UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2006 question paper

9709 MATHEMATICS

9709/03

Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

The grade thresholds for various grades are published in the report on the examination for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses.

CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2006 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.



Mark Scheme Notes

Marks are of the following three types:

- Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
 B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.



The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally	v acceptable)
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- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
- CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
- CWO Correct Working Only often written by a 'fortuitous' answer
- ISW Ignore Subsequent Working
- MR Misread
- PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)
- SOS See Other Solution (the candidate makes a better attempt at the same question)
- SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.



Page 4	Mark Scheme	Syllabus	Paper
	GCE A/AS LEVEL - OCT/NOV 2006	9709	03

1	EITHE	R: State or imply non-modular inequality $-0.5 < 3^x - 8 < 0.5$, or $(3^x - 8)^2 < (0.5)^2$, or	Di	
		corresponding pair of linear equations or quadratic equation	B1	
		Use correct method for solving an equation of the form $3^{N} = a$, where $a > 0$	MI	
		Obtain critical values 1.83 and 1.95, or exact equivalents State correct answer $1.83 \le x \le 1.95$	A1	
	OB	Use correct method for solving an equation of the form $3^x = a$, where $a > 0$		
	OR:	Obtain one critical value, e.g. 1.95, or exact equivalent	M1 A1	
		Obtain the other critical value 1.83, or exact equivalent	Al	
		State correct answer $1.83 \le x \le 1.95$	A1	4
		[Do not condone \leq for \leq . A!low final answer given in the form 1.83 \leq x, (and) x \leq 1.95.]		
		[Exact equivalents must be in terms of In or logarithms to base 10.]		
		[SR: Solutions given as logarithms to base 3 can only earn M1 and B1 of the first scheme.]		
2	EITHE	R: Use tan 2A formula and obtain a horizontal equation in tan x	MI	
		Simplify the equation to the form $3\tan^2 x = 1$, or equivalent	A1	
		Obtain answer 30°	A1	
		Obtain second answer 150° and no others in the range	A1	
	OR:	Use sin 24 and cos 24 formulae and obtain a horizontal equation in sin x or cos x	M1	
		Simplify the equation to $4\sin^2 x = 1, 4\cos^2 x = 3$, or equivalent	Al	
		Obtain answer 30°	A1	
		Obtain second answer 150° and no others in the range [Ignore answers outside the given range.]	A1	4
		[Treat answers in radians as a MR and deduct one mark from the marks for the angles.]		
		[Methods leading to an equation in cos 3x or cos 2x, or to the equality of two tangents		
		can also earn M1A1, and then A1 + A1 for 30° and 150° only.]		
		[SR: If the answer 30° is found by inspection or from a graph, and is exactly verified, award	d B2.	
		If a second answer 150° is found and verified, and no others stated, award B2.]		
3	(i) Stat	e derivative is $6 e^x - 3 e^{3x}$	B1	
		HER: Equate derivative to zero and simplify to an equation of the form $e^{2\pi} = a$	M1*	
			M1(de	ep*)
		Obtain answer $x = \frac{1}{2} \ln 2$, or equivalent (0.347, or 0.346, or 0.35)	A1	
	OR		M1*	
		Solve the linear equation for x	M1(de	(p*)
		Obtain answer $x = \frac{1}{2} \ln 2$, or equivalent (0.347, or 0.346, or 0.35)	A1	4
		ry out a method for determining the nature of a stationary point	M1	
	Sho	w that the point is a maximum with no errors seen	A1	2
4	Separat	e variables correctly and attempt to integrate one side	MI	
	Obtain terms $\frac{1}{2} \ln(1 + y^2)$ and x , or equivalent A1			
	or equi	to a constant or use limits $x = 0$, $y = 2$ with a solution containing terms $k \ln(1 + y^2)$ and x , valent	M1	
	-	any correct form of solution, e.g. $\frac{1}{2} \ln(1 + y^2) = x + \frac{1}{2} \ln 5$	A1	
		tge and obtain $y^2 = 5e^{2x} - 1$, or equivalent	A1	6

