



Cambridge International Examinations

Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	
MATHEMATICS			9709/31
Paper 3 Pure Mather	natics 3 (P3)	Oc	tober/November 2017
			1 hour 45 minutes
Candidates answer or	n the Question Paper.		
Additional Materials:	List of Formulae (MF9)		

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

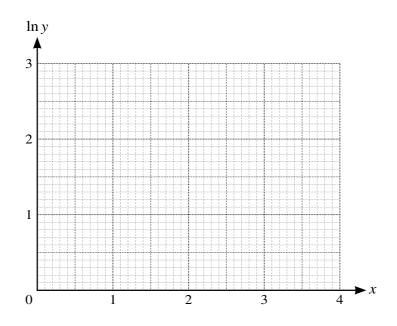


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Two variable quantities x and y are believed to satisfy an equation of the form $y = C(a^x)$, where C and a are constants. An experiment produced four pairs of values of x and y. The table below gives the corresponding values of x and y.

x	0.9	1.6	2.4	3.2
ln y	1.7	1.9	2.3	2.6

By plotting $\ln y$ against x for these four pairs of values and drawing a suitable straight line, estimate the values of C and a. Give your answers correct to 2 significant figures. [5]



Show by calculation that o	α lies between 2 and 3.	
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Two iterative formulae, A and B, derived from this equation are as follows:

$$x_{n+1} = (3x_n + 7)^{\frac{1}{3}},\tag{A}$$

$$x_{n+1} = (3x_n + 7)^{\frac{1}{3}},$$
 (A)
 $x_{n+1} = \frac{x_n^3 - 7}{3}.$ (B)

Each formula is used with initial value $x_1 = 2.5$.

(ii)	Show that one of these formulae produces a sequence which fails to converge, and use the othe formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

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(ii) Hence sketch the graph of $y = \tan(45^\circ + x) + \tan(45^\circ - x)$ for $0^\circ \le x \le 90^\circ$. [3]

5 The equation of a curve is $2x^4 + xy^3 + y^4 = 10$.

(i)	Show that $\frac{dy}{dx} = \frac{dy}{dx}$	$-\frac{8x^3 + y^3}{3xy^2 + 4y^3}.$				[4]
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and find the coordinates of these points.	[4

	The variables x and y satisfy the differential equation
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4\cos^2 y \tan x,$
	for $0 \le x < \frac{1}{2}\pi$, and $x = 0$ when $y = \frac{1}{4}\pi$. Solve this differential equation and find the value of x when $y = \frac{1}{3}\pi$.
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7	(a)	The complex number u is given by $u = 8 - 15i$. Showing all necessary working, find the two square roots of u . Give answers in the form $a + ib$, where the numbers a and b are real and exact. [5]

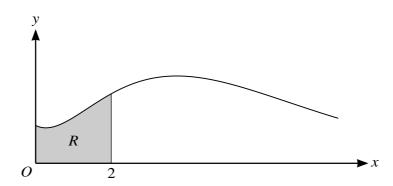
(b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z-2-\mathrm{i}| \leqslant 2$ and $0 \leqslant \arg(z-\mathrm{i}) \leqslant \frac{1}{4}\pi$. [4]

8	Let $f(r)$ =	$4x^2 + 9x - 8$
o	Let $I(x)$ –	$\frac{4x + 9x - 8}{(x+2)(2x-1)}$

(i) Express $f(x)$ in the f	$\operatorname{orm} A + \frac{B}{x+2} + \frac{C}{2x-1}.$	[4]

Hence show that $\int_{1}^{4} f(x) dx = 6 + \frac{1}{2} \ln(\frac{16}{7}).$	[5

9



The diagram shows the curve $y = (1 + x^2)e^{-\frac{1}{2}x}$ for $x \ge 0$. The shaded region R is enclosed by the curve, the x-axis and the lines x = 0 and x = 2.

Find the exact values of the <i>x</i> -coordinates of the stationary points of the curve.	[4]
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snow that the exact v	alue of the area of R is	$\frac{16-\frac{1}{e}}{e}$.	[5
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1)	Show that the lines do not intereset	
	Show that the lines do not intersect.	
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i)	Calculate the acute angle between the directions of the lines.	
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to both l and m . Give your answer in the form $ax + by + cz = d$.	
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