11. CLASS I METHOD FOR STABILITY AND CONTROL ANALYSIS

The purpose of this chapter is to present a method for rapidly determining whether or not the proposed configuration will have satisfactory stability and control characteristics. The method is presented as part of Step 11, p.d. sequence I as outlined in Chapter 2. Because the method is limited in scope as well as in accuracy, it should be used only in conjunction with preliminary design sequence I.

The method consists of 16 steps and deals with the following stability and control characteristics:

1. Static longitudinal stability (Longitudinal X-plot), see steps 11.1 through 11.7 in Section 11.1.

2. Static directional stability (Directional X-plot), see steps 11.8 through 11.11 in Section 11.2.

3. Minimum control speed with one engine out, see steps 11.12 through 11.16 in Section 11.3.

Example applications are given in Section 11.4.

Other important stability and control characteristics such as take-off rotation, cross-wind controllability, trim through the c.g. range and a variety of dynamic stability considerations are not covered by this Class I method. The reader should refer to Part VII (Ref.6) for a detailed discussion of these characteristics.

11.1 STATIC LONGITUDINAL STABILITY (LONGITUDINAL X-PLOT)

Step 11.1: Prepare a longitudinal X-plot for the airplane.

Figure 11.1 presents examples of longitudinal X-plots. Note that the two legs of the X are representative of:

1. The c.g. leg represents the rate at which the c.g. moves aft (fwd) as a function of horizontal tail (canard) area.

2. The a.c. leg represents the rate at which the a.c. moves aft (fwd) as a function of horizontal tail (canard) area.

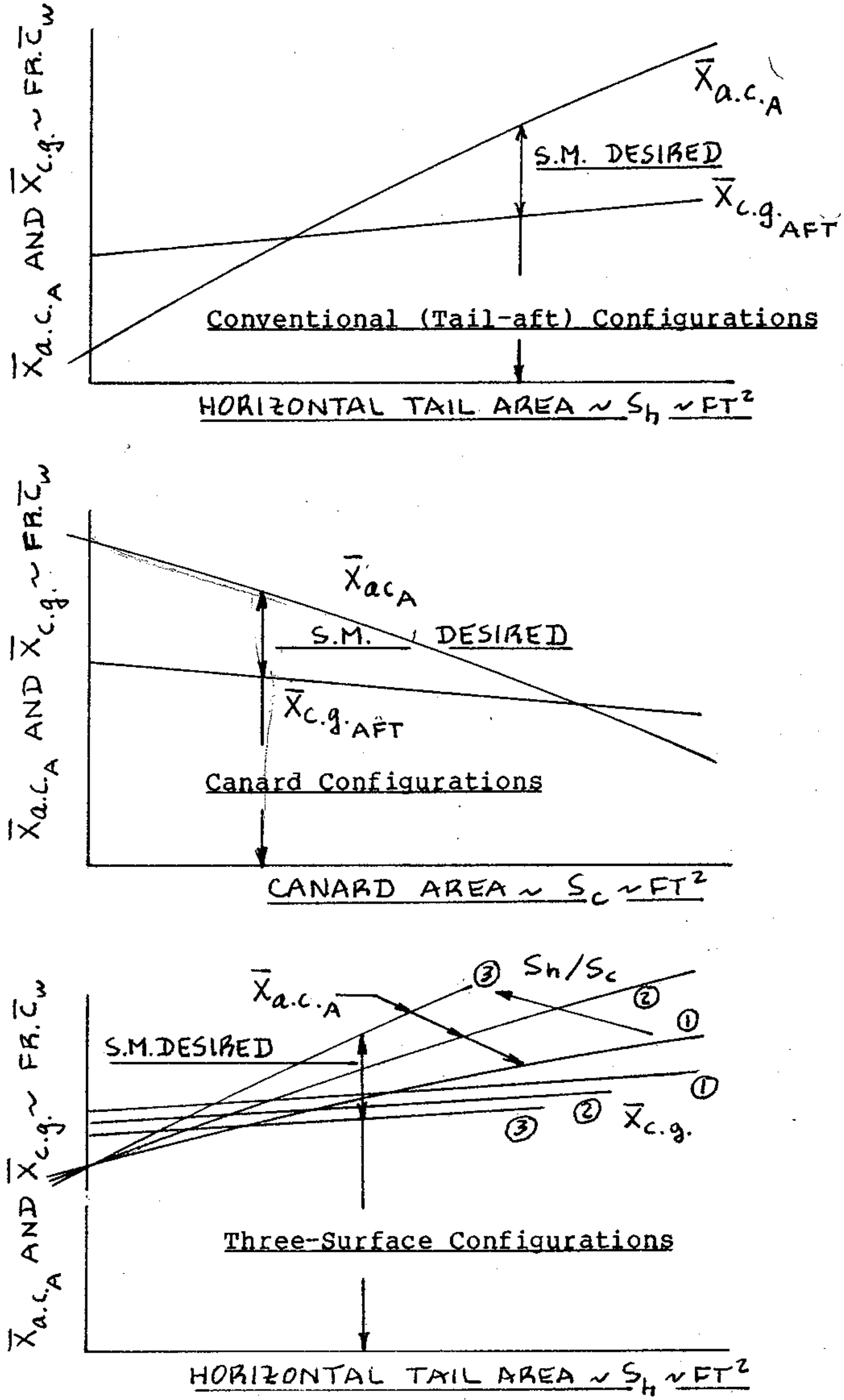


Figure 11.1 Examples of Longitudinal X-Plots

The c.g. leg is easily calculated with the help of the Class I weight and balance analysis of Step 10. From the Class I weight analysis the weight of the horizontal tail (canard) is known on a per ft^2 basis. Assuming this quantity to be independent of surface area, the c.g. can be found for any area of the horizontal tail (or canard).

The a.c. leg is calculated with the following equations:

$$\bar{X}_{ac_A} = [\bar{X}_{ac_{wf}} + \{C_{L_{\alpha_h}}(1 - d\varepsilon_h/d\alpha)(S_h/S)\bar{X}_{ac_h}$$
$$- C_{L_{\alpha_c}}(1 + d\varepsilon_c/d\alpha)\bar{X}_{ac_c}(S_c/S)\}/C_{L_{\alpha_{wf}}}]/F, \quad (11.1)$$

where:

$$F = [1 + \{C_{L_{\alpha_h}}(1 - d\varepsilon_h/d\alpha)(S_h/S) +$$
$$C_{L_{\alpha_c}}(1 + d\varepsilon_c/d\alpha)(S_c/S)\}/C_{L_{\alpha_{wf}}}] \quad (11.2)$$

Figure 11.2 defines the required geometric quantities in these equations. The aerodynamic quantities can be computed with methods presented in Part VI (Ref.5).

Note that Eqns.(11.1) and (11.2) apply to three types of airplanes in the following manner:

<u>For a tail-aft airplane:</u> set $S_c = 0$ and consider S_h as the independent variable.

<u>For a canard airplane:</u>set $S_h = 0$ and consider S_c as the independent variable.

<u>For a three-surface airplane:</u> freeze the ratio S_h/S_c and consider S_h as the independent variable.

For a three-surface airplane, the X-plot should be made for different ratios of S_h/S_c.

Both the c.g. leg and the a.c. leg of the 'X' can now be plotted as a function of area (hor.tail or canard). This completes the longitudinal X-plot.

Figure 11.1 presents conceptual X-plots for three types of airplanes.