Page 5	Mark Scheme	Syllabus	Paper
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- (i) Simplify product and obtain (1+x) (1-x) **B1** Complete the proof of the given result with no errors seen **B**1 2 (ii) Use correct method to obtain the first two terms of the expansion of $\sqrt{1+x}$ or $\sqrt{1-x}$ M1 EITHER: Obtain any correct unsimplified expansion of the numerator of the RHS of the identity up to the terms in x^3 A1 Obtain final answer with constant term $\frac{1}{2}$ A1 Obtain term $\frac{1}{16}x^2$ and no term in x A1 OR: Obtain any correct unsimplified expansion of the denominator of the LHS of the identity up to the terms in x^2 AI Obtain final answer with constant term 1/2 A1 Obtain term $\frac{1}{16}x^2$ and no term in x [Symbolic binomial coefficients are not sufficient for the M1. Allow two correct separate expansions to earn the first A1 if the context is clear and appropriate.] [Allow the use of Maclaurin, giving M1A1 for $f(0) = \frac{1}{2}$ and f'(0) = 0, A1 for $f''(0) = \frac{1}{8}$, and A1 for obtaining the correct final answer.]
- (i) State $2(3y^2)\frac{dy}{dx}$ as derivative of $2y^3$, or equivalent Bl State $3x \frac{dy}{dx} + 3y$ as derivative of 3xy, or equivalent **B**1 Solve for dy MI Obtain given answer correctly AI [The M1 is dependent on at least one of the B marks being obtained.] (ii) State or imply that the coordinates satisfy $y-x^2=0$ **B1** Obtain an equation in x (or in v) M1 Solve and obtain x = 1 only (or y = 1 only) A1Substitute x- (or y-)value in $y-x^2=0$ or in the equation of the curve M1 Obtain y = 1 only (or x = 1 only) A1 [SR: If B1 is earned and (1, 1) stated to be the only solution with no other evidence, award B2. If the point is also shown to lie on the curve award a further B2.]

