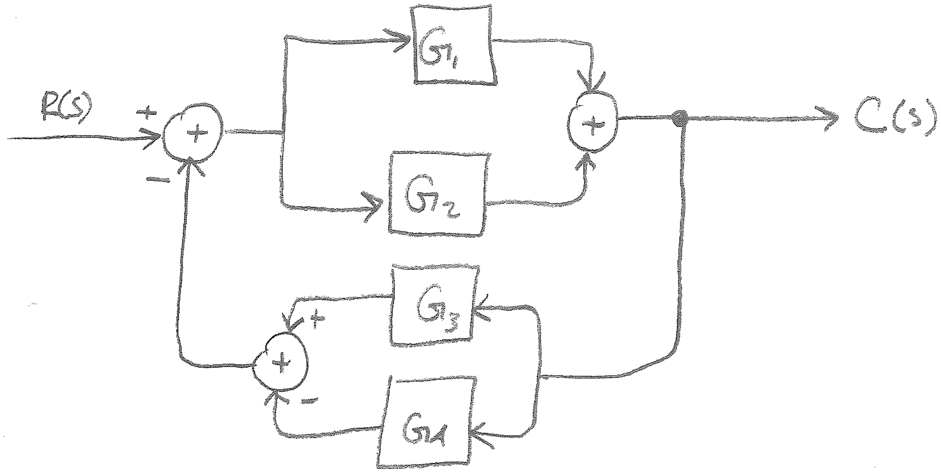
Brent Justice

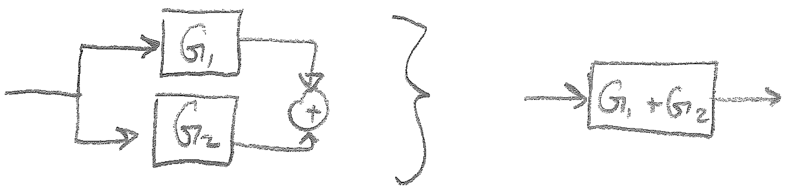
AAE 364

Hw # 4

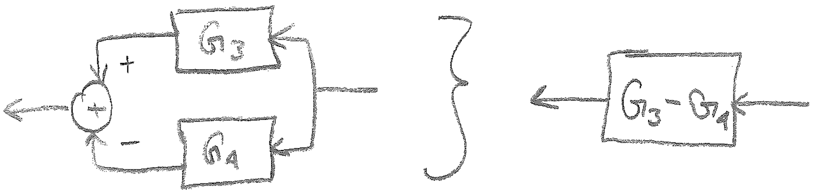
Problem B-2-1



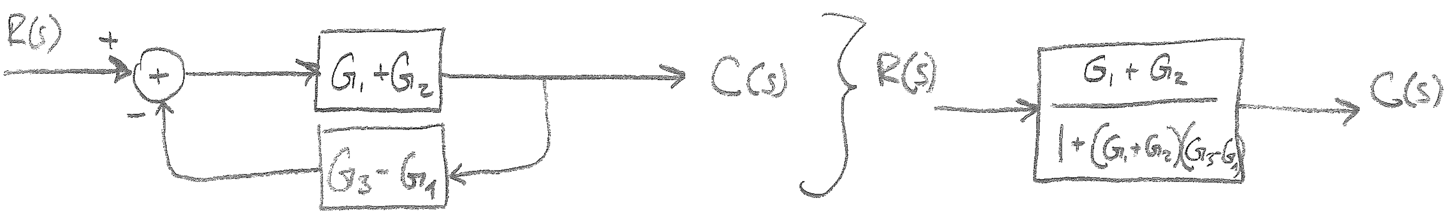
Parallel addition



Parallel addition

