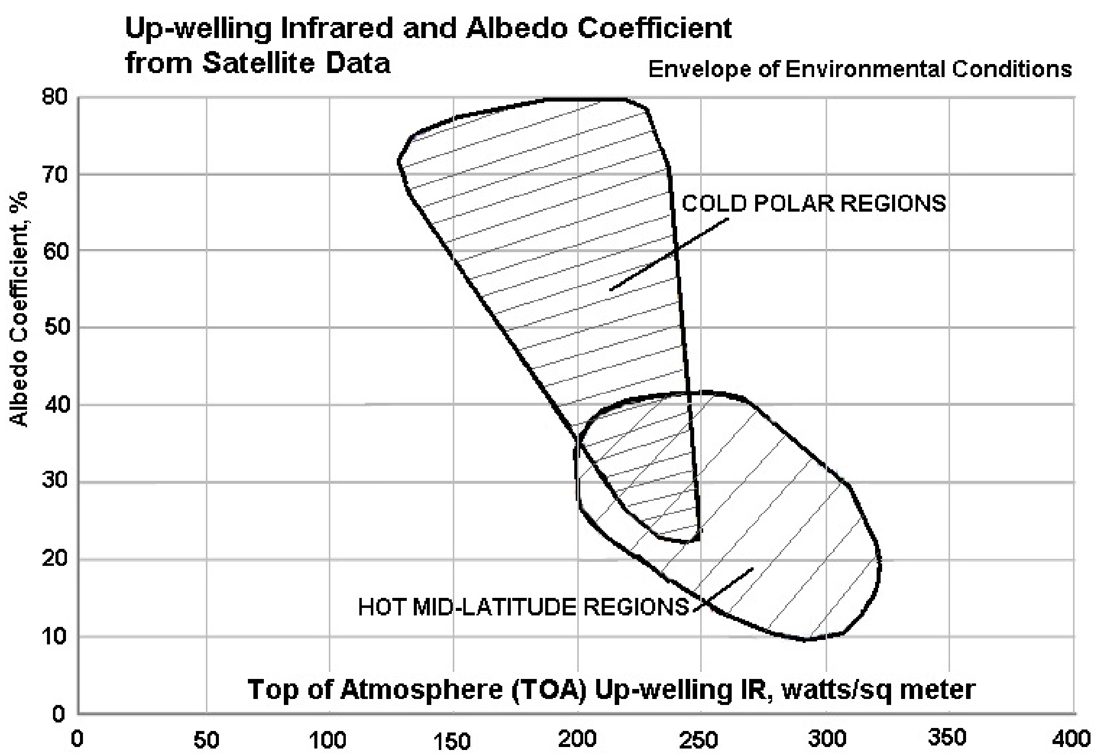


Figure 13.21. Thermal energy flux at top of atmosphere, original data from [3]

Figure 13.21 is a simplification of data from the ERBS spacecraft which mapped IR flux and albedo at the top of the atmosphere

(TOA). The max/min albedo and max/min IR flux is used to set temperature bounds on the balloon film. This affects performance and film stress.

As a design guide one can use the average values below:

Alice Springs, Australia, April	Upwelling IR = 267, albedo = 0.24
Circumglobal S. Hemisp., summer	Upwelling IR = 257, albedo = 0.23
Ft Sumner NM, Aug	Upwelling IR = 265, albedo = 0.25
Circumpolar Antarctica, Dec, Jan	Upwelling IR = 192, albedo = 0.63
Kiruna Sweden to Canada, June	Upwelling IR = 226, albedo = 0.45

13.5.1 Solar Flux

The transmissivity τ_{atm} of optical frequency irradiance thru the atmosphere follows a Beer-Lambert Law format of exponential decay [4], but we will only concern ourselves with values at the top of the atmosphere (TOA). Solar irradiance flux at the top of the atmosphere has a +/- 45 watts per sq meter variation over the course of a year. It can be expressed by the day number (1 thru 365, 1 starting on January 1):

$$I_{Sun} = 1358 + 45 \cdot \mathrm{Cos}(2\pi \cdot (\mathrm{Day}_{number} - 10) / 365) \qquad \mathrm{Watts/m^2}$$

Solar irradiance flux at the balloon altitude Z is then simply:

$$q_{Sun} = I_{sun} \cdot \tau_{atm}$$

where τ_{atm} is ~ 0.996 for high altitudes with solar elevation above 22 deg.

13.5.2 Albedo Flux

There is a simple model relating surface albedo and albedo flux. The assumption here is that the albedo number is the total specular + diffuse solar reflection, but treated as all diffuse for simplicity. The albedo flux is proportional to the solar irradiance at the top of the atmosphere and sine of the solar elevation angle Elv above the horizontal:

$$q_{albedo} = \text{Albedo} \cdot I_{sun} \cdot \text{Sin(Elv)} \qquad \text{Watts/m}^2$$

13.5.3 Infrared Flux

Top of the atmosphere upwelling IR fluxes are provided by satellite observations such as the Earth Observing System Aqua spacecraft:

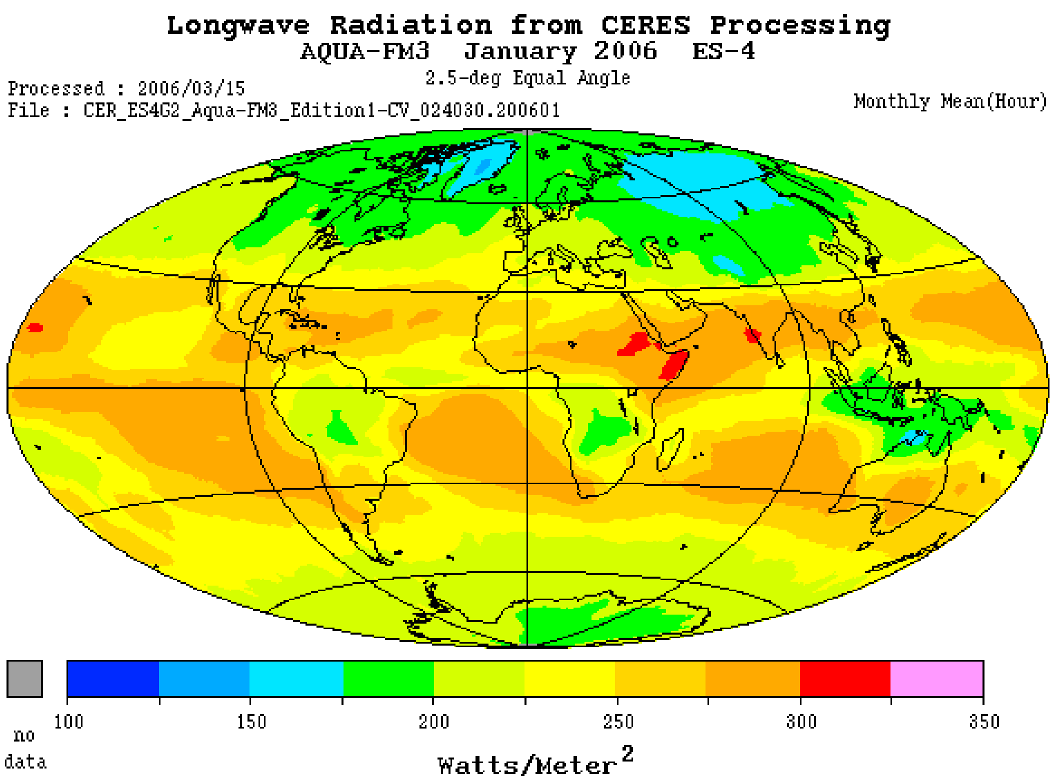


Figure 13.22. EOS satellite upwelling infrared data for January 2006, TOA

These are monthly averages which are useful, but for high-altitude scientific ballooning one can also come up with simple models to get the upwelling IR values at different altitudes based on the temperature at ground level. Just as direct solar is attenuated for thickness of the atmosphere, the same is applied to ground IR and cloud IR that has to pass through a certain amount of atmosphere to reach the balloon. The attenuation equation is based on a Beer-Lambert law of exponential decay. Note: IR at the top of the atmosphere (TOA) can vary drastically from one local zone to another due to the moisture content in the atmosphere which will block the IR. Thin cirrus clouds can have this effect on high-altitude balloons. Deserts will have the greatest day/night temperature swings due to little water acting as heat capacity storage. Somewhere on the order of 25ºC swing is not unreasonable. Oceans should have the least day/night swing in surface temperature. BEWARE! Most IR data on web sites is highly processed to reflect ground surface temperatures, not top-of-atmosphere (TOA) temperatures! A simple model [5, 7] for upwelling IR at altitude based on the ground temperature and a single attenuation factor follows.

Upwelling Infrared Environment

IR diffuse radiation at ground level with ground emissivity ε_{ground} (Stefan-Boltzman constant $\sigma = 5.67\times10^{-8}$ W/m^{2}K^4):

$$q_{IRground} = \varepsilon_{ground} \cdot \sigma \cdot T_{ground}^{\ \ 4} \quad \text{(W/m}^2\text{)}$$

Transmission factor of ground IR to account for atmospheric absorption below the balloon is approximated by Eq. (13.18).

$$Transmission_{IR} = A_{IR} \cdot \left[\frac{P_{air}}{P_{sealevel}} - 1 \right] + 1 \qquad (13.18)$$

Maximum attenuation factors A_{IR}:
A_{IR} = 0.45 for temperate air masses
A_{IR} = 0.35 for dry air masses
A_{IR} = 0.30 for very dry air masses (Antarctica)

Ground IR diffuse radiation at balloon altitude, W/m^2:

$$q_{IRgroundZ} = q_{IRground} \cdot Transmission_{IR}$$

Here is a suggested list of ground emissivity:
Desert = 0.85 (data suggests great variation within a region, as low as 0.8), Average ground = 0.95, Snow = 0.98. It is fortuitous that in the hottest places, the emissivity is low and in the coldest places the emissivity is high.

The down-welling sky IR environment is non-existent at high altitudes, and at the surface is about 250-300 watts/m^2.

13.6 Bulk Temperatures for Balloon Film and Gas

The high-altitude polyethylene scientific balloon is a thermal vehicle. The measured optical values for the typical zero pressure linear low density polyethylene StratoFilm 372 are scattered as one might expect with clear plastic. What is shown in the table are a mixture of Edward's mode transflectance measurements [6] and modifications due to flight experience. What is most important is the absorptivity to emissivity ratio, α/ε. At night there is no solar irradiance to absorb, so what determines balloon skin temperature in this case is the magnitude of the infrared environment, the view factor, and some atmospheric convection.

