Star Formation – HW #2

Due Wednesday Oct 3 at the beginning of class. Answer the following and show all work.

1. **Spherical Cows, I mean, Clouds**

a) Consider a spherical isothermal cloud at zero temperature with a constant density ρ_0. There is no magnetic field. If the cloud is a rest at $t = 0$, show that it will collapse to a point at a time:

$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2}. \tag{1}$$

b) Using the virial theorem, show that the maximum mass of an isothermal sphere of constant density is

$$M_{\rm max} = 1.77\frac{c_0^4}{\sqrt{G^3 p_0}}, \tag{2}$$

where c_0 is the isothermal sound speed and p_0 is the ambient pressure. If the unphysical assumption of constant density is dropped, one finds that the maximum stable mass has the same form as above, but with a coefficient of 1.18; $M_{\rm max}$ is now the Bonnor-Ebert mass.

c) Define the virial parameter as

$$\alpha \equiv \frac{5\sigma^2 R}{GM}, \tag{3}$$

where σ is the 1D velocity dispersion. This is convenient because each of the parameters can be estimated from observations. Let $v_A = \bar{B}/(4\pi\bar{\rho})^{1/2}$ be the Alfvén velocity in terms of the mean field and mean density of the cloud. Assuming the cloud is spherical, show that the Alfvén mach number can be written

$$\mathcal{M}_A \equiv \frac{\sigma}{v_A/\sqrt{3}} \equiv \frac{\sqrt{\alpha}}{1.98}\left(\frac{M}{M_\Phi}\right), \tag{4}$$

where $M_\Phi \equiv 0.12\Phi/\sqrt{G}$ is the magnetic critical mass. This identity shows that if the virial parameter is of order unity then $\mathcal{M}_A$ is as well, provided the cloud is slightly magnetically supercritical: $M \simeq 2M_\Phi$.

2. **Observing Real, Star-Forming Cows**

This problem uses real observations of different CO isotopologues of a nearby molecular cloud. I have uploaded ^{12}CO(1-0), 13CO(1-0), and C18O(1-0) maps of the protostellar cluster NGC1333 in the Perseus molecular cloud complex, obtained using the 14-m radio telescope at the Five College Radio Observatory (FCRAO), which used to be in MA. The ^{12}CO map was obtained as part of the COMPLETE survey of Perseus (Ridge et al. 2006 AJ, 131, 2921). The ^{13}CO and C^{18}O maps were obtained simultaneously and are part of a survey of the molecular gas surrounding a sample of protostellar clusters (Ridge et al. 2003, AJ, 126, 286).

First download thesw fits cubes from Canvas: ngc1333_12co.fits, ngc1333_13co.fits, and ngc1333_c18o.fits. Note the first cube is the "mystery" data cube from problem set 1.

Before starting this problem I suggest reading §15.4 in Chapter 15 of "Tools of Radio Astronomy" (by Tom Wilson et al.). This book is available in the Astronomy-Physics-Math library. For this problem you should assume the following:

- The efficiency of the FCRAO telescope is such that $T_A = 0.5$ TB
- The distance to NGC1333 is 280 pc (Zucker, C. et al. 2018 arXiv)
- The velocity axis shows the velocity with respect to the Local Standard of Rest ($v_{\rm LSR}$) and goes from -9.995 to 29.995 km/s, with channel widths of 0.133 km/s.
- Pixels are 25" x 25"

a) Make integrated intensity maps from the different line maps for the velocity range $3.5 < v_{\rm LSR} < 12.0$ km/s. Describe the maps in words and discuss their similarities and differences.

b) Obtain an excitation temperature map of the cloud using the peak emission of the ^{12}CO in each pixel. Discuss how you produced the map, your calculations, and findings.

c) Calculate the mass of the cloud for the same velocity range as above, using each of the three maps and assuming all lines are optically thin. Discuss your results. Compare your mass estimate using the ^{12}CO with the other two estimates. Why are they different or the same?

d) Now obtain the mass using the ^{13}CO(1-0) map, taking into consideration the lines optical depth, and compare any difference or similarities with the mass estimate obtained form the C^{18}O(1-0) map (above). Note that in the literature there are various ways to correct for opacities. Choose one technique and explain the assumptions and the method thoroughly.

e) Use your mass estimates and the velocity dispersion derived in HW 1 to compute the virial parameter under the optically thick and thin assumptions. Discuss the results.

The following papers may so be helpful:
Appendix of Bourke et al. 1997, ApJ, 476, 781
Appendix of Pineda et al. 2011, ApJ, 743, 201
Body and appendix of Dunham et al. 2014, 783, 29
Appendix of Zhang et al. 2016, ApJ, 832, 158