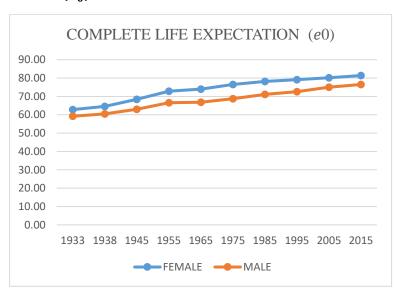
PART A

The comments below are based on the Life Table from USA.

COMPLETE LIFE EXPECTATION NEW BORN (e_0^0)

Year	FEMALE	MALE
1933	62.81	59.21
1938	64.56	60.47
1945	68.36	62.99
1955	72.81	66.59
1965	73.94	66.81
1975	76.46	68.73
1985	78.18	71.09
1995	79.09	72.59
2005	80.13	75.00
2015	81.35	76.50



Complete Life Expectancy (e₀) refers to the average number of years a newborn is expected to live. According to the data, there is a persistent gender gap in life expectancy in the United States. For instance:

- In **1933**, a newborn female was expected to live **62.81 years**, compared to **59.21 years** for males—a difference of **3.60 years**.
- By **2015**, female life expectancy rose to **81.35 years**, while males reached **76.50 years**—a gap of **4.85 years**.

This trend of females outliving males is consistent across decades. For example:

- In 2005, female life expectancy was 80.13 years versus 75.00 years for males, a 5.13-year difference.
- Similarly, in **1945**, females were expected to live **5.37 years longer** than males. Biological factors, among others, likely contribute to this disparity.

The data also reveals a **steady increase in life expectancy for both sexes from 1933 to 2015**:

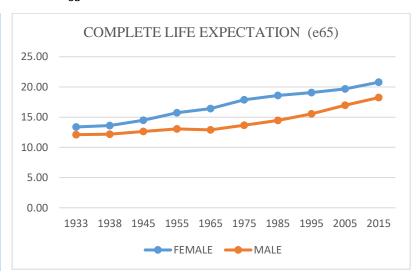
• Females: From 62.81 years (1933) to 64.56 years (1938), a 1.75-year increase over 5 years.

- Males: From 59.21 years (1933) to 60.47 years (1938), a 1.26-year increase over the same period.
- Over a 10-year span (2005–2015), female life expectancy rose by 1.22 years (80.13 to 81.35), while males gained 1.50 years (75.00 to 76.50).

This upward trajectory reflects advancements in healthcare, public health initiatives, and quality of life over time.

COMPLETE LIFE EXPECTATION (e_{65}^0)

Year	FEMALE	MALE
1933	13.38	12.11
1938	13.62	12.17
1945	14.49	12.64
1955	15.73	13.05
1965	16.42	12.91
1975	17.88	13.66
1985	18.58	14.45
1995	19.08	15.55
2005	19.68	16.98
2015	20.78	18.24



The table above represents the average number of additional years a person aged 65 is expected to live. The data highlights a persistent gender gap and a long-term upward trend:

• In 1933:

- o Females aged 65 had a remaining life expectancy of **13.38 years**.
- Males aged 65 had a remaining life expectancy of 12.11 years—a gender gap of 1.27 years.

• By 2015:

- o Females aged 65 could expect to live **20.78 years** (nearly 21 additional years).
- o Males aged 65 could expect **18.24 years**—a reduced gender gap of **2.54 years**.

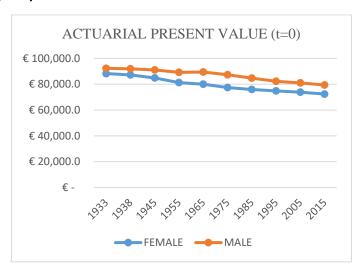
This trend mirrors the **upward trajectory of Complete Life Expectancy (e₀°)** from 1933 to 2015 for both sexes. Over this period:

- Female life expectancy at age 65 increased by **7.40 years** (from 13.38 to 20.78).
- Male life expectancy at age 65 rose by **6.13 years** (from 12.11 to 18.24).

