Shiftarea în fază/timp, filtrarea semnalelor periodice

Curs/Laborator 04

23.10.2024

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Exercițiul 1 – întârzierea un semnal în timp, prin modificarea spectrului său

$$s\left(t\right) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k t}{T}}$$

s(t) periodic cu perioada T

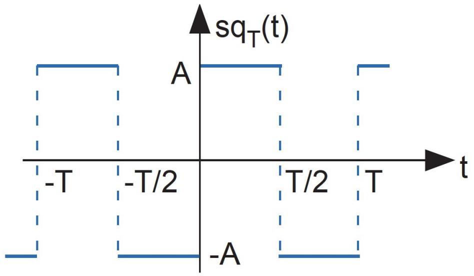
$$c_k = \frac{1}{T}\int_0^T s(t)e^{-j\frac{2\pi k t}{T}}\,dt$$

- c_k – coeficienți ai seriei Fourier complexe
- $\mathbf{s(t-\tau) \leftrightarrow c_k * e^{-j\frac{2\pi k\tau}{T}}}$

Exercițiul 1 – întârzierea un semnal în timp, prin modificarea spectrului său

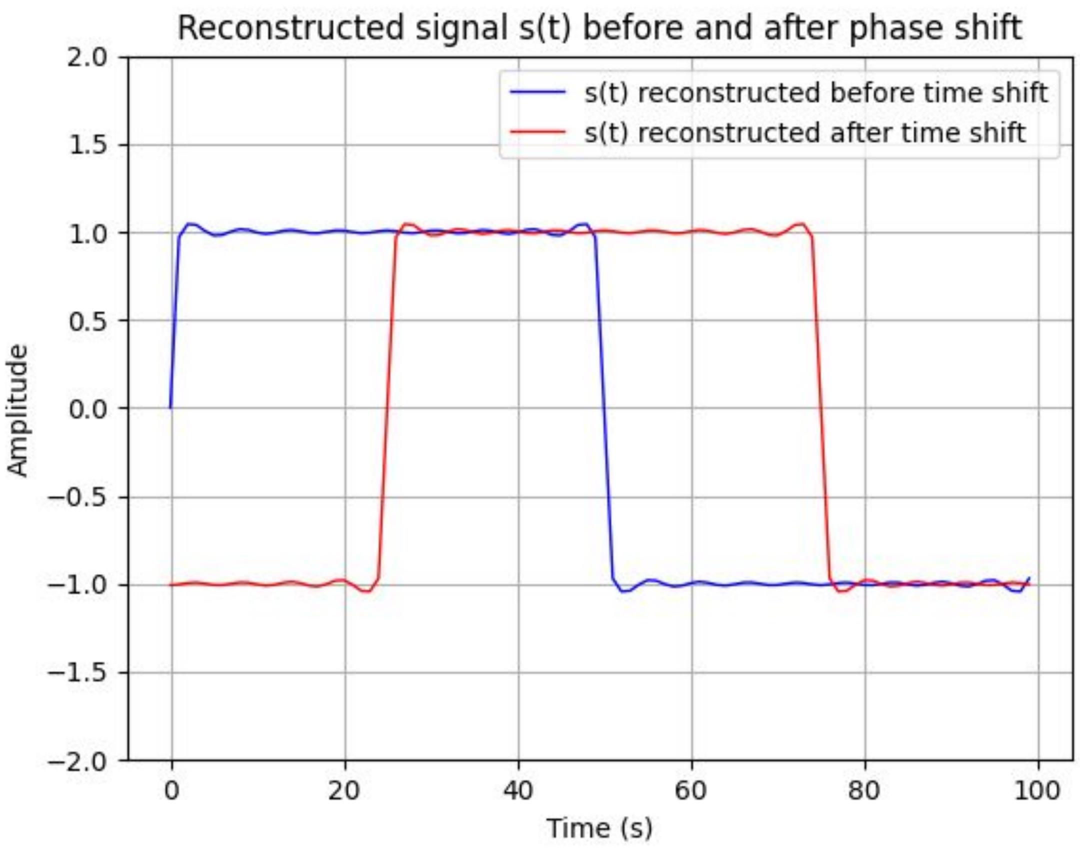
$$s(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k t}{T}}$$

- ck – coeficienti ai seriei Fourier complexe
- s(t)= $\begin{cases} A, & t \le T/2 \\ -A, & t > T/2 \end{cases}$ $, \mathrm{A} = 1, \mathrm{T} = 100$
- $c_k = \begin{cases} \frac{2}{j\pi k} A, & k\ impar \\ 0\ , & k\ par \end{cases}$
- $c_k' = c_k\ * e^{-j\frac{2\pi k\tau}{T}}, \tau = \frac{T}{4}$

