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Source: *The Review of Financial Studies*, Vol. 26, No. 8 (August 2013), pp. 2048-2094
Published by: Oxford University Press. Sponsor: The Society for Financial Studies.
Stable URL: https://www.jstor.org/stable/23470216
Accessed: 25-04-2019 21:31 UTC

Pricing Credit Default Swaps with Observable Covariates

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Observable covariates are useful for predicting default, but several studies question their value for explaining credit spreads. We introduce a discrete-time no-arbitrage model with observable covariates, which allows for a closed-form solution for the value of credit default swaps (CDS). The default intensity is a quadratic function of the covariates, specified such that it is always positive. The model yields economically plausible results in terms of fit, the economic impact of the covariates, and the prices of risk. Risk premiums are large and account for a smaller percentage of spreads for firms with lower credit quality. Macroeconomic and firm-specific information can explain most of the variation in CDS spreads over time and across firms, even with a parsimonious specification. These findings resolve the existing disconnect in the literature regarding the value of observable covariates for credit risk pricing and default prediction. (*JEL* G12)

Observable covariates are useful for predicting default, but several studies question their value for explaining credit spreads and the prices of credit risky securities.[1] This paper addresses this puzzle. Using data on corporate credit

We would like to thank Jeremy Berkowitz, Sreedhar Bharath, Joost Driessen, Tom George, Robert Jarrow, the editor (Pietro Veronesi), an anonymous referee, and seminar participants at Rice University, Georgia State University, the University of Toronto, York University, the Office of the Comptroller of the Currency, the European School for Management and Technology, and the FDIC Derivatives Conference for helpful comments. Ericsson would like to thank the Social Sciences and Humanities Research Council of Canada (SSHRC) and the Institut de Finance Mathematique de Montreal (IFM2) for financial support. Jacobs would like to thank the Bauer Chair at the University of Houston for financial support. Send correspondence to Stuart M. Turnbull, C.T. Bauer College of Business, 334 Melcher Hall, University of Houston, Houston, TX 77204-6021, USA; telephone: (713) 743-4767; fax: (713) 743-4622. E-mail: sturnbull@uh.edu.

1 Shumway (2001) finds that the excess stock return, stock return volatility, the ratio of net income to total assets, and the ratio of total liabilities to total assets can explain the probability of default. Duffie, Saita, and Wang (2007)

doi:10.1093/rfs/hht015 Advance Access publication April 5, 2013

default swaps (CDS), we show that observable covariates do play a major role in explaining the prices of credit risky instruments.

When pricing corporate bonds and CDSs, there are several approaches to identifying the observable determinants of credit spreads. One approach uses structural models of default, following Merton (1974).[2] In these models, the observable covariates are determined by the underlying theory. For example, in the simplest structural models, suggested by Merton (1974) and Black and Cox (1976), credit spreads are determined by interest rates, firm asset volatility, and firm leverage. However, several authors have come to the conclusion that structural models do a poor job of explaining credit spreads for corporate bonds and CDSs.[3] These findings cast doubt on the value of observable covariates for explaining credit risk.

There is a large body of literature that attempts to explain credit spreads, or credit spread changes, by regressing on observable covariates. Overall, this literature questions the explanatory power of observable covariates for credit spreads; see, for instance, Collin-Dufresne, Goldstein, and Martin (2001). The evidence from linear regressions, together with that from structural models, suggests a disconnect between the literature on default prediction, where observable covariates are highly successful, and the literature explaining credit spreads, where observable covariates are much less useful.

An alternative to the use of structural models for pricing corporate bonds and CDSs is the reduced-form approach, introduced by Jarrow and Turnbull (1992, 1995). Presumably in part because of the shortcomings of models with observable covariates, the reduced-form approach is most often implemented using latent factors.[4] Latent factor models usually provide a good in-sample fit. However, although the estimated latent factors can be compared to observables, they do not provide much intuition with respect to the economy-wide and firm-specific determinants of credit risk.

We follow Lando (1998) and assume that the default intensity in a reduced-form model is a Cox process depending on observable macroeconomic and firm-specific covariates.[5] We introduce a new discrete-time no-arbitrage model with observable covariates, where the dynamics for the stopping time are

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explain the probability of default using distance-to-default, the firm's stock return, the three-month Treasury-bill yield, and the one-year trailing S&P 500 index return. Chava, Stefanescu, and Turnbull (2011) examine the performance of different models using macroeconomic variables, firm-level variables, and sector frailties.

[2] See also Black and Cox (1976), Collin-Dufresne and Goldstein (2001), Cremers, Driessen, and Maenhout (2008), Geske (1977), Kim, Ramaswamy, and Sundaresan (1993), Leland (1994), Leland and Toft (1996), and Longstaff and Schwartz (1995).

[3] See, for example, Eom, Helwege, and Huang (2004) and Huang and Zhou (2008).

[4] For latent-factor reduced-form studies of corporate bonds, see, for example, Duffie and Singleton (1999), Duffee (1999), Driessen (2005), and Feldhutter and Lando (2007). For studies of credit default swaps, see Houweling and Vorst (2005), Chen et al. (2008), and Longstaff, Mithal, and Neis (2005).

[5] Bakshi, Madan, and Zhang (2006) provide a related analysis of observable determinants of corporate bond spreads within a no-arbitrage model.

described by a quadratic function of these covariates, and we derive a recursive closed-form solution for the pricing of CDSs. The advantage of our no-arbitrage model is that estimated spreads are positive by construction without restricting model coefficients, and that estimated spreads are internally consistent across maturities. It also allows us to estimate the prices of risk associated with different covariates.

The model is estimated using daily data for ninety-five firms for the period January 1, 2001 to December 31, 2010. We use 1-, 5-, and 10-year maturity CDS spreads in estimation. We show that observable covariates are adequate at explaining the variation in credit spreads over time and across firms. Our preferred model is a very parsimonious specification, with four covariates: two covariates extracted from the riskless term structure, firm leverage, and firm-level historical volatility. These covariates are suggested by the structural model of Merton (1974), and the estimated signs on the covariates in the resulting specification are therefore easily interpretable from a theoretical perspective. This model provides a good fit and, consistent with economic priors, volatility and leverage increase credit spreads for almost all firms. We estimate the prices of risk associated with the observable covariates, and they are intuitively plausible. We also provide estimates of the physical default probabilities based on CDS data, following Pan and Singleton (2008), and compare these estimates with risk-neutral default probabilities.[6] The estimated risk adjustment is quantitatively important. For five-year default probabilities, the ratio of the risk-neutral to physical probabilities is on average 3.14 across firms and time. When default probabilities increased in the financial crisis, the ratio of risk-neutral to physical default probabilities decreased, which means that the risk premium constitutes a smaller fraction of the spread in those periods. Consistent with this finding, the ratio of risk-neutral to physical default probabilities is smaller for firms with higher spreads. Model-implied physical default probabilities are somewhat higher than long-run unconditional averages from historical data.

We analyze the model errors to determine if other covariates have explanatory power for credit spreads. We find that several covariates are correlated with the errors, but firm option-implied volatility has by far the highest explanatory power, despite the inclusion of historical volatility in the parsimonious benchmark specification. This finding may be due to a component related to risk aversion and risk premiums in the implied volatility measure. It is also consistent with Carr and Wu (2011), who highlight the relationship between implied volatility and default probabilities. Including implied volatility as an additional covariate in the model leads to substantial improvements in fit. We

6 See Jarrow, Lando, and Yu (2005) and Pan and Singleton (2008) for a discussion of the distinction between physical default probabilities obtained from CDS data and physical default probabilities estimated using historical default data.

also investigate richer models with more covariates. Overall, the improvements on the more parsimonious specification are modest.

We conduct three types of out-of-sample exercises. The first is cross-sectional in nature: It values CDS contracts that are not used in estimation. The other two exercises are recursive time-series exercises. One is a hedging exercise, and the other is a rich versus cheap investment exercise. All three exercises confirm that the no-arbitrage model performs well out-of-sample, and substantially outperforms the linear regression approach.

In summary, this paper makes four contributions. First, it describes a new discrete-time, closed-form model with observable covariates for pricing credit risky assets. The intensity function is a quadratic function of the covariates, specified such that it is strictly positive without restrictions on the signs of the coefficients. Second, the no-arbitrage model yields sensible results in terms of fit and the economic impact of covariates on spreads. The methodological advantages of the no-arbitrage model and the consistency across maturities imposed in estimation do not come at the cost of empirical fit or statistical significance. Third, the empirical results provide strong evidence that observable covariates are useful in explaining credit spreads, which is consistent with the evidence regarding default prediction, therefore resolving a disconnect in the existing literature. Fourth, we provide evidence on the magnitude, the time-variation, and the cross-sectional variation of risk premiums in credit markets.

The paper proceeds as follows. In Section 1, we introduce the new discrete-time no-arbitrage model for CDSs. Both the term structure of interest rates and the process for the stopping time are described by quadratic functions of observable covariates. Section 2 presents a case study of the firm Nordstrom, Inc., to provide more intuition for the model's features. The data are described in Section 3. In Section 4, we present empirical results for ninety-five firms using a parsimonious specification with four observable covariates, and compare the estimates to those obtained using linear regression. In Section 5, we consider various robustness tests. Section 6 concludes.

1. Model Description

In this section, we describe the pricing models for the default-free term structure and for CDSs. We work in discrete time and assume that factors are described by compound autoregressive processes. See Gourieroux and Jasiak (2006) for a description of these processes.

1.1 Default-free bonds

The spot interest rate over a given period is assumed to be a quadratic function of the form

$$r_t = \left(\delta_0 + \sum_{k=1}^{n} \delta_k X_{k,t}^r \right)^2 ,$$

where $X^r_{k,t}$ are factors.[7] It is assumed that these can be described by the following $AR(1)$ dynamics under the risk-neutral measure:

$$X^r_t = \mu_r + \rho_r X^r_{t-1} + \Sigma_r e_t, \tag{1}$$

where X^r_t denotes a $(n,1)$ vector, $e_t \sim N(0,I)$, μ_r is a $(n,1)$ vector, and ρ_r and Σ_r are (n,n) matrices. The price of a default-free zero coupon bond is given by

$$B(t,t+h) = E\left[\exp\left(-\sum_{j=0}^{h-1} r_{t+j}\right) | r_t\right].$$

It is shown in Appendix A.1 that this can be written in the form

$$B(t,t+h) = \exp\left(A_h + B'_h X^r_t + X^{r\prime}_t C_h X^r_t\right), \tag{2}$$

where the explicit definitions of the coefficients A_h, B_h, and C_h are derived recursively.

1.2 Credit default swap valuation

We follow the discrete-time modeling of default described in Gourieroux, Monfort, and Polimenis (2006). A stopping time has an intensity process $\lambda(t)$. Given no default up to time t, the probability of no default over the next interval is $exp(-\lambda(t))$. A default time for an obligor generates a default process $N(t)$ that is zero before default and one after default. The probability for an obligor surviving until at least time h is given by

$$P_t[\tau > t+h] = E_t\left[\exp\left(-\sum_{j=0}^{h-1} \lambda_{t+j}\right)\right], \tag{3}$$

where τ denotes the time of default.

Default can arise from events that affect a particular sector or the whole economy or are unique to the obligor. For example, in the credit crisis, the fall in house prices was one of the major drivers of homeowner default. The fall in house prices occurred across many different states, eventually triggering default by mortgage originators and financial institutions. However, a particular institution's leverage and portfolio composition affected its chances of survival.[8] We assume that default for an obligor depends on a set of measurable covariates (see Lando 1994, 1998). The default intensity is assumed to depend on the same factors that affect the default-free term structure $X^r_{k,t}$

[7] See, for example, Longstaff (1989), Ahn, Dittmar, and Gallant (2002), Constantinides (1992), Brandt and Chapman (2002), Ang et al. (2011), and Leippold and Wu (2002) for quadratic term structure models. See Gourieroux and Monfort (2008) for an application of a discrete-time quadratic factor model to mortality intensity modeling.

[8] See Crouhy, Jarrow, and Turnbull (2008) for a description of the many different factors that contributed to the crisis.

and on other macro- and obligor-specific factors denoted by $X^d_{k,t}$ and is also assumed to be a quadratic function of these covariates

$$\lambda_t = \left(\alpha_0 + \sum_{k=1}^{n} \alpha^r_k X^r_{k,t} + \sum_{k=1}^{m} \alpha^d_k X^d_{k,t}\right)^2, \tag{4}$$

where n is the number of term structure factors and m the number of additional covariates. The advantage of a quadratic specification is that the intensity function is strictly positive. This is not the case for a linear specification if the state variables are assumed to be Gaussian.[9] If the state variables follow CIR processes, parameter restrictions are needed to ensure the positivity. Let

$$X_t \equiv \begin{bmatrix} X^r_t \\ X^d_t \end{bmatrix}$$

denote a $(q,1)$ vector, where $q = n + m$. It is shown in Appendix A.2 that

$$r_t + \lambda_t = \gamma_0 + \gamma_1' X_t + X'_{t+j} \Omega X_t, \tag{5}$$

where Ω is defined in the Appendix. We assume that the covariates X_t exhibit the following dynamics under the risk-neutral measure,

$$X_t = \mu + \rho X_{t-1} + \Sigma e_t, \tag{6}$$

where $e_t \sim N(0, I)$, μ is a $(q,1)$ vector, and ρ and Σ are (q,q) matrices that we specify to be diagonal. This assumption implies that X_t can be negative. Some of the covariates used in our empirical implementation are positive, and in these cases we specify X_t to be the natural logarithm of the covariate rather than its level.

For a CDS, we first consider the payments by the protection buyer. When entering into a contract, the protection buyer may possibly make an initial payment and a series of quarterly payments. In our sample the initial payment is zero, and we therefore ignore it in the pricing.[10] Let S denote the CDS spread. The protection buyer promises to make payments $S\Delta$ each quarter, conditional on no default by the reference obligor, where Δ is the time between payment dates. If a credit event occurs, the protection buyer receives a payment from the protection seller and the contract terminates. The present value of the payments by the protection buyer is

$$PB_t = E_t\left[S\Delta \sum_{j=1}^{h} 1_{(\tau > t+j)} A(t+j) \right], \tag{7}$$

9 See Bekaert, Cho, and Moreno (2006) and Ang and Piazzesi (2003) for applications of discrete-time affine Gaussian frameworks with observables.

10 Recently, changes in the CDS market make the upfront fee the pricing parameter. However, our data source provides us with the spreads.

where 1 denotes the indicator function and $A(t+j)$ is the riskless discount rate $\exp(-\sum_{j=0}^{h-1} r_{t+j})$. In Appendix A.2, we show that

$$E_t[1_{(\tau>t+j)}A(t+j)]=\exp(F_j+G_j'X_t+X_t'H_jX_t), \tag{8}$$

where the coefficients F_j, G_j, and H_j are derived recursively.

The protection seller will make a payment of $(1-R)$, where R is the recovery rate, if a default event occurs. We assume that if a default event occurs during the interval $(t+j-1, t+j)$, payment by the protection seller is made at the end of the interval. The present value of the promised payment by the protection seller is

$$PS_t=E_t\left[(1-R)\sum_{j=1}^{h} 1_{(t+j-1<\tau\leq t+j)}A(t+j)\right].$$

We assume that the recovery rate is known. We can relax this assumption, though identification becomes complex (e.g., see Pan and Singleton 2008). We can write the above expression in the form

$$PS_t=(1-R)\left(E_t\left[\sum_{j=1}^{h}(1_{(\tau>t+j-1)}A(t+j)\right]-E_t\left[\sum_{j=1}^{h}1_{(\tau>t+j)}A(t+j)\right]\right). \tag{9}$$

The second term of the right side of the above expression is given by expression (8). To evaluate the first term, consider

$$E_t\left[1_{(\tau>t+j-1)}A(t+j)\right]. \tag{10}$$

It is shown in Appendix A.2 that this also can be written in the form

$$E_t\left[1_{(\tau>t+j-1)}A(t+j)\right]=\exp\left(M_j+N_j'X_t+X_t'P_jX_t\right), \tag{11}$$

where the coefficients M_j, N_j, and P_j are derived recursively.

