UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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1 The first term of an arithmetic progression is 61 and the second term is 57. The sum of the first n terms is n. Find the value of the positive integer n. [4]

2 A curve is such that $\frac{dy}{dx} = -\frac{8}{x^3} - 1$ and the point (2, 4) lies on the curve. Find the equation of the curve. [4]

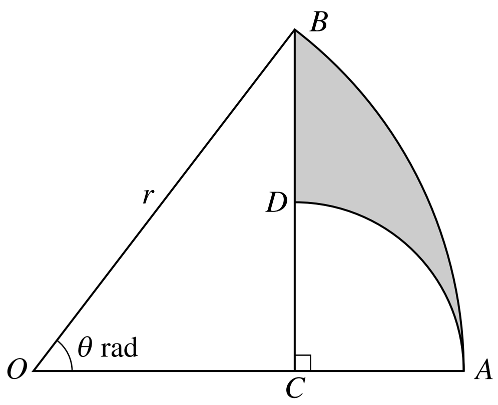
3 An oil pipeline under the sea is leaking oil and a circular patch of oil has formed on the surface of the sea. At midday the radius of the patch of oil is 50 m and is increasing at a rate of 3 metres per hour. Find the rate at which the area of the oil is increasing at midday. [4]

4 **(i)** Find the first 3 terms in the expansion of $(2x - x^2)^6$ in ascending powers of x. [3]

(ii) Hence find the coefficient of x^8 in the expansion of $(2 + x)(2x - x^2)^6$. [2]

5 A curve has equation $y = 2x + \frac{1}{(x-1)^2}$. Verify that the curve has a stationary point at $x = 2$ and determine its nature. [5]

6



The diagram shows a sector OAB of a circle with centre O and radius r. Angle AOB is θ radians. The point C on OA is such that BC is perpendicular to OA. The point D is on BC and the circular arc AD has centre C.

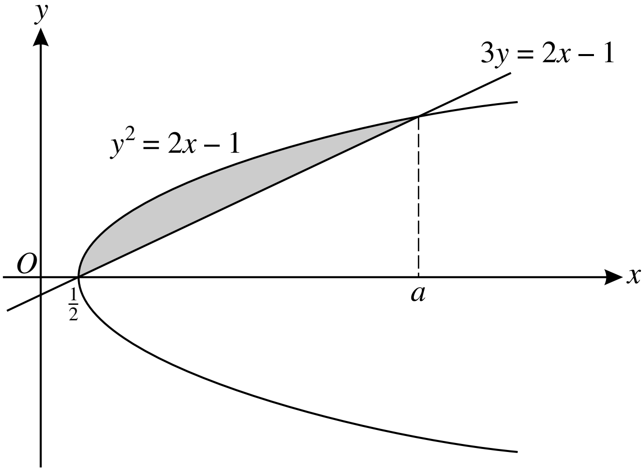
(i) Find AC in terms of r and θ. [1]

(ii) Find the perimeter of the shaded region ABD when $\theta = \frac{1}{3}\pi$ and $r = 4$, giving your answer as an exact value. [6]

7 **(i)** Solve the equation $2\cos^2\theta = 3\sin\theta$, for $0° \leqslant \theta \leqslant 360°$. [4]

(ii) The smallest positive solution of the equation $2\cos^2(n\theta) = 3\sin(n\theta)$, where n is a positive integer, is 10°. State the value of n and hence find the largest solution of this equation in the interval $0° \leqslant \theta \leqslant 360°$. [3]

8



The diagram shows the curve $y^2 = 2x - 1$ and the straight line $3y = 2x - 1$. The curve and straight line intersect at $x = \frac{1}{2}$ and $x = a$, where a is a constant.

(i) Show that $a = 5$. [2]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

9 The position vectors of points A and B relative to an origin O are $\mathbf{a}$ and $\mathbf{b}$ respectively. The position vectors of points C and D relative to O are $3\mathbf{a}$ and $2\mathbf{b}$ respectively. It is given that

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}.$$

(i) Find the unit vector in the direction of $\overrightarrow{CD}$. [3]

(ii) The point E is the mid-point of CD. Find angle EOD. [6]

10 The function f is defined by $\mathrm{f}(x) = 4x^2 - 24x + 11$, for $x \in \mathbb{R}$.

(i) Express $\mathrm{f}(x)$ in the form $a(x - b)^2 + c$ and hence state the coordinates of the vertex of the graph of $y = \mathrm{f}(x)$. [4]

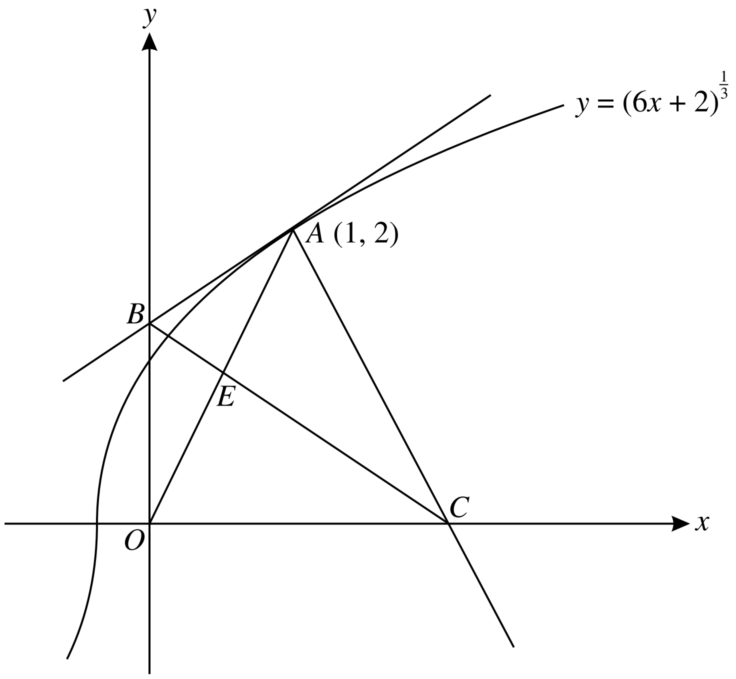
The function g is defined by $\mathrm{g}(x) = 4x^2 - 24x + 11$, for $x \leqslant 1$.

(ii) State the range of g. [2]

(iii) Find an expression for $\mathrm{g}^{-1}(x)$ and state the domain of g^{-1}. [4]

[Question 11 is printed on the next page.]

11



The diagram shows the curve $y = (6x + 2)^{\frac{1}{3}}$ and the point A (1, 2) which lies on the curve. The tangent to the curve at A cuts the y-axis at B and the normal to the curve at A cuts the x-axis at C.

(i) Find the equation of the tangent AB and the equation of the normal AC. [5]

(ii) Find the distance BC. [3]

(iii) Find the coordinates of the point of intersection, E, of OA and BC, and determine whether E is the mid-point of OA. [4]