UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/23   Paper 23, maximum raw mark 50

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M   Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
             B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF   Any Equivalent Form (of answer is equally acceptable)

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO   Correct Working Only - often written by a 'fortuitous' answer

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1   A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1   This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 State or imply $y \log 2.8 = x \log 13$ B1

Rearrange into form $y = \dfrac{\log 13}{\log 2.8} x$ or equivalent   B1

Obtain answer $k = 2.49$ B1 [3]

2 **(i)** State or imply correct ordinates 0.27067..., 0.20521..., 0.14936...   B1
Use correct formula, or equivalent, correctly with $h = 0.5$ and three ordinates   M1
Obtain answer 0.21 with no errors seen   A1 [3]

   (ii) Justify statement that the trapezium rule gives an over-estimate   B1 [1]

3 **EITHER** State or imply non-modular inequality $(2x-1)^2 < (x+4)^2$, or corresponding equation
   or pair of linear equations   M1
   Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations   M1
   Obtain critical values –1 and 5   A1
   State correct answer $-1 < x < 5$ A1 [4]
OR Obtain one critical value, e.g. $x = 5$, by solving a linear equation (or inequality) or
   from a graphical method or by inspection   B1
   Obtain the other critical value similarly   B2
   State correct answer $-1 < x < 5$ B1

4 **(a)** Obtain integral $a \sin 2x$ with $a = \pm \left(1, 2 \text{ or } \dfrac{1}{2}\right)$ M1

   Use limits and obtain $\dfrac{1}{2}$ (AG)   A1 [2]

   (b) Use $\tan^2 x = \sec^2 x - 1$ and attempt to integrate both terms   M1
   Obtain $3\tan x - 3x$ A1
   Attempt to substitute limits, using exact values   M1

   Obtain answer $2\sqrt{3} - \dfrac{\pi}{2}$ A1 [4]

5 **(i)** Use product rule   M1
   Obtain correct derivative in any form   A1
   Show that derivative is equal to zero when $x = 3$ A1 [3]

   (ii) Substitute $x = 1$ into gradient function, obtaining $2e^{-1}$ or equivalent   M1
   State or imply required y-coordinate is e^{-1} B1
   Form equation of line through $(1, e^{-1})$ with gradient found (NOT the normal)   M1
   Obtain equation in any correct form   A1 [4]

6 **(i)** Make a recognisable sketch of a relevant graph, e.g. $y = \ln x$ or $y = 2 - x^2$ B1
   Sketch a second relevant graph and justify the given statement   B1 [2]

   (ii) Consider sign of $\ln x - (2 - x^2)$ at $x = 1.3$ and $x = 1.4$, or equivalent   M1
   Complete the argument correctly with appropriate calculations   A1 [2]

(iii) Show that given equation is equivalent to $x = \sqrt{(2 - \ln x)}$ or *vice versa*   B1   [1]

(iv) Use the iterative formula correctly at least once   M1
Obtain final answer 1.31   A1
Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change
in the interval (1.305, 1.315)   B1   [3]

7   **(i)**   Substitute $x = 3$ and equate to 30   M1
Substitute $x = -1$ and equate to 18   M1
Obtain a correct equation in any form   A1
Solve a relevant pair of equations for a or for b M1
Obtain $a = 1$ and $b = -13$ A1   [5]

(ii)   Either show that f(2) = 0 or divide by $(x - 2)$, obtaining a remainder of zero   B1
Obtain quadratic factor $2x^2 + 5x - 3$ B1
Obtain linear factor $2x - 1$ B1
Obtain linear factor $x + 3$ B1
[Condone omission of repetition that $x - 2$ is a factor.]
[If linear factors $2x - 1$, $x + 3$ obtained by remainder theorem or inspection, award B2 + B1.]   [4]

8   **(i)**   Use correct $\sin(A - B)$ and $\cos(A - B)$ formulae   M1
Substitute exact values for $\sin 30°$ etc.   M1
Obtain given answer correctly   A1   [3]

(ii)   State $\sqrt{3} \sin x = \dfrac{1}{2} \sec x$ B1

Rearrange to $\sin 2x = k$, where k is a non-zero constant   M1

Carry out evaluation of $\dfrac{1}{2} \sin^{-1} \left(\dfrac{1}{\sqrt{3}} \right)$ M1

Obtain answer 17.6°   A1
Carry out correct method for second answer   M1
Obtain remaining 3 answers from 17.6°, 72.4°, 197.6°, 252.4° and no others in the
range   A1   [6]
[Ignore answers outside the given range]