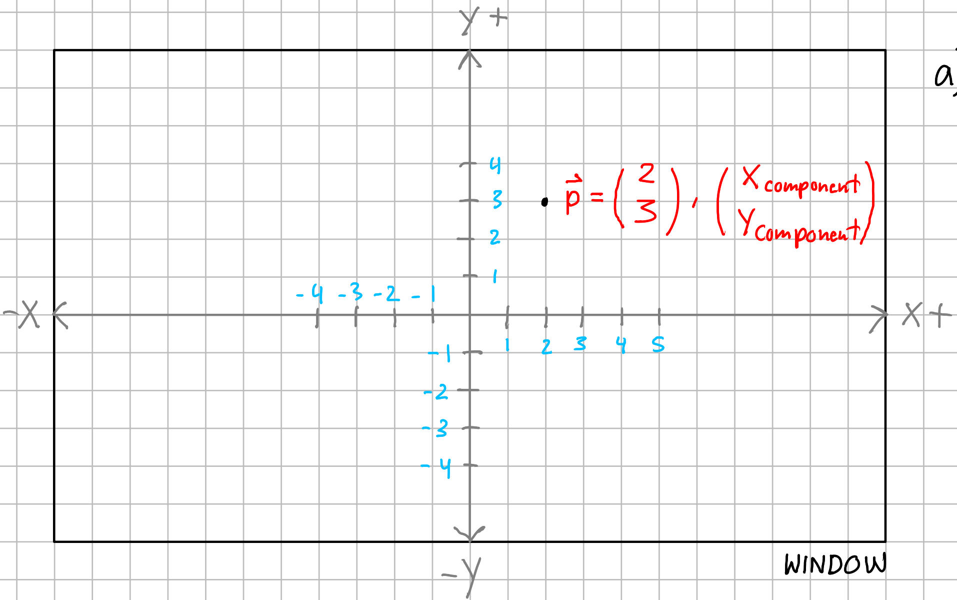
Whiteboard #1 (vectors and basic operations)

Vectors

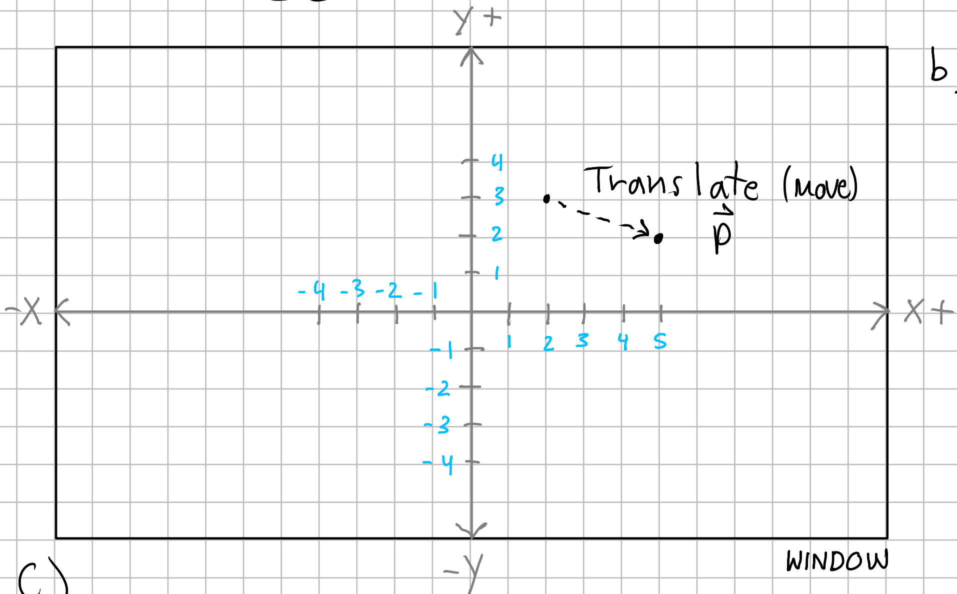


a) Related values grouped together e.g. X & Y are single values that describe how far along each axis we are.

$\vec{p}$ is an example of a vector. It represents a point in 2D space.

$\vec{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $x = 2$ and $y = 3$

Vector Operations



b) To move our point around in 2D space we just change our components inside $\vec{p}$.

Either manually setting each component, or by adding another vector to offset it.

$$\vec{p}' = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 2+3 \\ 3-1 \end{pmatrix}$$

$$\vec{p}' = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

There are many operations we can do on vectors much like our normal single valued numbers.

c) When we change where our points are in space, this is called a transformation.

Whiteboard #2 (defining geometry)

We can define geometry just by placing points around in Space. [Draw vertices of square]

Drawing lines from point to point will draw our square. Now with more complex shapes formed from combining our concepts of vectors, To move our new shap like what we saw before; we just apply our transform over all points.

We can use matrices to do lots of transformations at once.
But we need to extend the components in our vector.

✓ uneffected

$$\begin{array}{cc} & \;\; x \quad\; y \\ \text{row} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ & \text{col} \end{array} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + T_x \\ y + T_y \\ 1 \end{pmatrix}$$

Scaling

$$\begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} S_x X \\ S_y Y \\ 1 \end{pmatrix}$$

Rotation → just skewing in multiple directions

(x)

We can generalize these transformations the represent the state of our geometry

$M = T \cdot R \cdot S$

More on matix multiplication later it's very similar to our mat vec multipication component prod sum

Whiteboard #3

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \cdot x + 0 \cdot y + 0 \cdot z + 0 \cdot 1 \\ 0 \cdot x + 1 \cdot y + 0 \cdot z + 0 \cdot 1 \\ 0 \cdot x + 0 \cdot y + 1 \cdot z + 0 \cdot 1 \\ 0 \cdot x + 0 \cdot y + 0 \cdot z + 0 \cdot 1 \\ 0 \cdot x + 0 \cdot y + 0 \cdot z + 1 \cdot 1 \end{pmatrix}$$

Matrix Multiplication

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

Same multiplication process for 4x4.

if $M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ ← model matrix

if $P = \Big($ Perspective matrix $\Big)$

Multiplying is MP

Multiplying this against our verticies effects our geometry. lets see how

ORDER Of Multiplication maters: commutation
Commutative property