



Variational Quantum Amplitude Estimation

2023 QHack Open Hackathon

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Quantum Amplitude Estimation

Goal : Calculate the amplitude of “Good State”

- Exploits **Grover searching algorithm** and quantum phase estimation to **reduce the calls to the \hat{A}** compared to simple sampling

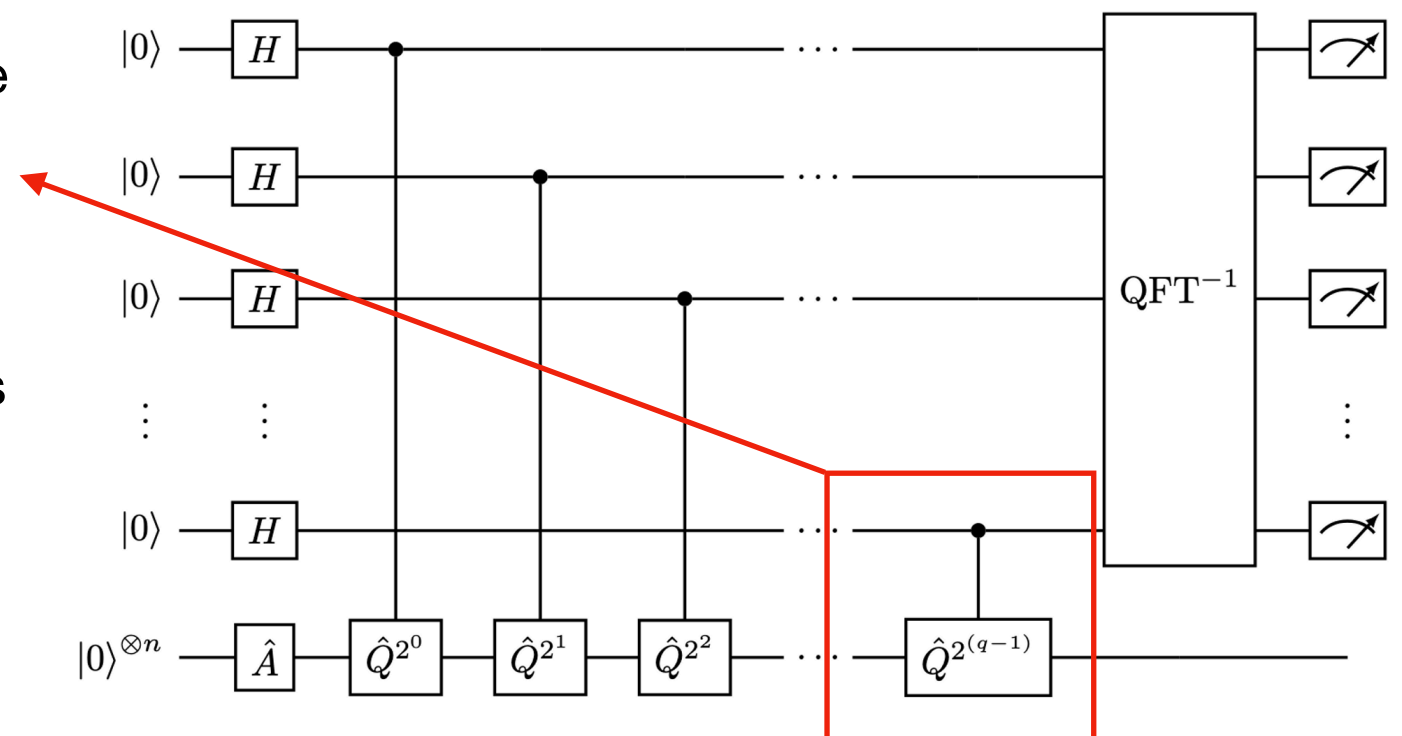
$$\begin{aligned} |\chi_0\rangle_{n+1} &= \hat{A} |0\rangle_{n+1} \\ &= \sqrt{1-a} |\psi_{\text{bad}}\rangle_n |0\rangle + \sqrt{a} |\psi_{\text{good}}\rangle_n |1\rangle \end{aligned}$$

Simple sampling : $\mathcal{O}(1/\epsilon^2)$ \longrightarrow QAE : $\mathcal{O}(1/\epsilon)$