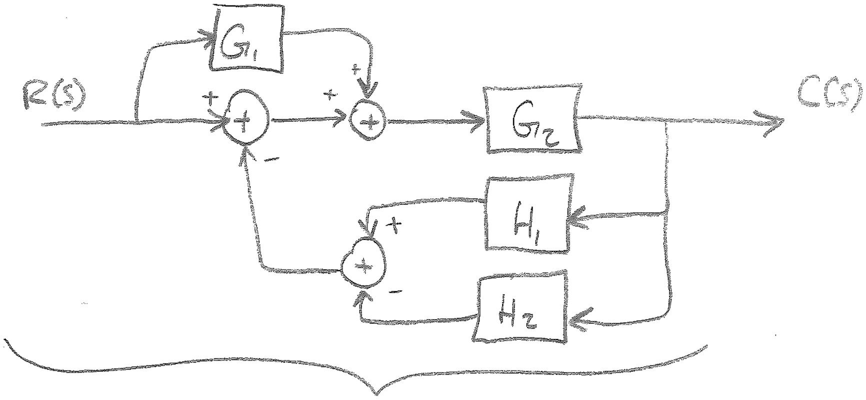


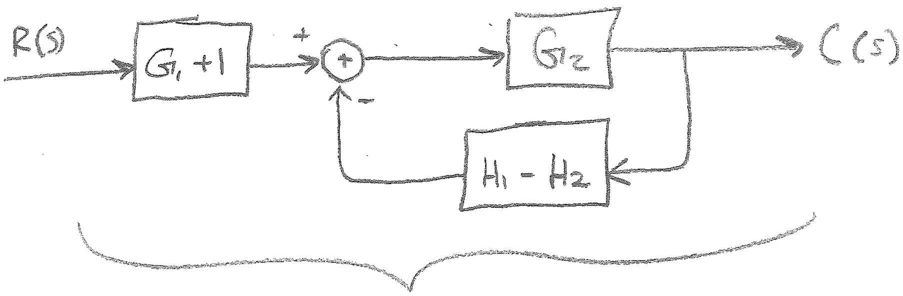
Closed loop Feedback

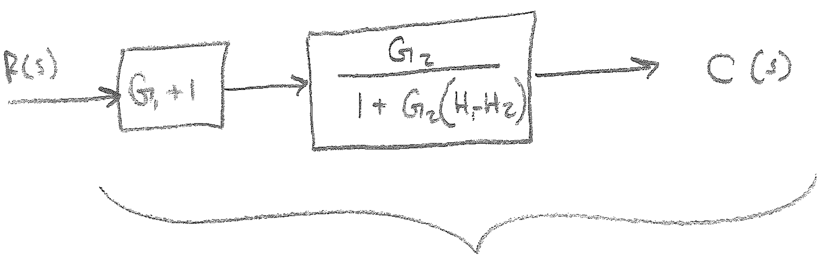


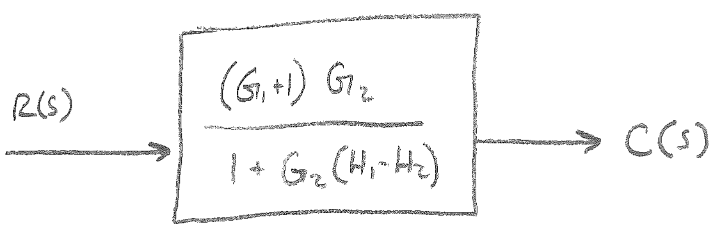
$$\boxed{\frac{Y(s)}{R(s)} = \frac{(G_1 + G_2)}{1 + (G_1 + G_2)(G_3 - G_4)}}$$

