MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

October/November 2006

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **3** printed pages and **1** blank page.

[Turn over

1 Find the set of values of x satisfying the inequality $|3^x - 8| < 0.5$, giving 3 significant figures in your answer. [4]

2 Solve the equation

$$\tan x \tan 2x = 1,$$

giving all solutions in the interval $0° < x < 180°$. [4]

3 The curve with equation $y = 6e^x - e^{3x}$ has one stationary point.

   (i) Find the x-coordinate of this point. [4]

   (ii) Determine whether this point is a maximum or a minimum point. [2]

4 Given that $y = 2$ when $x = 0$, solve the differential equation

$$y\frac{dy}{dx} = 1 + y^2,$$

obtaining an expression for y^2 in terms of x. [6]

5   **(i)** Simplify $\left(\sqrt{(1+x)} + \sqrt{(1-x)}\right)\left(\sqrt{(1+x)} - \sqrt{(1-x)}\right)$, showing your working, and deduce that

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}} = \frac{\sqrt{(1+x)} - \sqrt{(1-x)}}{2x}.$$ [2]

   (ii) Using this result, or otherwise, obtain the expansion of

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}}$$

   in ascending powers of x, up to and including the term in x^2. [4]

6 The equation of a curve is $x^3 + 2y^3 = 3xy$.

   (i) Show that $\dfrac{dy}{dx} = \dfrac{y - x^2}{2y^2 - x}$. [4]

   (ii) Find the coordinates of the point, other than the origin, where the curve has a tangent which is parallel to the x-axis. [5]

7 The line l has equation $\mathbf{r} = \mathbf{j} + \mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation $x + 2y + 3z = 5$.

   (i) Show that the line l lies in the plane p. [3]

   (ii) A second plane is perpendicular to the plane p, parallel to the line l and contains the point with position vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. [6]

8 Let $f(x) = \dfrac{7x + 4}{(2x + 1)(x + 1)^2}$.

    (i) Express $f(x)$ in partial fractions.   [5]

    (ii) Hence show that $\displaystyle\int_0^2 f(x)\,dx = 2 + \ln\frac{5}{3}$.   [5]

9 The complex number u is given by
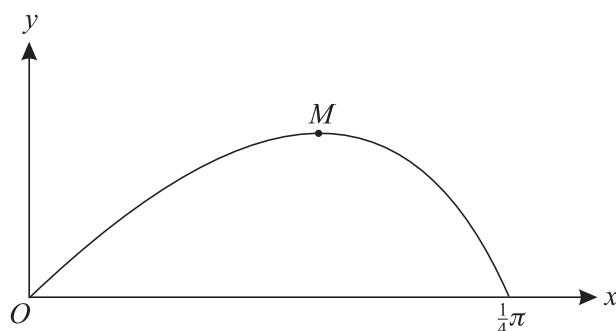
$$u = \frac{3 + i}{2 - i}.$$

    (i) Express u in the form $x + iy$, where x and y are real.   [3]

    (ii) Find the modulus and argument of u.   [2]

    (iii) Sketch an Argand diagram showing the point representing the complex number u. Show on the same diagram the locus of the point representing the complex number z such that $|z - u| = 1$.   [3]

    (iv) Using your diagram, calculate the least value of $|z|$ for points on this locus.   [2]

10

The diagram shows the curve $y = x \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$. The point M is a maximum point.

    (i) Show that the x-coordinate of M satisfies the equation $1 = 2x \tan 2x$.   [3]

    (ii) The equation in part **(i)** can be rearranged in the form $x = \frac{1}{2}\tan^{-1}\left(\dfrac{1}{2x}\right)$. Use the iterative formula

$$x_{n+1} = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x_n}\right),$$

    with initial value $x_1 = 0.4$, to calculate the x-coordinate of M correct to 2 decimal places. Give the result of each iteration to 4 decimal places.   [3]

    (iii) Use integration by parts to find the exact area of the region enclosed between the curve and the x-axis from 0 to $\frac{1}{4}\pi$.   [5]

4

BLANK PAGE