The spread of the CDS is set such that

$$PB_t=PS_t. \tag{12}$$

In what follows the price of default protection refers to the spread S.

1.3 The market prices of risk

So far, we have discussed the pricing model under the risk-neutral measure Q. Now we specify the market prices of risk. We apply this specification of the prices of risk to both the default-free term structure model as well as the credit default swap model. To change from the risk-neutral measure to the physical measure, we specify the Radon–Nikodym derivative to take the form

$$\frac{\Delta P}{\Delta Q}=\frac{\exp\left(-\Lambda_t'e_{t+1}\right)}{E_t[\exp(-\Lambda_t'e_{t+1})]}, \tag{13}$$

where Λ_t is an $N\times 1$ vector, with N the number of factors that are priced.

Given this assumption on the Radon-Nikodym derivative and the risk-neutral dynamic of the state vector (6), the dynamics of the state variables X_t under the physical measure can be written as

$$X_{t+1} = \mu + \rho X_t + \Sigma e_{t+1} - \Sigma \Lambda_t. \tag{14}$$

We follow the term structure literature and assume time-varying prices of risk that evolve with the state variables:[11]

$$\Lambda_t = \lambda_0 + \lambda_1 X_t, \tag{15}$$

where λ_0 is an $N \times 1$ vector and λ_1 is an $N \times N$ matrix. The dynamics of the state variables under the physical measure can therefore be written as

$$X_{t+1} = \mu^P + \rho^P X_t + \Sigma e_{t+1}, \tag{16}$$

where μ^P and ρ^P are given by

$$\mu^P = \mu - \Sigma \lambda_0 \tag{17}$$

$$\rho^P = \rho - \Sigma \lambda_1.$$

2. A Case Study: Nordstrom, Inc.

To illustrate the main findings and implications of our study, we begin with a case study of a single firm: Nordstrom, Inc., a company whose fortunes have varied significantly over the sample period. Table 1, Panel A, reports descriptive statistics. The average CDS spread over the 2001 to 2010 period is 66.9 basis points for one-year protection, 94.6 basis points for five-year protection, and 104.8 basis points for ten-year protection. Panels A and B of Figure 1 graph the market CDS spread for the five-year tenor together with model spreads. The other panels of Figure 1 depict the firm's leverage, historical volatility, stock price, and option-implied volatility. In the earlier part of our sample, the price of five-year default protection was around 150 basis points, and Standard & Poors assigned the company an A credit rating on their long-term debt. At the time, leverage and volatility were relatively high. Spreads started decreasing during 2003 and stayed low until the onset of the financial crisis in 2007. Spreads dramatically increased in the financial crisis, reaching almost 700 basis points. Subsequently, they decreased significantly but stayed somewhat higher than their precrisis levels.

Importantly, a visual inspection of Figure 1 suggests strong univariate relationships between the covariates and the CDS spread. Although it is of course important to confirm these impressions using other approaches that are both more formal and multivariate in nature, all suggested relationships

11 See, for example, Ang and Piazzesi (2003), Ang et al. (2011), and Dai, Le, and Singleton (2010).

Table 1
Nordstrom, Inc.: Summary statistics

Panel A: Descriptive statistics

	Mean	SD
1 yr. (bps)	66.9	122.5
3 yr. (bps)	79.6	119.0
5 yr. (bps)	94.6	111.3
7 yr. (bps)	96.5	104.1
10 yr. (bps)	104.8	98.6
Leverage	0.37	0.12
HVol	43.1%	16.4%
IV	44.4%	18.2%

Panel B: R^2s (%) and RMSEs

	Linear regression		No-arbitrage
Maturity	R^2 (%)	RMSE (bps)	RMSE (bps)
1 yr.	69.9	67.2	33.6
3 yr.	72.5	62.4	37.6
5 yr.	73.7	57.0	41.8
7 yr.	72.3	54.8	44.2
10 yr.	70.3	53.7	46.0

Panel A reports averages and standard deviations for the market spreads for Nordstrom, Inc. for 1-, 3-, 5-, 7-, and 10-year maturity CDS spreads, the leverage ratio (D/(D+E)), annualized historical volatility (HVol), and 30-day option-implied volatility (IV). Panel B reports the R^2 from the linear regression and the RMSE in basis points from the linear regression and no-arbitrage models. The data are for the period October 1, 2001 to December 31, 2010.

are consistent with available theory. Historical volatility, option-implied volatility, and leverage are positively associated with spreads. Stock prices are negatively associated with spreads, which can be interpreted directly as an implication of structural models, or alternatively can be explained by the robust negative correlation between stock returns and firm volatility. Because of space constraints, it is of course not possible to include figures for all firms used in the empirical analysis, but similarly strong associations between spreads and candidate covariates are apparent from visual inspection of the data for almost all firms.

Panels A and B of Figure 1 illustrate the performance of a parsimonious linear regression and the no-arbitrage model, respectively. In both cases, we limit the covariates to two term structure factors, leverage, and historical volatility. The data are discussed in more detail in Section 3.1. We estimate the no-arbitrage model using three tenors: one year, five years, and ten years. To save space, Figure 1 only reports on the five-year tenor. Table 1, Panel B, provides measures of fit for all tenors. The R^2 of the linear regression is high for all three tenors, around 71% on average. For the no-arbitrage model, we use three tenors jointly in estimation to impose consistency in pricing, and the fitting exercise is therefore more demanding than the regression approach. Despite this, the root-mean-squared errors (RMSEs) for the no-arbitrage model are substantially lower compared with the regression.

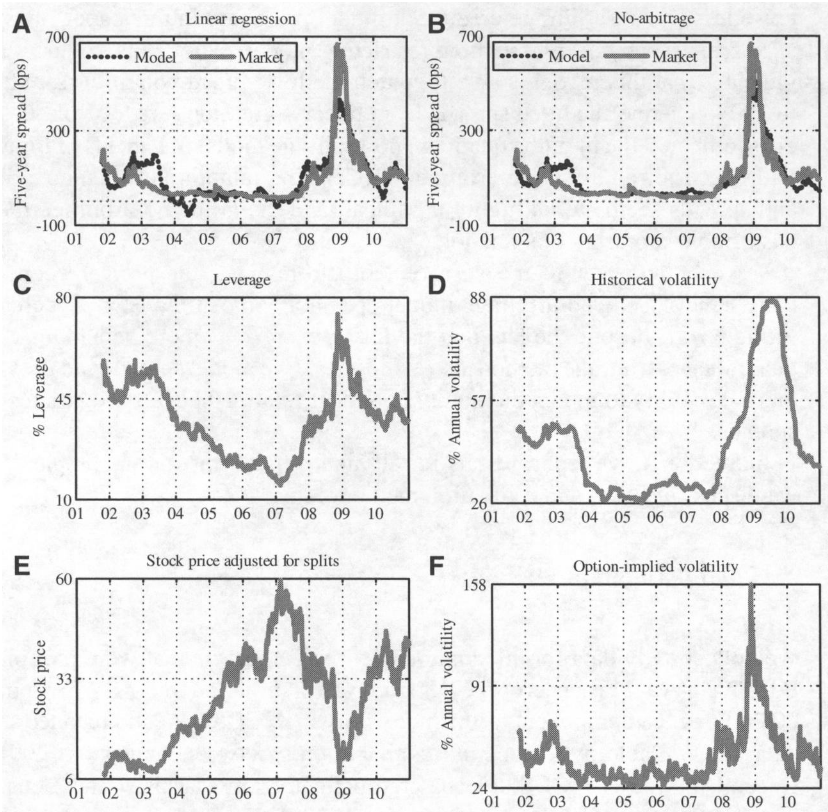

Figure 1
Nordstrom, Inc.: Model spreads, market spreads, and firm-specific covariates
Panels A and B show the time series of linear regression and no-arbitrage benchmark model spreads together with the market spread for the contract with five-year maturity. Panels C and D show the time series of the leverage and the annualized historical volatility. Panels E and F show the time series of the stock price adjusted for stock splits and the 30-day option-implied volatility.

Panels A and B of Figure 1 indicate that both the no-arbitrage and regression models perform well in pricing the CDS. The good fit of the no-arbitrage model obtains in spite of the discipline imposed by the no-arbitrage approach, which imposes consistency in pricing across maturities and avoids negative spreads. Note that during the period of relatively low market spreads, between 2004 to 2007, we observe several episodes of negative predicted spreads for the linear regressions. The most notable excursion into negative territory occurs in mid-2004, when the five-year spread approaches -60 basis points. The pattern is even more dramatic for the one- and three-year model spreads, which approach -200 basis points.

Negative spreads, in particular of such magnitudes, constitute arbitrage opportunities, and render the model useless for practical purposes during such episodes. The no-arbitrage credit risk model presented in this paper rules out such scenarios by design, without restricting model coefficients. Figure 1 also indicates that this model is able to match the low spread volatility from 2004 to 2008, whereas the fitted spreads from linear regressions are too volatile. We also compared the fit of the quadratic no-arbitrage model in Panel B of Figure 1 with the fit of an affine no-arbitrage model. The relation between covariates and spreads seems to be highly nonlinear, and the affine no-arbitrage model has trouble capturing this relation.

We conclude that in the case of Nordstrom, Inc., the greater economic consistency of the no-arbitrage model does not come at the cost of increased fitting errors, despite the fact that the fitting exercise is more demanding. The quadratic no-arbitrage method also avoids negative model spreads and provides more flexibility to model the highly nonlinear relation between covariates and spreads.

In Section 4, we report results for all ninety-five firms in our sample. First, we discuss our data and estimation methodology.

3. Data and Estimation Method

3.1 Data

We collect daily data for all single-name CDS contracts that were part of the DJ.CDX.NA.IG.1 (henceforth CDX) index, which started trading in September 2003. For each component of the index for which CDS data and balance sheet data are available, we collect all available data between January 1, 2001 to December 31, 2010. CDS spreads are obtained from Markit, which does not have CDS data available prior to January 1, 2001. From Markit we also obtain the number of contributors for the five-year maturity spread, which we use as a measure of CDS liquidity.[12] Balance sheet and other firm-specific data are obtained from the Center for Research in Security Prices (CRSP) and Compustat.

Of the 125 firms in the index, we were able to match the required CDS and balance sheet data for ninety-five separate firms. Panel A of Table A1 provides summary statistics for these ninety-five firms and also indicates data availability for each firm. Five of these firms experience credit events during our sample period: CIT Group, Delphi Corporation, Federal Home Loan Mortgage Corporation, Federal National Mortgage Association, and Washington Mutual. We include in the data all daily spreads available in Markit for these five firms and estimate the model on the entire sample, just as for the other firms.

We obtain CDS spreads for 1-, 3-, 5-, 7-, and 10-year maturities for all firms. The 1-, 5-, and 10-year maturities are used in estimation, whereas the 3- and 7-year maturities are used for an out-of-sample exercise. Panel A of Figure 2

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12 See Bongaerts, de Jong, and Driessen (2011) and Tang and Yan (2007) on liquidity in CDS markets.

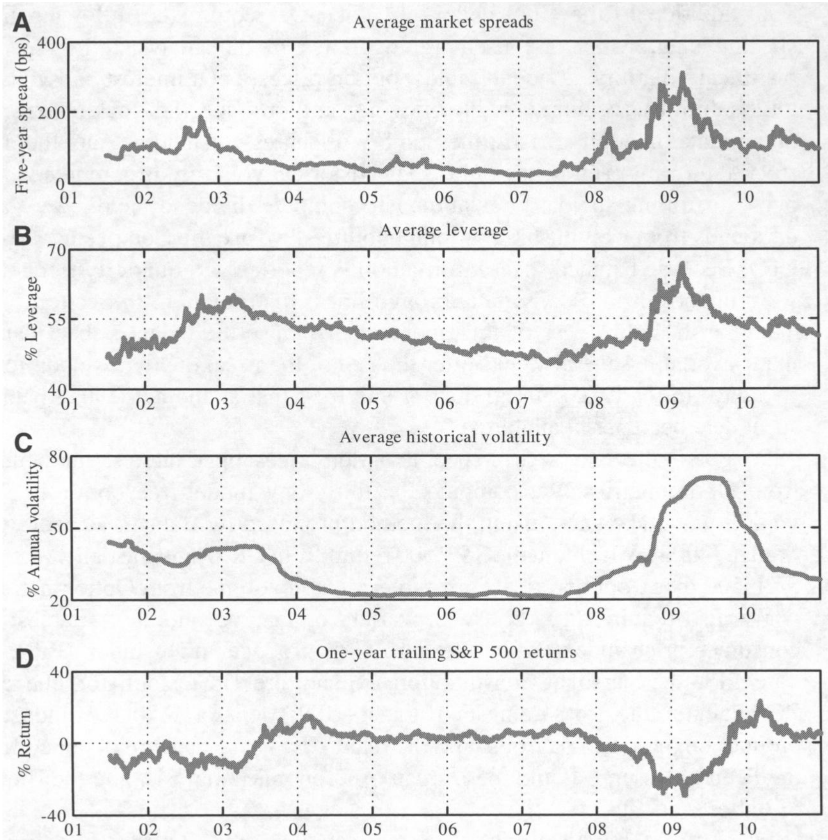

Figure 2
Average market spreads, average leverage, average historical volatility, and S&P 500 returns
We show the time series of the average market spread for the contract with five-year maturity, the time series of average leverage defined as the total liabilities scaled by the sum of total liabilities and market equity, the average historical volatility, and the one-year trailing S&P 500 return. Our sample period is from January 2001 to December 2010. We require at least five firms to compute the average, and therefore, the averages in Panels A–C start in June, 2001. Before June 2001, we do not have enough firms in our sample to compute the averages. For comparison, the graph for S&P 500 returns also starts in June 2001.

presents the average spread across all ninety-five firms for the five-year tenor. The variation over the sample for the other tenors is similar, but we do not include these figures to save space. The remaining panels show the average leverage, average historical volatility, and the S&P 500 return.

To estimate the risk-free term structure model, we use daily Libor rates with six-month maturity and interest swap rates with maturities of 1, 2, 3, 4, 5, 7, and 10 years. The Libor and interest swap rates are obtained from Bloomberg.[13]

[13] See Jagannathan, Kaplin, and Sun (2003), Li and Zhao (2006), and Duffie and Singleton (1997) for estimation of the risk-free term structure using Libor-swap rates.

In addition to the CDS and Libor data, we require the following firm-specific data, which are used as covariates in the analysis: firm-specific historical volatility, dividend yields, option prices, open interest and volume, total liabilities, the market value of equity, and firm-specific stock returns. We obtain data on equity prices, the number of shares outstanding, and the daily stock return for each firm from CRSP. Historical volatility is computed using one-year rolling standard deviations. To compute dividend yields, we obtain dividends from Compustat. For total liabilities, we use the Compustat variable LTQ. Because balance sheet information is reported at a quarterly frequency, we transform it into daily data through linear interpolation. In Section 5, we examine the robustness of this approach. Because the balance sheet data is made available several weeks after the end of the fiscal quarter, we add forty-five days to the end of fiscal quarter and treat that as the date when balance sheet data becomes available.

We obtain the data on firm-specific option prices, open interest, and volume from Optionmetrics. We compute the thirty-day model-free option-implied volatility and skewness using the method proposed by Bakshi, Kapadia, and Madan (2003). We obtain the S&P 500 return from CRSP and the daily VIX, the S&P 500 index option prices, open interest, and volume from Optionmetrics.

We also require data on a wide variety of macroeconomic variables: the consumer price index (CPI), the core CPI, producer price index (PPI), the core PPI, the personal consumption expenditure (PCE) deflator, the core PCE deflator, the gross domestic product (GDP) deflator, real GDP, industrial production, nonfarm payrolls, and the real PCE. These data are obtained from the Federal Reserve Bank, the Bureau of Economic Analysis, and the Bureau of Labor Statistics.