Problem 6-2-2



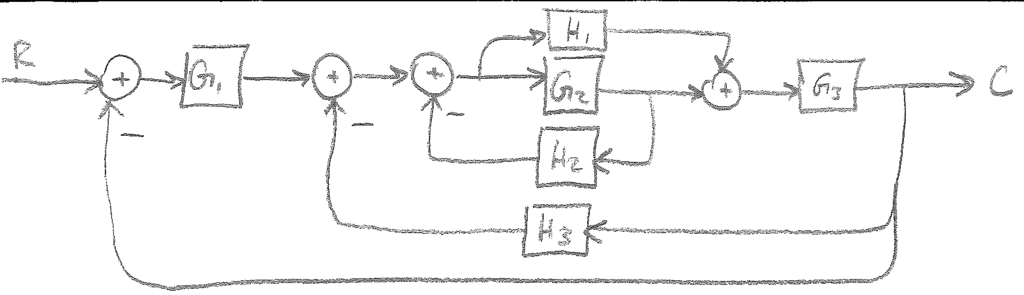


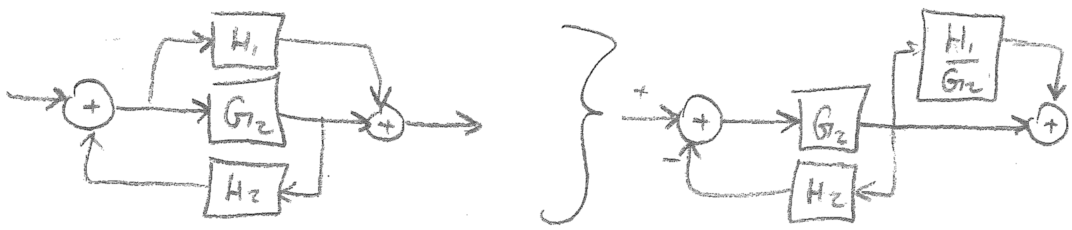


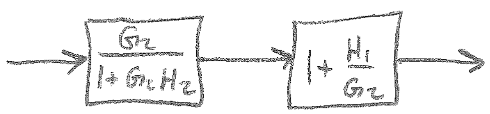


$$\frac{C(s)}{R(s)} = \frac{(G_1+1)\,G_2}{1+G_2(H_1-H_2)}$$

Problem B-2-3

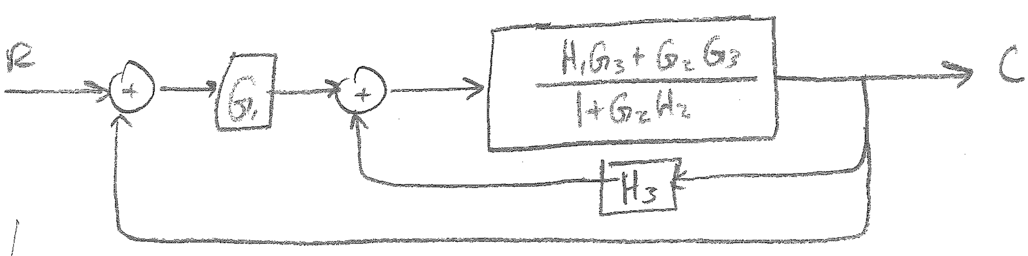






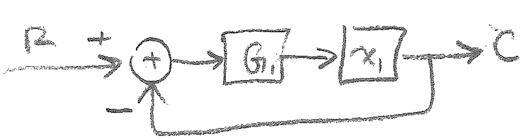
$$\left(\frac{G_2}{1+G_2H_2}\right)\left(1+\frac{H_1}{G_2}\right)$$

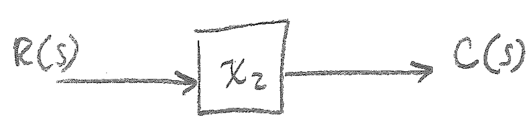
$$\frac{G_2}{1+G_2H_2}+\frac{H_1}{1+G_2H_2}=\frac{H_1+G_2}{1+G_2H_2}$$



$$\frac{\frac{H_1G_3+G_2G_3}{1+G_2H_2}}{1+\left(\frac{H_1G_3+G_2G_3}{1+G_2H_2}\right)H_3}=\frac{\frac{H_1G_3+G_2G_3}{1+G_2H_2}}{\frac{1+G_2H_2}{1+G_2H_2}+\frac{H_1H_3G_3+G_2G_3H_3}{1+G_2H_2}}=\frac{H_1G_3+G_2G_3}{1+G_2H_2}\cdot\frac{1+G_2H_2}{1+G_2H_2+H_1H_3G_3+G_2G_3H_3}$$

$$=\frac{H_1G_3+G_2G_3}{1+G_2H_2+H_1H_3G_3+G_2G_3H_3}=X_1$$





$$\frac{C(s)}{R(s)}=\frac{G_1X_1}{1+G_1X_1}=\frac{G_1G_3(H_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)+G_1G_3(H_2+H_1)}$$

$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases} \qquad R(t) = \begin{cases} 0 & t<0 \\ t & t>0 \end{cases}$$

