# Homework #3

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### Introduction

All code can be found under `src/`. The code was written in Python 3.6. `main.py` is the script that runs all parts of the assignment, `star.py` and `dust.py` are classes representing those two objects, and `util.py` is used for constants and useful utility functions. The `SHOW` array in `main.py` can be changed to adjust what output is displayed. All images can be found in `img/`.

The star we are interested in for this project is Fomalhaut. Fomalhaut has a mass of 1.92 $M_\odot$, a radius of 1.842 $R_\odot$, a temperature of 8590 K, and is 7.70 pc away from us. For the dust, we are interested in "astrosilicate" of 0.1, 1, 10, and 1000 microns with different $Q_{abs}$. The $Q_{abs}$ and the corresponding wavelengths are found in `0.1micron.txt`, `1micron.txt`, and `10micron.txt` (we consider the 1000 micron dust as a perfect absorber, thus $\forall\lambda.\ Q_{abs} = 1$), with the original file from Princeton available in `dust.txt` [1]. All emission spectra are shown in units of microns and Jansky (Jy); in SI units, this is $10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$.

### 1 Spectra of Fomalhaut

We first find the specific intensity through Planck's law, $B_\nu(\nu, T)$, using Fomalhaut's temperature and a specified frequency range (here $10^{13} - 10^{15}$ Hz was used). We then integrate over the solid angle (in this case, multiply by $\pi$) to convert this to flux density. Finally we multiply this by the square of the ratio of the stellar and orbital radii to find the flux density at those orbital radii (assuming a circular orbit). The resulting spectra are shown in Figure 1.

### 2 Power Absorbed by Dust

We create all of the dust instances and read in the relevant $Q_{abs}$ and wavelengths from the files. Here the input frequencies used were just the ones corresponding to the given wavelengths. The absorbed power is given by the equation

$$P_{in} = \pi r^2 \int F_{in,\nu} Q_{abs}(\nu) d\nu.$$

Here $r$ is the radius of the dust and $\nu$ is obtained from the relation $\lambda\nu = c$. $F_{in}$ is the absorbed flux from the star, which is obtained from the corresponding orbital radius/value from part 1. The resulting absorbed powers are shown in Figure 2.

## 3 Equilibrium Temperatures and Emission Spectra of Dust

If the dust is a perfect absorber, we can treat it like a blackbody and use the equation

$$P_{in} = P_{out} = 4\pi r^2 \sigma T^4$$

to calculate the equilibrium temperature. If it is not a perfect absorber, then we must consider physical effects. Due to the difficulty for dust to emit light at wavelengths comparable to its size, we know that the dust will have a higher equilibrium temperature than if it was a blackbody. Using the equation

$$P_{in} = P_{out} = 4\pi r^2 \int \pi B_\nu(\nu, T) Q_{abs}(\nu) d\nu,$$

we binary search over $T$ to find the value that correctly matches the incoming and outgoing powers to find the equilibrium temperature for imperfect dust. The resulting equilibrium temperatures are shown in Figure 2.

For perfect dust, we repeat the process we did in part 1 to produce the emission spectrum. For imperfect dust, we include the factor $Q_{abs}$ when integrating Planck's law over frequency. Here the orbital radius is taken to be the distance between us and Fomalhaut. The resulting spectra are shown in Figures 3 and 4.

## 4 Dust Count and Mass

The reference SED for Fomalhaut's two debris belts was obtained from Su et al. (2013) [2]. Qualitatively, it appears that the flux densities of the warm and cold debris belts (with temperatures $\sim 170$ and $\sim 50$ K) have maximums of $10^{2.5}$ and $10^4$ Jy, respectively. We divide these maxima by those of the individual grains from part 2 to find roughly the number of dust grains we need to match the observed SED (we assume that each orbital radius has only one type of dust). Assuming that the dust grains have density 2 $\mathrm{g\,cm^{-3}}$, we multiply this by the volume $\frac{4}{3}\pi r^3$ (assuming the dust is spherical) and amount to find the total dust mass. Based on both qualitative appearance from part 3 and from matching temperatures from part 3 with Su et al, the dust grain sizes that produce the best fits to the SED shape are 1 mm at 10 AU and 10 µ at 130 AU. The resulting number and masses are shown in Figure 5.