* ϵ : error

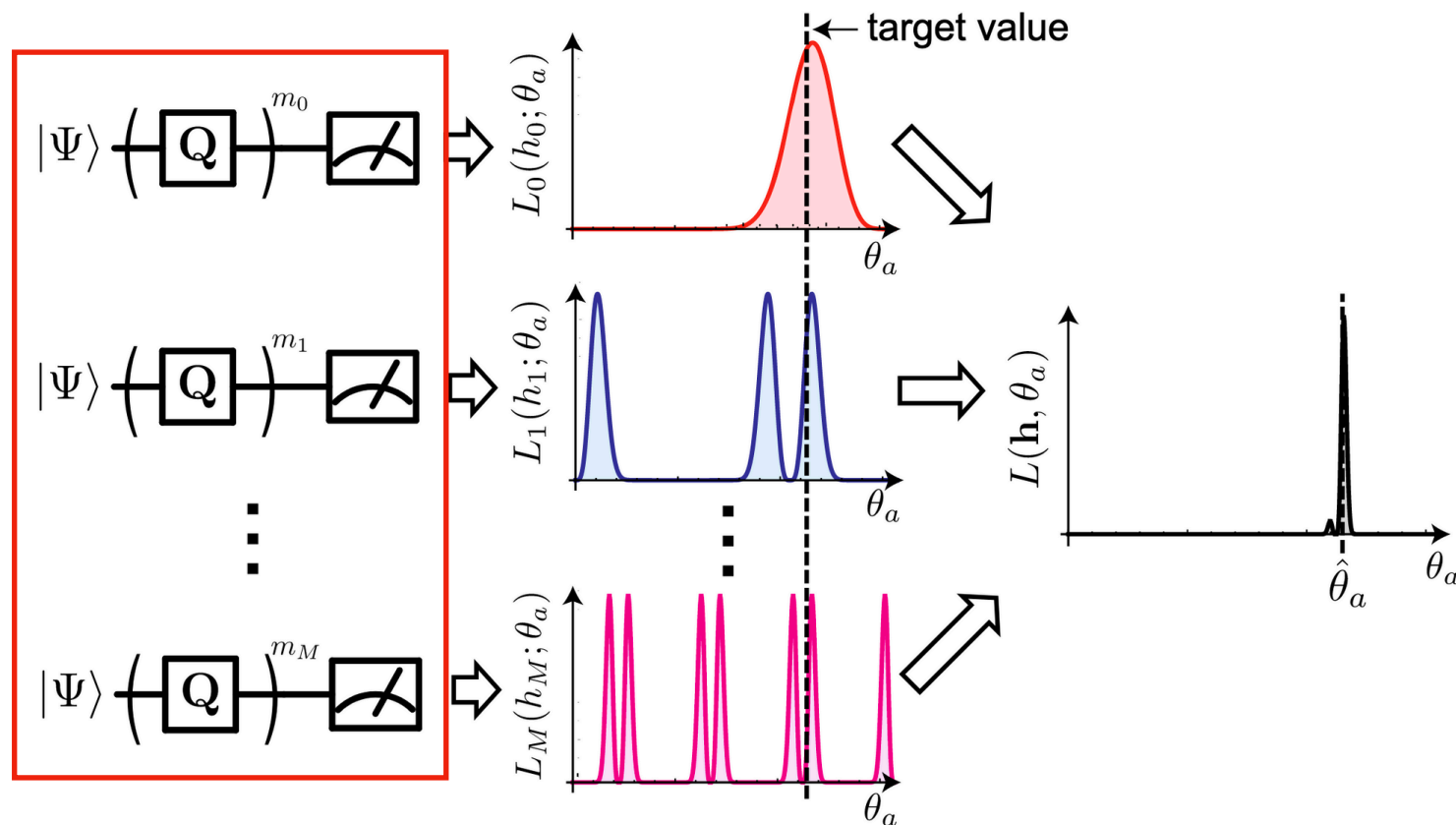
controlled-Q gates used in quantum phase estimation makes depth of the circuit **exponentially deep!**

-> **cannot be implemented on** noise intermediate scale quantum(**NISQ**) devices

