CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 Carry out division or equivalent at least as far as two terms of quotient M1
Obtain quotient $2x-4$ A1
Obtain remainder 8 A1 [3]

2 Obtain $1-x$ as first two terms of $(1+2x)^{-\frac{1}{2}}$ B1
Obtain $+\frac{3}{2}x^2$ or unsimplified equivalent as third term of $(1+2x)^{-\frac{1}{2}}$ B1
Multiply $1+3x$ by attempt at $(1+2x)^{-\frac{1}{2}}$, obtaining sufficient terms M1
Obtain final answer $1+2x-\frac{3}{2}x^2$ A1 [4]

3 State or imply correct form $\frac{A}{x}+\frac{Bx+C}{x^2+1}$ B1
Use any relevant method to find at least one constant M1
Obtain $A=2$ A1
Obtain $B=5$ A1
Obtain $C=-3$ A1 [5]

4 **(i)** Either State or imply non-modular equation $(4x-1)^2=(x-3)^2$ or pair of linear equations $4x-1=\pm(x-3)$ B1
Solve a three-term quadratic equation or two linear equations M1
Obtain $-\frac{2}{3}$ and $\frac{4}{5}$ A1

Or Obtain value $-\frac{2}{3}$ from inspection or solving linear equation B1
Obtain value $\frac{4}{5}$ similarly B2 [3]

(ii) State or imply at least $4^y=\frac{4}{5}$, following a positive answer from part (i) B1√
Apply logarithms and use $\log a^b=b\log a$ property M1
Obtain –0.161 and no other answer A1 [3]

5 **(i)** Use correct quotient rule or equivalent M1
Obtain $\frac{(1+e^{2x})2x-(1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent A1
Substitute $x=0$ and obtain $-\frac{1}{2}$ or equivalent A1 [3]

(ii) Differentiate y^3 and obtain $3y^2\frac{dy}{dx}$ B1
Differentiate $5xy$ and obtain $5y+5x\frac{dy}{dx}$ B1
Obtain $6x^2+5y+5x\frac{dy}{dx}+3y^2\frac{dy}{dx}=0$ B1

Substitute $x = 0, y = 2$ to obtain $-\frac{5}{6}$ or equivalent following correct work — B1 [4]

6 **(i)** State or imply A is (1, 4, –2) — B1
State or imply $\overrightarrow{QP} = 12\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$ or equivalent — B1
Use QP as normal and A as mid-point to find equation of plane — M1
Obtain $12x + 6y - 6z = 48$ or equivalent — A1 [4]

(ii) Either: State equation of PB is $\mathbf{r} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} + \lambda\mathbf{i}$ — B1
Set up and solve a relevant equation for λ. — M1
Obtain $\lambda = -9$ and hence B is (–2, 7, –5) — A1
Use correct method to find distance between A and B. — M1
Obtain 5.20 — A1

Or: Obtain 12 for result of scalar product of QP and $\mathbf{i}$ or equivalent — B1
Use correct method involving moduli, scalar product and cosine to find angle APB — M1
Obtain $35.26°$ or equivalent — A1
Use relevant trigonometry to find AB — M1
Obtain 5.20 — A1 [5]

7 **(a)** State or imply $3a + 3b\mathrm{i} + 2\mathrm{i}(a - b\mathrm{i}) = 17 + 8\mathrm{i}$ — B1
Consider real and imaginary parts to obtain two linear equations in a and b — M1*
Solve two simultaneous linear equations for a or b — M1 (dep*)
Obtain $7 - 2\mathrm{i}$ — A1 [4]

(b) Either: Show or imply a triangle with side 2 — B1
State at least two of the angles $\frac{1}{4}\pi, \frac{2}{3}\pi$ and $\frac{1}{12}\pi$ — B1
State or imply argument is $\frac{1}{4}\pi$ — B1
Use sine rule or equivalent to find r — M1
Obtain $6.69\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$ — A1

Or: State $y = x$. — B1
State $y = \frac{1}{\sqrt{3}}x + 2$ or $\frac{\sqrt{3}}{2} = \frac{x}{\sqrt{x^2 + (y-2)^2}}$ or $\frac{1}{2} = \frac{y-2}{\sqrt{x^2 + (y-2)^2}}$ — B1
State or imply argument is $\frac{\pi}{4}$ — B1
Solve for x or y. — M1
Obtain $6.69e^{\frac{1}{4}\pi i}$ — A1 [5]

8 **(a)** Carry out integration by parts and reach $ax^2 \ln x + b\int \frac{1}{2}x^2 \mathrm{d}x$ — M1*
Obtain $2x^2 \ln x - \int \frac{1}{x} . 2x^2 \, \mathrm{d}x$ — A1
Obtain $2x^2 \ln x - x^2$ — A1
Use limits, having integrated twice — M1 (dep*)
Confirm given result $56 \ln 2 - 12$ — A1 [5]

(b) State or imply $\frac{du}{dx} = 4\cos 4x$ B1

Carry out complete substitution except limits M1

Obtain $\int(\frac{1}{4} - \frac{1}{4}u^2)\,du$ or equivalent A1

Integrate to obtain form $k_1u + k_2u^3$ with non-zero constants k_1, k_2 M1

Use appropriate limits to obtain $\frac{11}{96}$ A1 [5]

9 **(i)** State or imply $R = 5$ B1

Use relevant trigonometry to find α M1

Obtain $\alpha = 0.6435$ A1 [3]

(ii) **(a)** Carry out appropriate method to find one value in given range M1

Obtain 1.80 A1

Carry out appropriate method to find second value in given range M1

Obtain 5.77 and no other value A1 [4]

(b) Express integrand as $k\sec^2(\theta - \text{their } \alpha)$ for any constant k M1

Integrate to obtain result $k\tan(\theta - \text{their } \alpha)$ A1

Obtain correct answer $2\tan(\theta - 0.6435)$ A1 [3]

10 **(i)** State $\frac{dV}{dt} = 80 - kV$ B1

Correctly separate variables and attempt integration of one side M1

Obtain $a\ln(80 - kV) = t$ or equivalent M1*

Obtain $-\frac{1}{k}\ln(80 - kV) = t$ or equivalent A1

Use $t = 0$ and $V = 0$ to find constant of integration or as limits M1 (dep*)

Obtain $-\frac{1}{k}\ln(80 - kV) = t - \frac{1}{k}\ln 80$ or equivalent A1

Obtain given answer $V = \frac{1}{k}(80 - 80e^{-kt})$ correctly A1 [7]

(ii) Use iterative formula correctly at least once M1

Obtain final answer 0.14 A1

Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145) A1 [3]

(iii) State a value between 530 and 540 cm^3 inclusive B1

State or imply that volume approaches 569 cm^3 (allowing any value between 567 and 571 inclusive) B1 [2]