Page 6	Mark Scheme	Syllabus	Paper
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Substitute in LHS of plane equation Verify that the equation is satisfied OR: State or imply the plane has equation \mathbf{r} . ($\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$) = 5, or equivalent B1 Substitute for \mathbf{r} in LHS and expand the scalar product M1 Verify that the equation is satisfied A1 OR: Verify that a point of I lies on the plane B1 Prind a second point or I and substitute its coordinates in the equation of p M1 Verify second point, e, e . ($1, -1, 2$) lies on the plane B1 Prind a second point or I and substitute its coordinates in the equation of p M1 Verify secand point, e, e . ($1, -1, 2$) lies on the plane B1 Promote B1 B1 Prind a second point or I and substitute its coordinates in the equation of p M1 Verify scalar product is zero and I is parallel to p M1 Verify scalar product of a direction vector of I with a vector normal to I M1 Verify scalar product of relevant vectors to form an equation in $a, b, c, e, e, e, a-2b+c=0$ or $a+2b+3c=0$ State two correct equations in a, b, c M1 M1 M1 M1 Substitute correctly in I	7	(i)	EITHER:	State or imply general point of I has coordinates $(s, 1-2s, 1+s)$, or equivalent	В1	
OR: State or imply the plane has equation \mathbf{r} . $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = 5$, or equivalent Substitute for \mathbf{r} in L18 and expand the scalar product \mathbf{r} or \mathbf{r} in L18 and expand the scalar product \mathbf{r} or \mathbf{r} in L18 and expand the scalar product \mathbf{r} or \mathbf{r} in L18 and \mathbf{r} in A1 OR: Verify that a point of I lies on the plane B1 Find a second point of I and substitute its coordinates in the equation of P M1 Verify second point, $e, e, (1, -1, 2)$ lies on the plane B1 Form scalar product of a direction vector of I with a vector normal to P M1 Verify scalar product of a direction vector of I with a vector normal to P M1 Verify scalar product is zero and I is parallel to P or I is a parallel to I or I is a parallel I or I in I is a parallel I or I is a parallel I or I in I in I is a parallel I or I in I in I in I in I is a parallel I or I in I in I is a parallel I in					MI	
Substitute for r in LHS and expand the scalar product Verify that the equation is satisfied OR: Verify that a point of I lies on the plane Find a second point or I and substitute its coordinates in the equation of P MI Verify second point of I lies on the plane Form scalar product of a direction vector of I with a vector normal to P MI Verify scalar product of a direction vector of I with a vector normal to P MI Verify scalar product of relevant vectors to form an equation in $a, b, c, e, e, a-2b+c=0$ State two correct equations in a, b, c Solve simultaneous equations and find one ratio, e.g. $a:b$ Obtain $a:b:c=4:1:-2$, or equivalent Substitute correctly in $4x+y-2z=d$ to find d Obtain $a:b:c=4:1:-2$, or equivalent Obtain $a:b:c=4:1:-2:-1$, or equivalent Obtain correct product, $a:b=1:-1$, $a:b=1:$					A1	
Verify that a point of I lies on the plane Find a second point of I lies on the plane Find a second point of I lies on the plane Find a second point of I and substitute its coordinates in the equation of P Verify second point, e.g., $(1, -1, 2)$ lies on the plane OR: Verify that a point of I lies on the plane Form scalar product of a direction vector of I with a vector normal to P Werify scalar product is zero and I is parallel to P (ii) EITHER: Use scalar product of relevant vectors to form an equation in $a, b, c, e.g.$, $a-2b+c=0$ or $a+2b+3c=0$ State two correct equations in a, b, c Solve simultaneous equations and find one ratio, e.g., $a:b$ Obtain $a:b:c=4:1:-2, c$ requivalent Substitute correctly in $4x+y-2z=1$ to find d Obtain $a:b:c=4:1:-2, c$ requivalent OR: Attempt to calculate vector product of relevant vectors, e.g., $(i-2j+k) \times (i+2j+3k)$ M2 Obtain $a:b:c=4:1:-2, c=4:1:-4$ Substitute correctly in $4x+y-2z=1$, or equivalent OStain correct product, $e.g.$, $e.g.$			OR:			
OR: Verify that a point of I lies on the plane Find a second point of I and substitute its coordinates in the equation of P Werify second point, e_{I} . (1, -1, 2) lies on the plane OR: Verify that a point of I lies on the plane Form scalar product of a direction vector of I with a vector normal to P Verify scalar product is zero and I is parallel to P (ii) EITHER: Use scalar product is zero and I is parallel to P State two correct equations in a, b, c Solve simultaneous equations and find one ratio, e.g. $a:b$ Obtain $a:b:c=4:1:-2$, or equivalent Substitute correctly in $4x+y-2z=d$ to find d Obtain $a:b:c=4:1:-2$, or equivalent OR: Attempt to calculate vector product of relevant vectors, e.g. $(i-2j+k) \times (i+2j+3k)$ Obtain $a:b:c=2-d$ in $a:b:c=2-d$ to find d Obtain correct product, e.g. $-8i-2j+4k$ Substitute correctly in $4x+y-2z=d$ to find d Obtain correct product, e.g. $-8i-2j+4k$ Substitute correctly in $4x+y-2z=d$ to find d Obtain correct product, e.g. $-8i-2j+4k$ Substitute correctly in $4x+y-2z=d$ to find d Obtain and in an d in						
Find a second point on I and substitute its coordinates in the equation of p I						
