Bond Prices, Default Probabilities and Risk Premiums[1]

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A feature of credit markets is the large difference between probabilities of default calculated from historical data and probabilities of default implied from bond prices (or from credit default swaps). Consider, for example, a seven-year A-rated bond. As we will see the average probability of default backed out from the bond's price is almost ten times as great as that calculated from historical data.

Why are the two estimates of the probability of default so different? The answer is that bond traders do not base their prices for bonds only on the actuarial probability of default. They build in an extra return to compensate for the risks they are bearing. The default probabilities calculated from historical data are referred to as *real-world* (or physical) default probabilities; those backed out from bond prices are known as *risk-neutral* default probabilities. Real-world default probabilities are usually less than risk-neutral default probabilities. This means that bond traders earn more than the risk-free rate on average from holding corporate bonds. Risk-neutral default probabilities are used when credit dependent instruments are valued. Real-world default probabilities are used in scenario analysis and in the calculation of bank capital under Basel II.

Altman (1989) was one of the first researchers to comment on the discrepancy between bond prices and historical default data. He showed that, even after taking account of the impact of defaults, an investor could expect significantly higher returns from investing in corporate bonds than from investing in risk-free bonds. As the credit rating of the corporate bonds declined, the extent of the higher returns increased.

Default Intensity Estimates

Table 1 shows estimates of the average seven-year risk-neutral and real-world default intensities per annum for bonds with different credit ratings. The default intensity (sometimes also called the hazard rate) is the probability of default per year conditional on no earlier default.

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We estimated the real-world default intensity from statistics on average cumulative default rates published by Moody's between 1970 and 2003.[2] Suppose that d is the cumulative default rate for T years and h is the average default intensity over the T years. The probability of the bond issuer surviving for T years is

$$e^{-hT} = 1 - d$$

It follows that

$$h = -\frac{1}{T}\ln(1-d)$$

In our analysis $T = 7$.

A good approximation for the risk-neutral default intensity per year for a bond is

$$\frac{y-r}{1-R} \tag{1}$$

where y is the bond's yield, r is the yield on a risk-free bond promising the same cash flows as the bond, and R is the recovery rate. In applying equation (1) we used the Merrill Lynch bond indices. These indices report the yield on U.S. dollar denominated corporate bonds with different credit ratings at the end of each month. The average bond maturities used in the construction of the indices over the period covered by our data (December 1996 to July 2004) ranged from 6.5 years for Baa bonds to 8.9 years for Ba bonds. We set the risk-free yield, r, equal to ten basis points less than the seven-year swap rate. The reason for this is explained in the next section. We also assumed that that the recovery rate, R, is 40%. (This is a common assumption of market participants.) The numbers we report in Table 1 are averages over the 92 months covered by our data.

Table 1 shows that the ratio of the risk-neutral to real-world default intensity decreases as the credit quality declines. However, the difference between the default intensities increases as credit quality declines. The size of the difference between the two default intensity estimates is sometimes referred to as the *credit spread puzzle*.

[2] See Hamilton et al (2004).

Table 1: Real World and Risk Neutral Default Intensities (1 basis point is 0.01%)

Rating	Real-world default intensity per yr (bps)	Risk-neutral default intensity (bps)	Ratio	Difference
Aaa	4	67	16.8	63
Aa	6	78	13.0	72
A	13	128	9.8	115
Baa	47	238	5.1	191
Ba	240	507	2.1	267
B	749	902	1.2	153
Caa and Lower	1690	2130	1.3	440

The Benchmark Risk-Free Rate

The idea underlying equation (1) is that the excess return of a corporate bond over a similar risk-free bond compensates the holder for the cost of defaults. However the estimates produced by the equation depend critically on the choice of the risk-free rate, *r*.

A natural choice for *r* is the Treasury rate. Treasury rates are yields on bonds that have no default risk and the bond yield spreads that are quoted in the market are usually spreads relative to a Treasury bond that has a similar maturity. However, Treasury rates tend to be lower than other rates that have a very low credit risk for a number of reasons:

1. Treasury bills and Treasury bonds must be purchased by financial institutions to fulfill a variety of regulatory requirements. This increases demand for these Treasury instruments driving the price up and the yield down.

2. The amount of capital a bank is required to hold to support an investment in Treasury bills and bonds is substantially smaller than the capital required to support a similar investment in other very low-risk instruments.