Panel A of Table A1 provides summary statistics for all ninety-five firms in the dataset.[14] The table includes the averages and standard deviations of spreads for the five-year tenor, as well as for leverage, defined as the ratio of debt over the sum of debt and market equity, and historical and option-implied volatility. The table also includes the firms' average rating over the sample period.[15] Most firms in the sample are rated A or BBB, which is not surprising given the composition of the CDX index. Despite the relative homogeneity of the firms in the sample in terms of ratings, the table indicates substantial differences in the average spread levels, as well as in the descriptive statistics for the observable covariates. The cross-sectional average of the five-year firm-averages of CDS spreads is just over 90 basis points, with a standard deviation of 70 basis points. The median average spread is 70 basis points, indicating positive skewness in the cross-section. Average historical volatility ranges between 18% and 63%, with a median of 33%. Firm option-implied volatility is on average slightly

[14] Panel B of Table A1 is discussed in Section 4.

[15] At each point in time, we assign a numerical code to the firm's rating. Subsequently, we average over time and map the average back to a rating category.

higher. Leverage is 53% on average, with a minimum of 15% and a maximum of 95%.

Financial firms, such as CIT Group, Inc. and American International Group, Inc., have among the largest leverage ratios and volatility and also the largest average spreads. Moreover, firms with average spread higher than 100 basis points have average leverage of 61% and average volatility of 41%, whereas firms with average spreads lower than 100 basis points have average leverage of 51% and average volatility of 31%. These statistics are suggestive of the a priori expected positive relationship between leverage and volatility on the one hand and spreads on the other hand. Our empirical investigation examines these relations in much more detail.

3.2 Estimation method

We now describe our estimation strategy, which proceeds in two steps. The first step is the estimation of the stochastic term structure factors, which we use as covariates. The dynamics of the risk-free term structure factors are estimated only once and assumed to be the same across all firms. These factors are latent state variables that need to be filtered from the data. In the second step, we estimate both the physical and risk-neutral dynamics of the covariates for each of the ninety-five firms, proceeding one firm at a time. This allows us to study the market prices of risk associated with the various covariates used to price the CDS.

The Kalman filter offers a convenient framework for the estimation of the stochastic term structure factors in the first step. For our application, the transition function is Gaussian, but the measurement function is highly nonlinear. In most term structure and credit risk applications, the nonlinearity in the measurement equation is addressed by the use of the extended Kalman filter, which approximates the nonlinearity using a Taylor expansion.[16] We instead use the unscented Kalman filter together with quasi-maximum likelihood, which directly allows for nonlinearities. The nonlinear state-space system in our case is specified as follows,

$$X_t^r = \mu_r^P + \rho_r^P X_{t-1}^r + \Sigma_r e_t \tag{18}$$

$$Y_t = Z(X_t) + u_t, \tag{19}$$

where Y_t is a D-dimensional vector of observables, $e_t \sim N(0, I)$ is the state noise, and $u_t \sim N(0, R)$ is the observation noise. R is a diagonal matrix. Note that the transition equation is linear, whereas the measurement equation contains a nonlinear transformation Z for our application. The log-likelihood

[16] See Chen and Scott (1995), Duan and Simonato (1999), and Duffee (1999) for applications of the extended Kalman filter to term structure models.

function for the data is obtained as a by-product of the Kalman filter recursions,

$$\log L = \sum_{t=1}^{T} -\frac{N_t}{2}\log(2\pi) - \frac{1}{2}\log|F_t| - \frac{1}{2}\left(Y_t - \overline{Y_t}\right)' F_t^{-1}\left(Y_t - \overline{Y_t}\right),$$

where T is the sample size, N_t indicates the number of available rates, F_t is the conditional covariance matrix obtained from the unscented Kalman filter recursion, and $\overline{Y_t}$ is the conditional expectation of Y_t. We use a particular implementation of the unscented Kalman filter, the square-root unscented Kalman filter proposed by Van der Merwe and Wan (2001), which we found to be numerically stable and computationally tractable.[17]

The second step does not require the estimation of latent state variables, and is therefore more straightforward. We estimate the credit risk model using the estimated default-free term structure factors, which we treat as observed, together with the other observable covariates. We proceed one firm at a time. The observable covariates follow the AR(1) process in (16). Based on the normality assumption for the AR(1) innovation, it is straightforward to write the resulting likelihood function. Because we observe the time series of covariates, we first estimate the dynamics of the covariates under the physical measure. Subsequently, we estimate the dynamics of the covariates under the risk-neutral measure and the loadings on the covariates using the term-structure of credit default swap spreads. The only parameters that are identical under the physical and risk-neutral measures are the standard deviations of the innovations. In estimation, they are fixed to the values estimated from the observed time series of covariates. We assume a constant recovery rate of 40% in estimation.

4. Empirical Results

In this section, we estimate the no-arbitrage model for all ninety-five firms in our sample using a parsimonious specification, with four covariates: two covariates extracted from the riskless term structure, firm leverage, and the firm's historical return volatility.[18] These covariates are suggested by a simple structural model, such as Merton (1974), and the estimated signs on the covariates are therefore easily interpretable from a theoretical perspective. We refer to this parsimonious specification as the benchmark specification. In the implementation, we use the natural logarithm of volatility and the natural logarithm of the firm's debt-to-equity ratio, to ensure that the support is unrestricted, consistent with the AR(1) assumption and the normality assumption on the innovations. We report on other specifications of the covariates in Section 5.4.

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17 See Chen et al. (2008) for an application of the unscented Kalman filter to credit risk models with latent factors. See Carr and Wu (2007) and Bakshi, Carr, and Wu (2008) for applications to equity options.

18 We repeat our analysis using two observable term structure factors instead of the stochastic factors: the level of the term structure, represented by the six-month yield, and the slope, represented by the difference of the ten-year and six-month yields. The results are very similar to the case of stochastic term structure factors, and we do not report them to save space.

Table 2
The risk-free term structure

Panel A: Risk-free term structure factor loadings and dynamics

	Factor 1	Factor 2
δ_0	0.043	
δ_1	0.342	
δ_2		0.980
ρ^Q	0.999	0.998
ρ^P	0.997	0.997
$\mu^Q \times 100$	−0.002	−0.004
$\mu^P \times 100$	−0.012	−0.006
$\sigma \times 100$	0.039	0.010

Panel B: Risk-free term structure model RMSE (bps) and measurement error standard deviation (bps)

	6 months	1 year	2 year	3 year	4 year	5 year	7 year	10 year
RMSE	20.63	7.28	8.96	7.81	7.05	7.43	8.64	12.93
ME SD	19.53	7.82	8.90	8.15	7.51	7.70	8.95	12.86

Panel A reports parameter estimates for the risk-free term structure factors. The two latent risk-free term structure factors are estimated using the unscented Kalman filter. The factor dynamics and short rate loadings for the risk-free term structure are estimated using the six-month Libor rate and 1-, 2-, 3-, 4-, 5-, 7-, and 10-year maturity swap rates. Panel B reports root-mean-squared errors (RMSEs) and measurement error standard deviations (ME SD) for the risk-free term structure.

4.1 The risk-free term structure

Table 2 reports on the estimation of the risk-free term structure. Panel A reports on the dynamics of the two latent risk-free term structure factors. The estimated parameters are reasonable and similar to those reported in the extant literature. Both factors are very persistent, and in both cases the risk-neutral persistence exceeds the persistence under the physical measure. Panel B reports the pricing errors as well as the measurement error standard deviation. The pricing errors for the swap curve are relatively small, ranging from about 7 basis points to 21 basis points across different maturities. The model seems to generate larger errors on the short and long end of the swap curve, consistent with the extant literature.

4.2 Firm-by-firm results

We now turn to a discussion of our findings for all ninety-five firms, using the parsimonious covariate specification with four observable covariates. The top panel of Figure 2 depicts the time series of the average CDS spread across firms for the five-year tenor. The average spread is approximately 100 basis points in 2001, and increases to just under 200 basis points in 2002. In the middle of the sample, from 2004 to early 2007, spreads are stable and low. Later in 2007, as the credit crisis develops, spread volatility increases and average spreads reach a peak of over 300 basis points in 2009. To save space, we do not depict the term structure. It is stable over most of the sample, but inverts for many firms in the financial crisis. Fitting the term structure is therefore very challenging.

Figure 2 also reports the time series of the cross-sectional averages of the two covariates, leverage, and historical volatility, as well as the one-year trailing return on the S&P 500, which we use in Section 5. Consistent with existing evidence, returns and volatility are negatively correlated. Our sample begins with a period of high volatility and negative returns. Between 2003 and mid-2008, volatility decreases and the stock market rallies. Starting in 2008, there is a sharp increase in volatility, together with a dramatic drop in the stock market. The cross-sectional average of leverage also changes significantly throughout our sample. Most notably, leverage dramatically increases during the financial crisis, which is due to the reduction in market equity.

Importantly, Figure 2 confirms the observation from Section 2 that there seems to be a strong association between candidate covariates and CDS spreads. The positive correlation between volatility in the third panel and the average spread in the top panel is quite apparent. Similarly, leverage and spreads move together. The bottom panel suggests a negative correlation between the S&P 500 return and the average spread.

Panel A of Table 3 presents the cross-sectional distribution of the parameters for the firm-specific covariate dynamics. Remember that the off-diagonal elements in ρ and Σ in (6) are assumed to be zero. The most important conclusion is that both covariate dynamics are very persistent under both the physical and risk-neutral measures.

Panel B of Table A1 reports estimation results on a firm-by-firm basis for the benchmark no-arbitrage model. To conserve space, we only report on the model's firm-by-firm fit for the five-year tenor. The qualitative conclusions for the other tenors are similar. Table 4 presents average summary statistics for all tenors. The average fit for the no-arbitrage benchmark specification is reported in the first row of Table 4, Panel A, and is referred to as "NA-benchmark". Tables A1 and 4 also report goodness of fit measures for the linear regression

$$S_t = \gamma + \beta_r X_t^r + \beta_d X_t^d + \varepsilon_t, \tag{20}$$

where S_t is the CDS spread at time t; X_t^r is the vector of term-structure factors at time t, which for this specification consists of the two term-structure factors; and X_t^d is the vector of other factors at time t, which for this specification consists of leverage and historical volatility, as in the no-arbitrage benchmark model.

Table A1, Panel B, indicates that for the five-year tenor, the regression (20), referred to as "Reg-benchmark", yields high R^2s on average, at 63.5%, ranging from 15% to 92% for individual firms. The third row of Table 4, Panel B, reports the average R^2 for all tenors, and all these averages indicate that the fit of the regressions is adequate. The second row of Table 4, Panel A, indicates that the average RMSE for the linear regression (20) is 45.2 basis points for the five-year tenor. The no-arbitrage model outperforms the linear regression on average, yielding an average RMSE of 42.6 basis points, as indicated in the first row of Panel A of Table 4. The no-arbitrage model also outperforms the regression approach for the one-year tenor, but slightly underperforms for the ten-year tenor. Panel A of Figure 3 graphically illustrates the fit of the no-arbitrage benchmark model, averaged across all ninety-five firms in the sample.

Table 3
Parameter distribution and deltas

Panel A: Parameter distribution

	Intensity loadings × 100					Covariates - Q-dynamics				Covariates - P-dynamics				SD × 100	
	Constant	α_{T_1}	α_{T_2}	$\alpha_{\ln(D/E)}$	$\alpha_{\ln(Hvol)}$	$\rho^{Q}_{\ln(D/E)}$	$\rho^{Q}_{\ln(HVol)}$	$\mu^{Q}_{\ln(D/E)} \times 100$	$\mu^{Q}_{\ln(HVol)} \times 100$	$\rho^{P}_{\ln(D/E)}$	$\rho^{P}_{\ln(HVol)}$	$\mu^{P}_{\ln(D/E)} \times 100$	$\mu^{P}_{\ln(HVol)} \times 100$	$\sigma_{\ln(D/E)}$	$\sigma_{\ln(HVol)}$
Mean	1.652	4.378	30.165	0.565	0.580	0.9986	0.9984	0.115	−0.068	0.9968	0.9999	0.013	−0.030	2.474	0.907
2.5%	−2.431	−18.691	−47.793	−0.277	−0.047	0.9894	0.9910	−0.863	−0.647	0.9868	0.9986	−1.284	−0.179	1.357	0.582
25%	0.276	−5.753	−3.464	0.236	0.260	0.9986	0.9986	−0.023	−0.074	0.9957	0.9994	−0.127	−0.091	1.977	0.689
50%	0.955	−0.088	10.295	0.517	0.421	0.9995	0.9992	0.006	−0.026	0.9975	0.9998	−0.013	−0.051	2.286	0.832
75%	2.471	11.173	53.977	0.785	0.831	0.9999	0.9998	0.124	−0.002	0.9986	0.9999	0.147	−0.020	2.759	1.020
97.5%	8.864	45.528	223.880	1.552	2.154	1.0007	1.0009	2.639	0.408	1.0043	1.0033	0.926	0.574	5.093	1.706
Std Dev	2.833	20.287	69.637	0.476	0.537	0.0027	0.0046	0.748	0.306	0.0046	0.0012	0.497	0.152	0.879	0.290

Panel B: Average deltas and percentage of positive deltas

	NA - ln(D/E)			NA - ln(HVol)			Regression - ln(D/E)			Regression - ln(HVol)		
	1 yr.	5 yr.	10 yr.	1 yr.	5 yr.	10 yr.	1 yr.	5 yr.	10 yr.	1 yr.	5 yr.	10 yr.
Sign (%)	93	92	92	91	93	93	97	97	97	89	91	79
Mean (bps)	0.69	0.61	0.50	0.76	0.64	0.51	1.73	1.33	1.17	0.53	0.51	0.33

Panel A reports the distribution of the model parameters. Panel B reports the percentage of positive signs for the deltas and the average deltas across all firms. In Panel B, NA indicates the no-arbitrage model, D/E is the debt-to-equity ratio, and HVol is the annualized historical volatility. The deltas for the no-arbitrage model are computed numerically, whereas the deltas for the regression model are the point estimates from the linear regression model. The deltas are reported for 1-, 5-, and 10-year maturity contracts.

Table 4
Model fit for various specifications

Panel A: Root-Mean-Squared Error (bps)

	In-sample			Out-of-sample	
Specification	1 yr.	5 yr.	10 yr.	3 yr.	7 yr.
NA-benchmark	50.9	42.6	42.0	44.1	41.2
Reg-benchmark	60.5	45.2	40.9		
NA affine-benchmark	60.7	46.8	43.6	50.3	43.9
NA-benchmark + IV	40.5	37.8	38.5	37.7	37.3
NA-extended	38.9	33.3	32.1	34.3	32.5
NA-DTD	56.1	46.1	45.7	48.1	45.6
NA-common component	50.4	43.5	43.0	44.2	42.3
NA-benchmark no interp.	51.8	42.7	42.2	44.1	41.4
Average spreads	70.0	90.9	102.1	79.7	95.1

Panel B: In-sample regression R^2s and RMSEs

			Average RMSEs (bps)		
Reg-benchmark	60.5	45.2	40.9	49.0	42.0
Diff. reg-benchmark	13.5	7.9	8.1	9.1	7.6
			Average R^2s (%)		
Reg-benchmark (%)	64.5	63.5	56.8	65.9	61.1
Diff. reg-benchmark (%)	3.0	7.2	5.2	5.9	6.6

Panel A reports the average RMSEs for the linear regression and no-arbitrage models. The first part of the model name indicates the type of model, whereas the second part of the name indicates the covariate specification. "Reg-benchmark" indicates the benchmark specification linear regression model; "NA-benchmark" indicates the no-arbitrage model; "NA affine-benchmark" is the affine no-arbitrage model with the benchmark covariate specification; NA-DTD uses the two default-free term structure factors and distance-to-default computed using the Merton model as covariates; "NA-benchmark + IV" indicates the benchmark model augmented by option-implied volatility; "NA-extended" indicates the benchmark specification augmented by implied volatility, the lagged one-year S&P 500 return, and the CDS liquidity measure; "NA-common component" indicates the model with three covariates filtered from twenty-three variables; and "NA-benchmark no interp." indicates the benchmark specification without interpolating the balance sheet data. For the no-arbitrage models, we use 1-, 5-, and 10-year maturity contracts for estimation, whereas 3- and 7-year maturity contracts are used for out-of-sample tests. For the regression models, we estimate the model one maturity at a time and report the corresponding RMSEs, that is all RMSEs for the regression models are in-sample. Panel B reports the average RMSEs and R^2s for the regression model and the difference regression model, both for the benchmark covariate specification.

