Algorithm to Accurately Determine Free Lift and Float Altitude For Zero Pressure and Super Pressure Balloons

Rodger Farley 7/23/2008

Inputs and Givens

An atmosphere model, 5 point or more to give Tair, Pair, and density at altitude Z.

Specific Gas Constants

$R_{air} := 287.1$ $R_{gas} := 2077.2$ $g := 9.807$ Acceleration of gravity, m/s/s

M_{gas} Mass of lift gas, kg

M_{gross} Gross mass, kg

$T_{AirLaunch}$ Ambient air temperature at launch, deg K

$P_{AirLaunch}$ Ambient air pressure at launch, Pascals

V_{design} Design volume at design super pressure, m^3

ΔP_{design} Design super pressure, Pa

EA_{tendon} Tendon stiffness, Newtons

N_{gore} Number of gores

ΔP_{launch} Initial assumed average deltaP at launch, Pa

ΔT_{launch} Assumed super temperature at launch, deg C

ΔT_{float} Assumed super temperature at float, deg C

Note:
Superpressure, deltaP, and differential pressure are used interchangeably. It refers to the average differential pressure for the volume of lift gas. Even a zero pressure balloon has differential pressure due to the aerostatic buoyancy gradient, and this does slightly compress the lift gas density making it have less buoyancy.

Launch Buoyancy, Pressure, and Free Lift

Ambient air density at launch, kg/m^3

$$\rho_{AirLaunch} := \frac{P_{AirLaunch}}{R_{air} \cdot T_{AirLaunch}}$$

Lift gas density at launch, kg/m^3

$$\rho_{GasLaunch} := \frac{\left(P_{AirLaunch} + \Delta P_{launch}\right)}{R_{gas} \cdot \left(T_{AirLaunch} + \Delta T_{launch}\right)}$$

$$V_{Launch} := \frac{M_{gas}}{\rho_{GasLaunch}}$$ Launch volume, m^3

$$Radius_{Launch} := \left(\frac{3}{4} \cdot \frac{V_{Launch}}{\pi}\right)^{\frac{1}{3}}$$ Spherical radius of launch bubble, m

Specific buoyancy at launch, N/m^3

$$b_{Launch} := g \cdot \left(\rho_{AirLaunch} - \rho_{GasLaunch}\right)$$

$$\Delta P := b_{Launch} \cdot Radius_{Launch}$$ average super pressure at launch, Pa due to aerostatic buoyancy pressure

$$\Delta P_{launch} := \Delta P$$ Iterate until agreement

$$GI := b_{Launch} \cdot V_{Launch}$$ Gross inflation, Newtons

$$FreeLift_{ratio} := \frac{GI}{g \cdot M_{gross}}$$ Free Lift Ratio

$$100 \cdot \left(FreeLift_{ratio} - 1\right) =$$ % Free Lift

Conditions at Float

Determine Un-Stretched Float Volume

Elasticity effects on the volume---

$$AbarT := \frac{\Delta P_{design} \cdot V_{design}^{\frac{2}{3}}}{g \cdot M_{gross}}$$ Non dimensional pressure head

$$TbarT := -0.528 + 1.604 \cdot AbarT$$ Non dimensional apex tension

$$Tension := TbarT \cdot g \cdot M_{gross}$$ Total tension at apex, N

$$Strain_{design} := \frac{Tension}{N_{gore} \cdot EA_{tendon}}$$ Design elastic strain

$$V_o := \frac{V_{design}}{\left(1 + Strain_{design}\right)^3}$$ Unstretched volume

Assume an Initial Float Volume Stretch Factor

The stretch factor is defined as actual float volume/design volume

$$Stretch_{factor} := 1.0017$$ Assume an initial volume elastic stretch factor: 1.0 for a ZPB, and 1.0017 for a pumpkin

Calculate the Spherical Radius Equivalent at Float

$$Radius_{float} := \left(\frac{3}{4} \cdot \frac{V_{design}}{\pi}\right)^{\frac{1}{3}}$$ Design equivalent spherical radius

Determine the Float Altitude:

OUTER LOOP (Do until stretch factor converges)

$$\rho_{system} := \frac{M_{gross} + M_{gas}}{V_{design} \cdot Stretch_{factor}}$$

$\rho_{air} := \rho_{system}$ System density at float = air density at float

$Z_{float} := -6317.12 \cdot \ln\left(\frac{\rho_{air}}{2.077608}\right)$ Approximate float altitude, m

$\frac{d}{dz}\rho_{air} = -\rho_{air} \cdot 0.0001175$ Approximate density gradient, kg/m^3 per meter

Call on Atmosphere Model for T_{air} and P_{air} at approximate altitude Z_{float}

$\Delta P_{float} := 2$ Initial guess for ZPB average buoyancy pressure, Pa

INNER LOOP (loop 25 times to converge altitude)