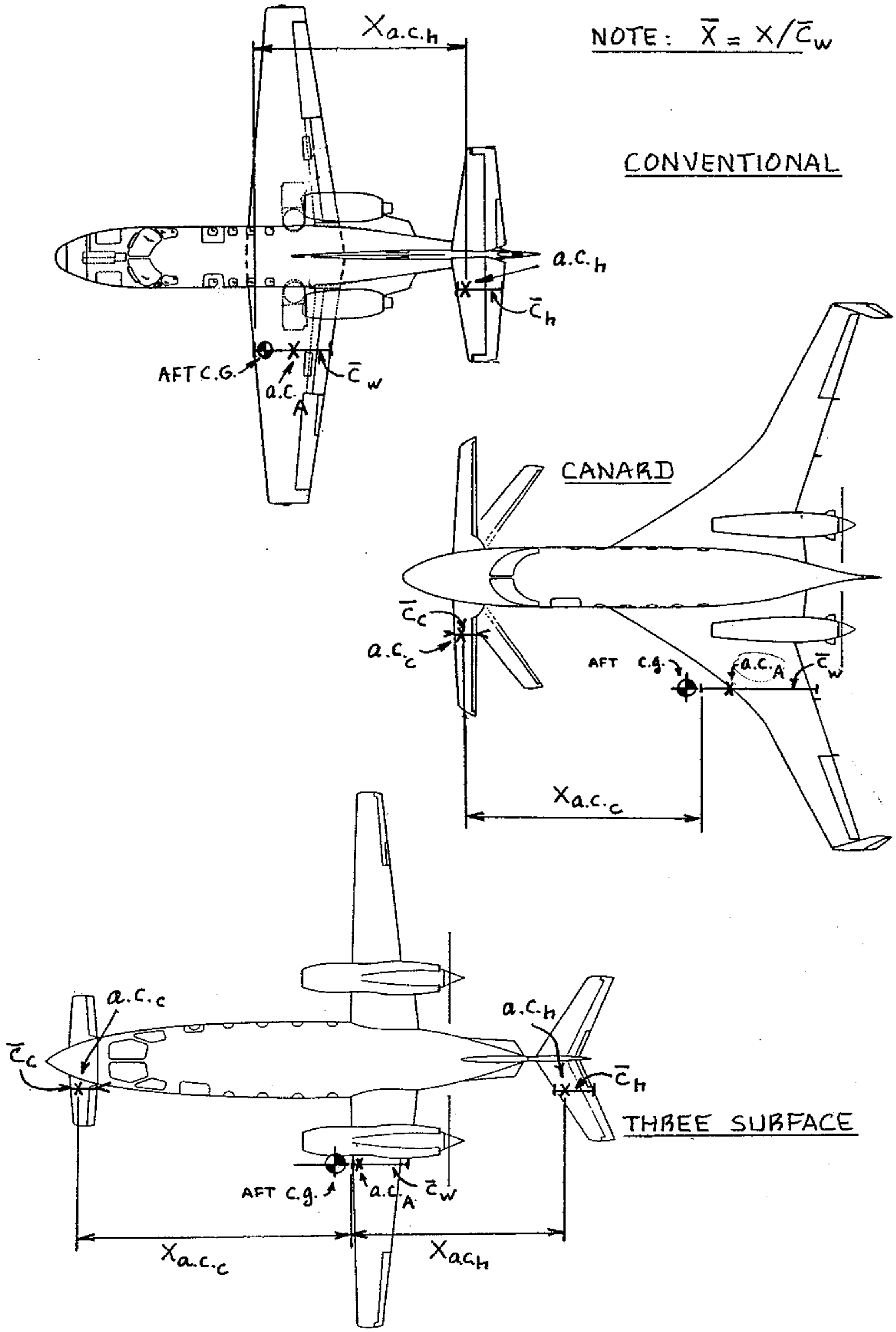


Figure 11.2 Geometric Quantities for A.C. Calculations

Step 11.2: Determine whether or not the airplane being designed needs to be 'inherently stable' or 'de-facto stable'.

Inherent stability is required of all airplanes which do not rely for their stability on a feedback augmentation system. If the airplane falls in this category proceed to Step 11.3.

De-facto stability is required of all airplanes which are stable only with a feedback augmentation system in place. If the airplane falls in this category proceed to Step 11.6.

Step 11.3: Determine whether the airplane being designed fits in any of the twelve categories listed on p.28 in Chapter 2.

If the airplane fits in categories 1-4, proceed to Step 11.4.

If the airplane fits in categories 5-12, proceed to Step 11.5.

Step 11.4: Using the 'aft' c.g. leg in Figure 11.1 find the empennage area required for a minimum static margin of 10 percent.

Reference 9 (Chapter 5) shows that for a static margin of 10 percent:

$$dC_m/dC_L = \bar{X}_{cg} - \bar{X}_{ac} = -0.10 \qquad (11.3)$$

Figure 11.1 shows how the empennage area follows from this. The required empennage area should be recorded.

Step 11.5: Using the 'aft' c.g. leg in Figure 11.1 find the empennage area required for a minimum static margin of 5 percent.

Reference 9 (Chapter 5) shows that for a static margin of 5 percent:

$$dC_m/dC_L = \bar{X}_{cg} - \bar{X}_{ac} = -0.05 \qquad (11.4)$$

Figure 11.1 shows how the empennage area follows from this. The required empennage area should be recorded.

Step 11.6: Using the 'aft' c.g. leg in Figure 11.1 find the SAS feedback gain required as a function of negative static margin.

This feedback gain is estimated from:

$$k_{\alpha} = (\Delta SM) C_{L_{\alpha}} / C_{m_{\delta_e}} , \qquad (11.5)$$

where:

$$C_{L_{\alpha}} = C_{L_{\alpha_{wf}}} + C_{L_{\alpha_h}} (1 - d\varepsilon/d\alpha)(S_h/S)$$
$$+ C_{L_{\alpha_c}} (1 + d\varepsilon/d\alpha) S_c/S \qquad (11.6)$$

The value of k_{α} thus computed should not exceed 5 deg. of elevator per degree of angle of attack.

Equation (11.5) is 'set up' in terms of angle of attack feedback to the elevator. If angle of attack is fed back to the stabilizer (in some fighters) or to the canard (as in the X29), the limit of 5 deg/deg also applies.

The value of incremental static margin, ΔSM in Eqn.(11.5) itself is obtained from the following 'de-facto' stability requirement:

$$\Delta SM = |\bar{X}_{ac} - \bar{X}_{cg} - 0.05| \qquad (11.7)$$

Values for $\bar{X}_{cg}$ and for $\bar{X}_{ac}$ follow from the X-plot in Figure 11.1 at any value of empennage area.

The highest level of static instability which is practical from a stability augmentation viewpoint is that which drives k_{α} to above 5 deg/deg. The corresponding empennage area is the smallest one allowable. This value should be recorded.

Step 11.7: The empennage area obtained from either Eqn.(11.4), (11.5) or (11.6) is the area to be used instead of that obtained from the $\bar{V}$-method, Eqn.(8.3).

If there is more than 10 percent difference between the empennage areas predicted from the $\bar{V}$-method and the method just described, the airplane weight and balance calculations of Chapter 10 should be reviewed and any necessary adjustments made.

11.2 STATIC DIRECTIONAL STABILITY (DIRECTIONAL X-PLOT)

Step 11.8: Prepare a directional X-plot for the airplane.