Zero pressure balloon (ZPB) Material SF372, 0.0008" thick

				polyester load tapes 25 mm			
	1 layer	2	3	4	LT 400	LT 150	LT 600
α	0.024	0.042	0.057	0.087	0.13	0.069	0.128
ε	0.134	0.234	0.314	0.475	0.793	0.618	0.743
τ	0.916	0.847	0.788	0.667	0.383	0.624	0.479
τ_{IR}	0.866	0.766	0.686	0.525	0.07	0.336	0.189
α/ε	0.176	0.180	0.182	0.184	0.16	0.11	0.17
0.8 mil film							

13.6.1 Steady State Translucent Spherical Balloon Temperature

If we model the balloon as a translucent sphere with a view factor of the earth equal to Fbe, the steady state equation boils down to this relationship for skin temperature [7]: (deg K) (Eq. 13.19)

$$\Delta T_{fa} := T_{film} - T_{air} \qquad \text{Film-air temperature differential, deg K}$$

$$T_{film} := \left[\frac{\frac{\alpha}{\varepsilon}\cdot q_{sun}\cdot(1+\tau\cdot(1+r))\cdot\left(\frac{1}{4}+Albedo\cdot\sin(Elv)\cdot F_{be}\right)+\left[q_{IR}\cdot F_{be}+q_{sky}\cdot\left(1-F_{be}\right)\right]\cdot\left[1+\tau_{IR}\cdot\left(1+r_{IR}\right)\right]-\frac{hc_{ext}\cdot\Delta T_{fa}}{\varepsilon}}{\sigma\cdot\left[1+\left(1+r_{IR}\right)\cdot\tau_{IR}\right]}\right]^{\frac{1}{4}}$$

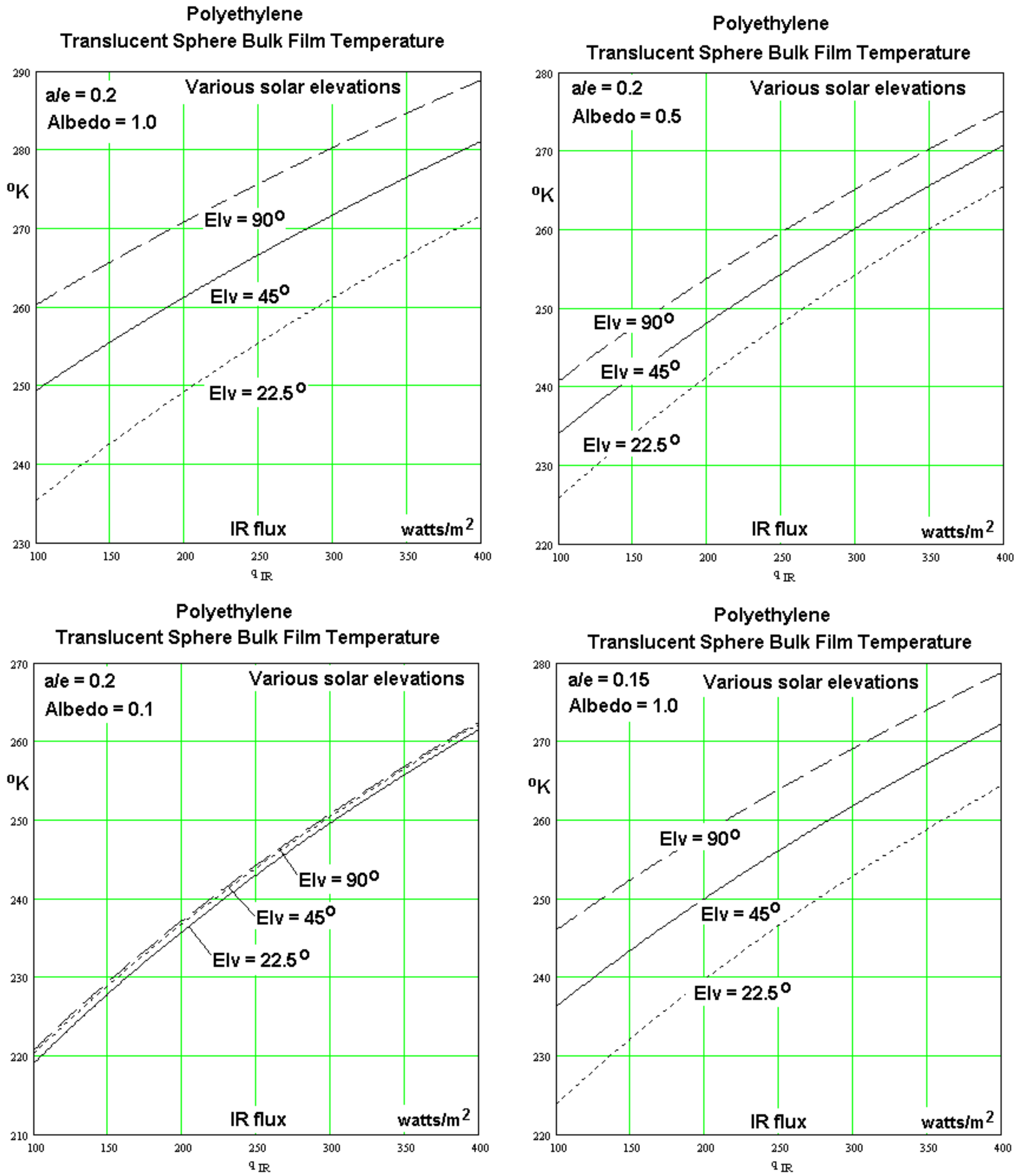
Where:
α = solar absorptivity
r = solar reflectance
r_{IR} = infrared reflectance
ε = infrared emissivity (= IR absorptivity)
τ = solar transmissivity
τ_{IR} = infrared transmissivity