Verify second point, e.g., $(1,-1,2)$ lies on the plane OR: Verify that a point of I lies on the plane Form scalar product of a direction vector of I with a vector normal to I lies of the plane Form scalar product is zero and I is parallel to I or I and I is parallel to I or I is a scalar product is zero and I is parallel to I or I is a scalar product is zero and I is parallel to I or I is a scalar product of relevant vectors to form an equation in I , I , I , I , I is a solor simultaneous equations and find one ratio, e.g. I is a solor simultaneous equations and find one ratio, e.g. I is a solor simultaneous equations and find one ratio, e.g. I is a solor I in I in I in I is a substitute correctly in I at I is I in I			OR:			
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Form scalar product of a direction vector of I with a vector normal to P Verify scalar product is zero and I is parallel to P Verify scalar product is zero and I is parallel to P Verify scalar product of relevant vectors to form an equation in a, b, c , e.g. $a-2b+c=0$ or $a+2b+3c=0$ State two correct equations in a, b, c A1 Solve simultaneous equations and find one ratio, e.g. $a:b$ P M1(dep)* Obtain $a:b:c=4:1:-2$, or equivalent A1 Substitute correctly in $A+y-D=2=1$ to find A Obtain equation $A+y-D=1$ and $A+y-$			-0.0			
(ii) EITHER: Use scalar product is zero and I is parallel to p or $a + 2b + 3c = 0$ or $a - 2b + 3c = 0$ State two correct equations in a, b, c All Solve simultaneous equations and find one ratio, e.g. $a : b$ M1(dep)* Obtain $a : b : c = 4 : 1 : -2$, or equivalent All Substitute correctly in $4x + y - 2z = 1$ to find d M1 Obtain equation $4x + y - 2z = 1$, or equivalent All Substitute correctly in $4x + y - 2z = 1$, or equivalent All Obtain 2 correct components of the product All Obtain 2 correct components of the product Obtain 2 correct product, e.g. $-8i - 2j + 4k$ All Substitute correctly in $4x + y - 2z = 1$ to find d M1 Obtain correct product, e.g. $-8i - 2j + 4k$ All Substitute correctly in $4x + y - 2z = 1$ to find d M1 Obtain correct product, e.g. $-8i - 2j + 4k$ All Substitute correctly in $4x + y - 2z = 1$ to find d M1 Obtain correct product, e.g. $-8i - 2j + 4k$ All Substitute correctly in $4x + y - 2z = 1$ to find d M1 Obtain correct product, e.g. $-8i - 2j + 4k$ All Substitute correctly in $4x + y - 2z = 1$ to find d M1 Obtain correctly in $4x + y - 2z = 1$ to find d M1 Substitute correctly in $4x + y - 2z = 1$ to find d M1 Substitute correctly in $4x + y - 2z = 1$ to find d M1 Substitute correctly in $4x + y - 2z = 1$ to find d M1 Substitute correctly in d All Public Public Substitute correctly in d All Public Substitute Correctly All Public Substitute Correctly in d All Public Substitute Correctly in d All Public Substitute Correctly All Public Substitute Cor			OR:			
(ii) EITHER: Use scalar product of relevant vectors to form an equation in $a, b, c, e.g.$ $a-2b+c=0$ or $a+2b+3c=0$ M1* State two correct equations in a, b, c A1 State two correct equations and find one ratio, e.g. $a:b$ M1(dep)* Obtain $a:b:c=4:1:-2$, or equivalent A1 Substitute correctly in $4x+y-2z=d$ to find d M1 Obtain equation $4x+y-2z=1$, or equivalent A1 OR: Attempt to calculate vector product of relevant vectors, e.g. $(i-2j+k)x(i+2j+3k)$ M2 Obtain 2 correct components of the product Obtain our correct product, e.g. $-8i-2j+4k$ A1 Substitute correctly in $4x+y-2z=d$ to find d M1 Obtain equation $4x+y-2z=1$, or equivalent IsR: If the outcome of the vector product is the negative of the correct answer allow the final mark to be available, i.e. M2AOAOMIA lis possible.] OR: Attempt to form 2-parameter equation for the plane with relevant vectors M2 State a correct equation, e.g. $x=2i+j+4k+\lambda(i-2j+k)+\mu(i+2j+3k)$ A1 State 3 equations in x, y, z, λ, μ Eliminate λ and μ Obtain equation $4x+y-2z=1$, or equivalent A1 State 3 equations in x, y, z, λ, μ Eliminate λ and μ Obtain equation $4x+y-2z=1$, or equivalent A1 Obtain one of the values $\lambda = 2, B=-1, C=3$ A1 Obtain the remaining two values A1 A1 Obtain the remaining two values A2 A2 A3 A4 Obtain one of the values $\lambda = 2, B=-1, C=3$ A1 Obtain the remaining two values A3 A4 Obtain one of the values $\lambda = 2, B=-1, C=3$ A1 Obtain the remaining two values A1 A4 Obtain the remaining two values A1 A4 Use any relevant method to obtain a constant Obtain one of the values $\lambda = 2, D=-1, E=2$ Obtain the remaining two values A1 A4 Use any relevant method to obtain a constant Obtain one of the values $\lambda = 2, D=-1, E=2$ Obtain the remaining two values A1 A4 Use limits correctly, having integrated all the partial fractions Obtain given answer following full and exact working The ft. is on λ , β , C etc.]						
State two correct equations in a, b, c Solve simultaneous equations and find one ratio, e.g. $a:b$ MI(dep)* Obtain $a:b:c=4:1:-2$, or equivalent A1 Substitute correctly in $4x+y-2z=d$ to find d Obtain equation $4x+y-2z=1$, or equivalent OR: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i}-2\mathbf{j}+\mathbf{k})\times(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ M2 Obtain 2 correct components of the product Obtain correct product, e.g. $-8\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ Substitute correctly in $4x+y-2z=d$ to find d Obtain equation $4x+y-2z=d$ to find d Obtain correct product, e.g. $-8\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ Substitute correctly in $4x+y-2z=d$ to find d Obtain equation $4x+y-2z=d$ to find d Obtain equation $4x+y-2z=d$ to find d Obtain correct product, e.g. $-8\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ Substitute correctly in $4x+y-2z=d$ to find d Obtain equation $4x+y-2z=1$, or equivalent [SR: If the outcome of the vector product is the negative of the correct answer allow the