Figure 11.3 shows an example of such an X-plot. The c.g. leg is again determined with the help of the Class I weight analysis of Step 10. The weight per ft^2 of the vertical tail is known from this weight analysis.

The C_{n_β} leg of the X-plot follows from:

$$C_{n_\beta} = C_{n_{\beta_{wf}}} + C_{L_{\alpha_v}}(S_v/S)(x_v/b) \tag{11.8}$$

The geometric quantities in Eqn.(11.8) are defined in Figure 11.4. The aerodynamic quantities on the right hand side of Eqn.(11.8) can be computed with the methods of Part VI (Ref.5).

Step 11.9: Determine whether or not the airplane being designed needs to have 'inherent' or 'de facto' directional stability.

If the airplane needs to be 'inherently' directionally stable, proceed to Step 11.10.

If the airplane needs to have 'de facto' directional stability proceed to Step 11.11.

Step 11.10: Assume that the overall level of directional stability must be:

$$C_{n_\beta} = 0.0010 \text{ per deg.} \tag{11.9}$$

Proceed to the X-plot of Figure 11.3 and find the value of S_v which produces this level of directional stability.

Step 11.11: Compute the required sideslip to rudder feedback gain from:

$$k_\beta = (\Delta C_{n_\beta})/C_{n_{\delta_r}} \tag{11.10}$$

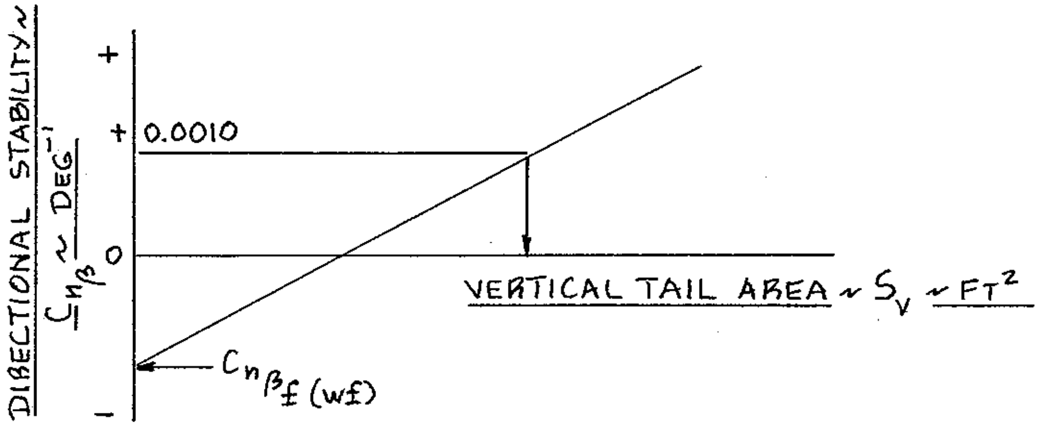


Figure 11.3 Example of Directional X-Plot

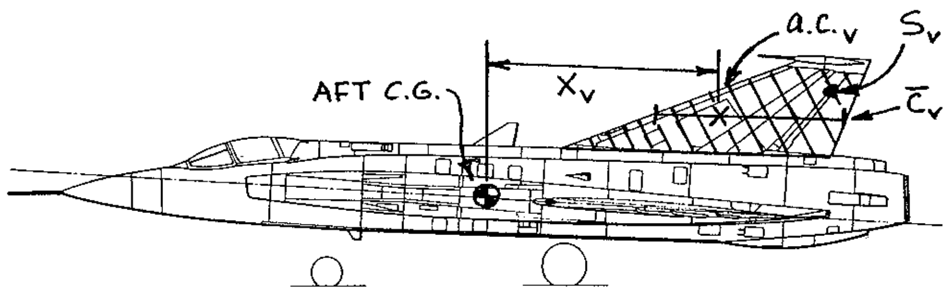


Figure 11.4 Geometric Quantities for Directional X-Plot

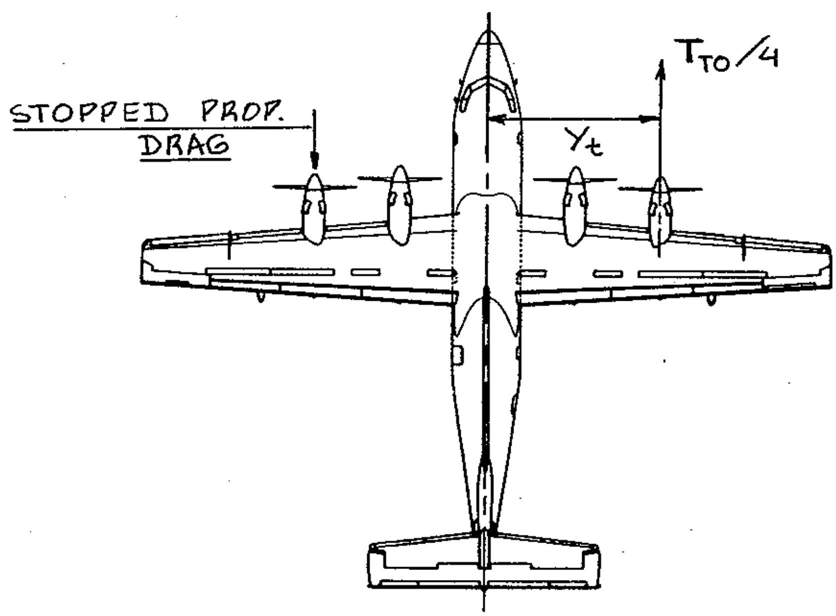


Figure 11.5 Geometry for Engine-out V_{mc} Calculation

The required value of ΔC_{n_β} follows from:

$$\Delta C_{n_\beta} = 0.0010 - C_{n_\beta} \tag{11.11}$$

The value of k_β thus computed should not exceed 5 deg/deg.

The vertical tail area resulting in the lowest value of inherent C_{n_β} which is consistent with Eqn.(11.11) is the smallest allowable vertical tail area. This empennage area should be recorded.

11.3 MINIMUM CONTROL SPEED WITH ONE ENGINE INOPERATIVE

Step 11.12: Determine the critical engine-out yawing moment from:

$$N_{t_{crit}} = T_{TO_e} y_t \tag{11.12}$$

The value of y_t corresponds to the lateral thrust moment arm of the most critical engine. Figure 11.5 illustrates y_t.

For a propeller driven airplane the known value of P_{TO} must be changed to the corresponding value of T_{TO}. Figure 3.8 of Part I (p.100) can be used to do this.

Step 11.13: Determine the value of drag induced yawing moment due to the inoperative engine from:

For a propeller driven airplane with fixed pitch propellers:

$$N_D = 0.75 N_{t_{crit}} \tag{11.13}$$

For a propeller driven airplane with variable pitch propellers:

$$N_D = 0.25 N_{t_{crit}} \tag{11.14}$$

For a jet driven airplane with a windmilling engine with low b.p.r.:

$$N_D = 0.15 N_{t_{crit}} \quad (11.15)$$

For a jet driven airplane with a wind-milling engine with high b.p.r.:

$$N_D = 0.25 N_{t_{crit}} \quad (11.16)$$

Step 11.14: Calculate the maximum allowable V_{mc} from:

$$V_{mc} = 1.2 V_s, \quad (11.17)$$

where V_s is the lowest stall speed of the airplane. This is usually the landing stall speed.