3. In the United States, Treasury instruments are given a favorable tax treatment compared with most other fixed-income investments because they are not taxed at the state level.

This leads many market participants to regard swap rates as better proxies for risk-free rates than Treasury rates.[3]

The credit default swap (CDS) market provides a way of estimating the benchmark risk-free rate used by participants in credit markets. If a five-year par yield corporate bond provides a yield of 6% and five-year protection can be bought against the issuer for 150 basis points a year, an investor can obtain an (approximate) risk-free return of 4.5% by buying the bond and buying credit protection. This suggests that the risk-free rate being used by market participants is 4.5%. Using this type of analysis across many corporations Hull et al (2004) estimate that the benchmark risk-free rate being used by market participants is the swap rate less 10 basis points.[4] This is similar to estimates that have been made by Moody's KMV.[5]

This explains why we set *r* equal to the seven-year swap rate minus 10 basis points in producing the results in Table 1. It is worth noting that if we instead chose *r* to be the seven-year Treasury rate the risk-neutral default intensities would be even higher making the difference between risk-neutral and real-world default intensities even more marked. For example the ratio of the risk-neutral to real world default intensity for A-rated companies would rise from 9.8 to over 15.

Risk Premiums

The results in Table 1 can be translated into the risk premiums earned by holders of corporate bonds. The expected excess return of corporate bonds over Treasuries has a number of components. One component is the difference between the Treasury yield and our estimate of the benchmark risk-free yield used by market participants. During the 92 months covered by our Merrill Lynch data this averaged 43 basis points. Another

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[3] A similar point is made forcefully by Duffee (1996). He argues that "Since the early 1980's [Treasury] bill yields have become increasingly irrelevant as a benchmark. This is not news to market participants…but nonetheless [is] likely a surprise to many academic economists."

[4] A financial institution can earn the five-year swap rate by making a sequence of six-month loans at LIBOR and swapping the proceeds for fixed. The 10 basis points can therefore be regarded as the credit risk in a six-month loan made at LIBOR.

[5] For example, Stephen Kealhofer's estimate of the risk-free rate in his presentation at the Moody's/New York University conference on Recent Advances in Credit Risk Research in May 2004 was very close to that of Hull et al (2004).

component is a spread to compensate for defaults. This is the real-world default intensity in Table 1 multiplied by 1–*R*. The final component is the extra risk premium earned by the holders of corporate bonds. Note that if the risk premium were zero, there would be no difference between real-world and risk-neutral default probabilities and vice versa.

Table 2: Excess Expected Returns Earned by Bond Traders

Rating	Bond Yield Spread over Treasuries (bps)	Spread of risk-free rate used by market over Treasuries (bps)	Spread to compensate for default rate in the real world (bps)	Extra Risk Premium (bps)
Aaa	83	43	2	38
Aa	90	43	4	43
A	120	43	8	69
Baa	186	43	28	115
Ba	347	43	144	160
B	585	43	449	93
Caa and Lower	1321	43	1014	264

Table 2 shows the split. For example, A-rated bonds earned 120 basis points more than Treasuries on average. 43 basis points of this is the difference between Treasuries and the market's risk-free benchmark. A further 8 basis points is necessary to cover defaults. The remaining 69 basis points is a risk premium earned by bondholders. We can see from Table 2 that as the quality of the bond declines from Aaa to Ba the risk premium increases. It then declines as we move from Ba to B and increases sharply as we move from Ba to Caa.[6] The extra risk premium reported in the last column of Table 2 can be considered as the expected return on a portfolio that is long a corporate bond and short a default-free bond.

As pointed out by Bernt et al (2004) the risk premiums have varied through time, peaking during the third quarter on 2002. We have calculated the risk premiums for individual years in the same way that they are calculated for the whole 1996 to 2003 in Table 2. We find that they increase steadily until 2002 and then decline sharply in 2003 and 2004. For

[6] These results are broadly similar to the results mentioned earlier that were calculated by Altman when he compared the return from portfolios of bonds with different credit ratings. Altman also found that the results for B-rated bonds ran counter to the overall pattern.

example, for the A-rating category the risk premium ranges from 35 basis points in 1997 to 119 basis points in 2002.

We now consider a number of the explanations that have been advanced for the average risk premiums in Table 2.

