

# Star Formation – HW #5

Due Tuesday Nov 20 by 6pm (\*before\* Thanksgiving break!). Answer the following questions. Show all work.

## 1. The $\alpha$ and the $\Omega$ .

Consider a disk having a dimensionless viscosity  $\alpha$ . The disk accretes at a steady rate  $\dot{M}$ , where  $\dot{M} = -3\pi\Sigma\nu$  and for an “alpha” disk with scale height  $h$ ,  $\nu = c_s h \alpha$ . The disk cools radiatively. Neglect the difference between the effective temperature of the disk  $T_{eff}$  (which is nothing more than a convenient way of stating what the emitted flux is) and the actual gas kinetic temperature  $T$ . Take the gas to have sound speed  $c_s$  and angular frequency  $\Omega$ , both of which vary with disk radius  $r$ .

You will need to make use of the relation between the power emitted per unit area and the accretion rate:

$$\sigma_{sb} T_{eff}^4 = \frac{3}{8\pi} \frac{GM_* \dot{M}}{r^3}. \quad (1)$$

- (a) Find how  $h/r$  scales with  $r$ , where  $h$  is the disk vertical scale height. Sketch how the disk looks.
- (b) Find how the surface density  $\Sigma$  scales with  $r$ .
- (c) Find how the disk mid-plane density  $\rho$  scales with  $r$ .
- (d) Find an approximate expression for how long it takes a pressure disturbance to equilibrate away. Call this time  $t_z$ , and express it as simply as possible.
- (e) Find an approximate expression for how long it takes a temperature disturbance to equilibrate away. Call this time  $t_{cool}$  and express in terms of  $\Omega$  and  $\alpha$ .
- (f) Find an approximate expression for how long it takes a mass disturbance (say, a local bunching of material) to viscously diffuse away. Call this time  $t_{visc}$ , and express in terms of  $\Omega$ ,  $\alpha$ ,  $h/r$ . Arrange  $t_z$ ,  $t_{cool}$  and  $t_{visc}$  in increasing order.
- (g) Find an expression for the critical radius  $r_{crit}$  beyond which Toomres  $Q < 1$ . Express the result in terms of  $\alpha$ ,  $M$  and  $\dot{M}$ . Alpha disks are generically unstable at large radii, leading some to surmise that the outer peripheries of quasar accretion disks / protostellar accretion disks are fertile breeding grounds for starbursts / binary companion stars or brown dwarfs.

## 2. Obsessing Over Outflows...

This problem uses CO(1-0) interferometric observations of a protostellar outflow to ex-

plore outflow launching and properties. The observations were carried out using the OVRO millimeter array of six 10.4 m telescopes. This source is a member of L1589, in the  $\lambda$  Orionis molecular shell. Assume  $d = 460$  pc, the beam size is  $4''$ , and the velocity channel  $\Delta v = 0.325$  km/s. The intensity units are (sadly) in Jy/beam.

(a) Produce an integrated intensity map of the outflow. Identify the velocity range of the blue and red-shifted lobes. What is the outflow extent and opening angle? Based on the data, do you think this outflow is launched by a younger or older source?

(b) Make a “Hubble diagram” (PV) of the outflow, and estimate the dynamical age of the outflow. Is this a good measure of the source’s actual age? What does the Hubble diagram suggest about the accretion history of the source?

(c) Use the CO emission to calculate the outflow mass. Offner & Chaban (2017) used simulations to show that molecular outflows contain about three times as much entrained core mass as is actually launched. Use this, together with the outflow mass, to estimate the source accretion rate and source mass. Discuss any assumptions you make.