Problem b-2-4

$k_p = 4 \quad T_i = 2$

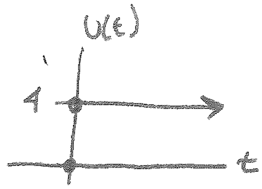
$k_i = 2 \quad T_d = 0.8$

Given $\dfrac{U(s)}{E(s)} = k_p = 4$, $e(t) = \text{Unit}(t)$

$U(s) = 4E(s)$

$U(t) = 4L^{-1}[E(s)]$

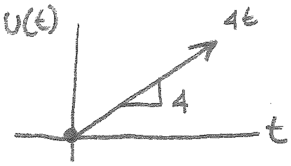
$U(t) = 4\,\text{Unit}(t)$



Given $\dfrac{U(s)}{E(s)} = k_p = 4$, $e(t) = R(t)$

$U(t) = 4\,e(t)$

$U(t) = 4R(t)$



Given $\dfrac{U(s)}{E(s)} = \dfrac{k_i}{s} = \dfrac{2}{s}$, $e(t) = \text{Unit}(t)$

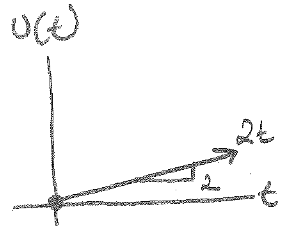
$E(s) = \dfrac{1}{s}$

$U(s) = \dfrac{2}{s}E(s)$

$U(s) = \dfrac{2}{s^2}$

$U(t) = L^{-1}\left(\dfrac{2}{s^2}\right) = 2L^{-1}\left(\dfrac{1}{s^2}\right) = 2t$

$U(t) = 2t$



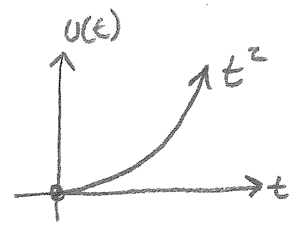
Given $\dfrac{U(s)}{E(s)} = \dfrac{2}{s}$, $e(t) = \text{ramp}(t) = t$ for $t>0$

$U(s) = \dfrac{2}{s}E(s)$ $\quad E(s) = \dfrac{1}{s^2}$

$U(s) = \dfrac{2}{s^3}$

$U(t) = 2L^{-1}\left(\dfrac{1}{s^3}\right) = 2\dfrac{t^2}{2} = t^2$

$U(t) = t^2$



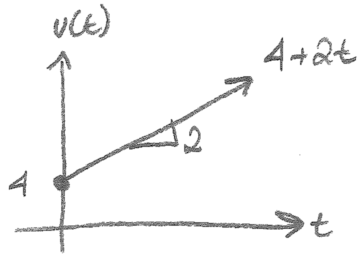
Given $\dfrac{U(s)}{E(s)} = k_p\left(1 + \dfrac{1}{T_i s}\right) = 4\left(1 + \dfrac{1}{2s}\right) = 4 + \dfrac{2}{s}$, $e(t) = \text{unit}(t)$

$E(s) = \dfrac{1}{s}$

$U(s) = \left(\dfrac{1}{s}\right)\left(4 + \dfrac{2}{s}\right) = \dfrac{4}{s} + \dfrac{2}{s^2}$

$U(t) = 4L^{-1}\left(\dfrac{1}{s}\right) + 2L^{-1}\left(\dfrac{1}{s^2}\right)$

$U(t) = 4\,\text{unit}(t) + 2t$



Given $\frac{U(s)}{E(s)} = 4 + \frac{2}{s}$, $e(t) = R(t) = t$ for $t > 0$

$E(s) = \frac{1}{s^2}$

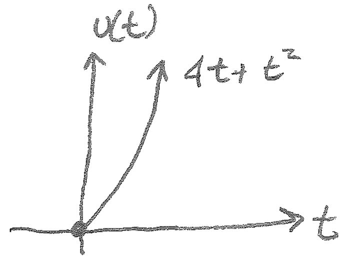
$$U(s) = \left(\frac{4}{s} + \frac{2}{s^2}\right)\left(\frac{1}{s^2}\right)$$

$$U(s) = \frac{4}{s^2} + \frac{2}{s^3}$$

$$u(t) = 4L^{-1}\left(\frac{1}{s^2}\right) + 2L^{-1}\left(\frac{1}{s^3}\right)$$

$$= 4t + 2\left(\frac{t^2}{2}\right)$$

$$u(t) = 4t + t^2$$

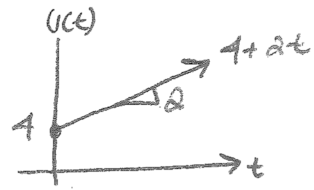


Given $\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = 4\left(1 + \frac{1}{2s} + 0.8s\right) = 4 + \frac{2}{s} + 3.2s$, $E(s) = \frac{1}{s}$