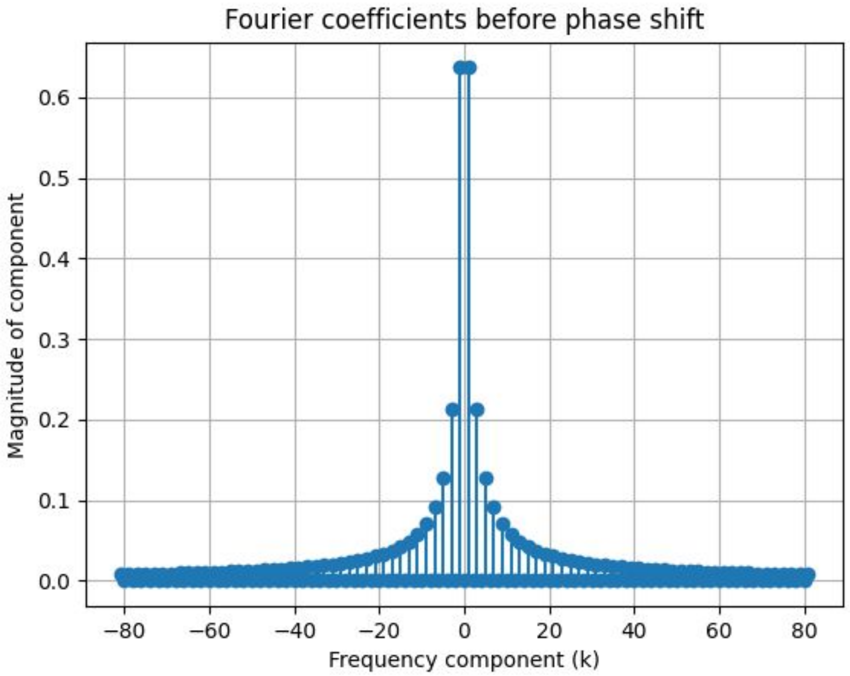


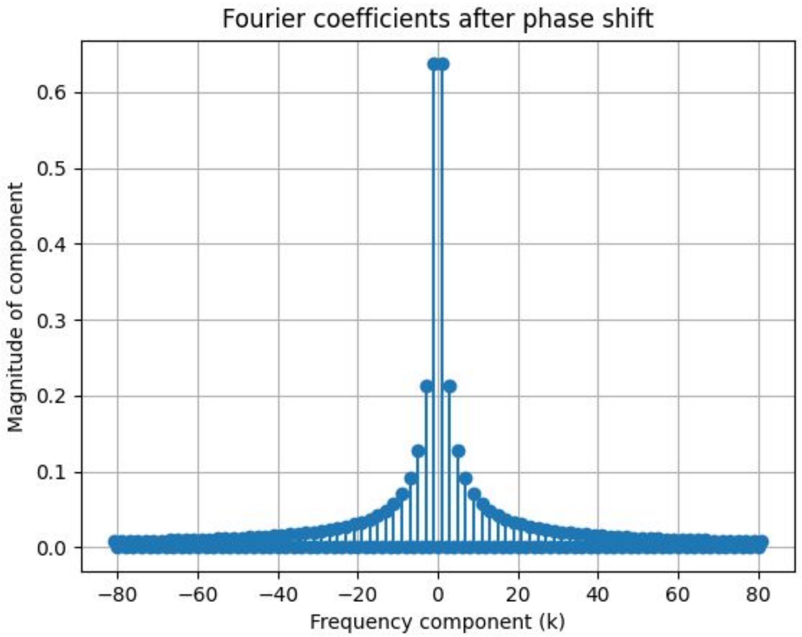
$$\mathrm{sq}_T(t) = \begin{cases} A & 0 < t < \frac{T}{2} \\ -A & \frac{T}{2} < t < T \end{cases}$$