Step 11.15: Calculate the rudder deflection required to hold the engine out condition at V_{mc} from:

$$\delta_r = (N_D + N_{t_{crit}})/\bar{q}_{mc} S b C_{n_{\delta_r}} \quad (11.18)$$

The value of the control power derivative $C_{n_{\delta_r}}$ may be computed with the methods of Part VI (Ref.5).

The rudder deflection resulting from Eqn.(11.18) should be no more than 25 degrees. If it is more, adjust the rudder size and/or the vertical tail size until this is satisfied. Record this vertical tail area.

Step 11.16: The largest vertical tail area which results from Steps 11.10, 11.11 or 11.15 is the vertical tail area required for the airplane. Determine this area.

If this vertical tail area differs by more than 10 percent from the one computed with the $\bar{V}$-method of Eqn.(8.4) it will be necessary to adjust the weight and balance calculations of Chapter 10.

11.4 EXAMPLE APPLICATIONS

Three example applications will now be discussed:

11.4.1 Twin Engine Propeller Driven Airplane: Selene
11.4.2 Jet Transport: Ourania
11.4.3 Fighter: Eris

11.4.1 Twin Engine Propeller Driven Airplane

Step 11.1: Figure 11.6 presents the longitudinal X-plot for the Selene.

Observe, that for the airplane to be 0.10 stable at its operating weight empty, a horizontal tail area of 58 ft^2 is required. This represents an increase of $58 - 37 = 21 \text{ ft}^2$. This larger horizontal tail has the effect of shifting the aft c.g. to $0.9\bar{c}_w$. This is still forward of the main landing gear. However, it may be necessary to move the main gear aft a bit to accomodate the longitudinal tip-over criterion of Chapter 9.

Note also from the X-plot, that the Selene will have to be restricted from flying at W_{OE} plus two aft passengers plus aft luggage. The airplane would become unstable in this flight condition. This is a rather common occurrence in this type of airplane.

Observe also from the X-plot, that with power-on the stability of the airplane is much better. This is a typical characteristic of pusher-propeller airplanes.

Step 11.2: The Selene must be an inherently stable airplane. Full time stability augmentation in this type of airplane is probably not affordable.

Step 11.3: The Selene fits into category 3: twin engine propeller driven airplanes.

Step 11.4: It was already decided in Step 11.1 that the horizontal tail area needs to be increased from 37 to 58 ft^2.

Steps 11.5 - 11.7: Not applicable.

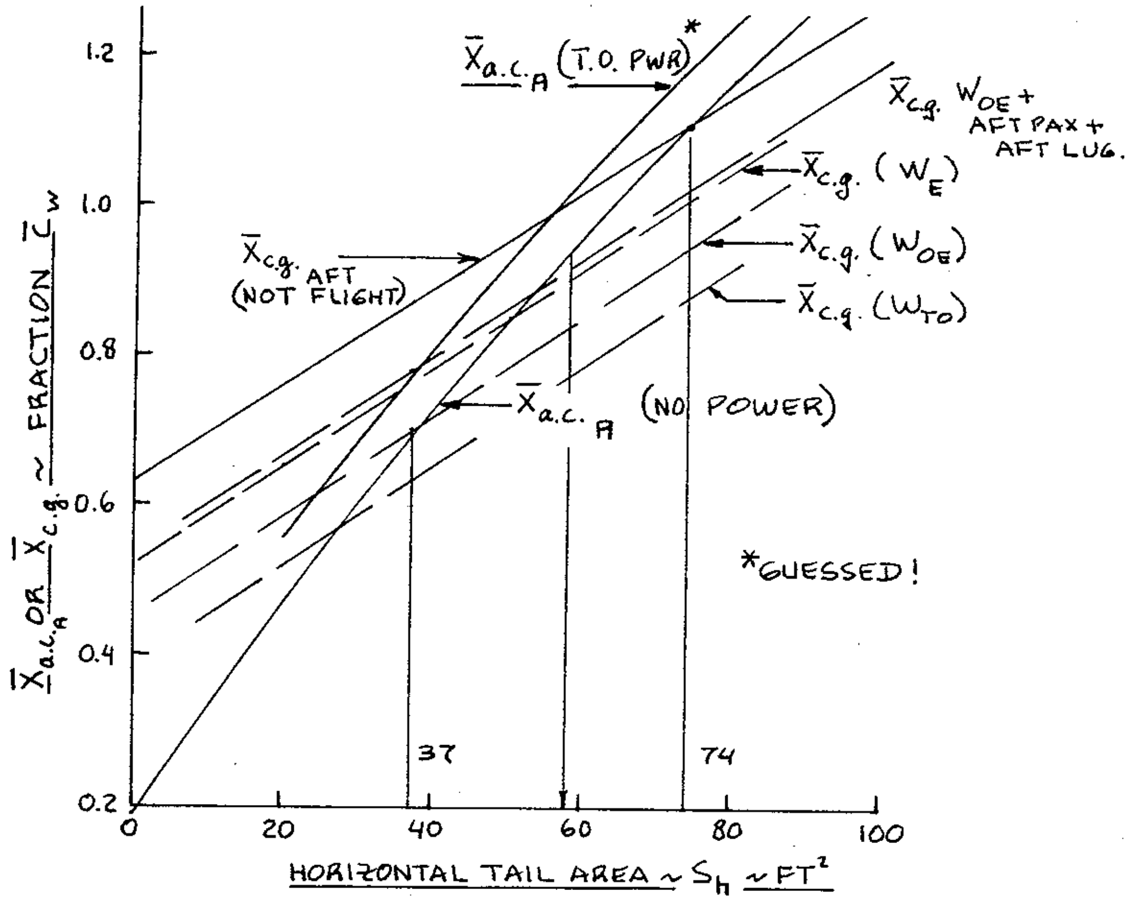


Figure 11.6 Selene: Longitudinal X-Plot

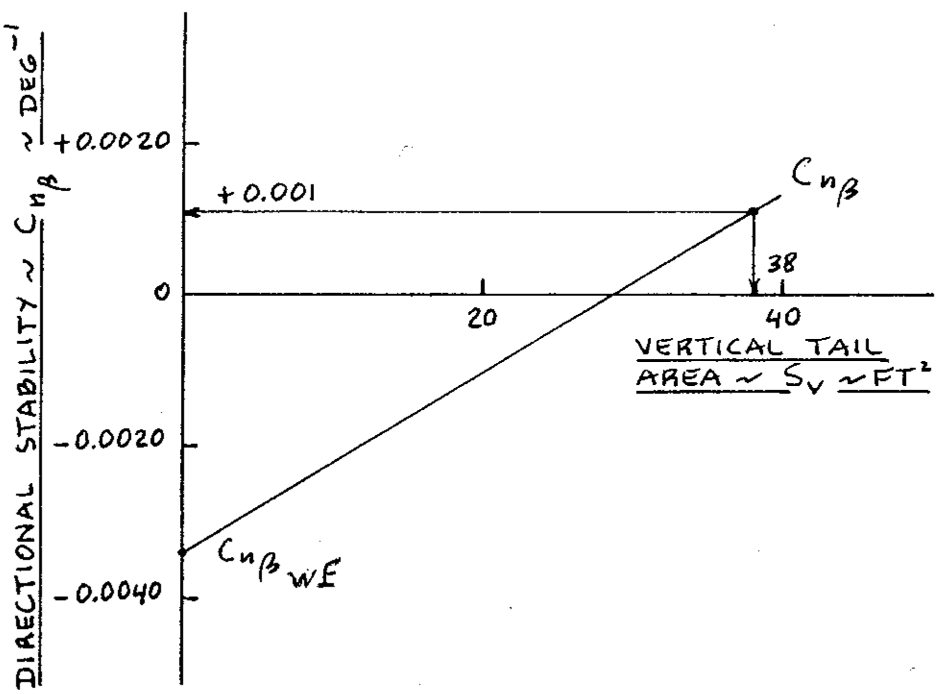


Figure 11.7 Selene: Directional X-Plot

Step 11.8: Figure 11.7 presents the directional X-plot for the Selene.