final mark to be available, i.e. M2AOA0M1A is possible.] OR: Attempt to form 2 -parameter equation for the plane with relevant vectors State a correct equation, e.g. $p=2\mathbf{i}+\mathbf{j}+4\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+\mathbf{k})+\mu(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ A1 State 3 equations in x, y, z, λ, μ Eliminate λ and μ Obtain equation $4x+y-2z=1$, or equivalent A1 Obtain equation $4x+y-2z=1$, or equivalent A1 Obtain equation $4x+y-2z=1$, or equivalent A1 Obtain the remaining two values A2 Obtain the remaining two values A3 A4 A5 A6 B1 OR: State or imply $\mathbf{f}(x) = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ Use any relevant method to obtain a constant Obtain one of the values $A=2$, $B=-1$, $C=3$ Obtain the remaining two values A1 Obtain one of the values $A=2$, $B=-1$, $C=3$ Obtain the remaining two values A1 A1 A1 Divident in the intermity of the values $A=2$, $A=1$, $A=1$ Obtain the remaining two values A1 A1 A1 Divident in the value in the valu		700	DETERM.			3
State two correct equations in a, b, c Solve simultaneous equations and find one ratio, e.g. $a:b$ $A1$ $A1$ $Obtain a:b:c = 4:1:-2, \text{ or equivalent} Obtain a:b:c = 4:1:-2, \text{ or equivalent} Otain equation 4x + y - 2z = d to find d Otain equation 4x + y - 2z = 1, \text{ or equivalent} OR: Attempt to calculate vector product of relevant vectors, e.g. (\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) OBtain 2 \text{ correct eomponents of the product} Obtain 2 \text{ correct product, e.g.} - 8\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} Substitute correctly in 4x + y - 2z = d to find d Obtain equation 4x + y - 2z = 1, \text{ or equivalent} ISR: If the outcome of the vector product is the negative of the correct answer allow the final mark to be available, i.e. M2\Delta0\Delta0M1A lis possible.] OR: \text{ Attempt to form } 2\text{-parameter equation for the plane with relevant vectors} State a correct equation, e.g., r = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) A1 State 3 \text{ equations in } x, y, z, \lambda, \mu Eliminate \lambda and \mu Obtain equation 4x + y - 2z = 1, \text{ or equivalent} A1 A2 A3 A4 A4 A5 A5 A5 A6 A6 A6 A7 A7 A7 A8 A8 A9 A9 A9 A9 A9 A9 A9 A9$		(II)	EITHER:			
Solve simultaneous equations and find one ratio, e.g. $a:b$ $A1$ Obtain $a:b:c=4:1:-2$, or equivalent $A1$ Substitute correctly in $4x+y-2z=d$ to find d $M1$ Obtain equation $4x+y-2z=1$, or equivalent $A1$ Obtain equation $4x+y-2z=1$, or equivalent $A1$ Obtain 2 correct components of the product $A1$ Obtain correct product, e.g. $-8i-2j+4k$ $A1$ Substitute correctly in $4x+y-2z=1$ of the find $A1$ Obtain correct product, e.g. $-8i-2j+4k$ $A1$ Substitute correctly in $4x+y-2z=1$ or equivalent $A1$ Obtain equation $4x+y-2z=1$ or equivalent $A1$ Substitute correctly in $4x+y-2z=1$ or equivalent $A1$ Substitute correct of the vector product is the negative of the correct answer allow the final mark to be available, i.e. $M2AOAOMIA1$ is possible.] OR: Attempt to form $2-p$ -parameter equation for the plane with relevant vectors $A1$ State 3 equations in $a=1$, $a=1$, $a=1$ and $a=1$ All Eliminate $a=1$ All Pobtain equation $a=1$ and $a=1$ All Obtain one of the values $a=1$ and $a=1$ All Obtain one of the values $a=1$ and $a=1$ All Obtain the remaining two values $a=1$ All Obtain one of the values $a=1$ and $a=1$ All Obtain one of the values $a=1$ and $a=1$ All Obtain one of the values $a=1$ and $a=1$ All Obtain one of the values $a=1$ and $a=1$ All Obtain one of the values $a=1$ and $a=1$ All Obtain one of the values $a=1$ and $a=1$ All Obtain one of the values $a=1$ and $a=1$ All Obtain one of the values $a=1$ and $a=1$ All Obtain one of the values $a=1$ and $a=1$						
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State 3 equations in x , y , z , λ , μ Eliminate λ and μ Obtain equation $4x + y - 2z = 1$, or equivalent 8 (i) EITHER: State or imply $f(x) = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ Use any relevant method to obtain a constant Obtain one of the values $A = 2$, $B = -1$, $C = 3$ Obtain the remaining two values [A correct solution starting with third term $\frac{Cx}{(x+1)^2}$ or $\frac{Cx+D}{(x+1)^2}$ is also possible.] OR: State or imply $f(x) = \frac{A}{2x+1} + \frac{Dx+E}{(x+1)^2}$ Use any relevant method to obtain a constant Obtain one of the values $A = 2$, $D = -1$, $E = 2$ Obtain the remaining two values (ii) Integrate and obtain terms $\frac{1}{2} \cdot 2 \ln(2x+1) - \ln(x+1) - \frac{3}{x+1}$, or equivalent Use limits correctly, having integrated all the partial fractions Obtain given answer following full and exact working [The f.t. is on A , B , C etc.]			OR:		M2	
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Obtain the remaining two values A1 + A1 5 (ii) Integrate and obtain terms $\frac{1}{2} \cdot 2 \ln(2x+1) - \ln(x+1) - \frac{3}{x+1}$, or equivalent B1 $\sqrt{10} + 10 \cdot 10$						
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Use limits correctly, having integrated all the partial fractions Obtain given answer following full and exact working [The f.t. is on A, B, C etc.]		(ii)	Integrate a			
Obtain given answer following full and exact working [The f.t. is on A, B, C etc.]				2.1	3.41	
[The f.t. is on A , B , C etc.]						
					Al	2
ISR: If B, C, or E are omitted, give B1M1in part (i) and B1/B1/M1 in part (ii): may 5/10.1				C, or E are omitted, give B1M1in part (i) and B1√B1√M1 in part (ii): max 5/10.]		

