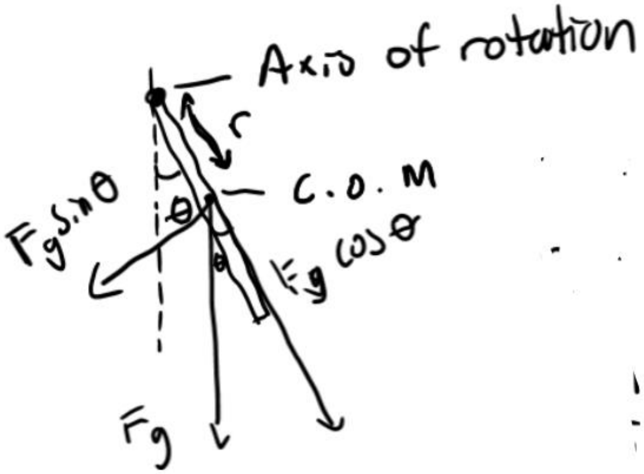
Appendix: Calculations

getPositionTimeFunction()/getPosition()

RodPendulum/Pendulum



We begin with Newton's Second Law for Rotation, which states that
$\Sigma\tau = I\alpha$

Substituting in the appropriate forces from the free body diagram (while taking CCW to be positive), and replacing $\alpha$with $\frac{d^2\theta}{dt^2}$, we get
$-mgrsin\theta = I\alpha$
$-mgrsin\theta = I(\frac{d^2\theta}{dt^2})$

We make use of the approximation $\sin\theta \approx \theta$. Note that the use of this approximation limits the range of valid inputs to $-0.5rad \leq \theta \leq 0.5rad$.
$mgr\theta = I(\frac{d^2\theta}{dt^2})$
$I(\frac{d^2\theta}{dt^2}) + mgr\theta = 0$
$\frac{d^2\theta}{dt^2} = \frac{-mgr}{I}\theta$

We define ω_0 to be the angular frequency of the pendulum. We obtain the angular frequency from taking the roots of the auxiliary equation.
$\omega_0 = \sqrt{\frac{mgr}{I}}$

Since we are dealing with oscillations, we will use the sinusoidal model for the solution to this differential equation. Therefore, the function that will satisfy the differential equation is given by