The model performs well throughout the sample, regardless of the level of market spreads. Its main weakness is that the volatility of the spread is too low and, more specifically, that it does not capture some short-lived increases in market spreads.

Our sample period includes the financial crisis, and the effect of the crisis on model fit is of significant interest. Panel A of Figure 3 suggests that model fit worsens during the financial crisis. To investigate this in more detail, we split up the sample in two periods: before the financial crisis, from January 1, 2001 to June 30, 2007, and during and after the financial crisis, from July 1, 2007 to December 31, 2010. For the five-year maturity, the RMSE before the financial crisis is 32.9 basis points, whereas the RMSE during and after the crisis is much higher, at 50.1 basis points. We do not discuss the other tenors to save space, but similar comments apply.

In summary, the no-arbitrage model provides a good fit, and it performs well compared with the regression approach. This is notable, because the

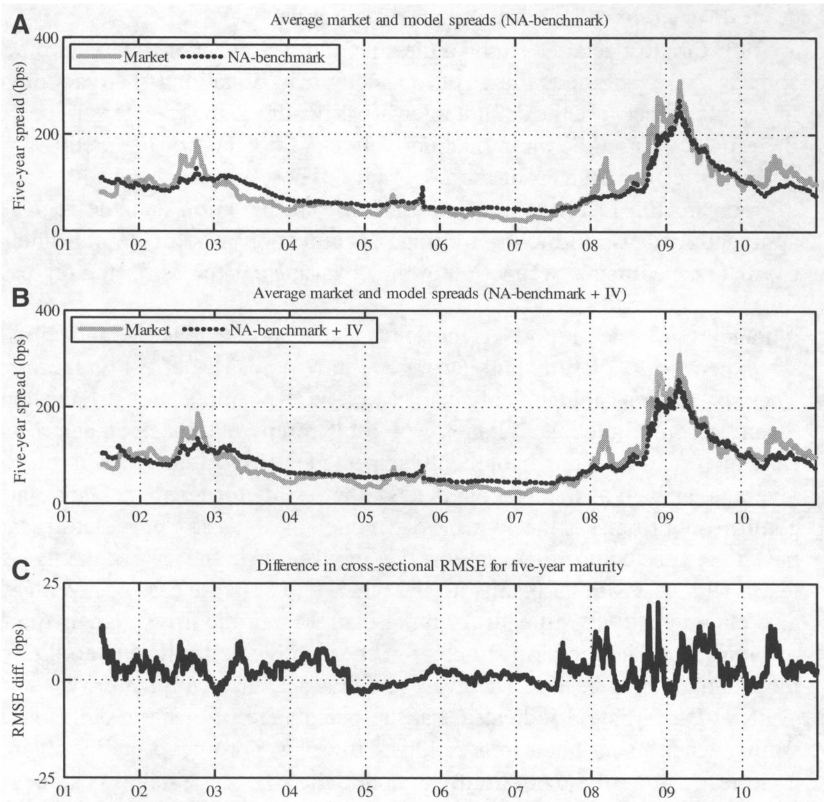

Figure 3
Model fit for various specifications
Panels A and B show the time series of average market spreads together with the time series of the average model spreads. In Panel A, NA-benchmark indicates the no-arbitrage model with the benchmark covariate specification. The benchmark covariate specification includes the default-free term structure factors, the natural log of total liabilities scaled by market equity, and the natural log of historical volatility. In Panel B, NA-benchmark + IV indicates the no-arbitrage model with the benchmark covariate specification augmented by option-implied volatility. Panel C shows the time series of the difference in cross-sectional RMSE between the specifications in Panels A and B.

model's enhanced economic consistency biases it toward a worse fit. Note however that a comparison of model fit is not the main focus of our study; the regressions simply serve as a benchmark to demonstrate that the no-arbitrage model performs adequately in terms of fit. More importantly, the no-arbitrage setup has two important methodological advantages. First, the no-arbitrage approach rules out negative spreads. Second, for the regression approach, there is no obvious way to impose consistency in the pricing across tenors, and regressions are implemented one tenor at a time, which also provides arbitrage opportunities. The most important conclusion is therefore that these methodological advantages of the no-arbitrage model do not cause model fit to worsen.

4.3 Observable covariates and credit spreads

We now turn to a detailed study of the quantitative impact of covariates on CDS spreads. Note that the loadings of the vector α in Equation (4) are not directly interpretable because the default intensity is quadratic in the state variables. We therefore focus on the numerical derivatives or "deltas" of the credit spreads with respect to changes in the covariates. These deltas also make it easier to compare the results of the no-arbitrage specification and the regression approach, because in the no-arbitrage specification it is the default intensity (4) that is specified as a function of the covariates, whereas for the regression (20) it is the credit spread.

Panel B of Table 3 reports on the deltas with respect to leverage and volatility. We expect both volatility and leverage to have a positive effect on the spread. These predictions hold not only in the Merton (1974) model but also in all more recent structural models.[19] Panel B of Table 3 reports the percentage of firms for which we obtain the theoretically expected positive deltas for volatility and leverage, as well as the average deltas. The results for leverage and volatility confirm our priors. For the no-arbitrage model, the deltas for the volatility factor have the expected positive sign for 93% of all firms in the case of the five-year tenor, with very similar results for the other tenors. In the case of leverage, we also obtain positive estimates for more than 90% of the firms. Panels A and B of Figure 4 depict the term structure of the average deltas for the no-arbitrage model and the regression. The average deltas decrease with maturity.

Panel B of Table 3 indicates that the percentage of positive deltas is very similar when using linear regressions. However, not unexpectedly given the nonlinear nature of the no-arbitrage model, the size of the deltas is sometimes very different for the no-arbitrage model and the regressions. For the five-year tenor, if the logarithm of the debt-to-equity ratio increases by 1%, the credit spread increases by 0.61 basis points on average across all firms for the no-arbitrage model. When using linear regressions, the estimated effect is larger on average, at 1.33 basis points. Panel C of Figure 4 graphs the average no-arbitrage leverage deltas against the regression leverage deltas for the cross-section of firms, together with 45° lines. For the majority of firms, the deltas are very similar, but there are a few outliers, most notably CIT Group and Cox Communications. Panel B of Table 3 shows that the average no-arbitrage and regression deltas are more similar in the case of historical volatility. Panel D of Figure 4 indicates that in the case of volatility, the only firm with very different results for both models is CIT Group.

Panel E of Figure 4 depicts the time path of the average deltas across firms implied by the no-arbitrage model, again focusing on the five-year tenor to save

[19] It is less clear what to expect from the term structure factors. Empirically, the link between the level of the risk-free term structure and credit spreads tends to be negative (see e.g. Duffee 1998; Collin-Dufresne, Goldstein, and Martin 2001). This empirical finding is often motivated by referring to the Merton (1974) model, but this is based on a comparative static, where asset value is taken to be exogenous. The evidence for the term structure factors is indeed quite mixed, and we do not report on it to save space.

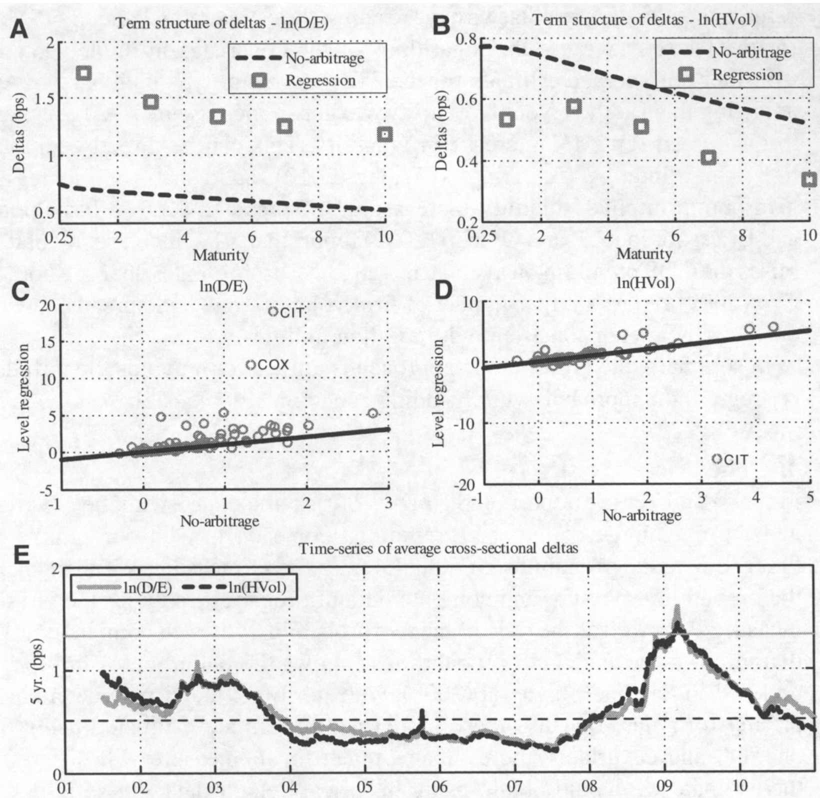

Figure 4
Deltas for benchmark no-arbitrage model and regression
Panels A and B show the term structure of deltas for a 1% change in the log covariates for both the debt-to-equity ratio (D/E) and historical volatility (HVol). Panels C and D show the scatter plot of the deltas for the debt-to-equity ratio and the historical volatility, for the benchmark linear regression and no-arbitrage models. The deltas are based on the contract with five-year maturity. Panel E shows the time series of the cross-sectional average of the deltas for the debt-to-equity ratio and the historical volatility, for the benchmark no-arbitrage model. For comparison, the horizontal lines show the average deltas from the linear regression models.

space. The deltas for the regression approach are provided as a benchmark. The most important conclusion is that the no-arbitrage model allows for substantial time-variation in the deltas. The deltas significantly increase in the crisis period for both covariates. The estimated delta for the regressions is near the maximum estimated by the no-arbitrage model in the case of leverage, whereas it is approximately equal to the average of the no-arbitrage estimates in the case of volatility. This result is due to the outliers indicated in Panels C and D of Figure 4.

The literature does not yet contain a wealth of evidence on the impact of macroeconomic and firm-specific variables on CDS spreads, even using simple regressions, but some results are available for volatility. To compare our estimates to existing results, we have to adjust the estimates in Panel B of Table 3 for the fact that we use the logarithms of the covariates in implementation. Consider our average estimate for the five-year maturity, which is 0.64. After adjusting this for the level of volatility, we get an increase of 1.72 basis points in the spread for a 1% increase in volatility. This can be directly compared to existing findings. Cao, Yu, and Zhong (2010) report that a 1% increase in firm option-implied volatility increases CDS spreads by 2 to 3 basis points, and Ericsson, Jacobs, and Oviedo (2009) report that a 1% increase in volatility raises the CDS premium on average by 0.8 to 1.5 basis points. Zhang, Zhou, and Zhu (2009) report a larger impact of firm volatility on CDS spreads. Overall, our estimates seem consistent with existing findings.

In summary, the no-arbitrage approach is attractive: It avoids the pitfalls of the regression approach, while yielding economically plausible results.

4.4 The prices of risk

Figure 5 and Table 5 report on the prices of risk associated with the covariates and the magnitude of the risk premiums. For each firm in our sample, we first estimate the probability of default under the Q measure. We then change the probability measure and compute default probabilities under the physical measure P, as well as the ratio of the two probabilities at each point in time. Our definition of the P default probabilities follows the definition of the spreads under P in Pan and Singleton (2008), who substitute the P parameters in the pricing formulas. Because these model parameters are estimated using data on CDS and covariates, the estimates under the P measure will differ from the estimate we would obtain using historical default data. These issues are explained in more detail in Jarrow, Lando, and Yu (2005) and Pan and Singleton (2008).

In Table 5, we document the cross-sectional properties of the default probabilities for the one-, the five-, and the ten-year tenor. Every day, we classify firms in four groups according to their spreads, and we then average the physical default probabilities and ratios of Q to P default probabilities in each group. In Panel A of Table 5, we report results for the entire sample. For all three tenors, the higher the spread, the higher the physical default probability, which is not surprising. More interestingly, the higher the spread, the lower the probability ratio. Risk premiums account for a smaller percentage of spreads for firms with lower credit quality.

Panel A of Table 5 indicates that the ratio of Q to P probabilities increases with the tenor. The P probabilities are high compared to unconditional historical default probabilities provided by rating agencies. For example, using S&P historical default rates for 1981–2011 for the rating composition of our sample, the weighted average cumulative five-year default probability for our sample is 1.86%, whereas the overall average from Table 5 is 3.29%. Our high estimates may be due to the difference in sample period, but more likely these results simply confirm that the P probabilities estimated from CDS spreads are

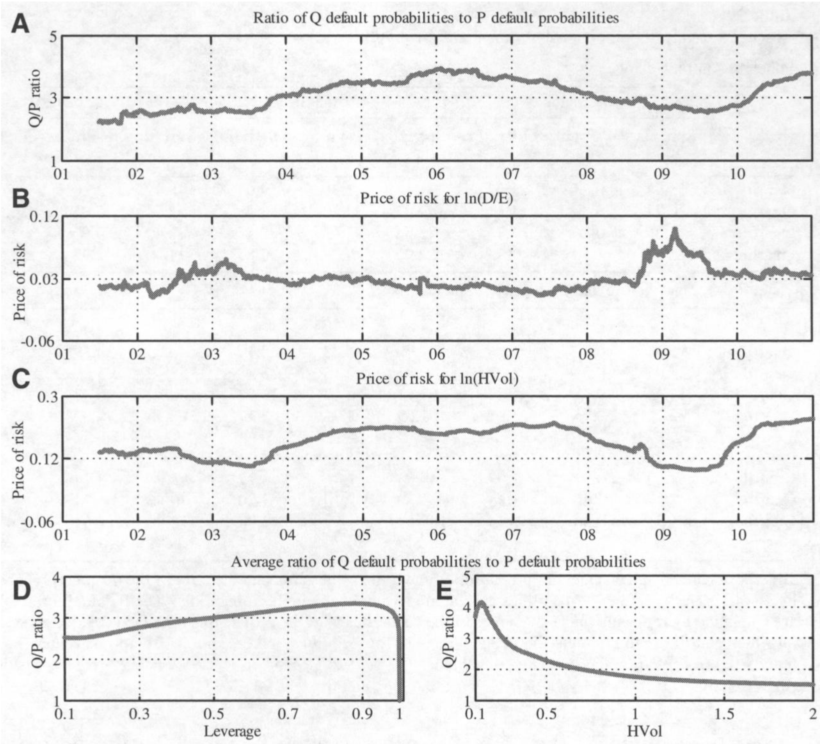

Figure 5
Market prices of risk for covariates and probability ratios. Benchmark model
Panel A shows the average ratio of Q default probabilities to P default probabilities for the five-year contract. The Q default probabilities are the model-implied default probabilities. The P default probabilities are computed by replacing the Q-dynamics of the covariates with the P-dynamics. Panels B and C graph the market price of risk ($\lambda_0+\lambda_1 X_t$) associated with the natural log of debt-to-equity ratio and the natural log of historical volatility for the benchmark specification. Panels D and E show the model-implied Q default probability to P default probability ratios for different values of leverage and annualized historical volatility (HVol). In Panel D, leverage is defined as D/(D+E). We convert each leverage level into the corresponding D/E number and use the natural log of D/E to compute the probabilities using the model estimates.

different from historical default data. This finding is of interest given that the role of ratings agencies has been questioned recently and that it has been argued that CDS data may replace rating agencies as indicators of credit quality.

Panel B of Table 5 reports results separately for the period before the financial crisis, from January 1, 2001 to June 30, 2007, and during and after the financial crisis, from July 1, 2007 to December 31, 2010. The average spreads are very different in both samples, but the same results obtain: The P probabilities are higher for the quantiles with higher spreads, and the probability ratio usually decreases, although the one-year spread is an exception in the precrisis period. Consistent with our priors, P probabilities increased during the crisis period.