Albedo = albedo coefficient of the ground and sky combination
Elv = solar elevation angle above a horizontal plane
Fbe = view factor that the balloon has of the earth, typically ~ 0.45
qsun = solar flux, watts/ m^2
qIR = up-welling infrared flux, watts/ m^2
qsky = down-welling IR, watts/ m^2

σ = Stefan - Boltzmann constant ($5.67\text{x}10^{-8}$ W/m^2K^4)
hc_{ext} is the air/film convection heat transfer coefficient, W / m^2 per deg K

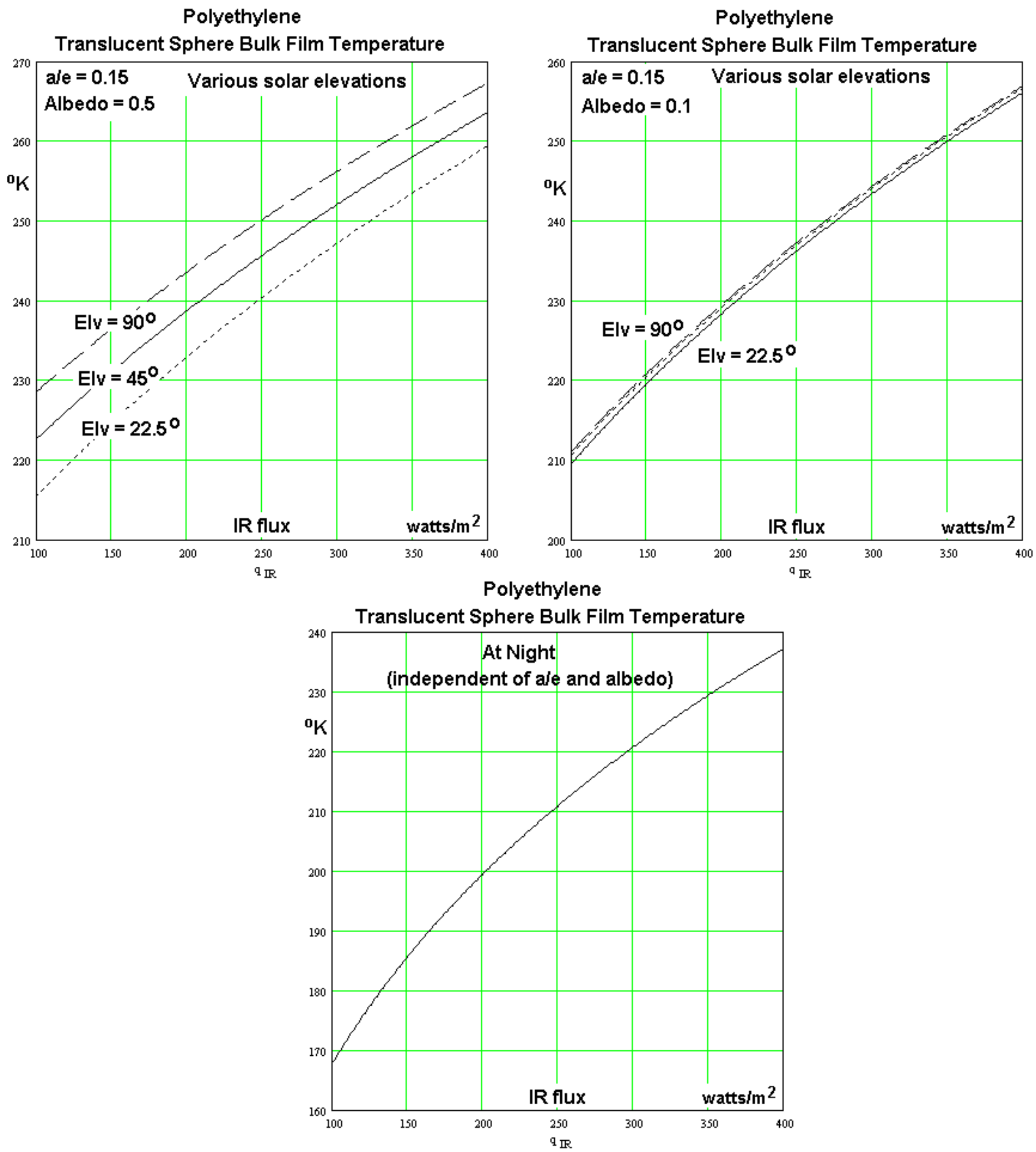
For preliminary design cases the external convection factor hc_{ext} can be ignored for establishing maximum balloon film temperatures. It should be noted that load tapes on the gores run hotter due to more absorption of radiant energy. Their effects can be approximated by modifying the skin optical properties with an area ratio approach of the load tape optical properties vs. skin area. Thus "global" properties can be used to get accurate bulk temperatures. More complex models modeling the balloon as a 6-sided rectangular box are used for roughing out the skin temperature gradients, or one can turn to software such as *Thermal Desktop*. Such gradients can run as high as 44C from coldest to hottest surface for typical balloon polyethylene film.

