MATH544
Dr. Miller
HW 2

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Problem 1 (1.4.10)**.** *A linear system over a field $\mathbb{F}$ is called upper triangular if the coefficient $a_{ij} = 0$ whenever $i > j$.*
Show that if $a_{ii} \neq 0$ then the system is consistent with a unique solution.

Proof. Base Case: $n = 1$.

$$a_{11}x_1 = b1$$

Trivially we can calculate

$$x_1 = \frac{b_1}{a_{11}}$$

Inductive Hypothesis: Given an (n+1) x (n+1) upper triangular system, and $a_{ii} \neq 0$. If we remove the bottom n rows, and make that into an n x n system with $x_1, ..., x_n$, then E_{n+1} will be the (n+1)$^{\text{th}}$ linear equation with (n+1) terms.

$$a_{11}x_1 + ... + a_{1n}x_n = b_1$$

$$\vdots$$

$$a_{nn}x_n = b_n$$

Let $j = n + 1$

$$a_{j1}x_1 + ... + a_{jn}x_n + a_{jj}x_j = b_j$$

Due to the base case, the n x n system has a single unique solution, $x_1, \ldots, x_n$. Because so,

$$x_j = \frac{1}{a_{jj}} \left(-(a_{j1}x_1 + ... + a_{jn}x_n) + b_j\right)$$

Thus an (n+1) x (n+1) linear system has a unique solution as well. □