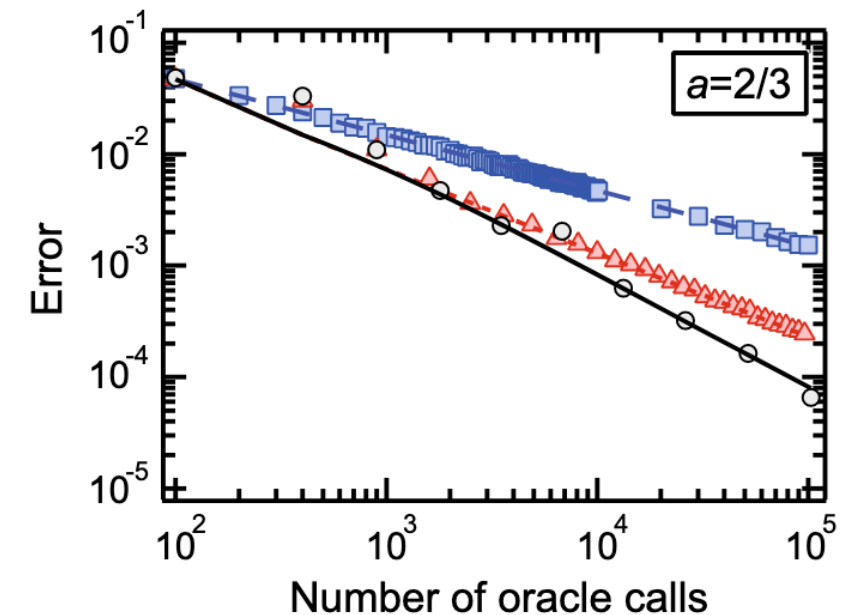


Circuit for conventional quantum amplitude estimation(QAE)[4]

Maximum Likelihood Amplitude Estimation



Maximum Likelihood Amplitude Estimation(MLAE)[2]



blue dots : Classical random sampling
red & black dots : MLAE[2]

- exploits classical **maximum-likelihood(ML) method** to estimate the amplitude
 - **simplified and relatively short circuit**
 - can be implemented on the real device
- Even though there isn't a rigorous mathematical proof, **quadratic speedup is achieved** in many of practical cases.

NVIDIA cuQuantum & QODA

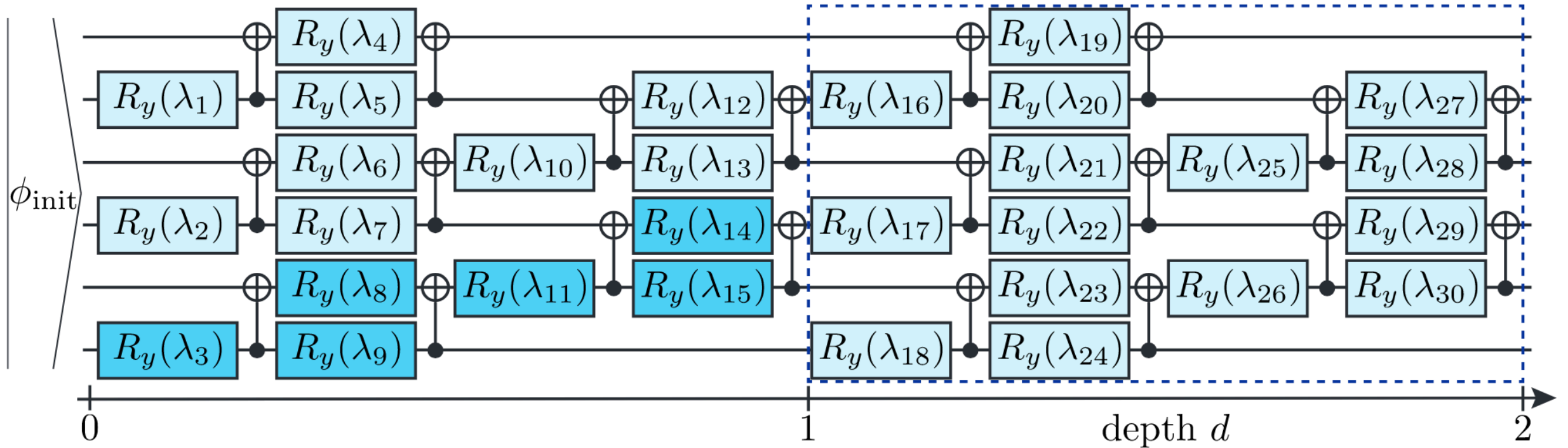
- We implemented GPU compatible quantum circuit simulation. All of our codes exploited cuQuantum and Quantum Optimized Device Architecture (QODA) provided by NVIDIA.
- Against expectation, cuQuantum and QODA showed inferior performance than CPU.