Taking the translucent sphere equation and graphing it for various α/ε ratios, albedo coefficients, and local solar elevation angles, one can see the expected pattern as a function of upwelling IR:



Figures 13.23

Entering the x-axis with a known IR flux and moving up to the proper solar elevation angle curve one can quickly determine a bulk film temperature, which for most purposes will also equal the lift gas temperature at float.



Figures 13.23 continued

Three albedo coefficients were graphed here (0.1, 0.5, 1.0), and two α/ε ratios (0.15, 0.20) typical of polyethylene film. These results reflect high altitude conditions (33.5 km) of some small amount of external convection. The graph of night time film temperatures are independent of α/ε and albedo, as no sun is shining. Film temperatures at night are completely dependent on the upwelling IR environment (view factor included), not the film optical properties.

13.7 Balloon Transient Thermal Behavior

Applying the 1st law to the balloon skin (plastic film) leads to these relationships [5, 7] (Eq. 13.20):

$$\overbrace{Q_{Sun} + Q_{Albedo} + Q_{IRplanet} + Q_{IRsky} + Q_{IRfilm} + Q_{ConvExt}}^{\textbf{Energy IN}} = \overbrace{Q_{ConvectionInternal} + Q_{IRout}}^{\textbf{Energy OUT}}$$

$$\overbrace{+ c_f \cdot M_{film} \cdot \frac{dT_{film}}{dt}}^{\textbf{Energy stored}}$$

The Q's are total energy exchanged with the film surface in watts from the environmental energy flux, plus the convection terms. The film material heat capacitance is c_f (watts/kg per deg K).

Applying the 1st law to the lift gas and taking into consideration that there could be some heat input from a burner ($\gamma = c_p/c_v$):

$$\frac{dT_{gas}}{dt} = \frac{(Q_{ConvectionInternal} + Q_{burner})}{c_v \cdot M_{gas}} + (\gamma - 1) \cdot \frac{T_{gas}}{\rho_{gas}} \cdot \frac{d\rho_{gas}}{dt} \qquad (13.21)$$

For a zero pressure assumption during ascent, one can express the equation as a function of the ascent rate dz/dt (m/s) and end up with this equation (z here is altitude, + going up):

$$\frac{dT_{gas}}{dt} = \left[\frac{(Q_{ConvectionInternal} + Q_{burner})}{M_{gas}} - g \cdot \frac{T_{gas}}{T_{air}} \cdot \frac{R_{gas}}{R_{air}} \cdot \frac{dz}{dt} \right] \cdot \frac{1}{c_p} \qquad (13.22)$$

There seems to have been a concern in the 1970's that the helium gas itself was absorbing infrared energy, but it's probable that those measurements were due to accidental water contamination of the lift gas. These equations assume the lift gas gets its heat energy supplied only by skin/gas convection or by direct heater input.

The energies exchanged with the balloon skin are listed below [5, 7]:

Absorbed direct sunlight heat:

$$Q_{Sun} = \alpha \cdot A_{projected} \cdot q_{Sun} \cdot \left[1 + \tau \cdot (1 + r_{effective})\right] \quad \text{Watts}$$

Absorbed albedo heat:

$$Q_{Albedo} = \alpha \cdot A_{surf} \cdot q_{Albedo} \cdot ViewFactor \cdot \left[1 + \tau \cdot (1 + r_{effective})\right] \quad \text{Watts}$$

Absorbed upwelling IR heat from the planet surface:

$$\mathrm{Q}_{\mathrm{IRplanet}} = \alpha_{IR} \cdot \mathrm{A}_{\mathrm{surf}} \cdot q_{IRplanet} \cdot ViewFactor \cdot \left[1 + \tau_{IR} \cdot (1 + r_{effectiveIR})\right] \text{Watts}$$

Absorbed IR from the sky:

$$\mathrm{Q}_{\mathrm{IRsky}} = \alpha_{IR} \cdot \mathrm{A}_{\mathrm{surf}} \cdot q_{IRsky} \cdot (1 - ViewFactor)\left[1 + \tau_{IR} \cdot (1 + r_{effectiveIR})\right] \quad \text{Watts}$$

Net emitted IR energy from both interior and exterior of the balloon skin:

$$\mathrm{Q}_{\mathrm{IRout}} = \sigma \cdot \varepsilon \cdot \mathrm{A}_{\mathrm{surf}} \cdot \left[1 + \tau_{IR} \cdot (1 + r_{effectiveIR})\right] \cdot T_{film}{}^{4} \quad \text{Watts}$$

A_{surf} refers to the external exposed surface area on the bubble, and $A_{projected}$ is the projected area of the bubble illuminated by direct solar exposure. The illuminated projected area of a natural-shape balloon varies with solar elevation angle Elv, and uses the top projected area as the reference. Here are some approximation formulas:

For a zero pressure shape:

$$Area_{projected} = Area_{top} \cdot [0.9125 + 0.0875 \cdot \mathrm{Cos}(\pi - 2 \cdot \mathrm{Elv})]$$

For a pumpkin shape:

$$Area_{projected} = Area_{top} \cdot [0.8219 + 0.1781 \cdot \mathrm{Cos}(\pi - 2 \cdot \mathrm{Elv})]$$

13.8 Meridionally-Lobed Membrane Structures

One can modify the classic membrane equations to incorporate weight from load tapes by making the film areal weight density a function of the geometry. One can do the same for a pumpkin balloon with bulging material, as well as adding transverse stress to alter the "natural" shape. The meridional tension T is a combination of the load tape and the film, each sharing a portion of this tension according to their stiffness, thermal contractions, and slack strain in the load tapes. The membrane equations consider it as a uniform structure, while structural equations can later separate the tension into the various components. The formulations that follow are generalized for a smooth shape, but with clever manipulations can also be used for shapes with lobed gores. The idea is that there is an equivalent smooth balloon underneath every bulged/lobed balloon.

Most references orient the Z axis up and integrate the differential equations from bottom to top. I do the opposite, as the boundary condition on the top is easy to calculate a priori. See the illustrations for definitions and the differential length element ds:

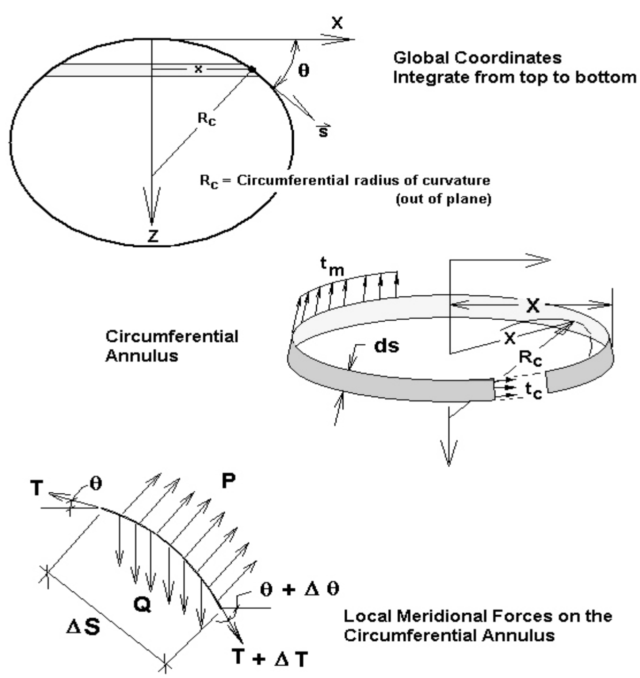


Figure 13.24. Membrane free body diagram of smooth annulus

Smooth axisymmetric membrane equations adopted for balloons:

t_c = circumferential film line load, N/m = $\text{stress}_c \cdot$ thickness
t_m = meridional film line load, N/m = $\text{stress}_m \cdot$ thickness
w = film weight per unit area, N/m^2 (can vary with location to account for load tape weight)
ΔP = differential pressure = $\Delta P_{apex} - b \cdot Z$, N/m^2 (Pascals)
T = total meridional tension = $2\pi \cdot X \cdot t_m$, N
b = specific buoyancy, N/m^3
X = radial coordinate, meters
Z = vertical coordinate, starting from the top and pointing down

The change in angle theta:

$$\frac{d\theta}{ds} = 2 \cdot \pi \cdot \frac{-t_c \cdot \sin(\theta) - X \cdot w \cdot \cos(\theta) + X \cdot \Delta P}{T}$$

The change in total meridional tension:

$$\frac{dT}{ds} = 2 \cdot \pi \cdot \left(t_c \cdot \cos(\theta) - X \cdot w \cdot \sin(\theta)\right)$$

Auxiliary geometric relationships:

$$\frac{dZ}{ds} = \sin(\theta) \text{ and } \frac{dX}{ds} = \cos(\theta)$$

The circumferential line load t_c can be a prescribed function to alter the shape, or it can be made zero to produce a natural shape. If $t_m = t_c$ = constant, and $w = 0$, then a sphere will result using a constant ΔP.

Integrate these equations from the top down. For a zero pressure design ΔP at the bottom = 0. The initial values of ΔP_{APEX} and T_{APEX} need to be adjusted until the solution constraints are satisfied (known as a point and shoot method). For a weightless apex plate the angle theta at the top = 0.

13.9 Preliminary Design and Performance Methodology

Designing a super pressure pumpkin for a particular set of requirements at high altitude is a daunting task. Instead, we will show the preliminary design steps useful for a natural-shaped zero pressure balloon, which is quite informative. It helps to have some approximation equations and some historical information to help in quick design studies of this sort. Much use is made from non-dimensional relationships.