Exercițiul 1

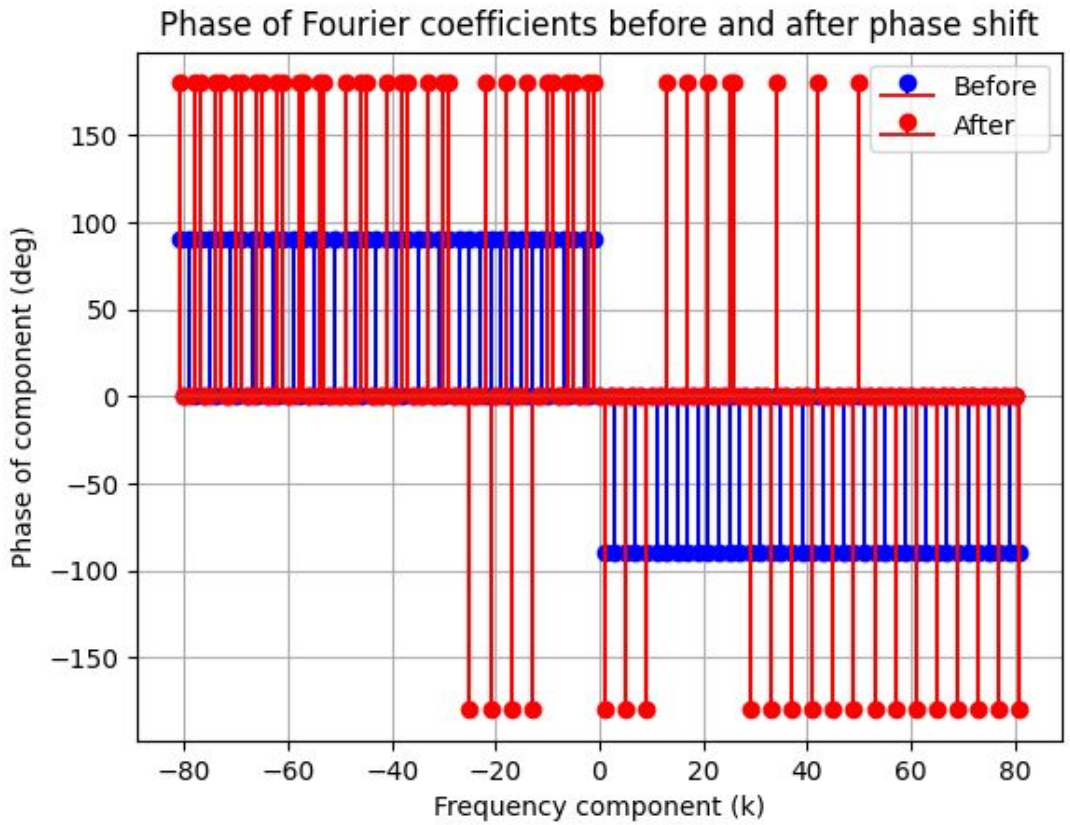


Exercițiul 1 - modulul





Exercițiul 1 - faza



Exercițiul 2 - filtrare

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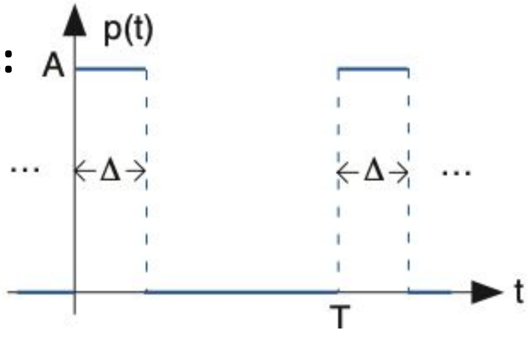
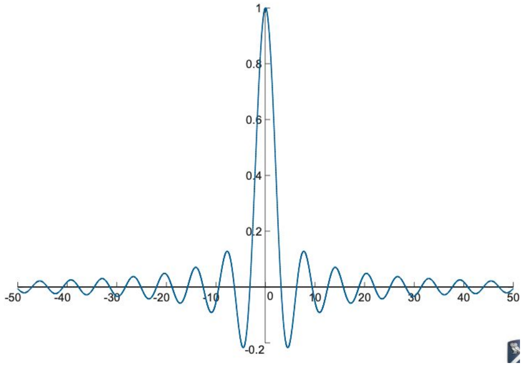
Semnal puls periodic:



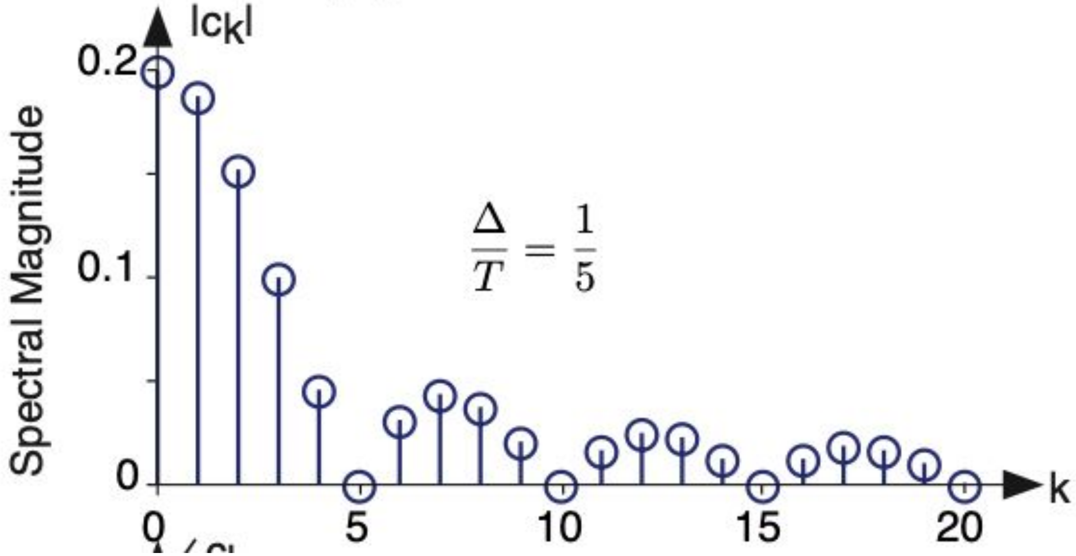
$$p(t) = \begin{cases} A, & 0 < t < \Delta \\ 0, & \Delta < t < T \end{cases}$$