Our finding that the probability ratio decreases with credit quality is consistent with the well-known stylized fact that expected loss explains a

Table 5
Average spreads, P default probabilities, and probability ratios

Panel A: Sample averages

	1-year maturity			5-year maturity			10-year maturity		
	Spreads (bps)	P-Prob (%)	Q/P ratio	Spreads (bps)	P-Prob (%)	Q/P ratio	Spreads (bps)	P-Prob (%)	Q/P ratio
Quartile 1	15.94	0.24	1.93	33.12	1.39	4.01	45.41	3.30	5.43
Quartile 2	26.78	0.38	1.61	47.97	2.10	3.43	61.74	5.07	4.66
Quartile 3	45.65	0.56	1.75	72.56	2.93	3.15	87.40	6.96	3.97
Quartile 4	126.54	1.50	1.73	162.90	6.72	2.85	174.40	12.92	3.36

Panel B: Averages before and during the financial crisis

					Precrisis				
Quartile 1	13.47	0.20	1.86	27.00	1.38	4.10	40.35	3.39	5.55
Quartile 2	23.04	0.35	1.57	40.01	2.13	3.48	55.49	5.27	4.76
Quartile 3	34.43	0.43	1.72	53.61	2.63	3.23	70.52	6.73	4.12
Quartile 4	72.78	0.75	1.92	101.69	4.31	3.17	122.78	9.92	3.75
					During crisis				
Quartile 1	22.30	0.31	1.97	43.53	1.46	3.77	52.25	3.14	4.96
Quartile 2	37.27	0.45	1.55	62.33	2.10	3.30	70.86	4.74	4.46
Quartile 3	75.51	0.86	1.75	107.54	3.59	2.98	112.90	7.61	3.70
Quartile 4	268.60	3.14	1.45	282.49	10.70	2.34	266.82	17.72	2.70

We report average spreads, default probabilities under the P-measure, and the ratio of the Q and P probabilities by quartile. The Q default probabilities are the model-implied default probabilities. The P default probabilities are computed by replacing the Q-dynamics of the covariates with the P-dynamics. On each day, we group firms into four quartiles and compute the average of the P-probabilities and the ratio of the probabilities. Panel A reports returns based on the sample period January 1, 2001 to December 31, 2010. Panel B reports results for two subsamples: from January 1, 2001 to June 30, 2007, and from July 1, 2007 to December 31, 2010.

larger percentage of the spread for lower rated bonds.[20] To the best of our knowledge, there is very limited direct evidence on credit risk premiums and Q to P probability ratios in the literature. Berndt et al. (2008) use corporate CDS data and default probabilities provided by KMV for three industries, and find an average ratio of risk-neutral to physical default intensities of 2.76. They also show that the ratios increase with the tenor, consistent with our findings.[21] Driessen (2005) estimates risk premia using corporate bond data and average historical default frequencies and finds an average ratio of risk-neutral to physical default intensities of 1.89. Pan and Singleton (2008) use a methodology similar to ours to compute ratios for sovereign CDS spreads using models with latent factors. They find ratios as high as 2.4 but as low as 0.8 for the one-year tenor. It is not obvious how to compare these numbers: The sample composition is different, results are tenor-dependent, and default intensities vary significantly over time, as evidenced by Figure 5. Moreover, some studies use historical default data, whereas others do not.

Panel A of Figure 5 presents the ratio of risk-neutral (Q) default probabilities to physical (P) default probabilities, where P default probabilities are again

20 See, for instance, Elton et al. (2001) and Huang and Huang (2012).

21 As an example, for Disney, they find an average intensity ratio of just over two, whereas one- and five-year ratios are approximately four and eight respectively.

computed using CDS data. We first compute the probability ratio for each firm in our sample and then compute the cross-sectional average ratio at each point in time. To save space, we focus on five-year default probabilities. The ratio is larger than one throughout the sample, indicating that the risk premium associated with the covariates in our model is economically significant. On average over the sample, the ratio is equal to 3.14. It varies between a minimum of 2.42 and a maximum of 3.95. Interestingly, the ratio decreases somewhat during the financial crisis. The Q probabilities go up significantly in the financial crisis, but P probabilities go up even more. The finding that the default probability ratio decreases when the spreads increase in the financial crisis is intuitively consistent with the finding in Table 5 that the ratio decreases for firms with lower credit quality.

Panels B and C of Figure 5 present the dynamics of the prices of risk $\lambda_0+\lambda_1 X_t$ associated with leverage and historical volatility.[22] Increases in leverage and volatility are negative shocks and can therefore be expected to increase marginal utility. The prices of risk associated with both covariates are indeed positive throughout the sample, and therefore, the price of a dollar increases when there is an increase in either of the two covariates. Interestingly, the price of risk associated with leverage increases in the financial crisis, but the price of risk associated with volatility decreases. We return to this finding in Section 5.4 when we consider alternative model specifications.

Panels D and E of Figure 5 indicate how the ratio of risk-neutral to physical default probabilities changes with the level of the covariates. These ratios are again computed firm-by-firm and subsequently averaged across firms. For leverage, the ratio increases until leverage reaches 88%, and subsequently decreases. This indicates that for distressed firms (high leverage), the increase in physical default probabilities is relatively larger than the increase in risk-neutral default probabilities. Note that as leverage approaches 100%, both probabilities will tend to one, and therefore the ratio tends to one. For volatility, the ratio increases for very low levels of volatility, and sharply decreases for higher levels.

4.5 Out-of-sample analysis

We conduct three types of out-of-sample analyses. The first exercise is cross-sectional: We take the in-sample parameter estimates, which are obtained using the 1-, 5-, and 10-year spreads, and use these estimates to value the 3- and 7-year contracts, which were not used in estimation. This analysis is conducted at the firm level, but for brevity we only report the resulting average RMSEs in Table 4. Importantly, the no-arbitrage model makes it very straightforward to value contracts that are not used in estimation, whereas such an exercise cannot be conducted for the regression approach without making additional

22 Because we estimate both the physical and risk-neutral dynamics of the covariates, we can use Equation (17) to obtain λ_0 and λ_1.

ad hoc assumptions. Panel A of Table 4 indicates that for the no-arbitrage model, the resulting out-of-sample RMSEs for the 3- and 7-year contracts are 44.1 and 41.2 basis points, respectively, of the same order of magnitude as the in-sample fit. To provide another comparison, Panel B of Table 4 indicates that for the regression approach the in-sample errors for the 3- and 7-year contracts are 49 and 42 basis points, respectively, larger than the out-of-sample errors for the no-arbitrage model.

To further demonstrate the importance of the no-arbitrage restrictions, we perform two time-series-based out-of-sample exercises that are implemented recursively. First, we perform a hedging exercise, where we hedge the exposure in a five-year CDS contract to the two firm-specific factors using the 3- and 7-year contract. The hedging exercise is performed as follows. The change in the value of a CDS contract is defined as

$$\Delta V(t,T) = \Delta S(t,T) \times PV1(t,T), \tag{21}$$

where $V(t,T)$ indicates the value of a CDS contract with maturity T; ΔS indicates the change in spreads; T indicates the maturity of the contract; and $PV1(t,T)$ is the present value of a 1 basis point spread.[23] The initial portfolio value is given by

$$L_P(t) = V(t,5) - [w_1 V(t,3) + w_2 V(t,7)] = 0, \tag{22}$$

where w_1 and w_2 indicate the positions in 3- and 7-year maturity contracts to hedge the exposure to the 5-year maturity contract. Note that when the CDS contracts are initiated, their value is zero. Our benchmark model has two interest rate risk factors and two firm-specific risk factors. The hedging exercise involves determining w_1 and w_2 such that we can hedge the exposure to the two firm-specific factors using 3- and 7-year maturity contracts. Therefore, we use the following two equations to solve for w_1 and w_2:

$$0 = \frac{\Delta L_P(t)}{\Delta X_3(t)} \quad \text{and} \quad 0 = \frac{\Delta L_P(t)}{\Delta X_4(t)}, \tag{23}$$

where $X_3(t)$ indicates the first firm-specific risk factor, leverage, and $X_4(t)$ indicates the second firm-specific risk factor, historical volatility. The derivatives in (23) can be computed using Equations (21) and (22), which in turn require the deltas for the covariates. In the case of the no-arbitrage model, the deltas at time t can be computed using numerical derivatives, whereas in case of the linear regression model the deltas are given by the regression coefficients.

Figure 6 presents the results of this hedging exercise. To keep the analysis manageable, we recursively estimate the model using three years of daily

[23] To facilitate the exposition, in (21), we do not account for the change in the maturity of the existing CDS contract between two periods.

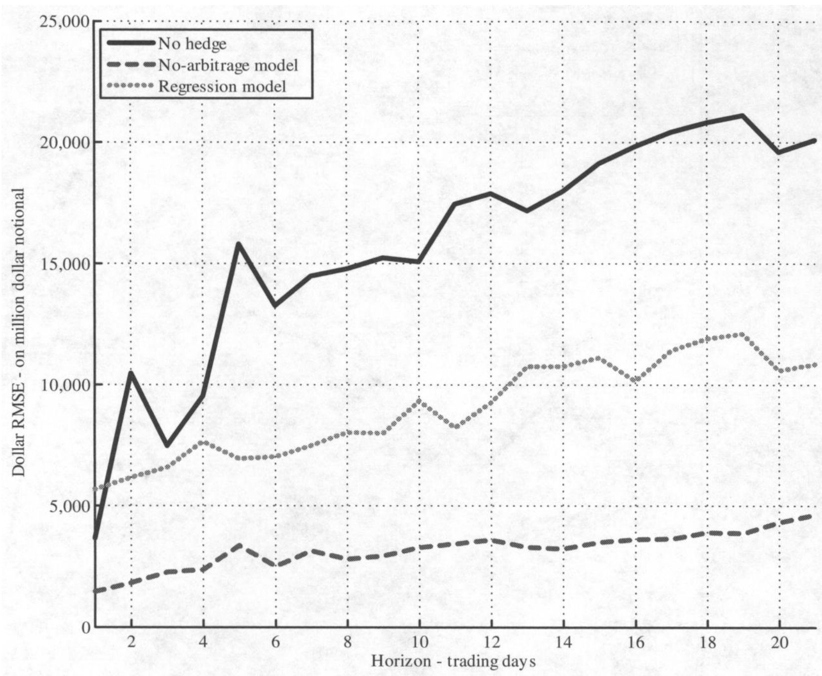

Figure 6
Out-of-sample hedging errors
We show the hedging errors for the no-arbitrage model and the linear regression model. We estimate the model recursively every month using three years of historical data, and we hedge a million dollar notional position in a five-year contract using the three-year and the seven-year maturity contracts. The no-hedge case indicates the scenario in which we do not hedge the position. The weights in the 3- and 7-year contracts to hedge the 5-year contract are determined by making the overall portfolio insensitive to the changes in the two firm-specific risk factors. The graph presents the average dollar root-mean-squared error across all firms in our sample for different horizons.

data, updating the data one month at a time. For each resulting estimation, we compute the results of the hedging exercise for the subsequent twenty-one business days. Figure 6 reports the resulting average hedging errors, defined in terms of the dollar root-mean-squared error of the change in value of the portfolio (22) on a one million dollar notional position. We compare the hedging performance of the no-arbitrage model with that of the linear regression model. We also report the results of not hedging the position, which is simply the change in value of the five-year contract. As expected, for each of these exercises the hedging error increases with the horizon. At every horizon, the no-arbitrage model significantly improves on the hedging performance of the regression model, thus illustrating the out-of-sample advantages of the no-arbitrage approach, presumably due to the no-arbitrage method's consistency when pricing different maturities.

Figure 7 reports on the third out-of-sample exercise. It depicts the out-of-sample performance of the no-arbitrage and regression approaches using a

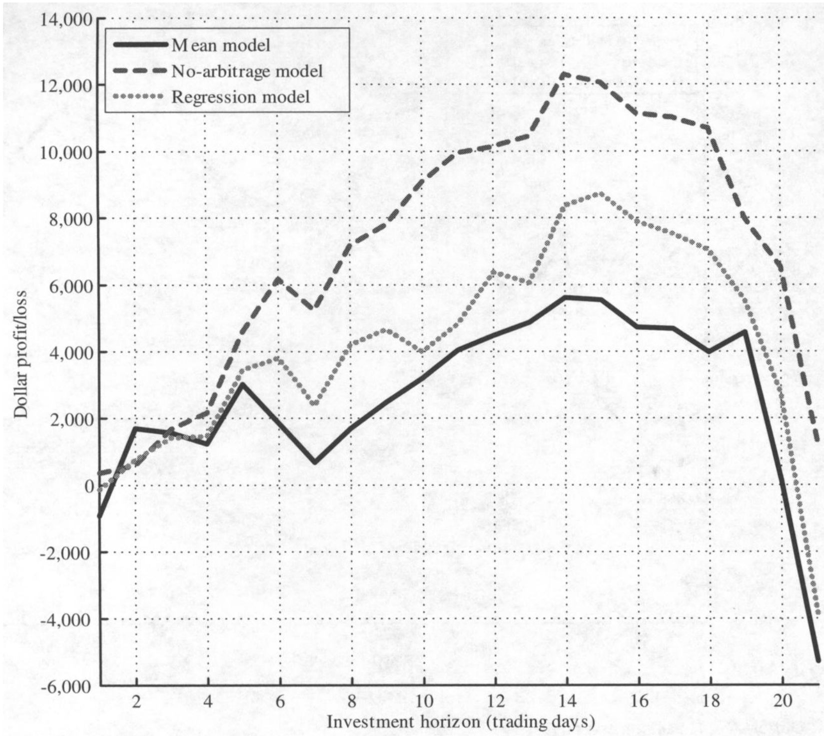

Figure 7
Out-of-sample investment exercise
We show the dollar profit of an out-of-sample investment exercise, which is based on the difference between the model and market spreads. We estimate the model recursively every month using three years of historical data and compute the difference between the model and market spread. Based on the difference, we create a portfolio where we buy protection on the firms with positive difference between model and market spreads and sell protection on the firms with negative difference between model and market spreads. The overall notional for the long position is one million dollars, and the overall notional for the short position is also one million dollars. The graph represents the average dollar profit over time for different investment horizons, with trading days on the x-axis.

rich versus cheap investment exercise, common in fixed income markets (see Tuckman and Serrat 2012) and recently applied to the CDS market by Bai and Wu (2011). As in the hedging exercise in Figure 6, we recursively estimate the model using three years of daily data, updating the data by one month at a time. We establish a long or short position in each underlying firm based on the deviation between model price and market price for the five-year maturity contract. If the deviation is positive (negative) the CDS is classified as "cheap" ("rich"). We buy cheap and sell rich CDS. The investment in each cheap (rich) CDS is proportional to the size of the difference between the model and market spreads. For a cheap CDS contract at time t, the size of the notional investment is

$$Not_{t,i} \propto \left(S_{t,i}^{Model} - S_{t,i}^{Market}\right),$$

where $Not_{t,i}$ is the notional exposure at time t for firm i, $S_{t,i}^{Model}$ is the model-implied spread at time t for firm i, and $S_{t,i}^{Market}$ is the market spread at time t for firm i. The larger the deviation between model and market spread for a given underlying, the larger the position in this contract. The total notional investment in all cheap CDS is set equal to one million

$$1 = \sum_i Not_{t,i} = C^+(t) \sum_i \left(S_{t,i}^{Model} - S_{t,i}^{Market} \right),$$

where $C^+(t)$ is the scaling constant such that the overall long position notional is one million dollars. The total notional investment in rich CDS is defined in a similar manner. We document the performance of the investment strategy for horizons of up to twenty-one business days.

Figure 7 yields two important conclusions. First, observable covariates are useful indicators for allocating capital in CDS markets, as both the regression and the no-arbitrage approaches generate positive profits for horizons up to twenty business days. For comparison, we also report on a third exercise in which long and short positions are created based on the deviation between the market spread and the historical average spread over the last three years. We refer to this as the "mean" model in Figure 7. The resulting profits are also positive for horizons up to eighteen business days but are substantially smaller. The second important conclusion is that the no-arbitrage model uses the information in the observable covariates in a much more efficient way than the regression approach, regardless of the investment horizon.

