$S = \{a_1 = 1;\ a_2 = 2; a_n = a_{n-1} + a_{n-2} | \ n \in \mathbb{N}\}$

$V(G) = S$

$ij \in E(G) \leftrightarrow ((i \in S \wedge j \in S \wedge (\exists k)(k \in S \rightarrow (i \cdot j - k) \in S))$

$E(G) = \{\{1,2\}..\{1,89\}..\{2,2\}..\{2,89\}..\{3,3\}..\{3..89..\}..\}$

$Q = \{1,2,3\}$

Conjectures:

$(\forall i \in S)(\forall j \in S)(i \in Q \vee j \in Q \wedge (i \neq 1 \vee j \neq 1) \rightarrow \{i,j\} \in E(G))$ Hopefully Proven

$(\forall i \in S)(\forall j \in S)(i \notin Q \wedge j \notin Q \rightarrow \{i,j\} \notin E(G))$ Hopefully there exists a proof

The program came up with(first 45 terms):

S={1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,1597,2584,4181,6765,10946,17711,28657,46368,75025,121393,196418,317811,514229,8320
40,1346269,2178309,3524578,5702887,9227465,14930352,24157817,39088169,63245986,102334155,165580141,267914296,433494437,70140
8733,1134903170,1836311903}

For a finite universe of the first 45 terms:

E(G) ={
{1,2},{1,3},{1,5},{1,8},{1,13},{1,21},{1,34},{1,55},{1,89},{1,144},{1,233},{1,377},{1,610},{1,987},{1,1597},{1,2584},{1,4181},{1,6765},{
1,10946},{1,17711},{1,28657},{1,46368},{1,75025},{1,121393},{1,196418},{1,317811},{1,514229},{1,832040},{1,1346269},{1,2178309},{1
,3524578},{1,5702887},{1,9227465},{1,14930352},{1,24157817},{1,39088169},{1,63245986},{1,102334155},{1,165580141},{1,267914296}
,{1,433494437},{1,701408733},{1,1134903170},{2,2},{2,3},{2,5},{2,8},{2,13},{2,21},{2,34},{2,55},{2,89},{2,144},{2,233},{2,377},{2,610}
,{2,987},{2,1597},{2,2584},{2,4181},{2,6765},{2,10946},{2,17711},{2,28657},{2,46368},{2,75025},{2,121393},{2,196418},{2,317811},{2,5
14229},{2,832040},{2,1346269},{2,2178309},{2,3524578},{2,5702887},{2,9227465},{2,14930352},{2,24157817},{2,39088169},{2,6324598
6},{2,102334155},{2,165580141},{2,267914296},{2,433494437},{2,701408733},{2,1134903170},{3,3},{3,5},{3,8},{3,13},{3,21},{3,34},{3,5
5},{3,89},{3,144},{3,233},{3,377},{3,610},{3,987},{3,1597},{3,2584},{3,4181},{3,6765},{3,10946},{3,17711},{3,28657},{3,46368},{3,7502
5},{3,121393},{3,196418},{3,317811},{3,514229},{3,832040},{3,1346269},{3,2178309},{3,3524578},{3,5702887},{3,9227465},{3,1493035
2},{3,24157817},{3,39088169},{3,63245986},{3,102334155},{3,165580141},{3,267914296},{3,433494437},{3,701408733}}

Proof for i=1

Want: $\{1,j\} \in E(g) \leftrightarrow (j \in S \wedge j > 1)$

So we need: $1j - k \in S$

1. $j = a_n$ $\because j \in S$
2. $1j - k$
3. $a_n - k$
4. $a_{n-1} + a_{n-2} - k$
5. let $k = a_{n-2}$ $\because k \in S$
6. $1 \bullet j - k = a_{n-1} + a_{n-2} - a_{n-2} = a_{n-1}$
7. $a_{n-1} \in S$ by definition $\therefore 1j - k \in S \therefore \{1,j\} \in E(g) \leftrightarrow (j \in S)$

Proof for i=2

Want: $\{2,j\} \in E(g) \leftrightarrow (j \in S)$

So we need: $2j - k \in S$

1. $j = a_n$ $\because j \in S$
2. $2j - k$
3. $2a_n - k$
4. let $k = a_n$ $\because k \in S$
5. $2j - k = 2a_n - a_n = a_n$
6. $a_n \in S$ by definition $\therefore 2j - k \in S \therefore \{2,j\} \in E(g) \leftrightarrow (j \in S)$

Proof for i=3

Want: $\{3,j\} \in E(g) \leftrightarrow (j \in S)$

So we need: $3j - k \in S$

1. $j = a_n$ $\because j \in S$
2. $3j - k$
3. $3a_n - k$
4. $3a_{n-1} + 3a_{n-2} - k$
5. $3a_{n-2} + 3a_{n-3} + 3a_{n-2} - k$
6. $6a_{n-2} + 3a_{n-3} - k$
7. let $k = a_{n-2}$ $\because k \in S$
8. $5a_{n-2} + 3a_{n-3}$
9. $3(a_{n-2} + a_{n-3}) + 2a_{n-2}$
10. $3a_{n-1} + 2a_{n-2}$
11. $2(a_{n-1} + a_{n-2}) + a_{n-1}$
12. $2a_n + a_{n-1}$
13. $a_n + (a_n + a_{n-1})$
14. $a_n + a_{n+1}$
15. a_{n+2}
16. $a_{n+2} \in S$ by definition
17. $3j - k = a_{n+2}$ $\therefore 3j - k \in S \therefore \{3,j\} \in E(g) \leftrightarrow (j \in S)$

I checked the first 500 fibonacci numbers and none have more than 3 edges except 1,2,3 which have infinitely many as proven through induction.

Program(Python) to look for edges not including 1,2, or 3 for z terms(technically the search would be $a_n < \sqrt{a_z}$ because i*j would mean to fully search it we would need to generate all the way to the a_n*a_n in the fibonacci sequence.)

```
fibs = [1,2] #If we start the sequence at 1,2 we don't need to worry about
two 1s, and we get the same sequence just shifted for n>1
z = 500
b = 100
a = z//b
edges = []
for i in range(z):
    fibs.append(fibs[-1]+fibs[-2])
print("Done generating fibonacci list")
for i in range(3,z): #Print only the ones not including 1,2,3 which have
infinite edges
    if(i%a==0): print("%.1f%s done" %(i*100/z,"%"))
    for j in range(i,z):
        for k in range(z):
            i1 = fibs[i]
            j1 = fibs[j]
            k1 = fibs[k]
            if(i1*j1-k1 in fibs):
                edges.append([i1,j1])
                break
print(edges)
```