Step 11.9: The Selene needs to have inherent directional stability: full time stability augmentation in this type of airplane is probably not affordable.

Step 11.10: Note from Figure 11.7 that the vertical tail of the Selene is slightly too large: an area of 36 ft^2 would be sufficient from a directional stability viewpoint.

Step 11.11: Not applicable.

Step 11.12: From the general arrangement drawing of the Selene (Fig.10.3) it follows that y_t= 6.3 ft. The maximum take-off power, P_{TO} was determined in Chapter 5 (p.135) as 449 hp. per engine. From Figure 3.8 in Part I (p.100) it is seen that at this power level, the take-off thrust is: T_{TO}= 1,200 lbs per engine.

The critical engine-out yawing moment it therefore: 1,200x6.3 = 7,560 ftlbs.

Step 11.13: The Selene will have variable pitch propellers. The value for N_D therefore is: 0.25x7,560 = 1,890 ftlbs.

Step 11.14: The landing stall speed is the lowest stall speed for the Selene. At the landing weight this is found to be: 99.3 kts.

The maximum allowable value for V_{mc} is therefore: 1.2x99.3 = 119 kts.

Step 11.15: From the vertical tail and rudder geometry definitions in Chapter 8 (p.210-211) and from the methods of Part VI the following value for rudder control power derivative is computed:

$$C_{n_{\delta_r}} = - 0.0027 \text{ deg}^{-1}$$

With Eqn.(11.18) this yields for the rudder deflection required at V_{mc} a value of 16.4 deg. This is well within the allowable value of 25 deg.

Step 11.16: The vertical tail size of the Selene is thus 'critical' from a directional stability viewpoint.

As seen in Step 11.10 the existing tail size of 38 ft^2 is sufficient.

11.4.2 Jet Transport

Step 11.1: Figure 11.8 presents the longitudinal X-plot for the Ourania.

Observe, that the Ourania is longitudinally stable without a horizontal tail. The cause for this is the too forward position of the wing on the fuselage. By moving the wing, together with the main landing gear 200 inches aft a more reasonable result is obtained. Note that now, at the nominal tail area of 254 ft^2 the Ourania has a level of instability of $0.085\bar{c}_w$. The reader will remember that the Ourania was to be configured as a 'relaxed' stability airplane.

Figure 11.9 shows how 'wing + main gear' movement affects the c.g. location on the mean geometric chord of the wing, $\bar{c}_w$.

Step 11.2: The Ourania must be a 'relaxed stability' airplane. The level of instability at aft c.g. should be the subject of a detailed study of the benefits in 'trimmed lift-to-drag ratio' which this confers on the airplane. For purposes of this p.d. study a level of instability of $0.085\bar{c}_w$ is arbitrarily selected.

Step 11.3: The Selene fits into category 7: jet transports.

Step 11.4: Not applicable.

Step 11.5: Not applicable.

Step 11.6: Using the 'aft' c.g. leg corresponding the 200 in. aft shift of the wing (Figure 11.8) it is found that the longitudinal stability augmentation system must generate a value of incremental static margin of:

$\Delta SM = 0.085 + 0.05 = 0.135.$

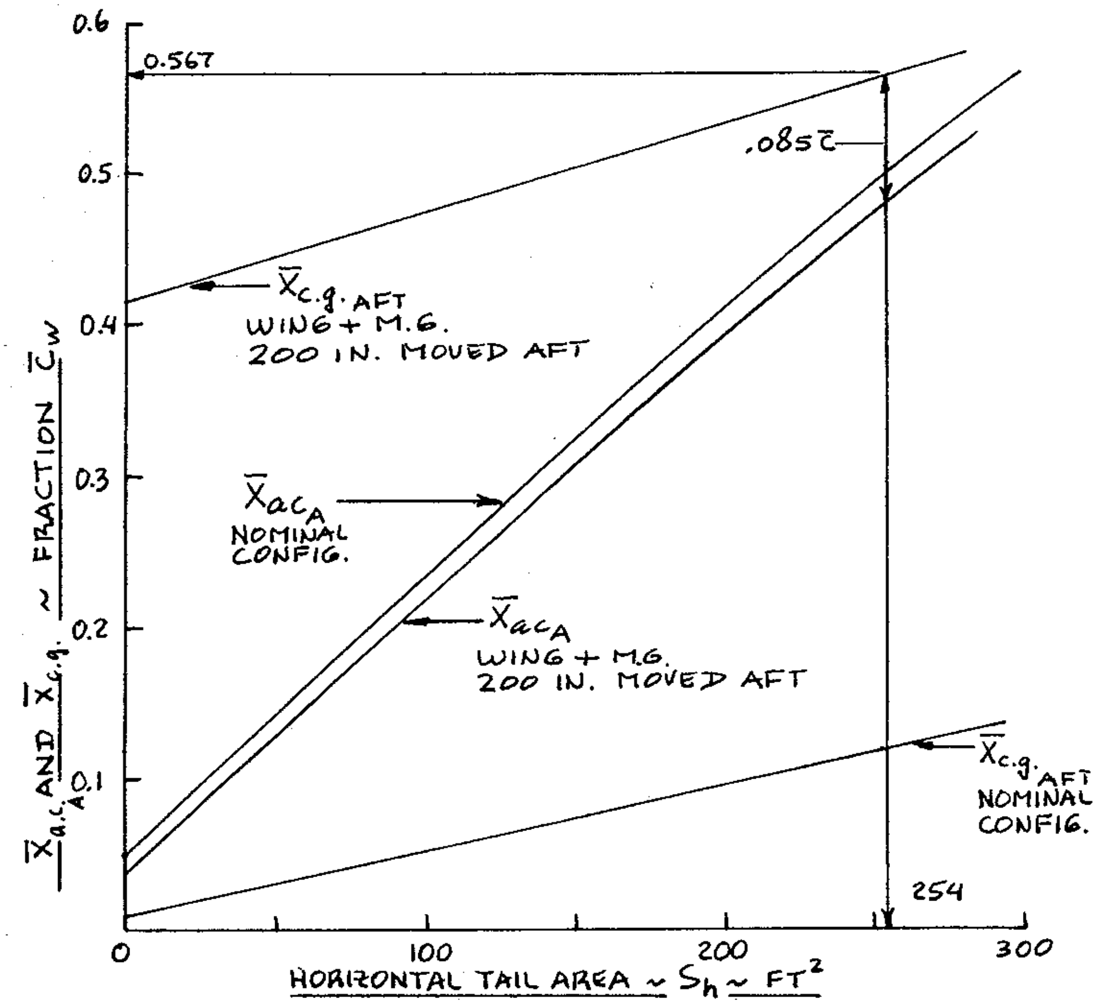


Figure 11.8 Ourania: Longitudinal X-Plot

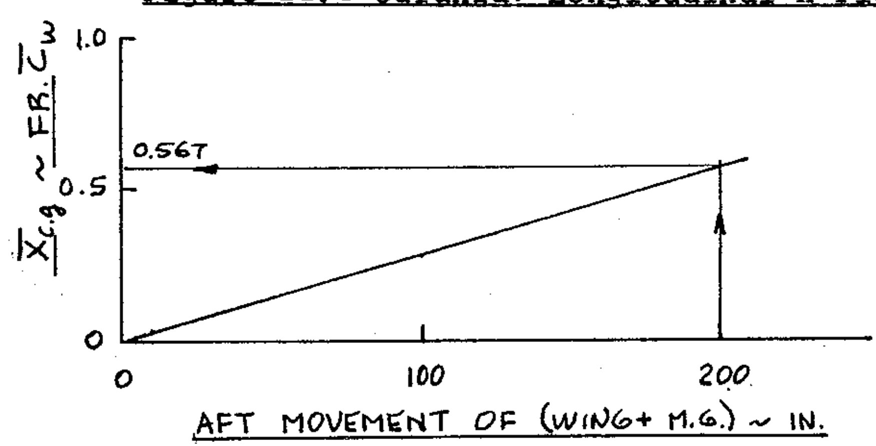


Figure 11.9 Ourania: Effect of 'Wing + Main Gear' Aft Movement on Airplane Center of Gravity

The total airplane lift curve slope was computed to be: $C_{L_\alpha} = 0.081\ \text{deg}^{-1}$. The value of the elevator control power derivative was found to be: $C_{m_{\delta_e}} = -\ 0.0251\ \text{deg}^{-1}$.

With these values and with Eqn.(11.5) it follows that: $k_\alpha = 0.44$ which is an acceptable value of feedback gain. It would appear that from this viewpoint the horizontal tail could be made smaller. At this point it is prudent not to do this. Class II methods may show that take-off rotation and trim at forward c.g. with the flaps down are more restrictive in tailplane design.

Step 11.7: The horizontal tail area will be maintained at 254 ft^2.

Step 11.8: Figure 11.10 presents the directional X-plot for the Ourania.

Step 11.9: The Ourania is to be a 'relaxed' stability airplane.

Step 11.10: Not applicable.

Step 11.11: Note from Figure 11.10 that the vertical tail of the Ourania already results in a level of directional instability of $C_{n_\beta} = -\ 0.0016$. Desired is a 'de-facto' level of 0.0010. The decrement of 0.0026 must be provided by the sideslip feedback system.

The rudder control power derivative of the Ourania was computed to be: $C_{n_{\delta_r}} = -\ 0.0012\ \text{deg}^{-1}$.

With the help of Eqn.(11.10) the feedback gain can be computed to be:

$$k_\beta = 0.0026/0.0012 = 2.2\ \text{deg/deg}.$$

This is acceptable. From this viewpoint then the vertical tail of the Ourania is not critical.

Step 11.12: From the general arrangement drawing of the Ourania (Fig.10.4) it follows that $y_t = 16.7$ ft. The

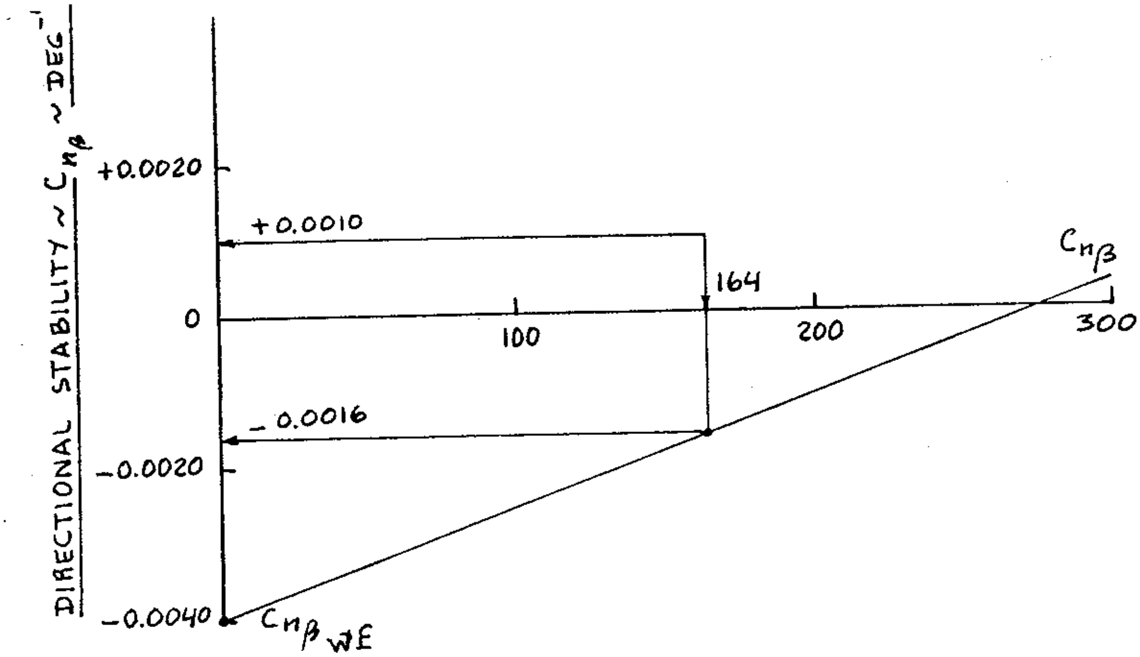
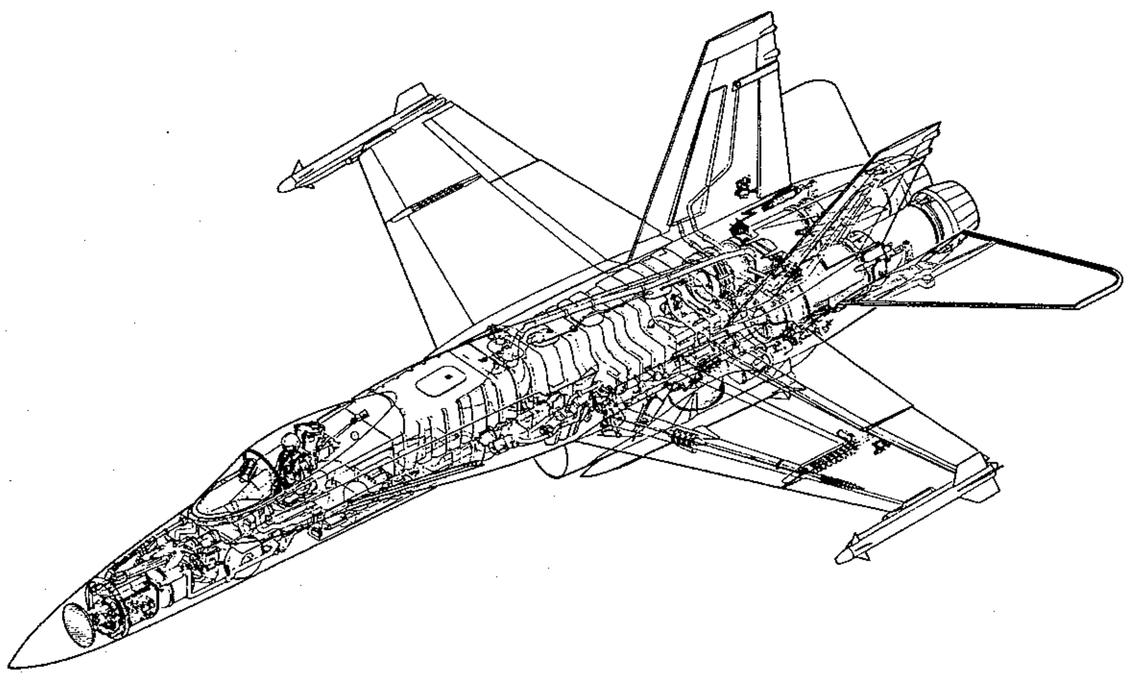


Figure 11.10 Ourania: Directional X-Plot



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maximum take-off thrust, T_{TO} was determined in Chapter 5 (p.138) as 24,000 lbs per engine.