5. Robustness

We now discuss the robustness of the empirical results in Section 4 by analyzing a variety of alternative specifications. These include alternative covariates and alternative modeling assumptions. We also discuss the importance of statistical assumptions to relate our findings to the existing literature.

5.1 Quadratic and affine models

It is not our objective to run a horse race between no-arbitrage and regression models, which are fundamentally different tools with different uses. The fit of the regression models is merely provided as a benchmark to verify that the fit of the no-arbitrage models is adequate. Our emphasis is on the methodological advantages of the no-arbitrage approach, and we provide the fit of regression models to investigate if these methodological advantages come at the cost of empirical fit.

However, the no-arbitrage model is quadratic, which affects the fit and the interpretation of the estimated coefficients and the deltas. We investigate the importance of the quadratic specification in two ways. First, we estimate a no-arbitrage model with an affine rather than a quadratic specification in the covariates. We choose the quadratic specification as our benchmark because it is straightforward to ensure positivity of the default intensity. However, an affine model provides a more natural comparison for a linear regression. Table 4, Panel A, presents results for model fit for the affine no-arbitrage model, which is referred to as the "NA affine-benchmark". The RMSE of the affine no-arbitrage model is very similar to that of the regression approach and is therefore higher than for the quadratic no-arbitrage model.

Our second approach is to use nonlinear least squares to estimate the regression specification

$$S_t = \left(\varrho + \phi_r X_t^r + \phi_d X_t^d\right)^2 + \varepsilon_t. \tag{24}$$

This specification precludes negative fitted spreads. The RMSE for this specification (data not reported) is very similar to that of the quadratic no-arbitrage model, but of course the disadvantage of the regression specification is that the no-arbitrage restrictions are not imposed across tenors.

In summary, we conclude that the no-arbitrage model allows us to impose no-arbitrage and consistency across tenors, and this methodological advantage comes at no cost in terms of empirical fit. The quadratic assumption improves empirical fit, and it is convenient to ensure the positivity of default probabilities.

5.2 Statistical assumptions and model fit

The empirical results for the benchmark linear regression models may seem surprising, because they suggest an adequate fit. Part of the existing literature concludes that observable covariates cannot explain much of the variation in credit spreads. We now comment on this apparent contradiction.

The specification of the no-arbitrage model in Section 1 defines the intensity—see (4)—in terms of the levels of the covariates. The linear regression specification (20) also takes the level of the spread as the dependent variable. However, the statistical specification of linear regression models of credit spreads, which dates back to at least as far as Fisher (1959), has been the subject of some debate. Collin-Dufresne, Goldstein, and Martin (2001), using monthly data on corporate bonds and regressions in differences rather than levels, argue that covariates suggested by economic theory have limited explanatory power. Other credit risk studies use levels regressions, and some authors report results using both specifications.

The choice between levels and difference regressions for credit spread analysis is a complex one, and no consensus has emerged in the literature. We do not attempt to resolve this issue here, but we want to briefly motivate our focus on levels specifications for daily CDS data, and to demonstrate that our results are consistent with the extant literature. From a statistical perspective, differencing is preferred if the dependent variable and/or regressors are characterized by stochastic trends and integrated, because regression analysis using integrated or nearly integrated variables may yield spurious regression results, in that R^2s and t-statistics may be misleading (Granger and

Newbold 1974). However, a stochastic trend may not be the most obvious representation of the variables used in (20). Although bond spreads, CDS spreads, and covariates such as volatility are typically highly autocorrelated, economic intuition suggests that they are stationary, in the sense that they are not inherently characterized by a positive drift like stock prices or aggregate consumption. This intuition is confirmed by the time series of aggregate spreads in Figure 2, the time series of spreads for Nordstrom, Inc., in Figure 1, and the graphs for other firms in the sample (data not reported). Spreads are not trending up or down through time in our sample period.

Moreover, there is an important potential cost to differencing, because the difference regression may be less statistically efficient than the levels regression. In many realistic scenarios, measurement error may also further lower the signal-to-noise ratio in a difference regression compared with a levels regression. Because from our perspective the economic impact of covariates on spreads is even more important than model fit, levels regressions may provide an advantage, as the estimates of economic impact are consistent.

Regardless of which specification is preferable, it is important to verify that our results are consistent with the literature. Panel B of Table 4 reports regression results for daily differences, referred to as the "Diff. reg-benchmark". The average R^2s are much lower than the R^2s for levels regressions, between 3% and 7%. This is consistent with the R^2s in Zhang, Zhou, and Zhu (2009), for example. Note that other existing studies report higher R^2s for difference specifications. This is due to data frequency. The results for bonds in Collin-Dufresne, Goldstein, and Martin (2001) and Avramov, Jostova, and Philopov (2007) are obtained using monthly data. Ericsson, Jacobs, and Oviedo (2009) and Blanco, Brennan, and Marsh (2005) use daily CDS data, but report gaps in the data, effectively yielding lower frequency data. We verified that using lower frequency differences indeed leads to higher R^2s.[24]

We conclude that our estimates for the benchmark regression models are consistent with the literature. We also believe that our focus on level specifications is well-motivated, because it leads to consistent estimates of the impact of covariates on spreads, and spreads and covariates seem to be mean reverting. However, both levels and difference specifications have advantages and disadvantages, and the trade-off needs to be carefully evaluated.

5.3 Model error analysis

To further address the performance of the covariates in the benchmark model in Section 4.2, we first regress the errors of the benchmark model on other candidate covariates. Table 6 presents the results of these regressions. We

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[24] It is not surprising that the explanatory power of observable covariates decreases with data frequency. As an example, in the limit, at frequencies investigated in the microstructure literature, fundamentals do not matter. This does not mean that those fundamentals do not matter at lower frequencies. We thank an anonymous referee for this observation.

Table 6
Analysis of model errors

Specification	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
One-year trailing S&P 500 ret.	57.9%											52.6%	57.9%			
VIX		80.0%										84.2%	85.3%	75.8%	52.6%	
Index put/call ratio			48.4%													
Index skewness				62.1%									32.6%			
Default premium					70.5%									38.9%		
Firm book to market						47.4%										
One-year trailing firm ret.							49.5%									49.5%
Firm option volatility								84.2%							43.2%	80.0%
Firm put/call ratio									63.2%							
Firm skewness										58.9%						51.6%
No. of CDS contributors											42.1%					
Adjusted R^2	5.6%	14.0%	1.5%	2.5%	8.6%	6.0%	5.1%	15.7%	4.8%	3.0%	2.2%	17.2%	17.9%	16.5%	20.0%	20.6%

We regress the model errors for the benchmark no-arbitrage specification on several combinations of covariates. The dependent variable is the error for the five-year tenor. Index and firm put/call ratio indicates the ratio of put open interest to call open interest for S&P 500 index options and individual firm options respectively. Index and Firm Skewness are the option-implied measures of skewness for S&P 500 index and individual firms, respectively. For each specification, we report the percentage of firms with significant loadings. The significance of the loadings is computed using Newey-West standard errors with ten lags. The last row indicates the average adjusted R^2 across firms.

consider univariate and multivariate regressions using a large number of covariates, motivated by covariates that have been used in existing work on credit spreads (see Cao, Yu, and Zhong 2010; Collin-Dufresne, Goldstein, and Martin 2001; Ericsson, Jacobs, and Oviedo 2009; Zhang, Zhou, and Zhu 2009). We present results for the five-year tenor, results for other maturities are similar.

Table 6 presents the averages of R^2s across firms and the percentage of firms with statistically significant loadings.[25] Regression results often differ between firms, but one important conclusion obtains. Risk-neutral information extracted from options explains an economically significant percentage of the variation in model residuals. Most notably, for the five-year maturity, the regression of model residuals on firm-level option-implied volatility results in an average R^2 of 15.7%. Even when using the market-wide implied volatility (VIX), the univariate regression has an average R^2 of 14%. Other option-implied information, such as option-implied skewness, further helps explain model residuals.

Our findings on firm-level and marketwide implied volatility are all the more noteworthy because the benchmark specification includes historical volatility. We therefore infer that the difference between the risk-neutral and the historical volatility, which is often referred to as the volatility risk premium and which is related to the pricing kernel and agents' risk aversion, contains additional useful information about credit spreads. This is consistent with Carr and Wu (2011), who show the theoretical and empirical relationship between CDS spreads and deep-out-of-the-money put option prices. We therefore use option-implied volatilities in the specification of some alternative models below.

5.4 Alternative covariate specifications

The analysis of alternative covariate specifications is of interest for several reasons. First, it is important to verify that the good performance of the no-arbitrage model in Section 4 extends to other specifications of the covariates. Second, the question arises by how much the fit can be improved by including additional covariates. Third, it is of interest to measure the economic impact of alternative covariates on spreads.

Because estimating the no-arbitrage model for many permutations of covariates is computationally costly, we mainly base model selection on the analysis of model errors in Section 5.3, which indicates that several variables are related to model errors but that option-implied information, and implied volatility in particular, are strongly associated with model errors. We therefore conduct several additional analyses. First, we augment the benchmark specification with option-implied volatility, which leads to five covariates: two term-structure factors, leverage, historical volatility, and option-implied

25 The standard errors are adjusted for autocorrelation using a Newey-West adjustment with ten lags.

volatility. This model is referred to as the "NA-benchmark + IV" in Panel A of Table 4. Second, we augment this specification with a number of other covariates that for some firms are correlated with model errors. We use a model with seven covariates: the four covariates of the benchmark model, option-implied volatility, the S&P 500 return, and liquidity, measured by the number of contributors to the Markit dataset. This richly parameterized model is referred to as "NA-extended" in Panel A of Table 4.

Finally, models with many covariates are difficult to analyze, in part because of the high correlation between some of the covariates. We therefore also analyze some more parsimonious models. First, rather than using historical volatility and leverage as separate covariates, we use distance-to-default as the sole covariate, following the logic of structual models. We compute distance-to-default following Duffie, Saita, and Wang (2007). This model is referred to as "NA-DTD" in Panel A of Table 4. Second, we follow Ang and Piazzesi (2003), Lu and Wu (2009), and Wu and Zhang (2008) and use a dimension reduction technique to limit the number of covariates. Specifically, we start with twenty-three candidate variables, which are divided in three categories, and use a hidden factor approach to extract one component each from the three categories of variables. The first category includes variables that are related to the overall macroeconomy. Following Wu and Zhang (2008), these variables include the CPI, the core CPI, the PPI, the core PPI, the PCE deflator, the core PCE deflator, the GDP deflator, real GDP, industrial production, nonfarm payrolls, real PCE, the slope of the risk-free term structure (defined as the ten year bootstrapped zero rate minus the six-month zero rate), and the level of the risk-free term structure (defined as the six-month zero rate). The second category includes variables that are related to the overall stock market. These variables include the trailing one-year S&P 500 index return, the VIX, 30-day index option-implied skewness, and the ratio of the open interest of index put options to the open interest of index call options. The third category includes variables that are related to firm-specific risk. These variables include the trailing one-year firm-specific return, debt scaled by equity, historical volatility, thirty-day firm option-implied volatility, thirty-day firm option-implied skewness, and the ratio of the open interest of firm put options to the open interest of firm call options. For each category, we use a Kalman filter to extract the common variation in the variables of a given category. The Kalman filter is able to take into account that the time series have different data frequencies. This model is referred to as the "NA-common component" in Panel A of Table 4.

Table 4, Panel A, contains summary statistics for the fit of these models. Adding implied volatility as an additional regressor substantially improves the fit, lowering the average RMSE to 37.8 basis points for the five-year tenor, as opposed to 45.2 basis points for the benchmark specification. Panel B of Figure 3 illustrates the fit of this model over time. Compared with the benchmark model with four covariates, this model better captures the data during the financial crisis. Panel C of Figure 3 depicts the difference in RMSE between the two models, and the improved performance of the extended model with implied volatility is very clear, especially during the financial crisis.

Panel A of Table 4 indicates that adding additional covariates to the model with implied volatility can further improve the fit, as illustrated by the NA-extended model, which further lowers the five-year average RMSE to 33.3 basis points. However, the improvement in fit is relatively modest, and it comes at a cost, because the parsimonious model more often yields the theoretically expected sign for the covariates (data not reported).

Finally, the more parsimonious model labeled "NA-DTD" significantly underperforms the benchmark model. When we reduce the dimension by grouping a large number of covariates in three groups, model fit also does not improve. The fit of the resulting model, which is referred to as the "NA-common component" model in Table 4, is about the same as that of the benchmark model but is not as good as the benchmark model expanded by the option-implied volatility. We conclude that the parsimonious specification suggested by Merton-type variables performs very well, and that option-implied volatility substantially improves the fit, which is consistent with the results of the model error analysis in Table 6.

Figure 8 further analyzes the importance and the economic implications of option-implied volatility and the "NA-benchmark + IV" model by presenting the ratio between risk-neutral and physical default probabilities for the five-year horizon and the prices of risk associated with the three firm-specific covariates. On average, the ratio of Q to P default probabilities in Panel A is remarkably close to the case of the benchmark model in Figure 5. The most important difference is that the ratio increases in the financial crisis, whereas in Figure 5 it decreases. The time paths of the prices of risk in Panels B–D of Figure 8 clearly indicate that this difference is due to the price of risk associated with implied volatility, which sharply increases during the financial crisis. The time paths of the prices of risk associated with historical volatility and leverage are similar to those for the benchmark model in Figure 5.

Panels E–G of Figure 8 indicate how the probability ratio changes with the level of the covariates. These ratios are again computed firm-by-firm and subsequently averaged across firms. For leverage and historical volatility, the pattern of the change in ratio with the increase in the level of covariates is very similar to the benchmark specification. For implied volatility, consistent with the evidence from the time-series graphs in Panels A and C, we find that the risk-neutral to physical probability ratio increases as implied volatility increases.

5.5 Balance sheet data

Following most of the available literature, we interpolate the balance sheet data between quarters. We investigate the robustness of our results to this assumption by instead not interpolating and considering the most recent available statement.

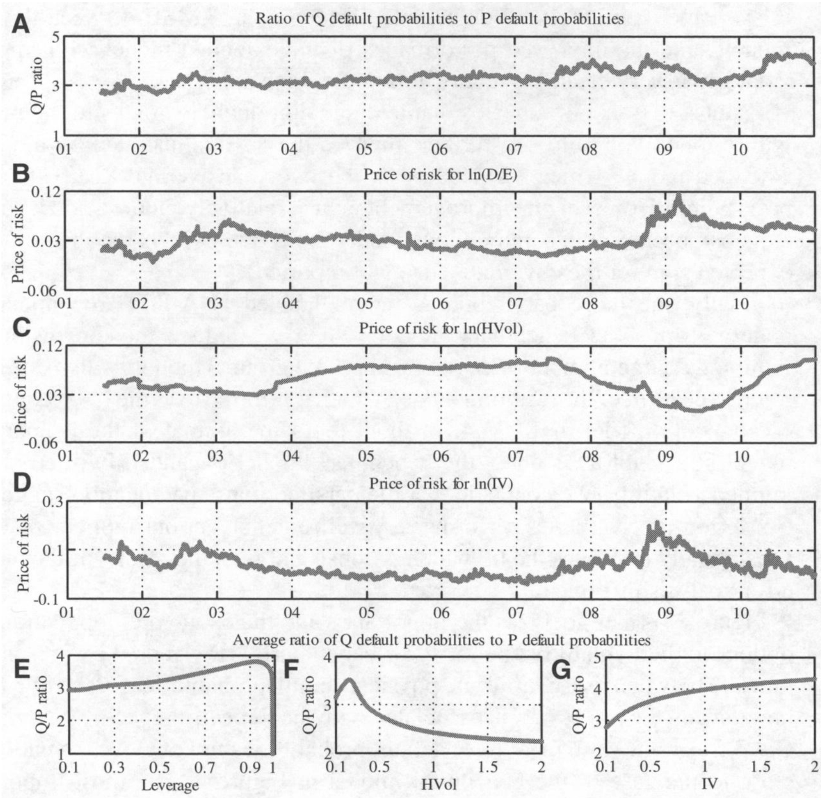

Figure 8
Market prices of risk for covariates and probability ratios. Benchmark model augmented with IV
Panel A shows the average ratio of Q default probabilities to P default probabilities for the five-year contract, for the benchmark model specification augmented by implied volatility. The Q default probabilities are the model-implied default probabilities. The P default probabilities are computed by replacing the Q-dynamics of the covariates with the P-dynamics. Panels B–D graph the market price of risk ($\lambda_0+\lambda_1 X_t$) associated with the natural log of the debt-to-equity ratio (D/E), the natural log of historical volatility (HVol), and the natural log of implied volatility (IV). Panels E–G show the average model-implied Q default probability to P default probability ratios for different values of leverage, annualized historical volatility (HVol), and annualized implied volatility (IV). In Panel E, leverage is defined as D/(D+E). We convert each leverage level into the corresponding D/E number and use the natural log of D/E to compute the probabilities using the model estimates.