$$U(s) = \frac{4}{s} + \frac{2}{s^2} + 3.2$$

$$u(t) = 4L^{-1}\left(\frac{1}{s}\right) + 2L^{-1}\left(\frac{1}{s^2}\right) + 3.2L^{-1}(1)$$

$$u(t) = 4 + 2t$$



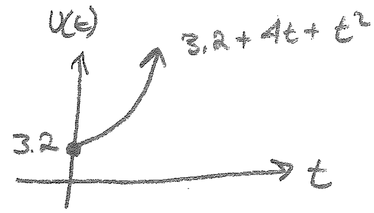
Given $\frac{U(s)}{E(s)} = 4 + \frac{2}{s} + 3.2s$, $E(s) = \frac{1}{s^2}$

$$U(s) = \frac{4}{s^2} + \frac{2}{s^3} + \frac{3.2}{s}$$

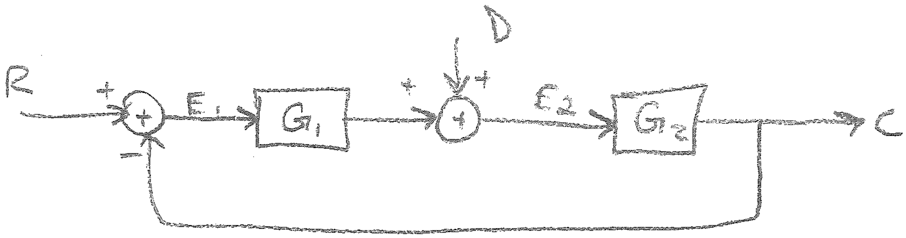
$$u(t) = 4L^{-1}\left(\frac{1}{s^2}\right) + 2L^{-1}\left(\frac{1}{s^3}\right) + 3.2L^{-1}\left(\frac{1}{s}\right)$$

$$= 4t + 2\left(\frac{t^2}{2}\right) + 3.2\,\text{unit}(t)$$

$$u(t) = 4t + t^2 + 3.2\,\text{unit}(t)$$



Problem 6-2-6



Find

$E_1 = R - G_2 E_2$

$E_2 = D + G_1 E_1$

$E_1 = R - G_2(D + G_1 E_1)$

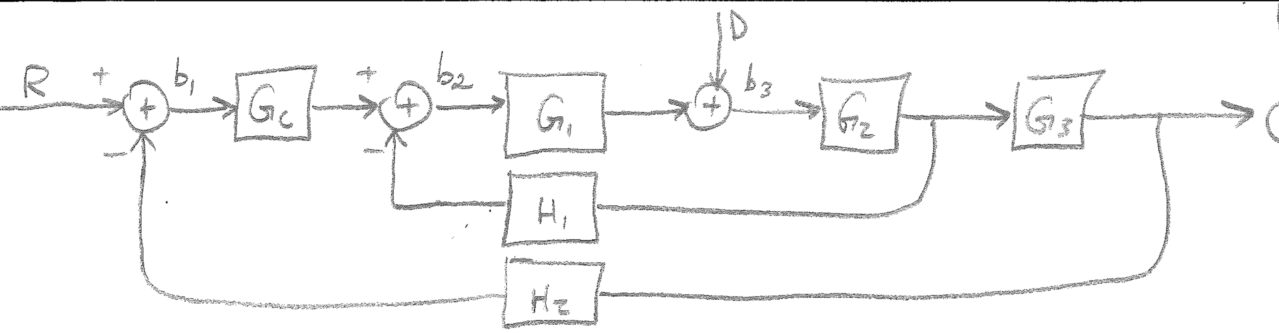
$E_1 = R - G_2 D - G_1 G_2 E_1$

$$E_1 = \frac{R - G_2 D}{1 + G_1 G_2}$$

$$\lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} (s)\left[\frac{R - G_2 D}{1 + G_1 G_2}\right]$$