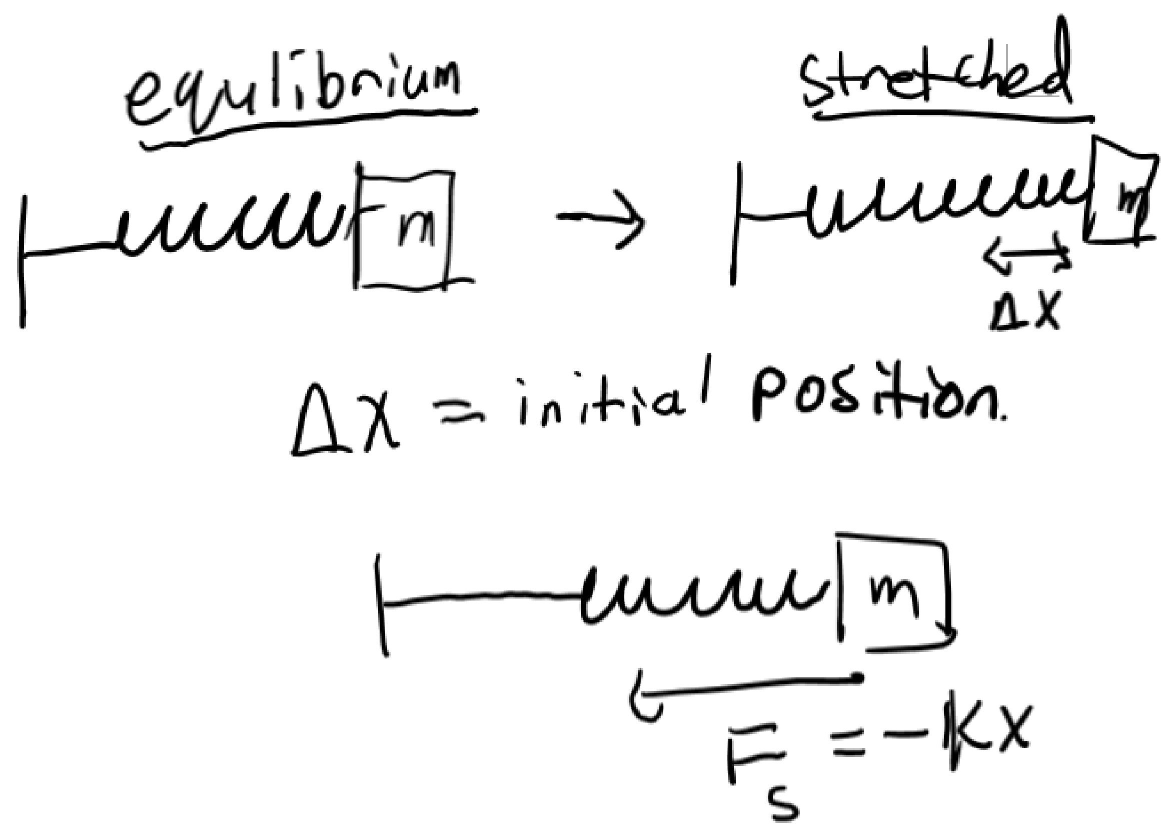
$\theta(t) = Acos(\omega_0 t)$

$\theta(t) = Acos(\sqrt{\frac{mgr}{I}}t)$

Note that for a physical pendulum, the point of rotation cannot be at the center of mass, or else no oscillation will happen.

For a normal string pendulum, simply replace the value of I with mr^2.

SpringOscillator



We begin with Newton's second law,

$\Sigma F = ma$

The only force acting on the spring-block system is the spring force.

$-kx = ma$

Replacing a with $\frac{d^2x}{dt^2}$, we get

$-kx = m\frac{d^2x}{dt^2}$

Finding the angular frequency once again, we get

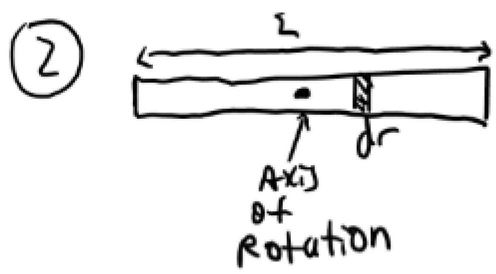
$$\omega_0 = \sqrt{\frac{k}{m}}$$

Solving the differential equation as shown above, we get

$$x(t) = Acos(\omega_0 t)$$

$$x(t) = Acos(\sqrt{\frac{k}{m}}t)$$

getMomentOfInertia()



For the following rod of length L with its axis of rotation at the center of mass, we can use integration to calculate the moment of inertia.

By definition, the moment of inertia of a rigid object is defined by

$$I = \int r^2\, dm$$

Assuming this rod has uniform mass distribution, for an infinitesimally small interval along the rod, the mass can be given by the expression $dm = \frac{M}{L}dr$, where $\frac{M}{L}$is the linear density of the rod.

Because the axis of rotation is directly in the center of the rod, we use the following lower and upper bounds of integration.

$$I = \int_{-L/2}^{L/2} r^2 dm$$

Substituting for dm and evaluating, we get

$$I = \frac{M}{L}\int_{-L/2}^{L/2} r^2 = \frac{M}{L}(\frac{L^3}{24} + \frac{L^3}{24}) = \frac{1}{12}ML^2$$

Suppose the axis of rotation is distance d from the center of mass. The parallel axis theorem states that for any axis of rotation parallel to one running through the center of mass, the moment of inertia of the parallel axis is given by

$$I = I_{cm} + Md^2$$

Where d is the distance from the center of mass.

getPeriod()

To get the period of the oscillator, we simply divide 2pi by the angular frequency.

$$T = \frac{2\pi}{\omega_0}$$

getKE()

Pendulum/RodPendulum

The pendulum travels following a circular motion. Thus, there is only rotational motion present, and there is no translational motion. Therefore, we only need to concern ourselves with the pendulum's rotational kinetic energy

By definition, the amount of rotational kinetic energy an object has is

$$E_{k_r} = \frac{1}{2} I\omega^2.$$

From above, we know that the position of the pendulum is given by the function

$$\theta(t) = Acos$$

$$\theta(t) = Acos(\omega_0 t)$$

To find the speed of the pendulum as a function of time, we simply differentiate with respect to time.