- $c_k = A * e^{-j\frac{\pi k \Delta}{T}} * \frac{\sin\left(\frac{\pi k \Delta}{T}\right)}{\pi k} = A * e^{-j\frac{\pi k \Delta}{T}} * \frac{\Delta}{T} * \mathrm{sinc}\left(\frac{\pi k \Delta}{T}\right)$

$$\mathrm{sinc}(x) \equiv \frac{\sin x}{x}$$



$$c_k = A e^{-j\frac{\pi k \Delta}{T}} \frac{\sin\left(\frac{\pi k \Delta}{T}\right)}{\pi k} = e^{-j\frac{\pi k \Delta}{T}} \frac{A\Delta}{T} \mathrm{sinc}\left(\frac{\pi k \Delta}{T}\right)$$



Exercițiul 2 – filtrare

- If $x(t) = e^{j2\pi ft},\ y(t) = H(f)e^{j2\pi ft}$
- If $x(t) = e^{j\frac{2\pi kt}{T}},\ y(t) = H\left(\frac{k}{T}\right)e^{j\frac{2\pi kt}{T}}$
- If $x(t) = c_{k_1}e^{j\frac{2\pi k_1 t}{T}} + c_{k_2}e^{j\frac{2\pi k_2 t}{T}}$

$$y(t) = H\left(\frac{k_1}{T}\right)c_{k_1}e^{j\frac{2\pi k_1 t}{T}} + H\left(\frac{k_2}{T}\right)c_{k_2}e^{j\frac{2\pi k_2 t}{T}}$$

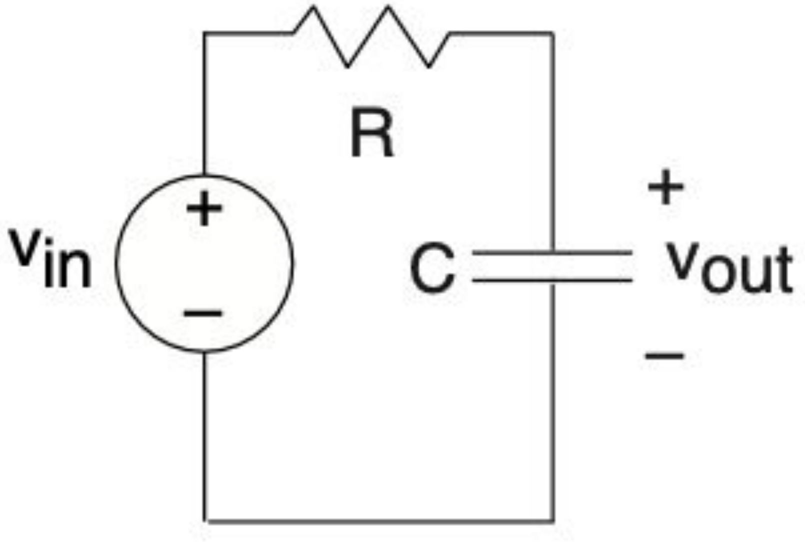
- If $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi kt}{T}},\ y(t) = \sum_{k=-\infty}^{\infty} H\left(\frac{k}{T}\right)c_k e^{j\frac{2\pi kt}{T}}$

$c_k^y = H\left(\frac{k}{T}\right) * \ c_k$, unde H este un sistem linear si invariant in timp

Exercițiul 2 – filtrare

- $H\left(f = \frac{k}{T}\right) = \frac{1}{1+j2\pi RC\frac{k}{T}}$
- $RC = \frac{1}{2\pi f_c}; f_c = \frac{1}{2\pi RC}$
- $c_k^y = \frac{1}{1+j2\pi RC\frac{k}{T}} * c_k$

$$y(t) = \sum_{k=-\infty}^{\infty} \frac{1}{1 + j2\pi RC\frac{k}{T}} c_k e^{j\frac{2\pi kt}{T}}$$



Filtru trece-jos

Exercițiul 2 – filtrare

T = 1ms (0.001s)

$f_T = 1 / T$