Assuming a standard atmosphere, one can generate an approximation formula for helium specific buoyancy b as a function of altitude for zero super pressure. Normal super temperatures change b by ~ 0.5%, so for this phase of study it is fine. Specific buoyancy b is N/m^3 and Alt is altitude in meters:

$$b(Alt) = 0.003 + 28 \cdot e^{-1.61 \cdot 10^{-4} \cdot (Alt+2600)} \qquad 11000 < \text{Alt} < 40000$$

$$b(Alt) = -1.2 + 19 \cdot e^{-0.9 \cdot 10^{-4} \cdot (Alt+5500)} \qquad 0 < \text{Alt} < 11000$$

Next is the film + load tape average areal density w from historic data:

$$w(Alt) = 0.77 - 1.25 \cdot 10^{-5} \cdot Alt \qquad N/m^2 \text{ for "light" designs}$$

$$w(Alt) = 1.43 - 2.7 \cdot 10^{-5} \cdot Alt \qquad N/m^2 \text{ for "heavy" designs}$$

13.9.1 **How big, how heavy?**

We come into this problem with a payload mass and a target float altitude. With the altitude we calculate the specific buoyancy b and the film areal density w. As a first guess we will assume that the ballast to suspended mass ratio is 25% ($Ballast_{ratio}$ = 0.25). That is load L is 25% ballast weight. If we know our payload mass (kg) which includes the gondola, then the suspended mass is simply:

$$Mass_{Suspended} = \frac{Mass_{Payload}}{(1 - Ballast_{ratio})} \tag{13.25}$$

The suspended load L = Mass $_{Suspended}$ · g where g = 9.807 m/s^2

Now we use a design formula Eq. 13.26 to easily determine the load ratio Lbar. This is easily solved with a few iterations starting with Lbar = 0.6. Remember κ = 0.541926, and the 2.682 factor is from section 13.4.5 balloon shape fundamentals (Eq. 13.17c).

$$Lbar = 1 - \frac{2.682}{\kappa} \cdot \frac{w}{b} \cdot \left(\frac{Lbar}{L} \cdot b \right)^{\frac{1}{3}} \tag{13.26}$$

Gross Weight = L / Lbar Newtons

At equilibrium gross weight = gross inflation GI

Volume = GI/b m^3

Balloon material mass, kg:

$$Mass_{Balloon} = \frac{GrossWeight}{g} - Mass_{Suspended} \tag{13.27}$$

Turning to non-dimensional relationships mapped out in the following approximation equations for a zero pressure shape, one can determine the gore length, diameter, height, surface area, number of gores, and load tape tension.

Determine the Σ_F number from equation 13.17c:

$$\Sigma_F = \frac{(1 - Lbar)}{2.682}$$

Determine the natural length λ_F

$$\lambda_F = Volume^{\frac{1}{3}}$$ meters

The gore length

$$Sbar = 1.994 - 0.336 \cdot \Sigma_F - 0.049 \cdot \Sigma_F{}^2$$

Gore length = Sbar · λ_F meters

The diameter

$$Dbar = 1.305 + 0.164 \cdot \Sigma_F + 0.479 \cdot \Sigma_F{}^2 - 1.478 \cdot \Sigma_F{}^3 + 3.667 \cdot \Sigma_F{}^4$$

Diameter = Dbar · λ_F meters

The balloon height and differential pressure at the apex

$$Hbar = 1.275 - 0.445 \cdot \Sigma_F - 1.496 \cdot \Sigma_F{}^2$$

Height = Hbar · λ_F meters

ΔPapex = Height · b N/m^2 (Pascals)

Surface area

$$Areabar = 4.913 + 2.598 \cdot (\Sigma_F - 0.05)^3$$

Area = Areabar · $\lambda_F{}^2$ meters^2

The number of gores can be determined by your manufacturing constraint, in that there is usually a maximum width per gore that can be handled, call that $Width_{max}$

$$Ngores = \frac{\pi \cdot Diameter}{Width_{\max}}$$

The maximum load tape tension is usually at the apex, not the base:

$$Tbar = 1.556 - 0.66 \cdot \Sigma_F - 1.59 \cdot \Sigma_F^{\ 2}$$

Total Tension at the apex = Tbar · λ_F^3 Newtons
Tension per load tape at the apex= Total Tension / Ngores

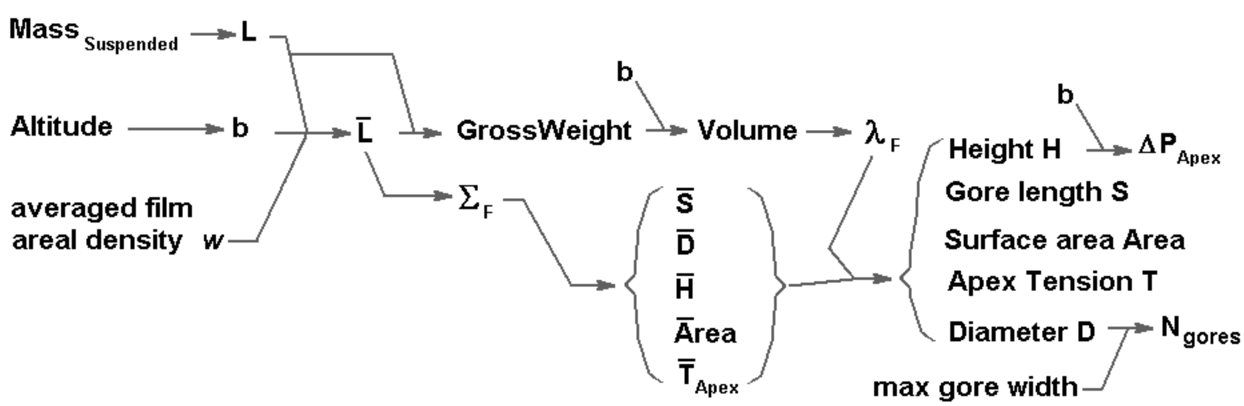


Figure 13.25. 1st pass zero pressure balloon design process

Example Problem

Figure 13.25 diagrams the zero pressure solution sequence.
With the basic size established, one can now work it directly with the apex tension, surface area and gore length to give better structural mass estimates. If the balloon mass is on target, then fine; if not then the parameter w needs to be adjusted until convergence.