$$\omega(t) = \frac{d\theta}{dt} = -A\omega_0 \sin(\omega_0 t)$$

Substituting this expression into the expression for rotational kinetic energy, we get

$$E_{k_r} = \frac{1}{2} IA^2 \omega_0{}^2 sin^2(\omega_0 t).$$

For simple string pendulums, use $I = mr^2$

SpringOscillator

By definition, the amount of translational kinetic energy an object has is

$$E_{k_T} = \frac{1}{2} mv^2.$$

From above, we know that the position of the pendulum is given by the function

$$x(t) = Acos$$

$$x(t) = Acos(\omega_0 t)$$

To find the speed of the pendulum as a function of time, we simply differentiate with respect to time.

$$\omega(t) = \frac{d\theta}{dt} = -A\omega_0 \sin(\omega_0 t)$$

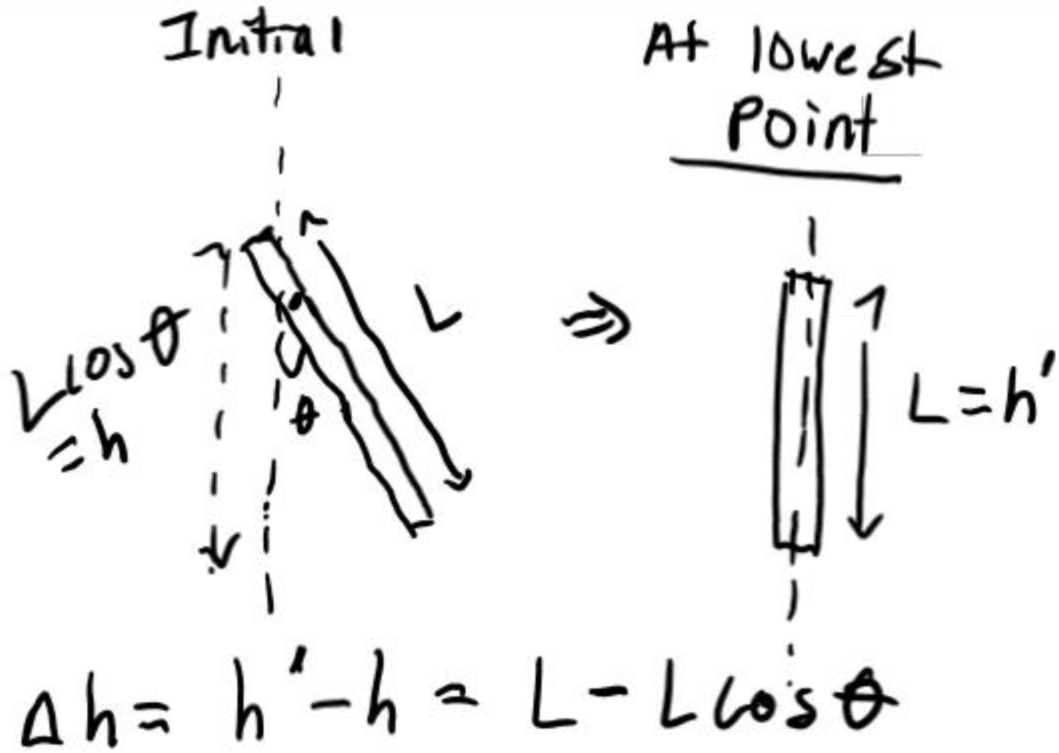
Substituting this expression into the expression for translational kinetic energy, we get

$$E_{k_r} = \frac{1}{2} mA^2 \omega_0{}^2 sin^2(\omega_0 t).$$

getPE()

Pendulum/RodPendulum

To find the amount of potential energy at a given time, we make use of conservation of energy. We can find the difference between the initial potential energy of the pendulum and the amount of kinetic energy present at the given time. This will give us the potential energy at the given time.



According to the diagram, the difference between the highest point and lowest point the bottom of the pendulum will reach is equal to $L - Lcos\theta$. Therefore, the initial potential energy of the system can be given by the expression

$E_{g_i} = mgL(1 - \cos\theta)$

Therefore,

$E_{g_t} = mgL(1 - \cos\theta) - E_{k_t}$

SpringOscillator

The amount of spring potential energy a system has is given by the expression

$E_s = \frac{1}{2}kx^2$

Where x is the compression or extension of the spring. We already have an expression for x,

$x(t) = Acos(\omega_0 t)$

Substituting this in, we get

$E_s = \frac{1}{2}kA^2cos^2(\omega_0 t)$