

**MATHEMATICS**

**9709/12**

Paper 1 Pure Mathematics 1 (P1)

**October/November 2014**

**1 hour 45 minutes**

Additional Materials:      Answer Booklet/Paper  
                                        Graph Paper  
                                        List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

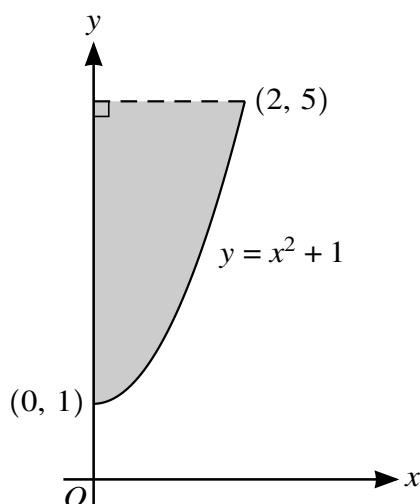
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

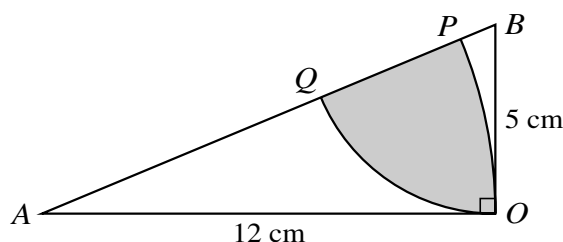
This document consists of 4 printed pages.

1



The diagram shows part of the curve  $y = x^2 + 1$ . Find the volume obtained when the shaded region is rotated through  $360^\circ$  about the **y-axis**. [4]

2



The diagram shows a triangle  $AOB$  in which  $OA$  is 12 cm,  $OB$  is 5 cm and angle  $AOB$  is a right angle. Point  $P$  lies on  $AB$  and  $OP$  is an arc of a circle with centre  $A$ . Point  $Q$  lies on  $AB$  and  $OQ$  is an arc of a circle with centre  $B$ .

(i) Show that angle  $BAO$  is 0.3948 radians, correct to 4 decimal places. [1]

(ii) Calculate the area of the shaded region. [5]

3 (i) Find the first 3 terms, in ascending powers of  $x$ , in the expansion of  $(1 + x)^5$ . [2]

The coefficient of  $x^2$  in the expansion of  $(1 + (px + x^2))^5$  is 95.

(ii) Use the answer to part (i) to find the value of the positive constant  $p$ . [3]

4 A curve has equation  $y = \frac{12}{3 - 2x}$ .

(i) Find  $\frac{dy}{dx}$ . [2]

A point moves along this curve. As the point passes through  $A$ , the  $x$ -coordinate is increasing at a rate of 0.15 units per second and the  $y$ -coordinate is increasing at a rate of 0.4 units per second.

(ii) Find the possible  $x$ -coordinates of  $A$ . [4]

- 5 (i) Show that the equation  $1 + \sin x \tan x = 5 \cos x$  can be expressed as

$$6 \cos^2 x - \cos x - 1 = 0. \quad [3]$$

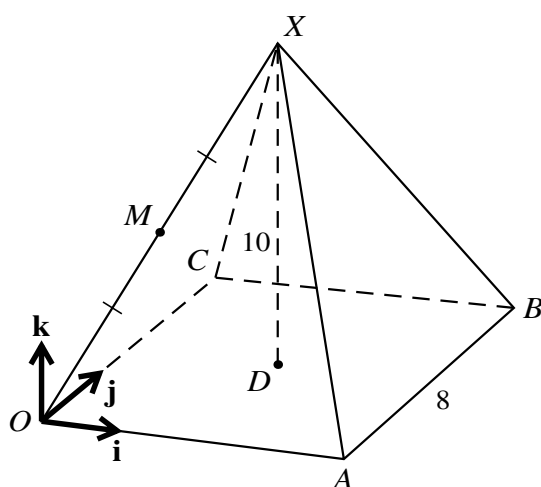
- (ii) Hence solve the equation  $1 + \sin x \tan x = 5 \cos x$  for  $0^\circ \leq x \leq 180^\circ$ . [3]

- 6 The equation of a curve is  $y = x^3 + ax^2 + bx$ , where  $a$  and  $b$  are constants.