The narrowing gender gap and overall improvements likely reflect advancements in healthcare, lifestyle changes, and socioeconomic factors.

WHOLE LIFE INSURANCE CONTRACT (Age 45)

Year	ı	FEMALE		MALE
1933	€	88,242.9	€	92,296.3
1938	€	87,160.7	€	91,911.0
1945	€	84,814.7	€	90,984.6
1955	€	81,224.3	€	89,150.7
1965	€	79,999.6	€	89,409.3
1975	€	77,430.2	€	87,329.0
1985	€	75,891.1	€	84,702.1
1995	€	74,753.8	€	82,255.0
2005	€	73,784.7	€	80,973.8
2015	€	72,388.0	€	79,397.6



Benefit: €150,000.00 paid to beneficiaries upon the insured's death. **Actuarial Present Value (APV) of Future Benefits** reflects the net single premium required to secure the policy.

Key Observations from the Table

- 1. Gender Disparity in APV:
 - o Males consistently pay higher premiums than females for the same benefit.
 - **2015**: Males pay €79,397.60 vs. females' €72,388.00 (difference: €7,009.60).
 - 2005: Males pay €80,973.80 vs. females' €73,784.70 (difference: €7,189.10).
 - o **Reason**: Males have shorter life expectancies, meaning the benefit is expected to be paid sooner, increasing the upfront premium.
- 2. **Temporal Trend (1933–2015)**:
 - o **APV declined for both sexes** due to rising life expectancy over time.
 - Females: APV fell from €88,242.90 (1933) to €72,388.00 (2015) a 17.9% decrease.
 - Males: APV fell from €92,296.30 (1933) to €79,397.60 (2015) —
 a 14.0% decrease.

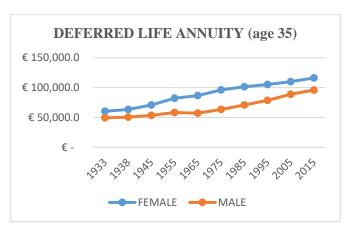
Example: By 2015, a policyholder paid €15,854.90 less (females) and €12,898.70 less (males) than in 1933 for the same €150,000 benefit.

Why Does APV Decrease Over Time?

- **Increasing Life Expectancy**: Longer lifespans delay the expected payout, reducing the present value of the benefit.
- **Improved Mortality Rates**: Advances in healthcare, lifestyle changes, and public health initiatives lower mortality risk over time.

DEFERRED LIFE ANNUITY (age 35)

Year		FEMALE		MALE
1933	€	60,344.6	€	49,372.0
1938	€	63,379.1	€	50,454.9
1945	€	70,766.6	€	53,443.3
1955	€	82,182.8	€	58,391.6
1965	€	86,609.3	€	57,360.5
1975	€	96,117.8	€	63,470.9
1985	€	101,543.3	€	70,805.8
1995	€	105,269.2	€	78,626.6
2005	€	109,981.2	€	88,881.1
2015	€	116,208.2	€	95,905.1



Contract Terms: Pays **€12,000 annually** starting at age 65 (after a 30-year deferral period) if the policyholder survives.

Actuarial Present Value (APV): Reflects the upfront premium required to fund future benefits.

Key Observations

- 1. Gender Disparity:
 - Females consistently require higher APVs than males due to their longer life expectancy.
 - **2015**: Female APV = €116,208.20 vs. Male APV = €95,905.10 (difference: €20,303.10).
 - **1933**: Female APV = €60,344.60 vs. Male APV = €49,372.00 (difference: €10,972.60).
 - **Reason**: Longer female lifespans increase the likelihood of payouts, requiring larger upfront reserves.
- 2. Upward Trend (1933–2015):

- o **APV rose significantly for both sexes** as life expectancy improved:
 - **Females**: APV increased by **92.6%** (from €60,344.60 to €116,208.20).
 - Males: APV increased by 94.3% (from €49,372.00 to €95,905.10).
- **Example**: By 2015, the APV for females was nearly **double** the 1933 value, reflecting decades of mortality improvements.

