UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

JC12 11_9709_12/RP

UNIVERSITY *of* CAMBRIDGE
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[Turn over

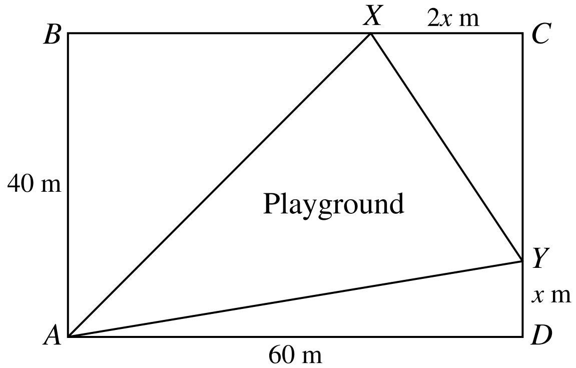
1 In the expansion of $\left(x^2 - \dfrac{a}{x}\right)^7$, the coefficient of x^5 is -280. Find the value of the constant a. [3]

2 A function f is such that $\mathrm{f}(x) = \sqrt{\left(\dfrac{x+3}{2}\right)} + 1$, for $x \geqslant -3$. Find

(i) $\mathrm{f}^{-1}(x)$ in the form $ax^2 + bx + c$, where a, b and c are constants, [3]

(ii) the domain of f^{-1}. [1]

3



The diagram shows a plan for a rectangular park $ABCD$, in which $AB = 40$ m and $AD = 60$ m. Points X and Y lie on BC and CD respectively and AX, XY and YA are paths that surround a triangular playground. The length of DY is x m and the length of XC is $2x$ m.

(i) Show that the area, A m^2, of the playground is given by

$$A = x^2 - 30x + 1200.$$ [2]

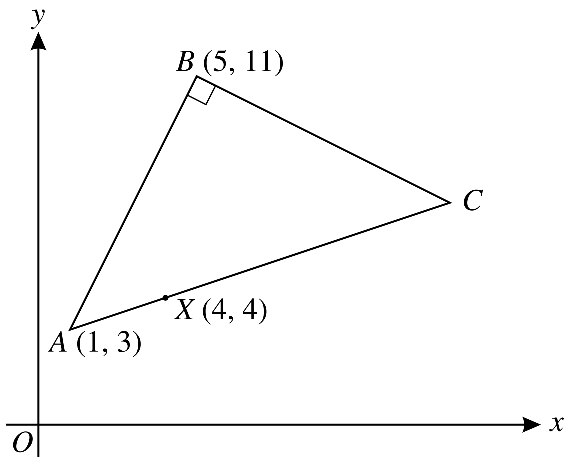
(ii) Given that x can vary, find the minimum area of the playground. [3]

4 The line $y = \dfrac{x}{k} + k$, where k is a constant, is a tangent to the curve $4y = x^2$ at the point P. Find

(i) the value of k, [3]

(ii) the coordinates of P. [3]

5



The diagram shows a triangle ABC in which A has coordinates $(1, 3)$, B has coordinates $(5, 11)$ and angle ABC is 90°. The point $X\ (4, 4)$ lies on AC. Find

(i) the equation of BC, [3]

(ii) the coordinates of C. [3]

6 **(i)** Show that the equation $2\cos x = 3\tan x$ can be written as a quadratic equation in $\sin x$. [3]

(ii) Solve the equation $2\cos 2y = 3\tan 2y$, for $0^\circ \leqslant y \leqslant 180^\circ$. [4]

7 The position vectors of the points A and B, relative to an origin O, are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} k \\ -k \\ 2k \end{pmatrix},$$

where k is a constant.

(i) In the case where $k = 2$, calculate angle AOB. [4]

(ii) Find the values of k for which $\overrightarrow{AB}$ is a unit vector. [4]

8 **(a)** In a geometric progression, all the terms are positive, the second term is 24 and the fourth term is $13\frac{1}{2}$. Find

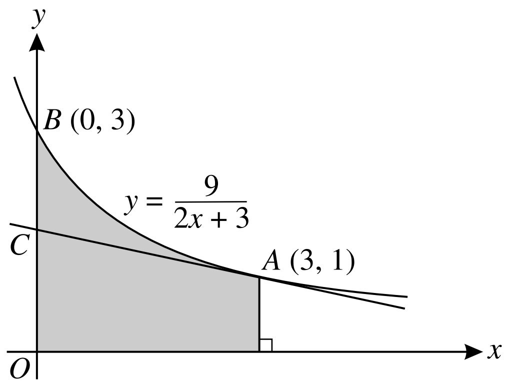
(i) the first term, [3]

(ii) the sum to infinity of the progression. [2]

(b) A circle is divided into n sectors in such a way that the angles of the sectors are in arithmetic progression. The smallest two angles are 3° and 5°. Find the value of n. [4]

[Questions 9, 10 and 11 are printed on the next page.]

9



The diagram shows part of the curve $y = \dfrac{9}{2x+3}$, crossing the y-axis at the point $B\,(0, 3)$. The point A on the curve has coordinates $(3, 1)$ and the tangent to the curve at A crosses the y-axis at C.

(i) Find the equation of the tangent to the curve at A. [4]

(ii) Determine, showing all necessary working, whether C is nearer to B or to O. [1]

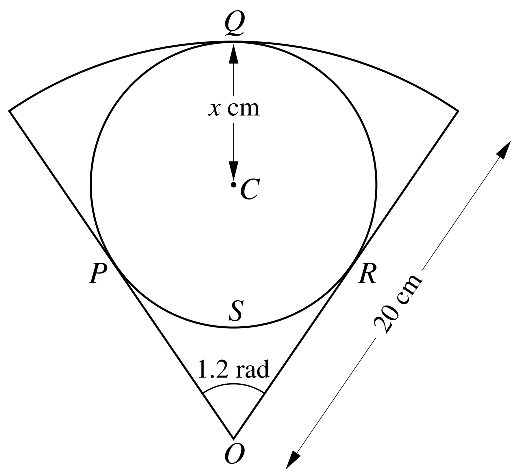
(iii) Find, showing all necessary working, the exact volume obtained when the shaded region is rotated through $360°$ about the x-axis. [4]

10 A curve is defined for $x > 0$ and is such that $\dfrac{dy}{dx} = x + \dfrac{4}{x^2}$. The point $P\,(4, 8)$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Show that the gradient of the curve has a minimum value when $x = 2$ and state this minimum value. [4]

11



The diagram shows a sector of a circle with centre O and radius 20 cm. A circle with centre C and radius x cm lies within the sector and touches it at P, Q and R. Angle POR = 1.2 radians.

(i) Show that $x = 7.218$, correct to 3 decimal places. [4]

(ii) Find the total area of the three parts of the sector lying outside the circle with centre C. [2]

(iii) Find the perimeter of the region $OPSR$ bounded by the arc PSR and the lines OP and OR. [4]