Processing Unit	CPU(PennyLane)	GPU(QODA)
Execution time (shallow circuit with 25 qubits)	21.8s	~3min

Even though we observed the longer simulation time in GPU code, it is expected that GPU can show better performance in the case of deep circuit.

For high level circuit simulation, more circuit gate such as controlled RX, RY, which are not provided by QODA, must be supported in future.

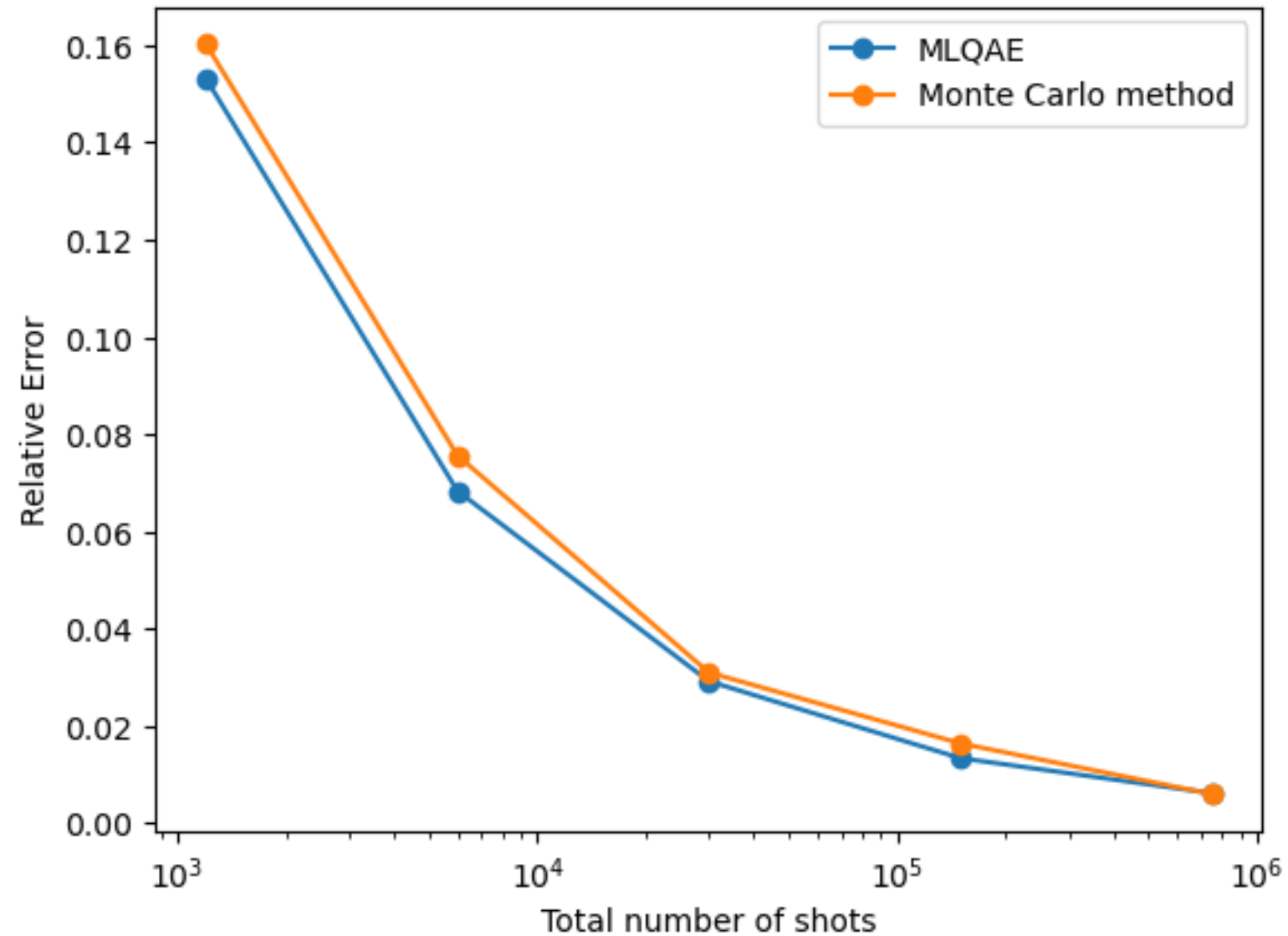
Variational Quantum Amplitude Estimation