The critical engine-out yawing moment it therefore: 24,000x16.7 = 400,800 ftlbs.

Step 11.13: The Ourania has high b.p.r engines. Eqn.(11.16) therefore applies in determining the windmilling drag induced yawing moment. The total yawing moment to be 'held' at V_{mc} is therefore 1.25x400,800 = 501,000 ftlbs.

Step 11.14: The landing stall speed is the lowest stall speed for the Ourania. At the landing weight it is found that V_{s_L} = 87 kts. This yields a V_{mc} = 105 kts.

Step 11.15: From the vertical tail and rudder geometry definitions in Chapter 8 (p.210-211) and from the methods of Part VI the following value for rudder control power derivative is computed:

$$C_{n_{\delta_r}} = -\ 0.0012\ \mathrm{deg}^{-1}$$

With Eqn.(11.18) this yields for the rudder deflection required at V_{mc} a value of 61 deg. This is clearly too much. The vertical tail of the Ourania is therefore too small.

If the vertical tail size is increased to 200 ft^2 while at the same time the rudder area ratio S_r/S_v is increased from 0.35 to 0.45 and the rudder is given a double hinge line (variable camber) so the rudder can be driven to 40 deg., a satisfactory solution can be obtained. The reader will realize that this will have to be verified with more detailed analyses and possibly a windtunnel test before the final decision on the vertical tail size can be made. However, for p.d. purposes it will be assumed that the vertical tail will have to be increased to 200 ft^2.

Step 11.16: It was already decided in the previous step to increase the vertical tail from 164 to 200 ft^2.

11.4.3 Fighter

Step 11.1: Figure 11.11 presents the longitudinal X-plot for the Eris.

Observe, that the Eris is longitudinally unstable without a horizontal tail. At the horizontal tail area of 93 ft^2 (determined from the V-method in Chapter 8) the level of instability is $0.133\bar{c}_w$. The reader should realize that the X-29 was designed to a level of instability of $0.350\bar{c}_w$ at its aft c.g!

Step 11.2: The Eris must be a negative stability airplane. The level of instability at aft c.g. and at forward c.g. should be the subject of a detailed study of the benefits in 'trimmed lift-to-drag ratio' and maneuvering performance which are conferred upon the airplane. For purposes of this p.d. study a level of instability of $0.133\bar{c}_w$ is arbitrarily selected.

Step 11.3: The Selene fits into category 9: fighters.

Step 11.4: Not applicable.

Step 11.5: Not applicable.

Step 11.6: Using the 'aft' c.g. leg of Figure 11.11 it is found that the longitudinal stability augmentation system must generate a value of incremental static margin of:

$$\Delta SM = 0.133 + 0.05 = 0.185.$$

The total airplane lift curve slope was computed to be: $C_{L_\alpha} = 0.078 \text{ deg}^{-1}$. The value of the elevator control power derivative was found to be: $C_{m_{\delta_e}} = - 0.0182 \text{ deg}^{-1}$.

With these values and with Eqn.(11.5) it follows that: $k_\alpha = 0.80$ which is an acceptable value of feedback gain. It would appear that from this viewpoint the horizontal tail could be made smaller. At this point it

