5.2 Methods of Curve Fittings

Suppose that we are given the points $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$ and we want to fit a function $y = xe^{Ax}, y = cx^A, y = \frac{1}{Ax+B}, etc.$

For these functions there are **two techniques** used
<u>(1) data linearization method</u> & **<u>(2) Nonlinear method using Matlab</u>**

<u>Data Linearization Method</u> (change of variable method)
The original (x_k, y_k) are transformed to new points (X_k, Y_k) & then we find least squares line $y = Ax + B$ for the new points we solve normal eqs & find A and B. After finding A & B, we need to write the function in terms of old variables (x_k, y_k). This technique is called data linearization.

Example: Consider the data

x_k	0	1	2	3	4
y_k	1.3	2.5	3.7	4.9	7.3

Use the data linearization method to find the least squares exponential fit $y = f(x) = ce^{Ax}$ for the data. Also find the root mean square error.

We want $y = ce^{Ax}$ in the form Y=Ax+B, so we need to get rid of the exponent.

Solution: $y = ce^{Ax}$

Taking *ln* on both sides.

$$\ln y = \ln(ce^{Ax}) \Rightarrow \ln y = \ln c + \ln e^{Ax}$$

$$\Rightarrow \underbrace{\ln y}_{Y} = \underbrace{\ln c}_{B} + A\underbrace{x}_{X}$$

The change of variables are $X = x, Y = \ln y$ & $\underbrace{B = \ln c}_{c = e^B}$

On exam will ask only up to here, the change of variables.

The normal eqs are

$$A\sum_{k=1}^{N} X_k^2 + B\sum_{k=1}^{N} X_k = \sum_{k=1}^{N} X_k Y_k$$

$$A\sum_{k=1}^{N} X_k + BN = \sum_{k=1}^{N} Y_k$$

	$X_k = x_k$	y_k	$Y_k = \ln y_k$	X_k^2	$X_k Y_k$
	0	1.3	$\ln(1.3) = 0.2624$	0	0
	1	2.5	$\ln(2.5) = 0.9163$	1	0.9163
	2	3.7	$\ln(3.7) = 1.3083$	4	2.6166
	3	4.9	$\ln(4.9) = 1.5892$	9	4.7676
	4	7.3	$\ln(7.3) = 1.9879$	16	7.9516
Sum	10		6.0641	30	16.2521

$$30A + 10B = 16.2521 \quad \leftarrow (1)$$
$$10A + 5B = 6.0641 \quad \leftarrow (2)$$

Multiplying eq**(2)** by 2 & subtracting from eq**(1)**

$$\begin{array}{rl} & 30A + 10B = 16.2521 \\ - & 20A + 10B = 12.1282 \\ \hline & 10A \quad\quad = 4.1239 \end{array} \quad \Rightarrow A = \frac{4.1239}{10} = 0.41239$$

Sub A into eq(2) $\Rightarrow 10(0.41239) + 5B = 6.0641$

$\Rightarrow 5B = 6.0641 - 4.1239 = 1.9402 \Rightarrow B = \dfrac{1.9402}{5} = 0.38804$

Ans. is in form Y=Ax+B, so we need back to form $y = ce^{Ax}$

So $c = eB = e^{0.38804} = 1.474088 \approx 1.4741$

Thus, the exponential fit is $y = f(x) = ce^{Ax} = 1.4741e^{0.41239x}$

Root Mean Square Error

The root mean square error is $E_2(f) = \left[\frac{1}{5}\sum_{k=1}^{5}\left(ce^{Ax} - y_k\right)^2\right]^{\frac{1}{2}}$

$$E_2(f) = \left[\frac{1}{5}\left\{(1.4741e^{0} - 1.3)^2 + (1.4741e^{0.41239} - 2.5)^2 + \left(1.4741e^{0.41239(2)} - 3.7\right)^2 + \left(1.4741e^{0.41239(3)} - 4.9\right)^2 + \left(1.4741e^{0.41239(4)} - 7.3\right)^2\right\}\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{5}\{0.0303 + 0.0748 + 0.1136 + 0.0322 + 0.13856\}\right]^{\frac{1}{2}}$$

=0.27909

One of these 7 will be asked on the exam.

Textbook has change of variable table for different functions on page 269.

when changing var., we want to get to this form
$y = Ax + B$

(1) $y = \frac{A}{x} + B$

Change of variables are $X = \frac{1}{x}, Y = y$

(2) $y = \frac{1}{Ax+B} \Rightarrow \underbrace{\frac{1}{y}}_{\text{Y}} = Ax + B$

Change of variables are $X = x$ & $Y = \frac{1}{y}$

(3) $y = \text{A} \underbrace{\ln x}_{\text{X}} + B$

Change of variables are $X = \ln x, Y = y$

(4) $y = (Ax + B)^{-2}$ or $y = \frac{1}{(Ax+B)^2}$

$$\frac{1}{y} = (Ax + B)^2 \Rightarrow \frac{1}{\sqrt{y}} = Ax + B$$

Change of variables are $X = x, Y = \frac{1}{\sqrt{y}}$ or $y^{-\frac{1}{2}}$

(5) $y = \frac{x}{A+Bx}$

$$\frac{1}{y} = \frac{A + Bx}{x} \Rightarrow \underbrace{\frac{1}{y}}_{\text{Y}} = \frac{A}{x} + \frac{B\cancel{x}}{\cancel{x}}$$

Change of variables are $\text{X} = \frac{1}{x}$ and $\text{Y} = \frac{1}{y}$

(6) $y = ce^{Ax}$

$\ln y = (ce^{Ax}) \Rightarrow \underbrace{\ln y}_{\text{y}} = \underbrace{\ln c}_{\text{B}} + \underbrace{Ax}_{\text{x}}$

Change of variables are $X = x$, Y=ln y & B = $\ln c$ or $c = e^B$

(7) $y = cx^A$

$\ln y = \ln(cx^A) \Rightarrow \ln y = \ln c + \ln(x^A) \Rightarrow \ln y = \ln c + A \ln x$

Change of variables are $X = \ln x, Y = \ln y\,, B = \ln c$ or $c = e^B$