If the balloon is a ZPB, then

$$\rho_{gas} = \frac{(P_{air} + \Delta P_{float})}{R_{gas} \cdot (T_{air} + \Delta T_{float})}$$ Gas density at float

$M_{gasFloat} := \rho_{gas} \cdot V_{design}$ Mass of lift gas

$$\rho_{system} := \frac{M_{gross} + M_{gasFloat}}{V_{design}}$$ System density with float gas mass

$b_{float} := g \cdot (\rho_{air} - \rho_{gas})$ Specific buoyancy at float

$\Delta P_{float} := 0.834 \cdot Radius_{float} \cdot b_{float}$ Better estimate on average buoyancy pressure

Call on Atmosphere Model for T_{air} and P_{air} and ρ_{air} at new altitude Z_{float}

$$\Delta Z := \frac{(\rho_{system} - \rho_{air})}{\frac{d}{dz}\rho_{air}}$$ Estimate of change of altitude needed

$Z_{float} = Z_{float} + \Delta Z$ New float altitude estimate

LOOP

Continue with the Loop to Determine Differential Pressure and Elastic Stretch…

To outer loop

Lift gas density, kg/m^3

$$\rho_{gas} := \frac{M_{gas}}{V_{design} \cdot Stretch_{factor}}$$

Specific buoyancy at float N/m^3

$$b_{float} := g \cdot \left(\rho_{air} - \rho_{gas}\right)$$

Average differential pressure at float Pa

$$\Delta P_{float} := \rho_{gas} \cdot R_{gas} \cdot \left(T_{air} + \Delta T_{float}\right) - P_{air}$$

If ZPB, then shape coefficient Cshape = 0.834
If Pumpkin, then Cshape = 0.693
If Sphere, then Cshape = 1.0

Super pressure values at Apex and Base, Pascals

$$\Delta P_{apex} := \Delta P_{float} + b_{float} \cdot Radius_{float} \cdot C_{shape}$$

$$\Delta P_{base} := \Delta P_{float} - b_{float} \cdot Radius_{float} \cdot C_{shape}$$

$$AbarT := \frac{\Delta P_{float} \cdot \left(V_{design} \cdot Stretch_{factor}\right)^{\frac{2}{3}}}{g \cdot M_{gross}}$$ Non dimensional pressure head

$$TbarT := -0.528 + 1.604 \cdot AbarT$$ Estimated Non dimensional apex tension

$$Tension := TbarT \cdot g \cdot M_{gross}$$ Total tension at apex, N

$$Strain_{float} := \frac{Tension}{N_{gore} \cdot EA_{tendon}}$$ Elastic strain under float differential pressure

$$V_{float} := V_{o} \cdot \left(1 + Strain_{float}\right)^{3}$$ Float Volume, m^3

$$Stretch_{factor} := \frac{V_{float}}{V_{design}}$$ New estimate on stretch factor

LOOP

The Gas Mass Calculator

FreeLift ratio = Gross Inflation / Gross Weight

Here are the formulas to determine the actual free lift ratio, and one based on the standard assumption of no super pressure or super temperature:

$$\mathrm{FreeLift}_{ratio} = \frac{M_{gas}}{M_{gross}} \cdot \left[\frac{\left(1 + \frac{\Delta T}{T}\right)}{\left(1 + \frac{\Delta P}{P}\right)} \cdot \frac{R_{gas}}{R_{air}} - 1 \right]$$

Free lift fraction expression as a function of ambient temperature T and pressure P, differential temperature dT and differential pressure dP, the mass of the gas and gross mass

$$M_{gas} = M_{gross} \cdot \frac{\mathrm{FreeLift}_{ratio}}{\left(\frac{R_{gas}}{R_{air}} - 1\right)}$$

Gas mass for a standard assumption of no super temperature and no super pressure

The Supply Tank Calculator

First, since under high pressure gases don't quite follow the ideal gas law, we need a correction factor. Such a correction factor is known as the Z factor, and this graph of Z factors describes the corrections for helium, oxygen, and air.

