MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

October/November 2003

1 hour 15 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
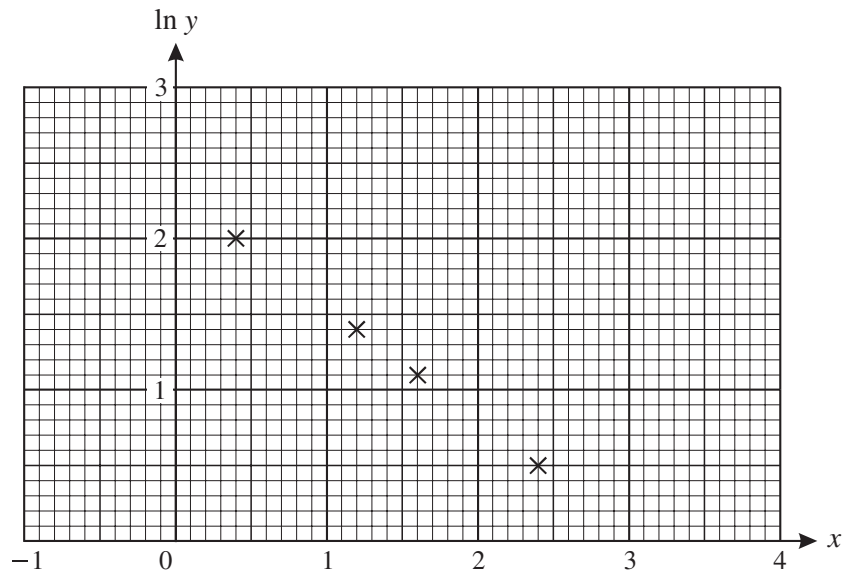The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

[Turn over

1 Find the set of values of x satisfying the inequality $|8 - 3x| < 2$. [3]

2

Two variable quantities x and y are related by the equation

$$y = k(a^{-x}),$$

where a and k are constants. Four pairs of values of x and y are measured experimentally. The result of plotting $\ln y$ against x is shown in the diagram. Use the diagram to estimate the values of a and k. [5]

3 The polynomial $x^4 - 6x^2 + x + a$ is denoted by $f(x)$.

   (i) It is given that $(x + 1)$ is a factor of $f(x)$. Find the value of a. [2]

   (ii) When a has this value, verify that $(x - 2)$ is also a factor of $f(x)$ and hence factorise $f(x)$ completely. [4]

4   **(i)** Express $\cos\theta + (\sqrt{3})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact value of α. [3]

   (ii) Hence show that one solution of the equation

$$\cos\theta + (\sqrt{3})\sin\theta = \sqrt{2}$$

is $\theta = \frac{7}{12}\pi$, and find the other solution in the interval $0 < \theta < 2\pi$. [4]

5 (i) By sketching a suitable pair of graphs, for $x < 0$, show that exactly one root of the equation $x^2 = 2^x$ is negative. [2]
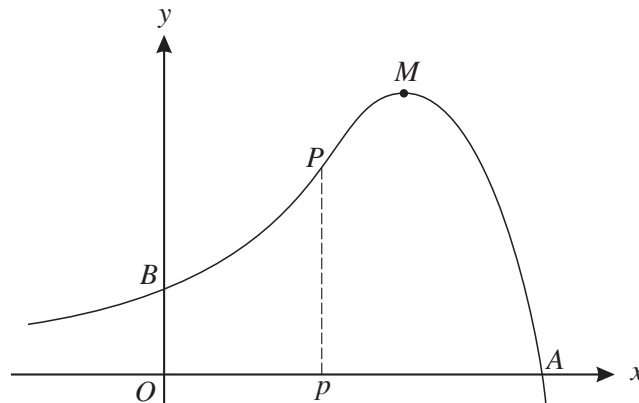
(ii) Verify by calculation that this root lies between -1.0 and -0.5. [2]

(iii) Use the iterative formula

$$x_{n+1} = -\sqrt{(2^{x_n})}$$

to determine this root correct to 2 significant figures, showing the result of each iteration. [3]

6

The diagram shows the curve $y = (4 - x)e^x$ and its maximum point M. The curve cuts the x-axis at A and the y-axis at B.

(i) Write down the coordinates of A and B. [2]

(ii) Find the x-coordinate of M. [4]

(iii) The point P on the curve has x-coordinate p. The tangent to the curve at P passes through the origin O. Calculate the value of p. [5]

7 (i) By differentiating $\dfrac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\dfrac{dy}{dx} = -\operatorname{cosec}^2 x$. [3]

(ii) Hence show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \operatorname{cosec}^2 x \, dx = \sqrt{3}$. [2]

By using appropriate trigonometrical identities, find the exact value of

(iii) $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \cot^2 x \, dx$, [3]

(iv) $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{1 - \cos 2x} \, dx$. [3]

BLANK PAGE