Steady State Translucent Spherical Balloon Temperature *Rodger Farley*

If we model the balloon as a translucent sphere with a view factor of the earth equal to Fbe, the steady state thermal balance equation boils down to this relationship for skin and gas bulk temperature (deg K):

$$T_{film} = \left[\frac{\frac{\alpha}{\varepsilon} \cdot q_{sun} \cdot \left[1 + \tau \cdot (1 + r_{eff})\right] \cdot \left[\frac{1}{4} + a \cdot \sin(Elv) \cdot F_{be}\right] + \left[q_{IR} \cdot F_{be} + qsky \cdot (1 - F_{be})\right] \cdot \left[1 + \tau_{IR} \cdot (1 + r_{effIR})\right] + \frac{HCext \cdot \Delta T}{\varepsilon}}{\sigma \cdot \left[1 + \tau_{IR} \cdot (1 + r_{effIR})\right]} \right]^{0.25}$$

Where:

α = solar absorptivity, ε = infrared emissivity, τ = solar transmissivity

τ_{IR} = infrared transmissivity, a = albedo coefficient, Elv = solar elevation angle above theoretical horizon

Fbe = view factor that the balloon has of the earth, typically ~ 0.45 at float altitudes

qsun = solar flux, watts/m^2, qIR = up-welling infrared flux, qsky = down-welling IR

σ = Steffan - Boltzman constant W / m^2K^4 = 5.67 x 10^{-8}

HCext is the convection heat transfer, W / m^2 per deg K

ΔT = Tair –Tfilm, which is the negative of super temperature if Tfilm = Tgas

This is an approximation formula for the air-to-skin convection heat transfer as a function of altitude (meters) and temperature differential that can be used for HCext for balloons in Earth's atmosphere:

$$HC_{sphere}(Alt, \Delta T) = e^{-0.00007 \cdot Alt} \cdot \left[0.38 + 0.7 \cdot \left(1 - e^{-0.06 \cdot |\Delta T|} \right) \right]$$

W / m^2 per deg K

At float altitudes, the down-welling infrared flux qsky is practically zero, except in places like Venus.

Definitions of reflectivity and effective reflectivity after 100 reflections for both the visible and IR wave lengths:

$$r := 1 - \alpha - \tau \qquad r_{IR} := 1 - \varepsilon - \tau_{IR}$$

$$r_{eff} := \sum_{i=1}^{100} r^{i} \qquad r_{effIR} := \sum_{i=1}^{100} r_{IR}^{\ i}$$

Equations for Solid sphere, Horizontal Plate, Vertical Plate

Solid Sphere, opaque (tau = 0, but reflectivity increases)

$$T_{solidsphere} = \left[\frac{\frac{\alpha}{\varepsilon}\cdot q_{sun}\cdot\left(\frac{1}{4}+a\cdot\sin(Elv)\cdot F_{be}\right)+q_{IR}\cdot F_{be}+q_{sky}\cdot\left(1-F_{be}\right)+\frac{HC_{ext}\cdot\Delta T}{\varepsilon}}{\sigma}\right]^{0.25}$$

Flat Horizontal Plate

$$T_{FilmFlatHoriz} = \left[\frac{\frac{\alpha}{\varepsilon}\cdot q_{sun}\cdot\sin(Elv)\cdot\left(1+2\cdot a\cdot F_{be}\right)+\left[q_{IR}\cdot 2\cdot F_{be}+q_{sky}\cdot\left(1-2\cdot F_{be}\right)\right]+\frac{HC_{ext}\cdot\Delta T}{\varepsilon}}{2\cdot\sigma}\right]^{0.25}$$

Flat Vertical Plate

$$T_{FilmFlatVert} = \left[\frac{\frac{\alpha}{\varepsilon}\cdot q_{sun}\cdot\left(\cos(Elv)+2\cdot a\cdot F_{be}\cdot\sin(Elv)\right)+\left[q_{IR}\cdot 2\cdot F_{be}+q_{sky}\cdot\left(1-2\cdot F_{be}\right)\right]+\frac{HC_{ext}\cdot\Delta T}{\varepsilon}}{2\cdot\sigma}\right]^{0.25}$$