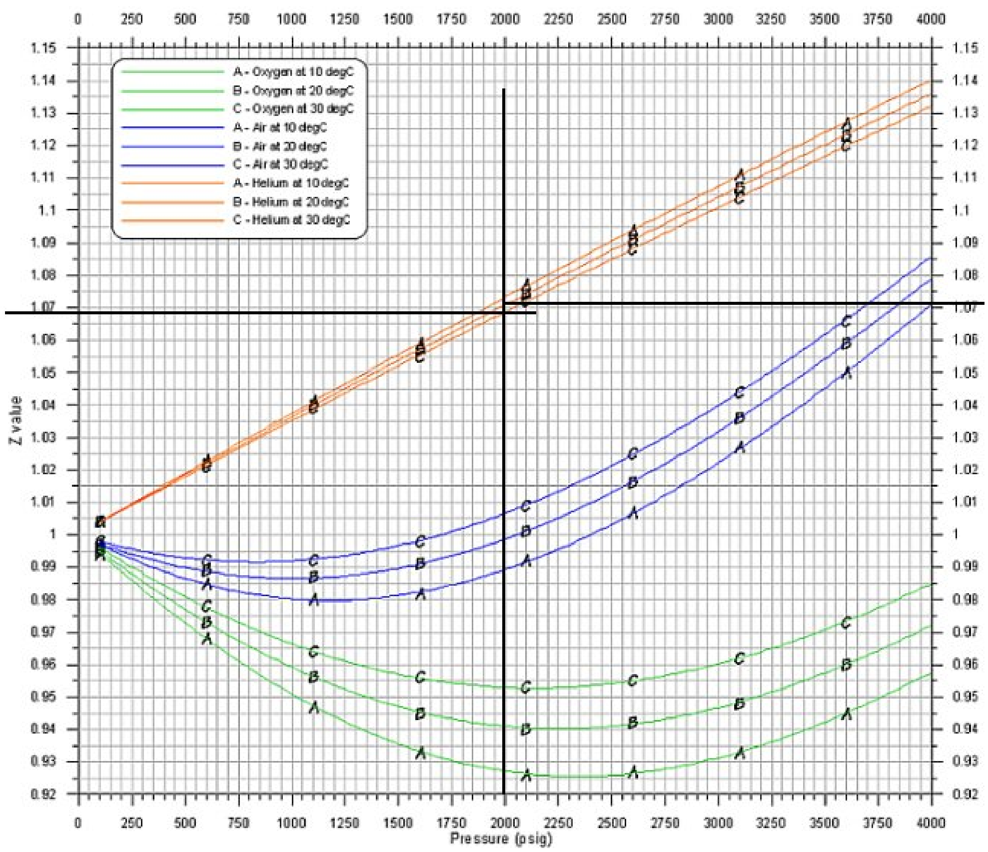
Recall, the Z factor accounts for differences with the ideal gas law:

Pressure $P = \rho \cdot R_{gas} \cdot T \cdot Z_{factor}$ Pascals

ρ is the density, kg/m^3

R_{gas} is the specific gas constant = 2077.2 for helium

T is the absoulte temperature, deg K



If you know the initial tank temperature and pressure, and final temperature and pressure:

V is the individual tank volume, m^3

N_{tank} is the number of gas bottles being used

The total trailer tank volume is referred as the "water volume", so the individual tank volume would simply be the trailer water volume divided by the number of tanks. For a PraxAir T-73 12 cylinder trailer, the individual volume = 78.592 cubic feet.

P_1 Initial absolute pressure, Pa

T_1 Initial temperature, deg K

Z_1 Z factor for the initial condition

P_2 Final absolute pressure, Pa

T_2 Final temperature, deg K

Z_2 Z factor for the final condition

$\rho_1 = \frac{P_1}{R_{gas} \cdot T_1 \cdot Z_1}$ Initial gas density in the tank, kg/m^3

$M_1 = \rho_1 \cdot V$ Initial gas mass in the individual supply tank, kg

$\rho_2 = \rho_1 \cdot \frac{P_2}{P1} \cdot \frac{T_1}{T_2} \cdot \frac{Z_1}{Z_2}$ Final gas density in the tank, kg/m^3

$M_2 = \rho_2 \cdot V$ Final gas mass in the individual supply tank, kg

$M_{gas} = N_{tank} \cdot (M_1 - M_2)$ Total lift gas mass delivered, kg

Conversion from PSIG to Pascals: Pascals = 6895* (PSIG+14.7)

Conversion from Pascals to PSIG: PSIG = Pascals/6895 - 14.7

If you know the initial tank temperature and pressure, and gas mass required:

P_1 Initial absolute pressure, Pa

T_1 Initial temperature, deg K

Z_1 Z factor for the initial condition

M_{gas} The total lift gas mass required, kg

$T2T1_{ratio}$ The assumed ratio of final temperature to initial temperature, deg K/deg K

$$M_1 = \frac{P_1 \cdot V}{R_{gas} \cdot T_1 \cdot Z_1}$$ Initial mass of lift gas in a single tank, kg

$$M_2 = M_1 - \frac{M_{gas}}{N_{tank}}$$ Final mass of lift gas in a single tank, kg

$$\rho_1 = \frac{M_1}{V}$$ Initial gas density in the tank, kg/m^3

$$\rho_2 = \frac{M_2}{V}$$ Final gas density in the tank, kg/m^3

$$T_2 = T2T1_{ratio} \cdot T1$$ Final gas temperature in the tank, deg K

LOOP 25 times to converge the final Z factor

$$P_2 = \rho_2 \cdot R_{gas} \cdot T_2 \cdot Z_2$$

Call subroutine to determine Z as function (P,T)

Loop

P_2 is the desired cut-off pressure, Pascals

Divide by 6895 then subtrack 14.7 to get gauge pressure PSIG