Liquidity

Part of the risk-premium in Table 2 is to compensate bondholders for liquidity risk. Estimates of liquidity risk premiums are difficult to obtain. Fleming (2001) looks at on-the run and off-the run bonds and concludes that the yield on off-the-run bonds for a five to ten year maturity is about 10 basis points higher than the yield on on-the-run bonds on average. The maximum spread observed was about 25 basis points.

Longstaff (2004) measures the size of what he terms the "flight to liquidity premium" on Treasury bonds by comparing yields on RefCorp and Treasury zero-coupon bonds. RefCorp bonds are bonds issued by the Resolution Funding Corporation (RefCorp), a government agency created by the Financial Institutions Reform, Recovery, and Enforcement Act of 1989 (FIRREA). RefCorp bonds have the same credit risk as Treasury bonds since their principal is fully collateralized by Treasury bonds and full payment of coupons is guaranteed by the Treasury under the provisions of FIRREA. RefCorp bonds receive the same tax treatment as U.S. Treasury bonds, but are less liquid. Longstaff finds that the spread of seven-year RefCorp bonds over seven-year Treasuries to be about 10 basis points on average. The highest spread observed was 35 basis points.

There is some evidence that the liquidity premium on corporate bonds may be higher than that on off-the-run Treasury bonds or RefCorp bonds. For example, Driessen (2004) decomposes the expected excess return over Treasuries from corporate bonds into several components and produces higher estimates for the average impact of liquidity than those produced by Fleming and Longstaff.

Based on the available evidence, a reasonable estimate of the average liquidity premium on corporate bonds would seem to be between 10 and 25 basis points. Liquidity is therefore likely to be an important component of the risk premium for bonds with relatively high credit ratings.

Trader's Expectations

Tables 1 and 2 assume that traders use historical default statistics from 1970 to 2003 to determine the probability that bonds will default in the future. In practice traders may be assigning positive subjective probabilities to depression scenarios that are much worse than any seen since 1970. If we use average default statistics for the whole 1920 to 2003 period we find that the historic default probabilities in Table 1 increase for investment grade bonds and decrease for non-investment grade bonds. The real-world default intensities (basis points per year) in Table 1 become 6 (Aaa), 22 (Aa), 30 (A), 77 (Baa), 217 (Ba), 518 (B), and 864 (Caa). The risk premiums (basis points per year) in Table 2 become 36 (Aaa), 34 (Aa), 59 (A), 97 (Baa), 174 (Ba), 231 (B), 760 (Caa).[7] It is plausible that part of the excess return in Table 2 for investment grade bonds arises from the fact that traders do not regard the last 35 years as a good indicator of the future.

Nondiversifiable Risk

Bonds do not default independently of each other. There are periods of time when default rates are very low and periods of time when they are very high. Evidence for this can be obtained by looking at the default rates in different years published by Moody's. Between 1970 and 2003 the default rate per year ranged from a low 0.09% in 1979 to a high of 3.81% in 2001. The average over the whole period was 1.27%. These results mean that there is systematic risk in bond returns that cannot be diversified away. Bond traders should demand an extra return for bearing this risk. Indeed this is the major theoretical justification for the excess returns in Table 2.

[7] It is interesting to note that B-bonds no longer run counter to the overall pattern when real-world probabilities are calculated using data over a longer time period.

One source of systematic risk is the systematic risk related to the overall performance of the economy that we see in equity markets. (We will refer to this as the "equity explanation"). As the quality of a bond declines it becomes more like equity. This is consistent with the Merton (1974) model and with real world behavior. As debt becomes more like equity the bond price is more affected by market variables that affect stock prices and the risk premium investors need to earn to compensate for non-diversifiable risks increases. Empirical results are consistent with these observations. Cornell and Green (1991) find low-grade bond portfolios more sensitive to stock market returns than high-grade bond portfolios.[8] For the period they consider (1960 to 1989) the beta for high-grade bonds is 0.25 compared with a beta for low-grade bonds of 0.52. Fama and French estimate betas for bonds with different credit ratings. Their estimates are 0.19 (Aaa), 0.20 (Aa), 0.21 (A), 0.22 (Baa), and 0.30 (lower than Baa).

