Fibonacci and Useless Graphs

Justin Baum

March 5, 2018 - May 31 2018

1 Introduction

We make a graph with the fibonacci sequence where each node is a number in the sequence, and every edge is any two nodes that when multiplied and subtracted with another(any node), give us another number in the fibonacci sequence. Then a set containing 1,2,3. We get some probably useless but neat properties.

$$S = \{a_1 = 1; a_2 = 1; a_n = a_{n-1} + a_{n-2} | n \in \mathbb{N}\}$$

$$V(G) = S$$

$$ij \in E(G) \leftrightarrow (i \in S \land j \in S \land ((\exists k \in S)(i \cdot j - k \in S))$$

$$Q = \{1, 2, 3\}$$

2 Conjectures

Conjecture 1:

$$(\forall i \in S)(\forall j \in S)((i \in Q \lor j \in Q) \land (i \neq 1 \lor j \neq 1) \rightarrow ij \in E(G)$$

Conjecture 2:

$$(\forall i \in S)(\forall j \in S)(i \notin Q \land j \notin Q \rightarrow ij \notin E(G))$$

Lemma 2.1.

$$(\forall i \in S)(\forall j \in S)((i \in Q \lor j \in Q) \land (i \neq 1 \lor j \neq 1) \rightarrow ij \in E(G)$$

Proof. i=1
Want: $\{1, j\} \in S$
Need: $j - k \in S$

1. $j = a_n \because j \in S$
2. $j - k$
3. $a_n - k$

4. $a_{n-1} + a_{n-2} - k$
5. $let\ k = a_{n-2} \because k \in S$
6. $j - k = a_{n-1} + a_{n-2} - a_{n-1} = a_{n-1}$
7. $a_{n-1} \in S$
8. $\therefore (\forall j \in S)(\exists k \in S)(j - k \in S) \rightarrow (\forall j \in S)(\{1, j\} \in E(G))$

□

Proof. i=2
Want: $\{2, j\} \in S$
Need: $2j - k \in S$

1. $j = a_n \because j \in S$
2. $2j - k$
3. $2a_n - k$
4. $let\ k = a_n \because k \in S$
5. $2j - k = 2a_n - a_n = a_n$
6. $\therefore (\forall j \in S)(\exists k \in S)(2j - k \in S) \rightarrow (\forall j)(\{2, j\} \in E(G))$

□

Proof. i=3
Want: $\{3, j\} \in S$
Need: $3j - k \in S$

1. $j = a_n \because j \in S$
2. $3j - k$
3. $3a_n - k$
4. $3a_{n-1} + 3a_{n-2} - k$
5. $3a_{n-2} + 3a_{n-3} + 3a_{n-2} - k$
6. $6a_{n-2} + 3a_{n-2} - k$
7. $let\ k = a_{n-2} \because k \in S$
8. $5a_{n-2} + 3_a n - 3$
9. $3(a_{n-2} + a_{n-3}) + 2a_{n-2}$
10. $3a_{n-1} + 2a_{n-2}$
11. $2(a_{n-1} + a_{n-2}) + a_{n-1}$

12. $2a_n + a_{n-1}$
13. $(a_n + a_{n-1}) + a_n$
14. $a_{n+1} + a_n$
15. a_{n+2}
16. $\therefore (\forall j \in S)(\exists k \in S)(3j - k \in S) \rightarrow (\forall j \in S)(\{3, j\} \in E(G))$ $\square$

$\square$

3 Conjecture 2: Exhaustive Search

For a finite universe of the first 45 terms of the fibonacci sequence:

$$|E(G)| = 128$$

The second conjecture on these results also remained true.
I also ran a test through the first 1000 terms, and the second conjecture held true. However this is not a proof, and my professor Dr. Cooper believes I need to hash out some number theory to fully prove this conjecture.

4 Conclusion

Useless graph with neat properties, the second conjecture most likely needs some number theory, so hopefully this is completed at some point. I was unable to find any papers with this conjecture, but then again I only used Google Scholar.