Why Does APV Increase Over Time?

- **Longer Lifespans**: Rising life expectancy increases the probability of surviving the deferral period and receiving payouts.
- **Higher Payout Likelihood**: Insurers must reserve more funds upfront to cover anticipated longer payment durations.
- **Economic and Health Advances**: Improved healthcare, living standards, and reduced mortality rates over the 20th/21st centuries.

PART B: Whole Life Insurance Contract (Age 30)

Key Contract Terms

- Death Benefit: \$100,000.00
- Interest Rate: 2%
- Premium Payment Term: Level annual premiums for 30 years
- Expense Structure:
 - Year 1: 30% of initial premium + 1% of face amount + \$30.00 per policy
 - Years 2–10: 10% of each premium + 0.5% of face amount + \$10.00 per policy
 - After Year 10: 5% of premium + 0.2% of face amount + \$5.00 per policy
- Settlement Expense: \$100.00
- Cash Value: 80% of reserve
- Life Table: 2015 USA Female Life Table

Initial Results

Metric	Value
Level Annual Gross Premium	\$2,145.99
Level Annual Net Premium	\$1,629.02
Expected Profit (NPVEP)	\$1,742.76
Risk Discount Rate (RDR)	8%
Profit Margin	21.11%
Internal Rate of Return (IRR)	10.58%

Sensitivity Analysis

1. Risk Discount Rate (RDR)

RDR	NPV
5%	\$6,705.30
8%	\$1,742.76
9%	\$928.88
10.58%	\$0.00
11%	(\$65.13)
13%	(\$798.88)
15%	(\$1,331.55)

Key Insight: NPV turns negative when RDR exceeds the IRR (10.58%), indicating losses for the insurer.

2. Market Interest Rate

Interest Rate	NPV
2%	(\$3,305.69)
3%	(\$781.46)
3.31%	\$1.05
4%	\$1,742.76
7%	\$9,315.44

Key Insight: The insurer must earn $\ge 3.31\%$ on investments (vs. the guaranteed 2% to policyholders) to achieve a positive NPV.

3. Policy Benefit Impact

Benefit	NPV	Gross Premium
\$1,000.00	(\$2.81)	\$34.08
\$10,000.00	\$155.87	\$226.07
\$50,000.00	\$861.16	\$1,079.37
\$100,000.00	\$1,742.76	\$2,145.99
\$150,000.00	\$2,624.37	\$3,212.62
\$400,000.00	\$2,239.77	\$8,545.75

Key Insight: Higher benefits increase gross premiums and NPV up to a point (\$150k), after which reserve requirements may offset gains.

4. Life Table Variations

Year	NPV (Female)	NPV (Male)
2015	\$1,742.76	\$1,807.40
2005	\$1,808.14	\$1,881.53
1985	\$1,875.92	\$1,977.78
1975	\$1,886.10	\$1,997.12

Key Insight: Older mortality tables (e.g., 1975) yield higher NPVs due to shorter historical life expectancies. Males consistently show higher NPVs than females, reflecting lower life expectancy.

5. Cash Value Impact

Cash Value (% of Reserve)	NPV
50%	\$1,985.26
70%	\$1,823.59
80%	\$1,742.76

Cash Value (% of Reserve) NPV

100% \$1,581.10

Key Insight: Lower cash value payouts (e.g., 50% of reserve) increase profitability by reducing insurer liabilities.

Summary of Key Drivers

- 1. Profitability Threshold: RDR must stay below IRR (10.58%).
- 2. Investment Returns: ≥3.31% required to offset guaranteed 2% policyholder rate.
- 3. Benefit Scaling: Larger benefits increase premiums but require careful reserve management.
- 4. Mortality Assumptions: Updated life tables reduce NPV due to longer life expectancy.
- 5. Cash Value Policy: Retaining reserves improves profitability.