## 5 Radiation Pressure and Poynting-Robertson Drag

The force from radiation pressure is calculated using $F_{RP} = \frac{1}{c} P_{in}$. The force from Poynting-Robertson drag is calculated using $F_{PR} = \frac{v}{c^2} P_{in} = \frac{\sqrt{GM_\star/a}}{c^2} P_{in}$. Here $P_{in}$ is just the absorbed power from the star calculated in part 2. For characteristic timescales, we can get an approximation using the $\beta$ value, the square of the ratio of the forces from radiation pressure and gravity. If $\beta > 1$, we know that the force from radiation pressure is greater than the force of gravity and the dust will immediately be blown out of the system. Otherwise the timescale for Poynting-Robertson drag to remove the dust from the system is given by the equation [3]

$$\tau = \frac{1}{4} \left( \frac{GM_\star \beta}{c} \right)^{-1} \left( a^2 - R_\star^2 \right).$$

The forces and the removal scales are shown in Figure 6.

## References

[1] B. T. Draine, "Optical properties of interstellar dust grains."

[2] K. Y. L. Su, G. H. Rieke, R. Malhotra, K. R. Stapelfeldt, A. M. Hughes, A. Bonsor, D. J. Wilner, Z. Balog, D. M. Watson, M. W. Werner, and K. A. Misselt, "Asteroid belts in debris disk twins: Vega and fomalhaut," *The Astrophysical Journal*, vol. 763, no. 2, p. 118, 2013.

[3] J. Klaka and M. Kocifaj, "Times of inspiralling for interplanetary dust grains," *Monthly Notices of the Royal Astronomical Society*, vol. 390, no. 4, pp. 1491–1495, 2008.

## Appendix

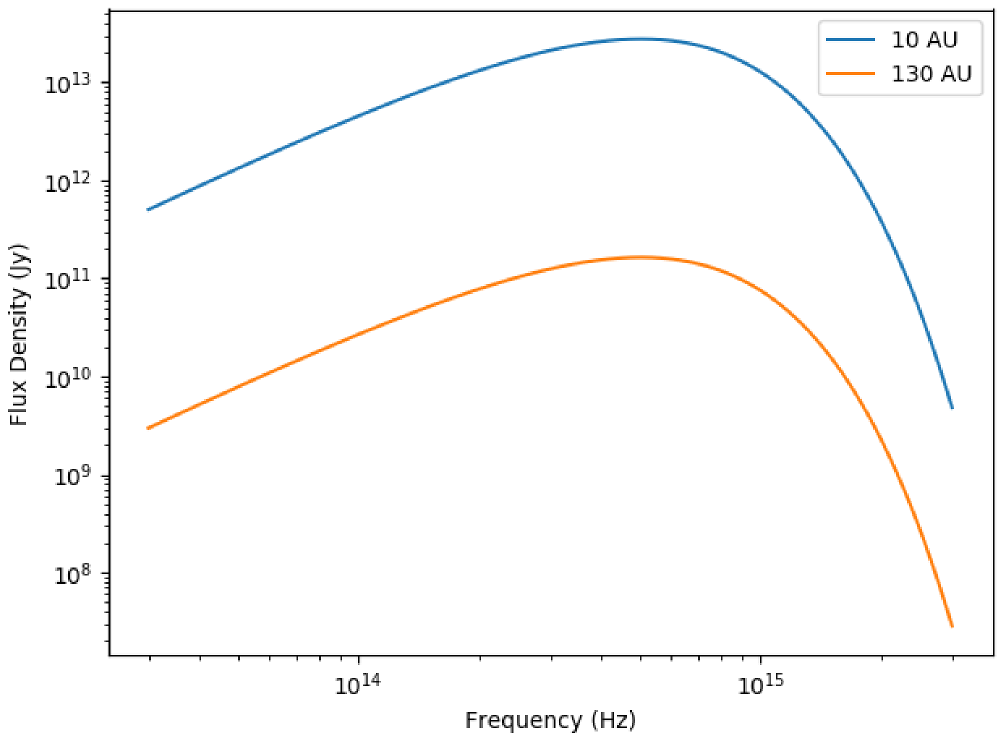


Figure 1: The SED of Fomalhaut.