Variational quantum circuit ansatz of depth d for 6 qubits [1]

- With MLAE, the depth of $|\chi_m\rangle_{n+1} = \mathcal{Q}^m |\chi_0\rangle_{n+1}$ scales with $O(2m + 1)$ [1]
- The advantage of VQAE over MLAE is that the maximum circuit depth of VQAE is independent of the total number of MLAE steps, whereas in MLAE this depth grows linearly with the number of MLAE steps.

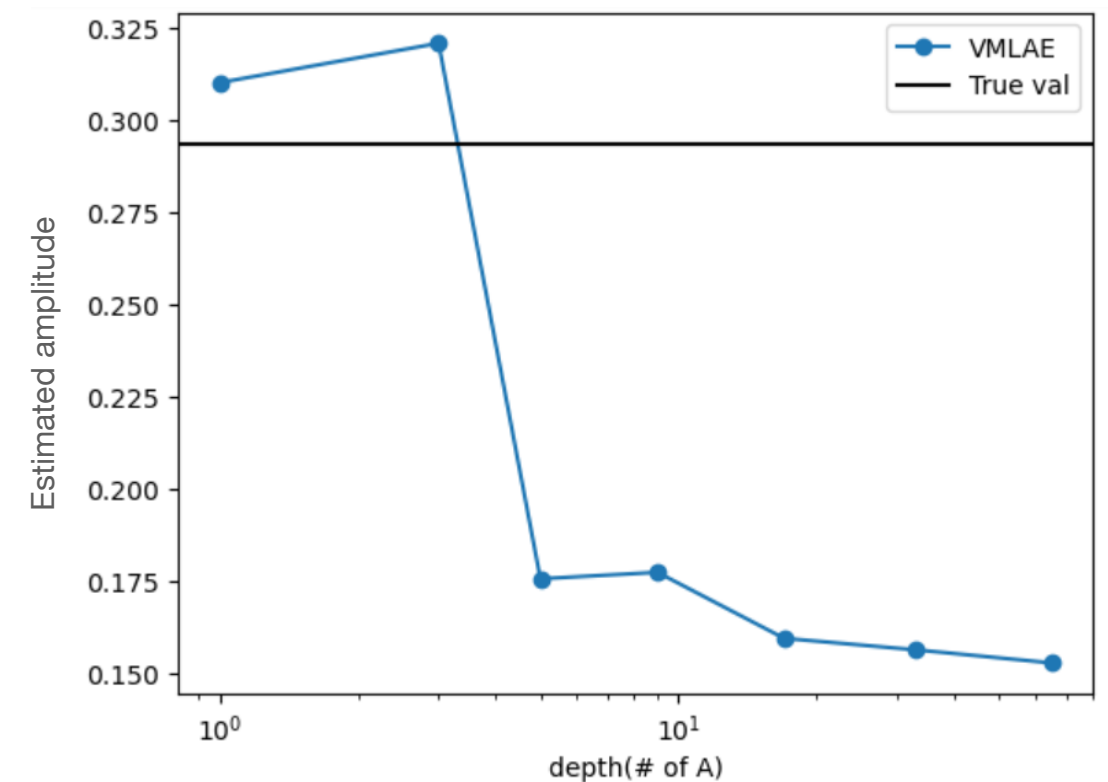
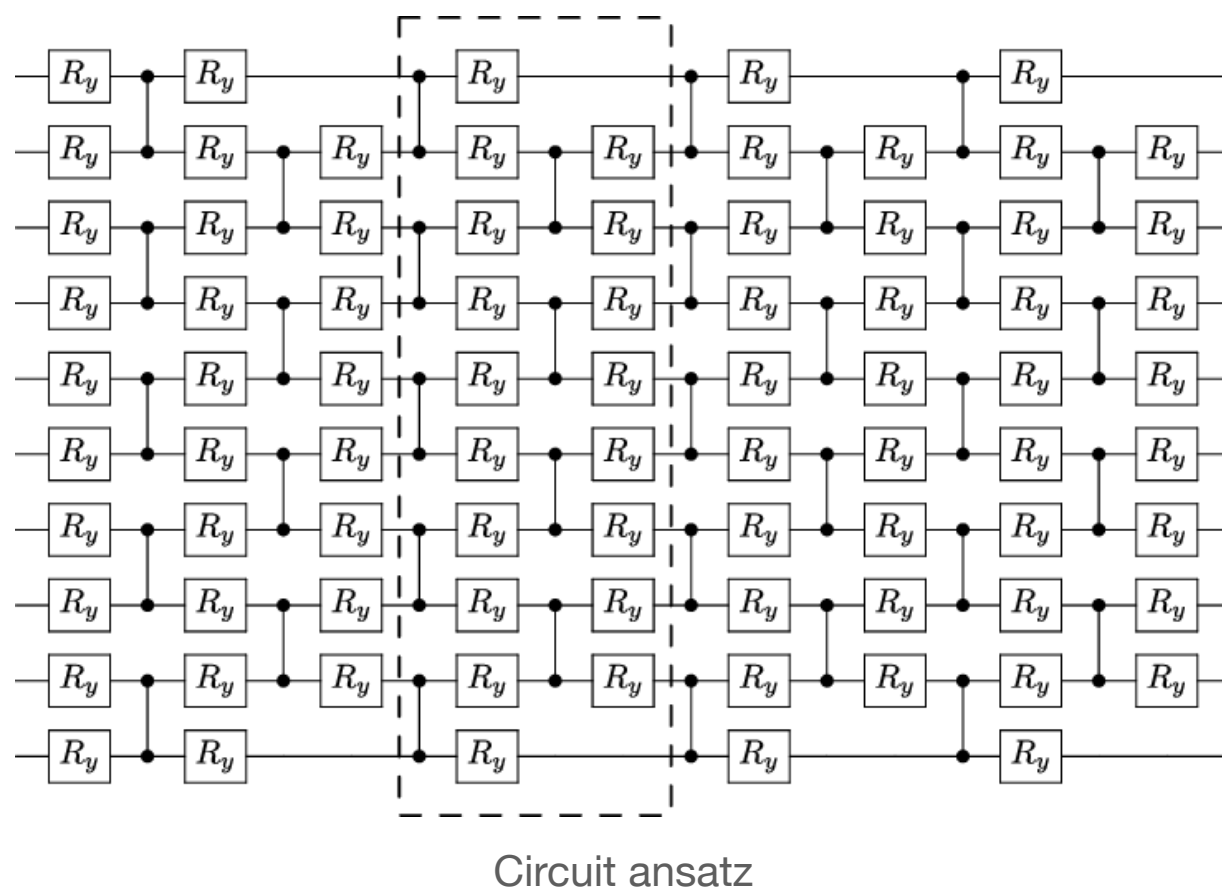
Results



- We observed that MLAE has lower error than Naive Monte Carlo method.

Results

- We used a hardware efficient ansatz different from the ansatz in reference[1]
- Optimization partially successes in the case of the number of $A = 1$ and 3. However, as the number of A become higher, our ansatz did not converge and showed a poor result.



References

- [1] K. Plekhanov, M. Rosenkranz, M. Fiorentini, and M. Lubasch, *Variational Quantum Amplitude Estimation*, Arxiv (2021).
- [2] Y. Suzuki, S. Uno, R. Raymond, T. Tanaka, T. Onodera, and N. Yamamoto, *Amplitude Estimation without Phase Estimation*, Quantum Inf Process 19, 75 (2020).
- [3] G. Brassard, P. Høyer, M. Mosca, and A. Tapp, *Quantum Computation and Information*, Contemp Math 53 (2002).
- [4] A. Callison and D. E. Browne, *Improved Maximum-Likelihood Quantum Amplitude Estimation*, Arxiv (2022).