= Steady state response for Error

Problem B-2-7



$$C = G_3 G_2 b_3$$

$$b_3 = D + G_1 b_2$$

$$b_2 = G_c b_1 - H_1 G_2 b_3$$

$$b_1 = R - G_3 G_2 b_3 H_2$$

$$b_2 = G_c (R - G_3 G_2 b_3 H_2) - H_1 G_2 b_3$$

$$b_2 = G_c R - G_c G_3 G_2 b_3 H_2 - H_1 G_2 b_3$$

$$b_3 = D + G_1 (G_c R - G_c G_3 G_2 b_3 H_2 - H_1 G_2 b_3)$$

$$b_3 = D + G_1 G_c R - G_1 G_c G_3 G_2 b_3 H_2 - G_1 H_1 G_2 b_3$$

$$b_3 = \frac{D + G_1 G_c R}{1 + G_1 G_c G_3 G_2 H_2 + G_1 H_1 G_2}$$

$$C = \frac{G_3 G_2 D + G_3 G_2 G_1 G_c R}{1 + G_1 G_c G_3 G_2 H_2 + G_1 H_1 G_2}$$

Assume $D(s) = 0$

$$\frac{C}{R} = \frac{G_3 G_2 G_1 G_c}{1 + G_1 G_c G_3 G_2 H_2 + G_1 H_1 G_2}$$

Assuming $R = 0$,

$$\frac{C}{D} = \frac{G_2 G_3}{1 + G_1 G_2 G_3 G_c H_2 + G_1 H_1 G_2}$$

Problem B-3-6

$$X_2(s) = X_1(s)\,\frac{k_3}{ms^2 + b_2 s + k_2 + k_3}$$

$$(ms^2 + b_1 s + k_1 + k_3)\,X_1(s) = U(s) + k_3 X_1(s)\,\frac{k_3}{ms^2 + b_2 s + k_2 + k_3}$$

$$X_1(s)\left[ms^2 - b_1 s + k_1 k_3 - \frac{k_3^2}{ms^2 + b_2 s + k_2 + k_3}\right] = U(s)$$

$$\frac{X_1(s)}{U(s)} = \left[\frac{(ms^2 - b_1 s + k_1 k_3)(ms^2 + b_2 s + k_2 + k_3) - k_3^2}{ms^2 + b_2 s + k_2 + k_3}\right]^{-1}$$

$$X_1(s) = \frac{U(s) + k_3 X_2}{ms^2 + b_1 s + k_1 + k_3}$$

$$(ms^2 + b_2 s + k_2 + k_3)\,X_2 = \frac{k_3 U + k_3^2 X_2}{ms^2 + b_1 s + k_1 + k_3}$$

$$X_2 = \frac{k_3 U + k_3^2 X_2}{(ms^2 + b_1 s + k_1 + k_3)(ms^2 + b_2 s + k_2 + k_3)}$$

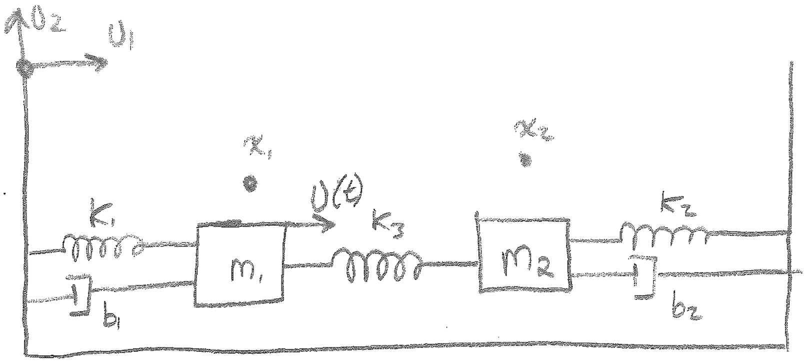
$$X_2 = \frac{k_3 U}{(ms^2 + b_1 s + k_1 + k_3)(ms^2 + b_2 s + k_2 + k_3)} \cdot \Big(1 -$$

$$\frac{X_2}{X_1} = \frac{k_3}{ms^2 + b_2 s + k_2 + k_3}$$

$$\frac{X_2}{U} = \frac{X_1}{U} \cdot \frac{X_2}{X_1}$$

$$\frac{X_2}{U} = \left(\frac{ms^2 + b_2 s + k_2 + k_3}{(ms^2 - b_1 s + k_1 k_3)(ms^2 + b_2 s + k_2 + k_3) - k_3^2}\right) \cdot \frac{k_3}{ms^2 + b_2 s + k_2 + k_3}$$