| r (µ) | a (AU) | Absorbed Power (W) | Equilibrium Temp (K) |
|---|---|---|---|
| 0.1 | 10 | $1.466 * 10^{-12}$ | 259.8 |
| 1 | 10 | $6.251 * 10^{-10}$ | 212.6 |
| 10 | 10 | $6.599 * 10^{-8}$ | 159.1 |
| 1000 | 10 | $7.117 * 10^{-4}$ | 177.8 |
| 0.1 | 130 | $8.674 * 10^{-15}$ | 97.8 |
| 1 | 130 | $3.699 * 10^{-12}$ | 83.4 |
| 10 | 130 | $3.905 * 10^{-10}$ | 48.5 |
| 1000 | 130 | $4.211 * 10^{-6}$ | 49.3 |

Figure 2: The absorbed power and equilibrium temperatures of the different dust.

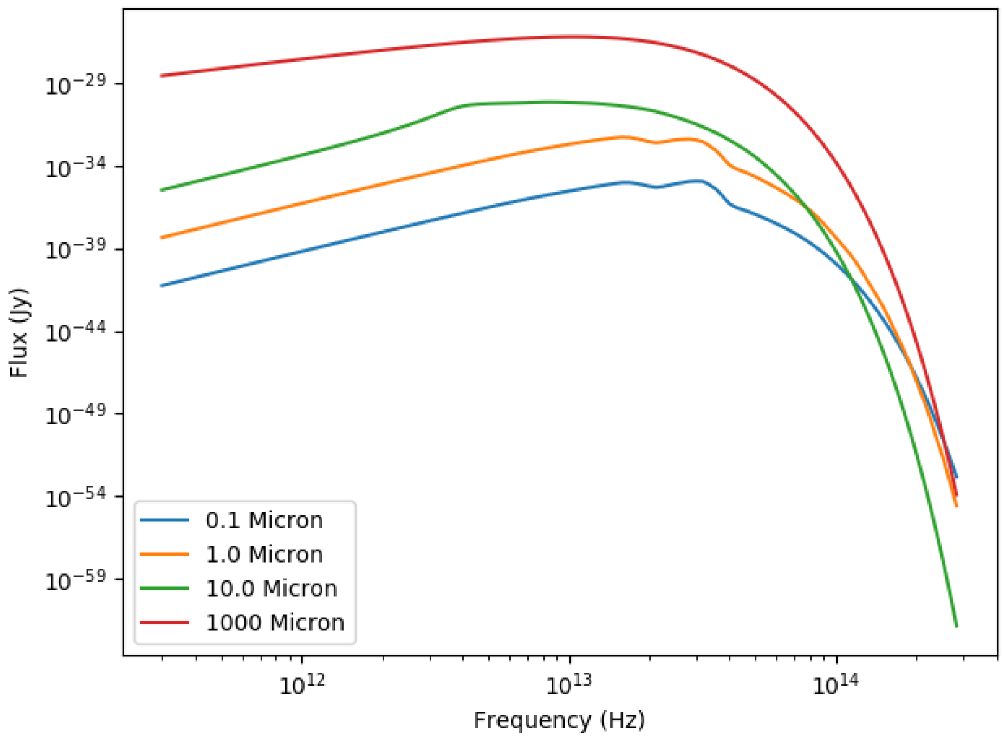


Figure 3: The SEDs of the individual dust grains at 10 AU

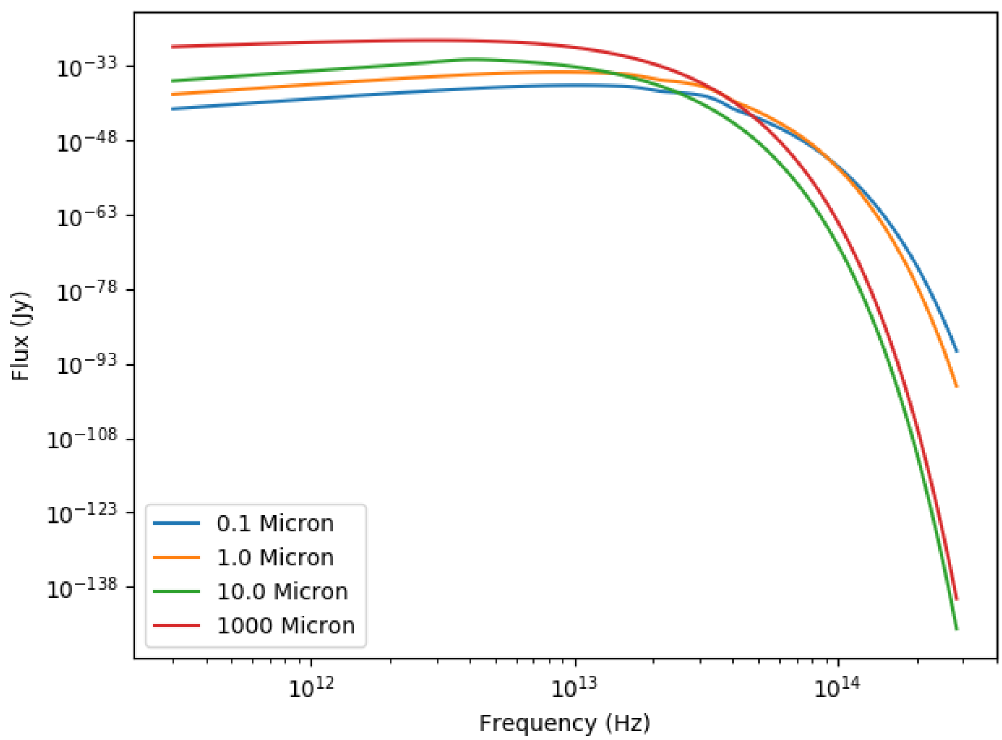


Figure 4: The SEDs of the individual dust grains at 130 AU.

| r (µ) | a (AU) | Number of Dust Grains | Total Mass (kg) |
|---|---|---|---|
| 0.1 | 10 | $2.807 * 10^{34}$ | $2.352 * 10^{17}$ |
| 1 | 10 | $6.241 * 10^{31}$ | $5.229 * 10^{17}$ |
| 10 | 10 | $4.702 * 10^{29}$ | $3.939 * 10^{18}$ |
| 1000 | 10 | $5.336 * 10^{25}$ | $4.470 * 10^{20}$ |