- (i) In the case where the curve has no stationary point, show that  $a^2 < 3b$ . [3]

- (ii) In the case where  $a = -6$  and  $b = 9$ , find the set of values of  $x$  for which  $y$  is a decreasing function of  $x$ . [3]

7



The diagram shows a pyramid  $OABCX$ . The horizontal square base  $OABC$  has side 8 units and the centre of the base is  $D$ . The top of the pyramid,  $X$ , is vertically above  $D$  and  $XD = 10$  units. The mid-point of  $OX$  is  $M$ . The unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are parallel to  $\overrightarrow{OA}$  and  $\overrightarrow{OC}$  respectively and the unit vector  $\mathbf{k}$  is vertically upwards.

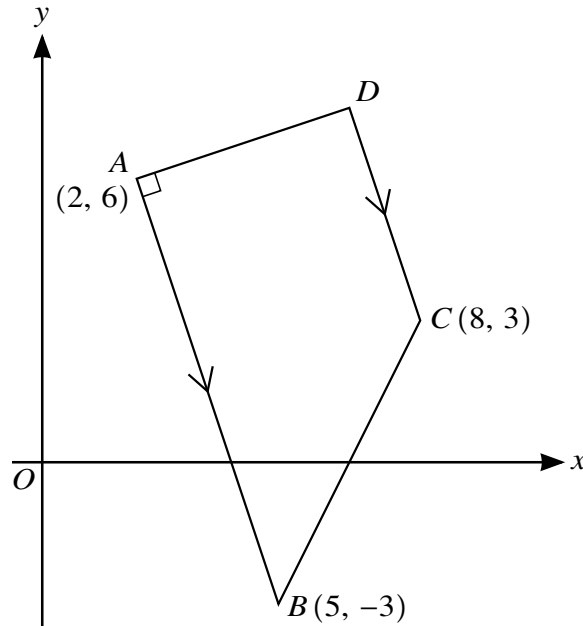
- (i) Express the vectors  $\overrightarrow{AM}$  and  $\overrightarrow{AC}$  in terms of  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ . [3]

- (ii) Use a scalar product to find angle  $MAC$ . [4]

- 8 (a) The sum,  $S_n$ , of the first  $n$  terms of an arithmetic progression is given by  $S_n = 32n - n^2$ . Find the first term and the common difference. [3]

- (b) A geometric progression in which all the terms are positive has sum to infinity 20. The sum of the first two terms is 12.8. Find the first term of the progression. [5]

[Questions 9, 10 and 11 are printed on the next page.]



The diagram shows a trapezium  $ABCD$  in which  $AB$  is parallel to  $DC$  and angle  $BAD$  is  $90^\circ$ . The coordinates of  $A$ ,  $B$  and  $C$  are  $(2, 6)$ ,  $(5, -3)$  and  $(8, 3)$  respectively.

(i) Find the equation of  $AD$ . [3]

(ii) Find, by calculation, the coordinates of  $D$ . [3]

The point  $E$  is such that  $ABCE$  is a parallelogram.

(iii) Find the length of  $BE$ . [2]

10 A curve is such that  $\frac{d^2y}{dx^2} = \frac{24}{x^3} - 4$ . The curve has a stationary point at  $P$  where  $x = 2$ .

(i) State, with a reason, the nature of this stationary point. [1]

(ii) Find an expression for  $\frac{dy}{dx}$ . [4]

(iii) Given that the curve passes through the point  $(1, 13)$ , find the coordinates of the stationary point  $P$ . [4]

11 The function  $f : x \mapsto 6 - 4 \cos\left(\frac{1}{2}x\right)$  is defined for  $0 \leq x \leq 2\pi$ .

(i) Find the exact value of  $x$  for which  $f(x) = 4$ . [3]

(ii) State the range of  $f$ . [2]

(iii) Sketch the graph of  $y = f(x)$ . [2]

(iv) Find an expression for  $f^{-1}(x)$ . [3]

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