MATHEMATICS   **9709/72**

Paper 7 Probability & Statistics 2 **(S2)**   **May/June 2012**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
                          Graph Paper
                          List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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1   The number of new enquiries per day at an office has a Poisson distribution. In the past the mean has been 3. Following a change of staff, the manager wishes to test, at the 5% significance level, whether the mean has increased.

    (i) State the null and alternative hypotheses for this test. [1]

The manager notes the number, N, of new enquiries during a certain 6-day period. She finds that $N = 25$ and then, assuming that the null hypothesis is true, she calculates that $P(N \geqslant 25) = 0.0683$.

    (ii) What conclusion should she draw? [2]

2   A population has mean 7 and standard deviation 3. A random sample of size n is chosen from this population.

    (i) Write down the mean and standard deviation of the distribution of the sample mean. [2]

    (ii) Under what circumstances does the sample mean have

        (a) a normal distribution, [1]

        (b) an approximately normal distribution? [1]

3   In a sample of 50 students at Batlin college, 18 support the football club Real Madrid.

    (i) Calculate an approximate 98% confidence interval for the proportion of students at Batlin college who support Real Madrid. [4]

    (ii) Give one condition for this to be a reliable result. [1]

4   Bacteria of a certain type are randomly distributed in the water in two ponds, A and B. The average numbers of bacteria per cm^3 in A and B are 0.32 and 0.45 respectively.

    (i) Samples of $8\,cm^3$ of water from A and $12\,cm^3$ of water from B are taken at random. Find the probability that the total number of bacteria in these samples is at least 3. [3]

    (ii) Find the probability that in a random sample of $155\,cm^3$ of water from A, the number of bacteria is less than 35. [5]

5   Fiona and Jhoti each take one shower per day. The times, in minutes, taken by Fiona and Jhoti to take a shower are represented by the independent variables $F \sim N(12.2, 2.8^2)$ and $J \sim N(11.8, 2.6^2)$ respectively. Find the probability that, on a randomly chosen day,

    (i) the total time taken to shower by Fiona and Jhoti is less than 30 minutes, [4]

    (ii) Fiona takes at least twice as long as Jhoti to take a shower. [4]

6 At a certain shop the weekly demand, in kilograms, for flour is modelled by the random variable X with probability density function given by

$$f(x) = \begin{cases} kx^{-\frac{1}{2}} & 4 \leqslant x \leqslant 25, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

  (i) Show that $k = \frac{1}{6}$. [2]

  (ii) Calculate the mean weekly demand for flour at the shop. [3]

  (iii) At the beginning of one week, the shop has 20 kg of flour in stock. Find the probability that this will not be enough to meet the demand for that week. [2]

  (iv) Give a reason why the model may not be realistic. [1]

7 The weights, X kilograms, of bags of carrots are normally distributed. The mean of X is μ. An inspector wishes to test whether $\mu = 2.0$. He weighs a random sample of 200 bags and his results are summarised as follows.

$$\Sigma x = 430 \qquad \Sigma x^2 = 1290$$

  (i) Carry out the test, at the 10% significance level. [6]

  (ii) You may now assume that the population variance of X is 1.85. The inspector weighs another random sample of 200 bags and carries out the same test at the 10% significance level.

    (a) State the meaning of a Type II error in this context. [1]

    (b) Given that $\mu = 2.12$, show that the probability of a Type II error is 0.652, correct to 3 significant figures. [7]

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