Similar to the benchmark specification, we assume that the data becomes available forty-five days after the fiscal quarter ends. Table 4 presents the results from this exercise, referred to as the "NA-benchmark no interp.". This assumption does not affect the results much.

6. Concluding Remarks

This paper makes four contributions. First, we introduce a no-arbitrage model with observable covariates, which allows for a closed-form solution for the value of CDSs. We specify the default intensity as a quadratic function of the covariates such that the intensity function is always positive. Our approach enables us to study the effects of observable covariates, while maintaining the discipline of a no-arbitrage model and imposing pricing consistency across maturities.

Our second contribution is empirical. We demonstrate that macroeconomic and firm-specific information can explain most of the variation in CDSs over time and across firms. A parsimonious model with four covariates suggested by theory performs very well. The model provides plausible results from an economic perspective: The impact of covariates such as volatility and leverage on CDSs is entirely consistent with economic intuition and the logic of structural credit risk models, such as Merton (1974). Moreover, we find that requiring our no-arbitrage model to simultaneously fit CDS prices for different maturities does not come at the cost of empirical fit. We also characterize the prices of risk corresponding to the different covariates, and we find that the estimates are intuitively plausible. We also investigate richer models with variables commonly used in the literature. We find that option-implied volatility has substantial explanatory power, but we find a limited role for other covariates.

Third, the finding that observable covariates are very useful to explain credit spreads resolves a disconnect in the existing literature, making valuation results consistent with the evidence regarding default prediction. Fourth, we provide evidence on risk premia in credit markets. The cross-sectional evidence indicates that the size of the risk premium as a fraction of the total spread decreases for firms with lower credit quality.

Appendix

A.1 Risk-free bond pricing

If ϵ is a $(n,1)$ vector described by a multivariate normal distribution $\epsilon \sim N(0,\Gamma)$, Γ being nonsingular, then the Laplace transform of a quadratic form

$$Q=\epsilon' A\epsilon + a'\epsilon + d \tag{A1}$$

is given by[26]

$$E[\exp(tQ)]=\exp\left(-\frac{1}{2}\ln(\det(I-2t\,\Gamma\, A))+td+\frac{1}{2}ta'\left(\Gamma^{-1}-2tA\right)^{-1}at\right). \tag{A2}$$

We want to price a default-free zero coupon bond

$$E_t[\exp(-r_t-\ldots-r_{t+h-1})]\equiv L_{t,h}.$$

Let $r_{t+j}=(\delta_0+\delta' X^r_{t+j})^2$, where X^r_{t+j} and δ are $(n,1)$ vectors, $j=0,1,\ldots,h-1$. We can rewrite this in the form

$$r_{t+j}=\left(\delta_0+\delta' X^r_{t+j}\right)'\left(\delta_0+\delta' X^r_{t+j}\right)=\delta_0^2+2\delta_0\delta' X^r_{t+j}+X^{r\prime}_{t+j}\delta\delta' X^r_{t+j}. \tag{A3}$$

[26] The proof is given in Mathai and Provost (1992, p. 40).

Assume that

$$X_t^r = \mu_r + \rho_r X_{t-1}^r + \Sigma_r e_t, \tag{A4}$$

where $e_t \sim N(0, I)$, μ_r is a $(n, 1)$ vector and ρ_r and Σ_r are (n, n) matrices.

First, consider

$$L_{t+h-1,1} \equiv E_{t+h-1}[\exp(-r_{t+h-1})] = \exp(-r_{t+h-1}).$$

Substituting expression (A3), we have

$$L_{t+h-1,1} = \exp\left(A_1 + B_1' X_{t+h-1}^r + X_{t+h-1}^{r\prime} C_1 X_{t+h-1}^r\right),$$

where

$$\begin{aligned} A_1 &= -\delta_0^2 && \text{scalar} \\ B_1 &= -2\delta_0 \delta && \text{a } (n,1) \text{ vector} \\ C_1 &= -\delta\delta' && \text{a } (n,n) \text{ matrix.} \end{aligned}$$

To determine $L_{t,h}$, we use iterative expectations. We first consider $L_{t+h-2,2}$

$$L_{t+h-2,2} = \exp\left(-\delta_0^2 - 2\delta_0 \delta' X_{t+h-2}^r - X_{t+h-2}^{r\prime} \delta\delta' X_{t+h-2}^r\right) E_{t+h-2}\left[L_{t+h-1,1}\right]$$

and then use expression (A2) and simplify. This process is repeated to give, after much simplification

$$L_{t,h} = \exp(A_h + B_h' X_t + X_t' C_h X_t),$$

where for $k = 2, \ldots, h$

$$A_k = -\delta_0^2 + \left(A_{k-1} + B_{k-1}' \mu_r + \mu_r' C_{k-1} \mu_r\right) + \frac{1}{2}\left(\Sigma_r' B_{k-1} + 2\Sigma_r' C_{k-1} \mu_r\right)' \left(I - 2\Sigma_r' C_{k-1} \Sigma_r\right)^{-1}$$

$$\left(\Sigma_r' B_{k-1} + 2\Sigma_r' C_{k-1} \mu_r\right) - \frac{1}{2} \ln\left[\det\left(I - 2\Sigma_r' C_{k-1} \Sigma_r\right)\right]$$

$$B_k' = -2\delta_0 \delta' + (B_{k-1} + 2C_{k-1}\mu_r)' \rho_r + 2\left(\Sigma_r' B_{k-1} + 2\Sigma_r' C_{k-1} \mu_r\right)' \left(I - 2\Sigma_r' C_{k-1} \Sigma_r\right)^{-1} \Sigma_r' C_{k-1} \rho_r$$

and

$$C_k = -\delta\delta' + \rho_r' C_{k-1} \rho_r + 2\left(\Sigma_r' C_{k-1} \rho_r\right)' \left(I - 2\Sigma_r' C_{k-1} \Sigma_r\right)^{-1} \left(\Sigma_r' C_{k-1} \rho_r\right).$$

A.2 Default intensity modeling

The intensity function is also a quadratic function of the form

$$\begin{aligned} \lambda_{t+j} &= \left(\alpha_0 + \alpha' X_{t+j}^r + \alpha_d' X_{t+j}^d\right)' \left(\alpha_0 + \alpha' X_{t+j}^r + \alpha_d' X_{t+j}^d\right) \\ &= \alpha_0^2 + 2\alpha_0 \alpha' X_{t+j}^r + 2\alpha_0 \alpha_d' X_{t+j}^d + 2X_{t+j}^{r\prime} \alpha \alpha_d' X_{t+j}^d + X_{t+j}^{r\prime} \alpha\alpha' X_{t+j}^r + X_{t+j}^{d\prime} \alpha_d \alpha_d' X_{t+j}^d, \end{aligned}$$

where X_{t+j}^d and α_d are $(m, 1)$ vectors, $j = 0, 1, \ldots, h-1$. The sum of the interest rate plus the intensity is given by

$$r_{t+j} + \lambda_{t+j} = \left(\delta_0 + \delta' X_{t+j}^r\right)' \left(\delta_0 + \delta' X_{t+j}^r\right) + \left(\alpha_0 + \alpha' X_{t+j}^r + \alpha_d' X_{t+j}^d\right)' \left(\alpha_0 + \alpha' X_{t+j}^r + \alpha_d' X_{t+j}^d\right),$$

which can be written in the form

$$r_{t+j}+\lambda_{t+j}=\delta_0^2+\alpha_0^2+2\left(\delta_0\delta'+\alpha_0\alpha'\right)X_{t+j}^r+2\alpha_0\alpha_d'X_{t+j}^d \\ +2X_{t+j}^{r\prime}\alpha\alpha_d'X_{t+j}^d+X_{t+j}^{r\prime}\left(\delta\delta'+\alpha\alpha'\right)X_{t+j}^r+X_{t+j}^{d\prime}\alpha_d\alpha_d'X_{t+j}^d. \tag{A5}$$

Define

$$\gamma_0\equiv\delta_0^2+\alpha_0^2$$

a scalar. Let $q=n+m$ and define

$$\gamma_1\equiv\begin{bmatrix}2(\delta_0\delta+\alpha_0\alpha)\\ 2\alpha_0\alpha_d\end{bmatrix}$$

a $(q,1)$ vector. Define

$$\Omega\equiv\begin{bmatrix}\delta\delta'+\alpha\alpha' & \alpha\alpha_d'\\ \alpha_d\alpha' & \alpha_d\alpha_d'\end{bmatrix}$$

a (q,q) matrix and

$$X_{t+j}\equiv\begin{bmatrix}X_{t+j}^r\\ X_{t+j}^d\end{bmatrix}$$

a $(q,1)$ vector, then

$$r_{t+j}+\lambda_{t+j}=\gamma_0+\gamma_1'X_{t+j}+X_{t+j}'\Omega X_{t+j}. \tag{A6}$$

It is assumed that

$$X_t=\mu+\rho X_{t-1}+\Sigma e_t, \tag{A7}$$

where $e_t\sim N(0,I)$, μ is a $(q,1)$ vector and ρ and Σ are (q,q) matrices.

The derivation of expression (8) requires evaluating

$$E_t\left[\exp\left(-\sum_{j=0}^{h-1}r_{t+j}+\lambda_{t+j}\right)\right]. \tag{A8}$$

This expression is isomorphic to expression (A3), so the derivation follows that given in Appendix A.1. Expressions (A6) and (A7) are similar to expressions (A3) and (A4) in Appendix A.1.

First, consider

$$L_{t+h-1,1}\equiv E_{t+h-1}[\exp(-r_{t+h-1}-\lambda_{t+h-1})]=\exp(-r_{t+h-1}-\lambda_{t+h-1}).$$

Substituting expression (A6), we have

$$L_{t+h-1,1}=\exp\left(F_1+G_1'X_{t+j-1}+X_{t+h-1}'H_1X_{t+j-1}\right),$$

where

$$\begin{array}{ll} F_1=-\gamma_0 & \text{scalar}\\ G_1=-\gamma_1 & \text{a } (q,1) \text{ vector}\\ H_1=-\Omega & \text{a } (q,q) \text{ matrix.}\end{array}$$

Repeating the logic used in Appendix A.1 gives

$$L_{t,h}=\exp\left(F_h+G_h'X_t+X_t'H_hX_t\right),$$

where for $k=2,\dots,h$

$$F_k=-\gamma_0+\left(F_{k-1}+G_{k-1}'\mu+\mu'H_{k-1}\mu\right)+\frac{1}{2}\left(\Sigma'G_{k-1}+2\Sigma'H_{k-1}\mu\right)'\left(I-2\Sigma'H_{k-1}\Sigma\right)^{-1}$$

$$\left(\Sigma'G_{k-1}+2\Sigma'H_{k-1}\mu\right)-\frac{1}{2}\ln\left[\det\left(I-2\Sigma'H_{k-1}\Sigma\right)\right]$$

$$G_k'=-\gamma_1'+(G_{k-1}+2H_{k-1}\mu)'\rho+2\left(\Sigma'G_{k-1}+2\Sigma'H_{k-1}\mu\right)'\left(I-2\Sigma'H_{k-1}\Sigma\right)^{-1}\Sigma'H_{k-1}\rho$$

and

$$H_k=-\Omega+\rho'H_{k-1}\rho+2\left(\Sigma'H_{k-1}\rho\right)'\left(I-2\Sigma'H_{k-1}\Sigma\right)^{-1}\left(\Sigma'H_{k-1}\rho\right).$$

The Derivation of Expression (10)

First, consider

$$L_{t+h-1,1}=\exp(-r_{t+h-1}).$$

Substituting expression (A3), we have

$$L_{t+h-1,1}=\exp\left[-\left(\delta_0^2+2\delta_0\delta'X_{t+h-1}^r+X_{t+h-1}^{r\prime}\delta\delta'X_{t+h-1}^r\right)\right]$$

$$\equiv\exp\left(M_1+N_1'X_{t+h-1}^r+X_{t+h-1}^{r\prime}P_1X_{t+h-1}^r\right),$$

where

$$M_1=-\delta_0^2 \qquad \text{scalar}$$

$$N_1=-2\delta_0\delta \qquad \text{a } (n,1) \text{ vector}$$

$$P_1=-\delta\delta' \qquad \text{a } (n,n) \text{ matrix.}$$

Next, consider

$$L_{t+h-2,2}=\exp(-r_{t+h-2}-\lambda_{t+h-2})E_{t+h-2}[L_{t+h-1,1}].$$

Using expressions (A2) and (A5), we have

$$L_{t+h-2,2}=\exp\left(M_2+N_2'X_{t+h-2}+X_{t+h-2}'P_2X_{t+h-2}\right), \tag{A9}$$

where

$$M_2\equiv-\left(\delta_0^2+\alpha_0^2\right)+\left(M_1+N_1'\mu_r+\mu_r'P_1\mu_r\right)+\frac{1}{2}\left(\Sigma_r'N_1+2\Sigma_r'P_1\mu_r\right)'\left(I-2\Sigma_r'P_1\Sigma_r\right)^{-1}$$

$$\left(\Sigma_r'N_1+2\Sigma_r'P_1\mu_r\right)-\frac{1}{2}\ln\left[\det\left(I-2\Sigma_r'P_1\Sigma_r\right)\right]$$

$$N_2'\equiv\begin{bmatrix}(N_1+2P_1\mu_r)'\rho_r+2\left(\Sigma_r'N_1+2\Sigma_r'P_1\mu_r\right)'\left(I-2\Sigma_r'P_1\Sigma_r\right)^{-1}\Sigma_r'P_1\rho_r-2\left(\delta_0\delta'+\alpha_0\alpha'\right)\\ -2\alpha_0\alpha_d'\end{bmatrix}$$

$$P_2\equiv\begin{bmatrix}\rho_r'P_1\rho_r+2\rho_r'P_1\Sigma_r\left(I-2\Sigma_r'P_1\Sigma_r\right)^{-1}\Sigma_r'P_1\rho_r-\left(\delta\delta'+\alpha\alpha'\right) & -\alpha\alpha_d'\\ -\alpha_d\alpha & -\alpha_d\alpha_d'\end{bmatrix}.$$

From this point, the analysis is similar to the derivation of expression (8).