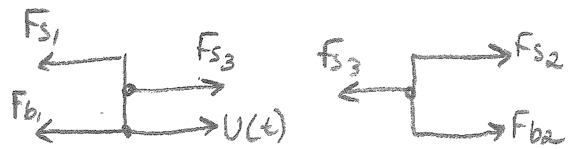
$$\frac{X_2}{U} = \frac{k_3}{(ms^2 - b_1 s + k_1 k_3)(ms^2 + b_2 s + k_2 + k_3) - k_3^2}$$



Problem B-3-6

Find $\frac{X_1(s)}{U(s)}$, $\frac{X_2(s)}{U(s)}$

Assume @ $x_1 = x_2 = 0$, springs are NOT stretched



$F_{s_1} = -x_1 k_1 U_1$ $\qquad$ $F_{s_3} = -x_2 k_3 U_1$

$F_{s_3} = -x_1 k_3 U_1$ $\qquad$ $F_{s_2} = -x_2 k_2 U_1$

$F_{b_1} = -\dot{x}_1 b_1 U_1$ $\qquad$ $F_{b_2} = -\dot{x}_2 b_2 U_1$

$\Sigma F_{U_1} = m\ddot{x}_1$

$-x_1 k_1 - x_1 k_3 - \dot{x}_1 b_1 = m\ddot{x}_1$

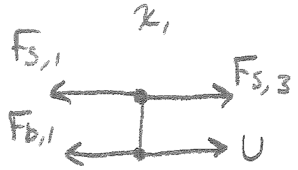
$m\ddot{x}_1 + \dot{x}_1 b_1 + x_1(k_1 + k_3) = U$

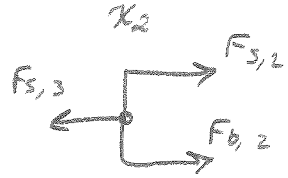
let $\left.\begin{aligned} L[x_1(t)] &= X_1(s) \\ L[\dot{x}_1(t)] &= sX_1(s) \\ L[\ddot{x}_1(t)] &= s^2 X_1(s) \end{aligned}\right\}$ since initial conditions $= 0$

$(ms^2 + b_1 s + k_1 + k_3)X_1(s) = L[U(t)]$

$$\frac{X_1(s)}{U(s)} = \frac{1}{ms^2 + b_1 s + k_1 + k_3}$$

FBD





Problem B-3-6

For x_1,

$F_{s,1} = -x_1 k_1 U_1$

$F_{b,1} = -\dot{x}_1 b_1 U_1$

$U = U$

$F_{s,3} = -(x_1 - x_2) k_3 U_1$

For x_2,

$F_{s,3} = -(x_2 - x_1) k_3 U_1$

$F_{b,2} = -\dot{x}_2 b_2 U_1$

$F_{s,2} = -x_2 k_2 U_1$

$$\Sigma F_{x_1, U_1} = m\ddot{x}_1$$

$$U - x_1 k_1 - \dot{x}_1 b_1 - (x_1 - x_2) k_3 = m\ddot{x}_1$$

$$m\ddot{x}_1 + b_1 \dot{x}_1 + x_1 k_1 + (x_1 - x_2) k_3 = U$$

$$m\ddot{x}_1 + b_1 \dot{x}_1 + x_1 (k_1 + k_3) = U + x_2 k_3$$

let $L[x_1(t)] = X_1(s)$

$L[\dot{x}_1(t)] = s X_1(s)$

$L[\ddot{x}_1(t)] = s^2 X_1(s)$

since initial conditions $= 0$

$$\boxed{(ms^2 + b_1 s + k_1 + k_3) X_1(s) = U(s) + k_3 X_2(s)}$$

$$\Sigma F_{x_2, U_1} = m\ddot{x}_2$$

$$-(x_2 - x_1) k_3 - \dot{x}_2 b_2 - x_2 k_2 = m\ddot{x}_2$$

$$m\ddot{x}_2 + \dot{x}_2 b_2 + x_2 k_2 + (x_2 - x_1) k_3 = 0$$

$$m\ddot{x}_2 + b_2 \dot{x}_2 + x_2 (k_2 + k_3) = x_1 k_3$$

Laplace

$$\boxed{(ms^2 + b_2 s + k_2 + k_3) X_2(s) = k_3 X_1(s)}$$