Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8253559189

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **February/March 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

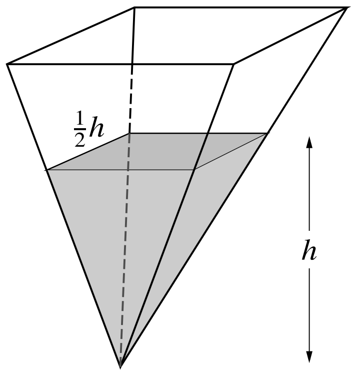
JC17 03_9709_12/RP

CAMBRIDGE International Examinations

1 Find the set of values of k for which the equation $2x^2 + 3kx + k = 0$ has distinct real roots. [4]

2 In the expansion of $\left(\frac{1}{ax} + 2ax^2\right)^5$, the coefficient of x is 5. Find the value of the constant a. [4]

3



The diagram shows a water container in the form of an inverted pyramid, which is such that when the height of the water level is h cm the surface of the water is a square of side $\frac{1}{2}h$ cm.

(i) Express the volume of water in the container in terms of h. [1]

[The volume of a pyramid having a base area A and vertical height h is $\frac{1}{3}Ah$.]

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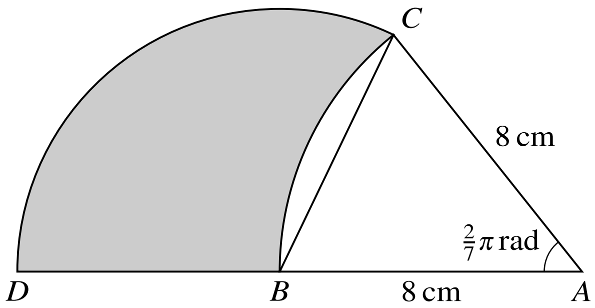
Water is steadily dripping into the container at a constant rate of $20\,\text{cm}^3$ per minute.

(ii) Find the rate, in cm per minute, at which the water level is rising when the height of the water level is 10 cm. [4]

4



In the diagram, $AB = AC = 8$ cm and angle $CAB = \frac{2}{7}\pi$ radians. The circular arc BC has centre A, the circular arc CD has centre B and ABD is a straight line.

(i) Show that angle $CBD = \frac{9}{14}\pi$ radians. [1]

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(ii) Find the perimeter of the shaded region. [5]

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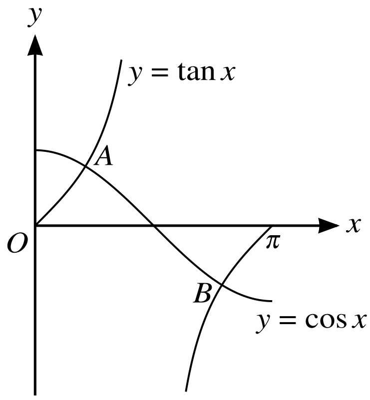

The diagram shows the graphs of $y = \tan x$ and $y = \cos x$ for $0 \leqslant x \leqslant \pi$. The graphs intersect at points A and B.

(i) Find by calculation the x-coordinate of A. [4]

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(ii) Find by calculation the coordinates of B. [3]

6 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = 7\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}.$$

(i) Use a scalar product to find angle OAB. [5]

(ii) Find the area of triangle *OAB*. [2]

7 The function f is defined for $x \geqslant 0$ by $\mathrm{f}(x) = (4x+1)^{\frac{3}{2}}$.

(i) Find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$. [3]

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The first, second and third terms of a geometric progression are respectively $\mathrm{f}(2)$, $\mathrm{f}'(2)$ and $k\mathrm{f}''(2)$.

(ii) Find the value of the constant k. [5]

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8 The functions f and g are defined for $x \geqslant 0$ by

$$\mathrm{f} : x \mapsto 2x^2 + 3,$$
$$\mathrm{g} : x \mapsto 3x + 2.$$

(i) Show that $\mathrm{gf}(x) = 6x^2 + 11$ and obtain an unsimplified expression for $\mathrm{fg}(x)$. [2]

(ii) Find an expression for $(\mathrm{fg})^{-1}(x)$ and determine the domain of $(\mathrm{fg})^{-1}$. [5]

(iii) Solve the equation $\mathrm{gf}(2x) = \mathrm{fg}(x)$. [3]

9 The point $A\,(2,\ 2)$ lies on the curve $y = x^2 - 2x + 2$.

(i) Find the equation of the tangent to the curve at A. [3]

The normal to the curve at A intersects the curve again at B.

(ii) Find the coordinates of B. [4]

The tangents at A and B intersect each other at C.

(iii) Find the coordinates of C. [4]

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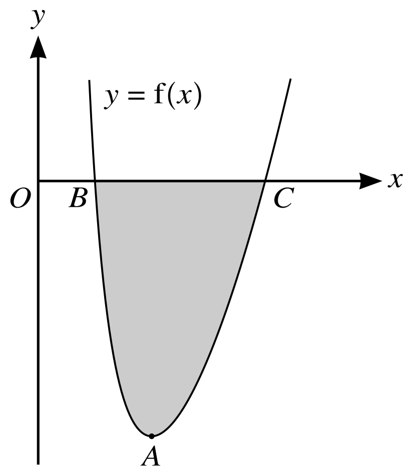

The diagram shows the curve $y = \mathrm{f}(x)$ defined for $x > 0$. The curve has a minimum point at A and crosses the x-axis at B and C. It is given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - \frac{2}{x^3}$ and that the curve passes through the point $\left(4, \frac{189}{16}\right)$.

(i) Find the x-coordinate of A. [2]

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(ii) Find $\mathrm{f}(x)$. [3]

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(iii) Find the x-coordinates of B and C. [4]

[Question 10 (iv) is printed on the next page.]

(iv) Find, showing all necessary working, the area of the shaded region. [4]