In interpreting these numbers we should bear in mind that the holders of bonds bear interest rate risk because when interest rates rise (fall) the present value of the bond's cash flows falls (rises) and a negative (positive) return is realized. We will refer to this as the *present value* or *PV* interest rate risk. Interest rates are negatively related to stock market returns and so the PV interest rate risk gives rise to a positive beta. For high quality bonds it is likely that PV interest rate risk is almost the sole explanation of the betas observed. However, the risk premiums in Table 2 do not reflect this risk because they are calculated by comparing two long-term yields.[9]

It seems likely that the equity explanation for the systematic risk of bond returns is an important factor in explaining the excess returns of low quality bonds, but is not likely to be important for Aa and Aaa bonds. To get a very rough indication of the potential magnitude of the effect for non-investment grade bonds we note that the beta of non-investment grade bonds was 0.11 higher in the Fama and French study than the beta of Aaa bonds.[10] If we attribute all of this to the equity explanation of systematic risk and

[8] Low-grade bond portfolios were portfolios that were more than two thirds invested in bonds rated Baa or less. High-grade bonds results were based on portfolios constructed by Ibbotson & Associates.
[9] An interesting point here is that as yields rise, interest-rate sensitivity falls. High-yield bonds have a smaller PV interest-rate risk than do high quality bonds. This effect lowers the betas of high yield (low quality) bonds.
[10] Cornell and Green's estimate of the beta difference is higher.

assume an excess return on the market over the risk-free rate of, say, 5% we have explained 55 basis points of the excess return in non-investment grade bonds.

Collin-Dufresne et al (2001) find a significant relation between changes in interest rates and changes in credit spreads. This is consistent with Merton's (1974) model since a higher interest rate increases the risk neutral drift of the firm's value process and this in turn reduces the risk neutral probability of default and thus the credit spreads. The credit spreads' sensitivity to interest rate tends to increase as the credit quality declines. This is a different type of interest risk from the PV interest rate risk and should be reflected in the risk premium in Table 2. Since interest rates are negatively correlated with market returns but positively correlated with reducing credit spreads this effect should act to lower the betas of low quality bonds.

Collin-Dufresne et al (2003*)* distinguish between *direct jump* risk premium and *contagion risk* premium*.* The *direct jump* risk premium is associated with the risk that a non-negligible number of firms in the economy default at the same time. *Contagion risk* is the risk that the default of one firm triggers an increase in the risk of default of other firms. Collin-Dufresne et al estimate that contagion risk premium is several times higher than direct jump risk premium. They test empirically the impact of credit events on returns of the Lehman corporate bond index.[11] Their empirical results suggest that large firms' credit events cause a market wide increase in bond spreads. This may indicate that credit events command a risk premium as they have a 'contagious' effect on credit spreads.

Diversifiable Risks

Another explanation for the credit spread puzzle has been proposed by Amato and Remolona (2003). They argue that it is more difficult to diversify risks in a corporate bond portfolio than in an equity portfolio. This is because the returns from corporate bonds are highly skewed with limited upside portfolio. In practice, many bond portfolios are far from being fully diversified and as a result diversifiable risk may be priced. A related point is that there may be agency costs in that the incentives facing bond portfolio

[11] A credit event is defined as an increase in a bond spread of more than 200 bps.

managers may lead them to be less than fully diversified--particularly if this means increasing expected default losses. Also, there may be supply/demand effects. A large new issue of a telecommunications bond for example will increase the supply of telecommunications bonds and may increase the yield on that bond and all other telecommunications bonds.

Conclusions

In comparing Tables 1 and 2 we see that large differences between real-world and risk-neutral default probability estimates translate into relatively modest (but non-negligible) premiums demanded by bond traders for the risks they are bearing. The risk premiums for non-investment grade bonds can be interpreted as compensation for the non-diversifiable risks being born by bondholders. The risk premiums for investment grade bonds are more difficult to explain and seem high relative to the (real-world) expected losses on these bonds. There are a number of possible explanations. The market may be recognizing and pricing the contagion risk identified by Collin-Dufresne et al (2003). The liquidity risk premium on corporate bonds may be higher than some of the estimates that have been made (perhaps over 30 basis points for Aaa bonds and even more for lower grade bonds). Diversifiable risk may be priced. There may be agency costs, supply/demand effects or other institutional factors. There is currently a great deal of research being carried out in this area and hopefully a clear picture of the relative importance of different factors will emerge in the future.

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