| 0.1 | 130 | $9.062 * 10^{37}$ | $7.592 * 10^{20}$ |
| 1 | 130 | $1.899 * 10^{35}$ | $1.591 * 10^{21}$ |
| 10 | 130 | $5.693 * 10^{32}$ | $4.770 * 10^{21}$ |
| 1000 | 130 | $7.912 * 10^{28}$ | $6.628 * 10^{23}$ |

Figure 5: The number of dust grains and subsequent mass needed to match the reference SED.

| r (µ) | a (AU) | Radiation Pressure (N) | P-R Drag (N) | Removal Time (yr) |
|---|---|---|---|---|
| 0.1 | 10 | $4.890 * 10^{-21}$ | $2.219 * 10^{-25}$ | 0 |
| 1 | 10 | $2.085 * 10^{-18}$ | $9.077 * 10^{-23}$ | 0 |
| 10 | 10 | $2.201 * 10^{-16}$ | $9.582 * 10^{-21}$ | $9.045 * 10^{4}$ |
| 1000 | 10 | $2.374 * 10^{-12}$ | $1.033 * 10^{-16}$ | $8.387 * 10^{6}$ |
| 0.1 | 130 | $2.893 * 10^{-23}$ | $3.494 * 10^{-28}$ | 0 |
| 1 | 130 | $1.234 * 10^{-20}$ | $1.490 * 10^{-25}$ | 0 |
| 10 | 130 | $1.302 * 10^{-18}$ | $1.573 * 10^{-23}$ | $1.529 * 10^{7}$ |
| 1000 | 130 | $1.405 * 10^{-14}$ | $1.696 * 10^{-19}$ | $1.417 * 10^{9}$ |

Figure 6: The absorbed power and equilibrium temperatures of the different dust.