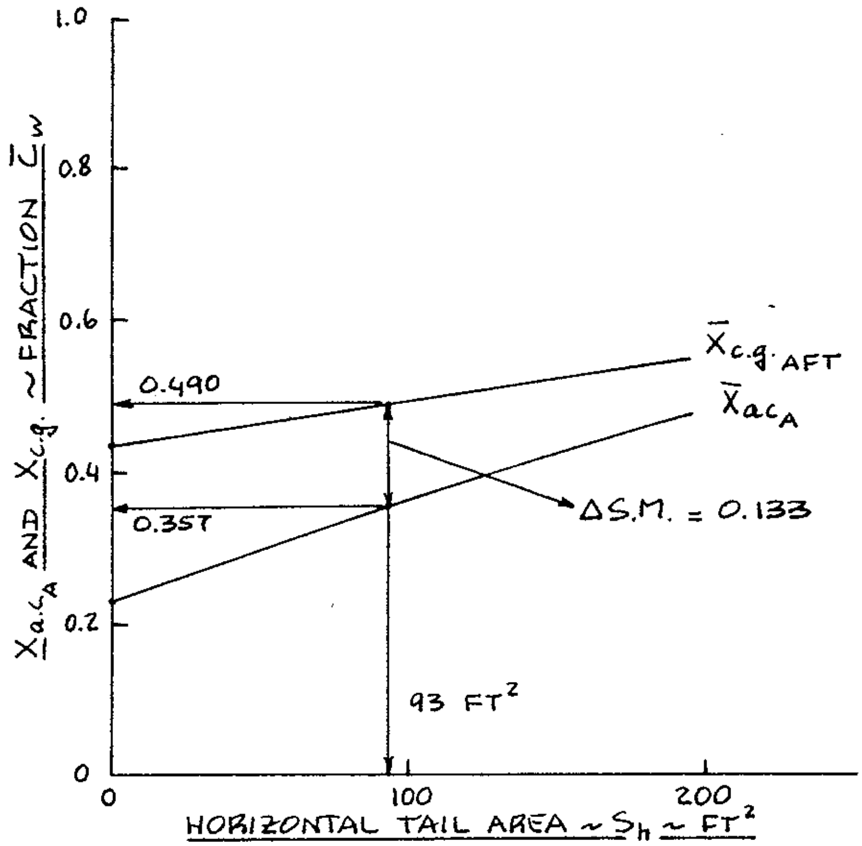


Figure 11.11 Eris: Longitudinal X-Plot

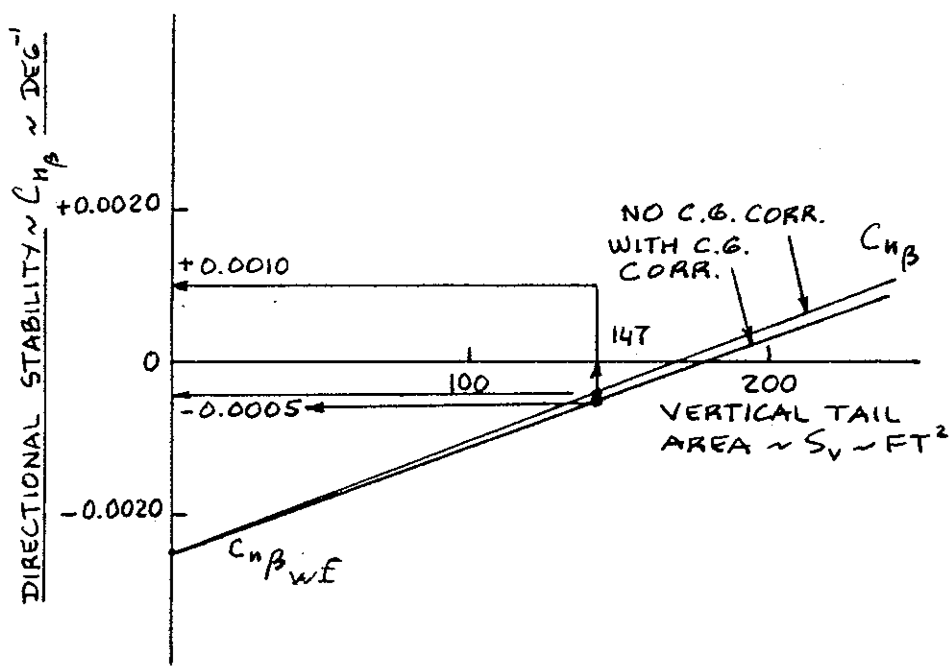


Figure 11.12 Eris: Directional X-Plot

is prudent not to do this. Class II methods may show that take-off rotation and trim at forward c.g. with the flaps down are more restrictive in tailplane design.

Step 11.7: The horizontal tail area of 93 ft^2 will be kept.

Step 11.8: Figure 11.12 presents the directional X-plot for the Eris.

Step 11.9: The Eris is to be a 'negative' stability airplane.

Step 11.10: Not applicable.

Step 11.11: Note from Figure 11.12 that the vertical tail of the Eris renders the airplane directionally unstable at a level of $C_{n_\beta} = -0.0005$. Desired is a 'de-facto' level of 0.0010. The decrement of 0.0015 must be provided by the sideslip feedback system.

The rudder control power derivative of the Eris was computed to be: $C_{n_{\delta_r}} = -0.0007 \text{ deg}^{-1}$.

With the help of Eqn.(11.10) the feedback gain can be computed to be:

$k_\beta = 0.0015/0.0007 = 2.1$ deg/deg.

This is acceptable. From this viewpoint then the vertical tail of the Eris is not critical.

Step 11.12: From the general arrangement drawing of the Eris (Fig.10.5) it follows that $y_t = 1.7$ ft. The maximum take-off thrust, T_{TO} was determined in Chapter 5 (p.140) as 16,000 lbs per engine.

The critical engine-out yawing moment it therefore: 12,000x1.7 = 20,400 ftlbs.

Step 11.13: The Eris has low b.p.r engines. Eqn.(11.16) therefore applies in determining the windmilling drag induced yawing moment. The total yawing moment to be 'held' at V_{mc} is therefore 1.15x20,400 = 23,460 ftlbs.

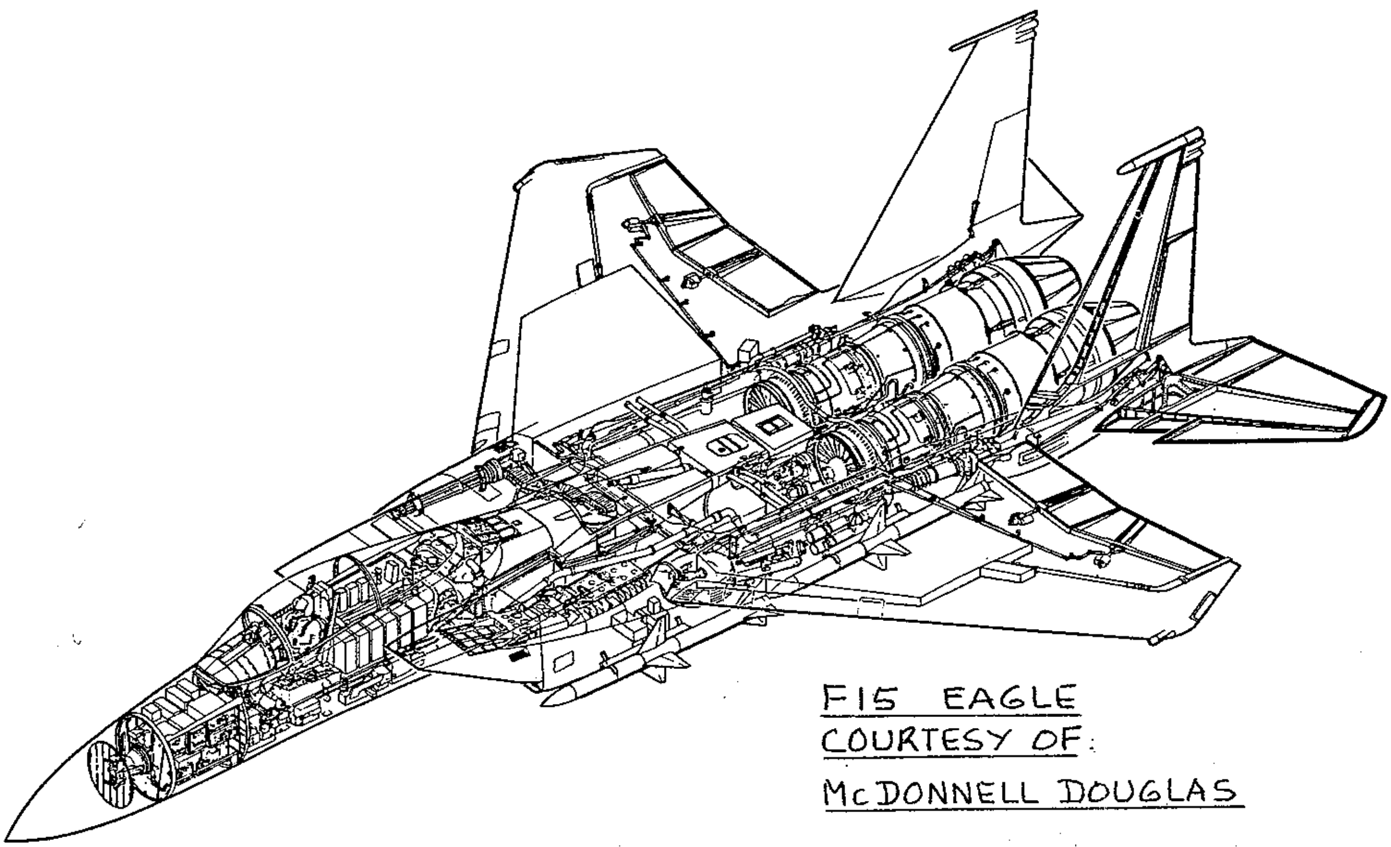
Step 11.14: The landing stall speed is the lowest stall speed for the Eris. At the landing weight this is 131 kts. This yields a V_{mc} = 158 kts.

Step 11.15: From the vertical tail and rudder geometry definitions in Chapter 8 (p.214-215) and from the methods of Part VI the following value for rudder control power derivative is computed:

$$C_{n_{\delta_r}} = - 0.00074 \text{ deg}^{-1}$$

With Eqn.(11.18) this yields for the rudder deflection required at V_{mc} a value of 9.3 deg. This is acceptable. The vertical tail of the Eris is therefore not critical from a viewpoint of engine-out control.

Step 11.16: It was already decided in the previous step to keep the vertical tail size at 147 ft^2.



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