Star Formation – HW #1

Due Wednesday Sept. 19 at the beginning of class. Answer the following questions. Show all work. See HW1 folder for additional HW1 files.

1. **Eat Your Vegetables: Radiative Transfer for the CO Lines**

a) Write down the steady-state rate equation describing the population of the rotational levels of CO. Treat the radiation field by assuming that some fraction $\epsilon \leq 1$ of the photons produced following spontaneous decay, escape from the cloud containing the emitting molecules. In this approximation, the radiative term

$$n_k A_{kj} + (n_k B_{kj} - n_j B_{ij}) J_\nu \tag{1}$$

can be replaced by $n_k \epsilon A_{kj}$.[1]

The level populations are now a function of three parameters: T, $n(\mathrm{H_2})$ (the collision partner density) and ϵ (the escape probability). The physical process under consideration is referred to as radiation trapping.

Write the population rate equation in matrix form and solve for the level populations. Check that the number of equations and the number of unknowns is sufficient to find a solution.

b) Plot level curves of the fractional level populations, $n(J)/n$, for $J = 0 - 5$ as a function of temperature $(0 < T < 200)$ K and $\mathrm{H_2}$ density $(10^3 < n(\mathrm{H_2}) < 10^5)$ cm^{-3} for $\epsilon = 0.01$ and 1.0.

c) Explain how the trapping of radiation modifies the level populations and suggests a new definition of critical density.

d) Compute the optical depth at line center for the CO $J = 1 - 0$ and CO $J = 2 - 1$ transitions for a typical giant molecular cloud with $A_v = 10$ and a CO/$\mathrm{H_2}$ abundance ratio of 10^{-4}. Assume a Doppler broadened line with a 1D velocity dispersion of $\sigma = 3$ km/s.

e) What is the relation between ϵ and line optical depth?

2. **Stirring Things Up**

As we discussed in class, turbulence is often studied using numerical simulations. Data cubes of simulated density (g cm^{-3}), and x, y, z velocity (cm/s) have been uploaded to Canvas. The simulated region has $T = 10$ K and $L = 5$ pc on a side.

[1] For the higher transitions the Einstein A coefficient can be approximated by $A_{J,J-1} = \frac{3J^4}{2J+1} A_{10}$. The rate coefficient can be roughly expressed as $\gamma_{J,J-1} = 10^{-11}\sqrt{T}\mathrm{cm^3 s^{-1}}$.

a) Plot the gas density probability distribution function (PDF). What does this suggest about the velocity dispersion? Does the simulation include gravity?

b) Use the velocity data to compute and plot the velocity power spectrum. Determine the slope, energy injection scale and dissipation scale. What do you expect for the injection and dissipation scales for a real cloud?

c) Bonus: Given this data, do you think this is modeling a super-Alfvénic or sub-Alfvénic cloud? Why?

3. **Mystery Region**

A ^{12}CO (1-0) data cube of a mystery region has been uploaded to Canvas. Use what you've learned so far about CO, turbulence and clouds to figure out some information about this region: Is this cloud star-forming? What is the cloud velocity dispersion and Mach number? Is it a "low-mass" or "high-mass" region? Justify your answers.