Table A1
Descriptive statistics

Firm name	Sample period	Rating	Panel A: Firm–specific descriptive statistics								Panel B: Model fit (5 yr.)		
			Mkt spr. (5 yr.)		HVol		FIV		LEV		Linear regr.		NA
			Mean	SD	Mean	SD	Mean	SD	Mean	SD	R^2 (%)	RMSE	RMSE
Honeywell Intl Inc	01/02–12/10	A	37.70	23.88	0.31	0.12	0.32	0.12	0.42	0.06	59.9	15.11	15.30
AT&T Corp.	05/01–11/05	BBB	175.39	130.53	0.39	0.12	0.38	0.14	0.67	0.05	35.1	105.12	109.47
E I du Pont de Nemours & Co	10/01–12/10	A	36.49	30.34	0.28	0.12	0.28	0.11	0.39	0.07	75.7	14.96	13.31
Eastman Kodak Co	06/01–12/10	BB	385.75	400.03	0.45	0.22	0.48	0.21	0.65	0.12	92.5	109.75	100.13
Goodrich Corp	10/01–12/10	BBB	72.35	54.56	0.35	0.11	0.35	0.12	0.50	0.09	63.9	32.77	25.60
Ingersoll Rand Co	06/01–12/10	BBB	45.43	22.95	0.36	0.14	0.35	0.13	0.44	0.13	58.5	14.78	19.68
Intl Business Machs Corp	10/01–12/10	A	34.42	20.77	0.25	0.10	0.28	0.10	0.34	0.04	61.2	12.94	13.34
Altria Gp Inc	02/01–12/10	BBB	104.93	62.08	0.26	0.09	0.27	0.09	0.35	0.08	47.7	44.89	45.28
ConocoPhillips	10/02–12/10	A	37.07	21.40	0.30	0.13	0.30	0.12	0.48	0.07	63.2	12.98	13.09
Amgen Inc.	02/02–12/10	A	41.38	28.18	0.31	0.10	0.32	0.10	0.15	0.07	52.7	19.37	19.99
The Kroger Co.	06/01–12/10	BBB	68.48	24.09	0.28	0.07	0.30	0.08	0.53	0.05	54.6	16.22	16.70
Gen Mls Inc	01/02–12/10	BBB	43.53	18.90	0.18	0.06	0.21	0.06	0.40	0.04	60.5	11.88	12.32
Caterpillar Inc	06/01–12/10	A	52.76	56.07	0.33	0.11	0.35	0.12	0.56	0.07	82.2	23.65	18.69
Deere & Co	10/01–12/10	A	46.90	34.28	0.35	0.14	0.37	0.14	0.59	0.06	67.7	19.48	19.69
Bristol Myers Squibb Co	04/02–12/10	A	30.49	14.11	0.27	0.09	0.30	0.10	0.26	0.03	48.5	10.12	10.67
Dow Chem Co	10/01–12/10	A	92.25	100.02	0.35	0.16	0.36	0.17	0.49	0.09	88.1	34.47	30.20
Lockheed Martin Corp	05/01–12/10	BBB	39.66	18.28	0.26	0.09	0.28	0.10	0.44	0.07	57.3	11.94	12.29
MeadWestvaco Corp	02/02–12/10	BBB	94.01	52.71	0.34	0.15	0.37	0.15	0.57	0.06	48.7	37.73	39.14
Intl Paper Co	06/01–12/10	BBB	118.03	118.91	0.35	0.20	0.35	0.17	0.59	0.08	84.6	46.61	42.84
FirstEnergy Corp	05/02–12/10	BBB	104.73	73.28	0.26	0.11	0.26	0.12	0.63	0.08	67.1	42.03	42.02
Progress Engy Inc	11/01–12/10	BBB	56.22	33.18	0.21	0.08	0.21	0.09	0.63	0.03	33.6	27.03	27.62
Halliburton Co	11/01–12/10	BBB	102.00	129.96	0.45	0.19	0.44	0.17	0.32	0.14	58.0	84.24	52.62
Amern Elec Pwr Co Inc	02/02–12/10	BBB	75.67	84.38	0.27	0.14	0.25	0.12	0.69	0.05	68.3	47.52	34.33
Constellation Engy Gp Inc	10/01–12/10	BBB	114.30	103.56	0.31	0.15	0.32	0.15	0.63	0.08	70.7	56.01	58.41
Alcoa Inc.	10/01–12/10	A	122.79	176.74	0.42	0.20	0.42	0.18	0.48	0.12	86.0	66.07	40.38
Northrop Grumman Corp	04/02–12/10	BBB	44.43	25.72	0.24	0.10	0.26	0.10	0.47	0.06	50.2	18.14	18.72
Raytheon Co	02/01–12/10	BBB	63.36	44.32	0.28	0.12	0.28	0.12	0.45	0.09	75.1	22.11	21.54
Campbell Soup Co	10/01–12/10	A	28.85	10.45	0.20	0.06	0.22	0.07	0.32	0.04	35.3	8.40	8.94
Whirlpool Corp	10/01–12/10	BBB	93.71	90.16	0.38	0.15	0.40	0.14	0.61	0.07	79.9	40.36	26.77
Walt Disney Co	06/01–12/10	A	48.91	28.55	0.32	0.13	0.32	0.12	0.35	0.05	71.3	15.29	15.42
Hewlett Packard Co	06/01–12/10	A	43.30	29.37	0.37	0.14	0.36	0.11	0.36	0.06	47.3	21.31	18.82
Baxter Intl Inc	10/01–12/10	A	33.45	15.42	0.27	0.10	0.28	0.09	0.26	0.07	45.1	11.42	12.04

(*continued*)

Table A1
Continued

Firm name	Sample period	Rating	Panel A: Firm–specific descriptive statistics								Panel B: Model fit (5 yr.)		
			Mkt spr. (5 yr.)		HVol		FIV		LEV		Linear regr.		NA
			Mean	SD	Mean	SD	Mean	SD	Mean	SD	R^2 (%)	RMSE	RMSE
Duke Engy Corp	06/01–03/06	BBB	62.69	38.49	0.32	0.13	0.32	0.17	0.64	0.07	59.2	24.56	26.29
Arrow Electrs Inc	10/01–12/10	BBB	131.82	98.75	0.38	0.11	0.41	0.11	0.57	0.08	74.3	50.02	45.73
Omnicom Gp Inc	12/01–12/10	A	74.31	71.75	0.29	0.14	0.32	0.16	0.46	0.06	61.9	44.30	42.08
Wells Fargo & Co	10/01–12/10	AA	71.12	80.21	0.35	0.33	0.34	0.26	0.82	0.05	84.1	31.98	31.24
Weyerhaeuser Co	10/01–12/10	BBB	103.44	64.47	0.35	0.17	0.36	0.15	0.58	0.06	70.7	34.87	37.49
Computer Sciences Corp	10/01–12/10	A	64.54	34.18	0.35	0.12	0.36	0.11	0.48	0.08	14.8	31.53	31.64
McDONALDS Corp	10/01–12/10	A	30.11	12.97	0.25	0.07	0.27	0.08	0.24	0.06	55.3	8.67	9.36
Un Pac Corp	03/02–12/10	BBB	47.68	20.19	0.29	0.12	0.30	0.12	0.50	0.08	48.3	14.52	14.73
Target Corp	10/01–12/10	A	45.37	41.33	0.34	0.13	0.35	0.15	0.37	0.08	78.5	19.14	17.73
Fed Natl Mtg Assn	10/01–09/08	AAA	38.80	41.69	0.32	0.18	0.40	0.31	0.94	0.02	85.5	16.49	16.91
Pulte Homes Inc	02/02–03/10	BBB	155.94	108.33	0.51	0.20	0.54	0.23	0.52	0.09	75.3	53.88	54.04
Wal Mart Stores Inc	07/01–12/10	AA	27.65	22.02	0.23	0.08	0.25	0.09	0.27	0.07	72.6	11.52	11.23
ConAgra Foods Inc	10/01–12/10	BBB	45.02	16.68	0.21	0.06	0.23	0.07	0.42	0.04	48.3	11.99	12.55
Nordstrom Inc	10/01–12/10	A	94.57	111.33	0.43	0.16	0.44	0.18	0.37	0.12	73.7	57.04	41.75
Southwest Airls Co	10/01–12/10	A	97.41	76.74	0.37	0.12	0.39	0.13	0.41	0.12	77.1	36.74	36.19
Amern Express Co	10/01–12/10	A	86.18	111.25	0.37	0.23	0.36	0.22	0.71	0.06	78.3	51.82	53.18
Chubb Corp	10/01–12/10	A	43.59	27.78	0.28	0.13	0.28	0.13	0.68	0.03	49.0	19.83	20.59
Newell Rubbermaid Inc	06/01–12/10	BBB	74.33	49.87	0.32	0.15	0.33	0.15	0.45	0.09	84.4	19.69	20.13
CSX Corp	06/01–12/10	BBB	62.60	35.02	0.35	0.12	0.36	0.13	0.58	0.10	40.2	27.09	27.50
Cigna Corp	08/02–12/10	BBB	78.49	60.44	0.41	0.20	0.39	0.19	0.82	0.08	73.8	30.93	30.19
Norfolk Sthn Corp	06/01–12/10	BBB	45.03	21.01	0.36	0.10	0.35	0.12	0.52	0.08	50.6	14.76	15.06
Dominion Res Inc	10/01–12/10	BBB	54.15	28.16	0.22	0.09	0.23	0.10	0.59	0.03	42.2	21.41	20.30
Amern Intl Gp Inc	10/02–12/10	AA	270.52	509.65	0.59	0.60	0.42	0.35	0.87	0.09	67.8	289.07	249.95
Anadarko Pete Corp	06/01–12/10	BBB	78.58	90.47	0.38	0.16	0.40	0.15	0.49	0.09	49.2	64.50	61.09
Carnival Corp	10/01–12/10	A	90.76	78.17	0.35	0.15	0.37	0.14	0.33	0.08	68.4	43.91	41.14
Fed Home Ln Mtg Corp	02/02–09/08	AAA	34.98	43.08	0.36	0.26	0.50	0.42	0.95	0.02	91.7	16.48	18.45
Safeway Inc	06/01–12/10	BBB	66.88	21.26	0.31	0.08	0.34	0.09	0.49	0.07	65.1	12.56	13.47
Jones Apparel Gp Inc	04/02–10/10	BBB	178.21	197.83	0.46	0.33	0.43	0.23	0.37	0.12	87.7	69.33	66.59
ACE Ltd	01/03–12/10	A	61.37	32.49	0.32	0.16	0.32	0.14	0.77	0.03	73.5	16.71	17.82
Transocean Inc	10/01–12/10	A	85.00	91.16	0.42	0.12	0.44	0.13	0.31	0.13	52.7	62.65	54.01
Allstate Corp	08/01–12/10	A	59.30	62.64	0.30	0.21	0.29	0.19	0.82	0.04	70.6	33.93	27.01
Eastman Chem Co	10/01–12/10	BBB	69.28	31.90	0.32	0.12	0.32	0.12	0.53	0.08	41.7	24.36	24.13

(*continued*)

Table A1
Continued

Firm name	Sample period	Rating	Panel A: Firm–specific descriptive statistics								Panel B: Model fit (5 yr.)		
			Mkt spr. (5 yr.)		HVol		FIV		LEV		Linear regr.		NA
			Mean	SD	Mean	SD	Mean	SD	Mean	SD	R^2 (%)	RMSE	RMSE
Simon Ppty Gp Inc	10/02–12/10	A	114.67	144.28	0.37	0.28	0.36	0.24	0.51	0.07	82.1	61.02	55.54
WA Mut Inc	03/02–08/08	A	128.35	477.87	0.24	0.10	0.28	0.17	0.89	0.03	85.7	52.13	26.82
Hartford Financial Services	11/01–12/10	A	136.65	191.87	0.52	0.56	0.43	0.38	0.94	0.02	91.2	56.84	59.65
Valero Engy Corp	10/01–08/05	BBB	92.06	62.29	0.36	0.06	0.39	0.09	0.62	0.12	59.2	39.79	42.17
Marriott Intl Inc	10/01–12/10	BBB	106.70	107.69	0.34	0.15	0.37	0.16	0.35	0.09	83.4	43.88	34.10
Sempra Engy	10/01–12/10	BBB	68.10	46.89	0.26	0.10	0.28	0.12	0.65	0.06	48.5	33.65	30.59
XL Cap Ltd	02/02–06/10	A	156.16	213.02	0.50	0.50	0.46	0.39	0.80	0.08	76.9	102.42	73.33
Devon Engy Corp	10/01–12/10	BBB	58.80	39.92	0.37	0.14	0.39	0.13	0.42	0.11	48.2	28.72	27.65
MetLife Inc	03/02–12/10	A	127.13	173.75	0.40	0.31	0.37	0.27	0.93	0.01	84.4	68.66	76.10
Aetna Inc.	12/01–12/10	A	69.69	54.30	0.37	0.13	0.39	0.14	0.69	0.09	59.8	31.97	29.52
Kraft Foods Inc	06/02–12/10	A	49.27	25.90	0.22	0.06	0.24	0.07	0.57	0.09	55.1	17.95	18.72
CIT Gp Inc	07/03–11/09	A	450.57	928.59	0.63	0.63	0.55	0.47	0.89	0.06	79.9	445.29	360.64
Albertsons Inc	10/01–05/06	BBB	90.31	63.34	0.30	0.06	0.31	0.09	0.55	0.06	62.8	38.61	48.26
Alcan Inc.	03/02–11/07	A	33.53	10.53	0.33	0.06	0.32	0.08	0.50	0.08	43.5	7.91	7.91
BellSouth Corp	10/01–11/06	A	41.68	28.55	0.28	0.12	0.27	0.11	0.38	0.04	43.2	21.52	22.53
Burlington Nthn Santa Fe Corp	02/02–02/10	BBB	41.28	19.89	0.29	0.08	0.29	0.10	0.51	0.09	43.8	14.91	15.22
Cendant Corp	06/01–07/06	BBB	144.31	120.53	0.35	0.13	0.38	0.17	0.59	0.04	70.9	65.04	59.78
CENTEX CORP	10/01–08/09	BBB	154.99	144.83	0.48	0.23	0.55	0.27	0.69	0.06	58.3	93.58	95.46
Cox Comms Inc	10/01–12/04	BBB	232.61	207.55	0.36	0.11	0.39	0.17	0.44	0.02	65.5	121.82	202.76
Delphi Corp	06/01–10/05	BB	256.12	324.95	0.39	0.09	0.46	0.32	0.77	0.07	87.8	113.27	132.00
Electr Data Sys Corp	06/01–08/08	BBB	125.67	84.44	0.37	0.21	0.37	0.16	0.45	0.10	73.5	43.43	41.79
Fedt Dept Stores Inc	06/01–05/07	BBB	58.76	28.13	0.33	0.08	0.35	0.08	0.52	0.07	66.2	16.35	13.51
Loews Corp	10/01–12/10	A	70.04	43.69	0.30	0.17	0.30	0.15	0.80	0.06	62.9	26.60	27.43
Maytag Corp	12/01–02/06	BBB	107.98	80.16	0.42	0.09	0.38	0.10	0.62	0.07	60.2	50.79	44.85
MBNA Corp	10/01–11/05	BBB	91.25	60.04	0.39	0.13	0.36	0.14	0.61	0.03	53.9	40.26	37.81
Motorola Inc	06/01–12/10	BBB	143.91	127.08	0.46	0.17	0.44	0.15	0.39	0.11	70.2	69.34	71.87
Rohm & Haas Co	06/01–02/09	BBB	41.44	33.93	0.31	0.14	0.31	0.17	0.39	0.06	46.1	18.54	18.13
Sprint Corp	06/01–08/05	BBB	240.61	286.86	0.43	0.16	0.44	0.20	0.47	0.07	56.9	188.18	167.04
May Dept Stores Co	06/01–08/05	BBB	54.03	14.12	0.34	0.05	0.33	0.09	0.50	0.05	41.4	10.80	10.94
Time Warner Inc	10/03–12/10	BBB	89.60	48.65	0.30	0.18	0.30	0.14	0.48	0.07	63.7	29.29	31.57
Wyeth	06/01–08/09	A	41.23	28.42	0.30	0.11	0.30	0.10	0.27	0.04	50.4	20.17	21.30
Average across Firms			90.90	90.85	0.34	0.15	0.35	0.15	0.54	0.07	63.5	45.20	42.63

We report summary statistics for the ninety-five firms in the dataset. CDS statistics are based on the five-year maturity credit default swap spreads in basis points. HVol indicates the annualized historical volatility; FIV indicates 30-day option-implied volatility; LEV indicates the leverage defined as the total liabilities scaled by the sum of the total liabilities and firm equity. For each firm, we also report the R^2 from the linear regression and the RMSEs in basis points from the benchmark linear regression and no-arbitrage models. The sample period for each firm is indicated in *mm/yy format. The covariates include two stochastic term structure factors, historical volatility, and leverage (debt scaled by debt plus the market value of equity)*. NA stands for the no-arbitrage model.

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