Page 7	Mark Scheme	Syllabus	Paper
	GCE A/AS LEVEL - OCT/NOV 2006	9709	03

9	(i)	EITHER:	Multiply numerator and denominator by $2 + i$, or equivalent Simplify numerator to $5 + 5i$ or denominator to 5	M1 A1	
		OR:	Obtain answer 1 + i Obtain two equations in x and y, and solve for x or for y	A1 M1	
		OK.	Obtain $x = 1$	A1	
			Obtain y = 1	A1	
		OR:	Using correct processes express u in polar form	M1	
			Obtain $u = \sqrt{2}$ (cos 45° + i sin 45°), or equivalent	A1	,
			Obtain answer 1 + i	A1	3
	(ii)		the modulus is $\sqrt{2}$ or 1.41	B1√	
		State that	the argument is 45° or $\frac{1}{4}\pi$ (or 0.785)	B1√	2
	(iii)	Show the	point representing u in a relatively correct position	BI√	
			ircle with centre at the point representing u	BI√	
			or imply the radius is 1	B1	3
	[NB: If the Argand diagram has unequal scales the locus is not circular in appearance, but an ellipse with centre u and equal axes parallel to the axes of the diagram earns B1√, and B1 i both semi-axes are indicated or implied to be equal to 1. In such a situation only award B1√ for a circle with centre u and a horizontal or vertical radius indicated or implied to be 1.]				
	(iv)		complete strategy for calculating min z for the locus	M1	
			swer $\sqrt{2}-1$ (or 0.414) s on the value of u .]	A1√	2
		[
10	(i)	Use prod	uct rule	M1	
			orrect derivative $\cos 2x - 2x \sin 2x$	A1	
	(10)		erivative to zero and obtain given answer correctly	A1	3
	(11)		nal answer 0.43	M1 A1	
			fficient iterations to at least 3d.p. to justify its accuracy to 2 d.p., or show there is a sign	74.1	
			the interval (0.425, 0.435)	A1	3
	(iii)	Attempt	integration by parts and obtain $\pm kx \sin 2x \pm \int l \sin 2x dx$, where $k, l = \frac{1}{2}, 1$, or 2	M1*	
		Obtain 1	$x \sin 2x - \int_{\frac{1}{2}} \sin 2x dx$	A1	
		Obtain ir	definite integral $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x$	A1	
			as $x = 0$ and $x = \frac{1}{4}\pi$ having integrated twice	M1(de	p)*
		Obtain as	nswer $\frac{1}{8}\pi - \frac{1}{4}$, or exact equivalent	A1	5