Let's say our suspended mass is 2500 kg, our design float altitude is 37.2 km (122042 ft), and our averaged combined areal density w = 0.3705 N/m^2. At that altitude the specific buoyancy b = 0.04916 N/m^3. The suspended load L is therefore 24518N. Iterating the Lbar equation gives us a load ratio of Lbar = 0.6027; that is 60.27% of the gross weight will be suspended weight. Next the gross weight = L/Lbar = 40679 N, and volume = gross weight / b = 827454.8 m^3 = 29.22 million cubic feet. The characteristic length λ_F = cube root of volume = 93.882 meters. Moving into the non-dimensional parameters, SigmaF = 0.1481 which allows the calculations of the nondimensional gore length = 1.943, diameter = 1.337, height = 1.176, surface area = 4.915, and meridional tension = 1.423. Combining with the characteristic length LambdaF, the actual sizes are thus: gore length = 182.427 m, diameter = 125.498 m, height = 110.429 m, surface area = 43323.8 m^2, total apex tension = 57901 N. If for manufacturing the max gore width is limited to 2.478 m (97.5"), then the number of gores is 159. The apex differential pressure is 5.429 Pa.

13.9.2 **How much ballast?**

To answer this question we must first discover the film and gas temperatures at solar noon for the hottest daytime temperature and the coldest gas temperature at night. First approximate the solar declination angle for the date (by day number, Day_{number}), then the solar elevation angle at local noon:

$$Declination = 23.452 \cdot Sin\left[2 \cdot \pi \cdot \frac{284 + Day_{number}}{365}\right]$$

$$Elevation_{noon} = Declination + 90 - Latitude$$

These are in degrees with north latitudes +, and south latitudes -. In section 13.5.1 use the equation to determine the solar flux q_{sun}. Select from your environment sources what the max daytime upwelling IR will be, and the albedo coefficient. Use the table in 13.6 to select the bulk optical properties for your balloon. Use the equation for a translucent sphere and determine the bulk film temperature at noon (assume = gas temperature). Make the q_{sun} = 0 for nighttime and determine the night gas temperature. Select a nighttime parking altitude acceptable to the mission and flight safety; maybe 15000 ft (4.6 km) lower than daytime altitudes. From your atmosphere model determine the air temperatures for both day altitude and night altitude. With these temperatures one can calculate the super temperatures at noon at the daytime altitude and super temperature at the nighttime parking altitude.

$$\Delta T_{day} = Tfilm_{day} - Tair_{day}$$
$$\Delta T_{night} = Tfilm_{night} - Tair_{night}$$

Then, the nominal gas mass with zero super temperature is:

$$M_{gas_{nominal}} = \frac{M_{gross}}{\frac{R_{gas}}{R_{air}} - 1}$$

For launch multiply by the free lift ratio, usually 1.1, to get the launch gas mass required.

Reconfiguring the free lift ratio equation (section 13.4.3) for a zero pressure balloon at equilibrium to get the daytime gas mass (remember the super temperature vents out some of the gas):

$$M_{gas_{day}} = \frac{M_{gross}}{\left[\left(1 + \frac{\Delta T_{day}}{T_{air_{day}}}\right) \cdot \frac{R_{gas}}{R_{air}} - 1\right]}$$

The gross mass capacity at night using the daytime gas mass:

$$M_{gross_{night}} = M_{gas_{day}} \cdot \left[\left(1 + \frac{\Delta T_{night}}{T_{air_{night}}}\right) \cdot \frac{R_{gas}}{R_{air}} - 1\right]$$

Finally, the ballast that needs to drop to maintain the selected parking altitude:

$$M_{ballast} = M_{gross} - M_{gross_{night}}$$

If the ballast ratio ($M_{ballast}$ / $M_{suspend}$) is less than the originally assumed amount, then the parking altitude and other design parameters are viable. Generally one will find that ~ 10% of the gross weight needs to dropped every night, and over a cold storm the upwelling IR can be so blocked as to rapidly bring down the balloon where there is just not enough ballast aboard. If one can accept a lower parking altitude at night, then the ballast requirement will go down, 5%-7% of the gross weight being a good target.

Suspended Mass Capability

With your new design (or old one) a graph of the maximum suspended mass capacity of any balloon can be made if you have the balloon envelope material mass (everything above the hook point on the base plate), call it $M_{balloon}$. With your atmosphere model one can make air and lift gas density a function of altitude Alt.

$$M_{suspended}(Alt) = Volume \cdot \left(\rho_{air}(Alt) - \rho_{gas}(Alt)\right) - M_{balloon}$$

Or, with the definition of specific buoyancy b:

$$M_{suspended}(Alt) = Volume \cdot \frac{b(Alt)}{g} - M_{balloon}$$

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