MATH 3940-1 Numerical Analysis for Computer Scientists

Problem Set 1: Systems of Linear Equations

Note: You can use Octave or Matlab for the questions that says to use Matlab.

1. Consider the following system

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 6 \\ 3x_1 + x_2 &= 5 \\ 2x_1 + x_2 + x_3 &= 3 \end{aligned}$$

   (a) Solve the system using Gaussian elimination method with no pivoting.

   (b) Solve the system using Gaussian elimination method with partial pivoting.

2. Consider the system of linear equations

$$\begin{aligned} -x_1 + 2x_2 + 2x_3 + 5x_4 + x_5 &= 7 \\ 3x_2 + x_3 + 2x_4 + x_5 &= 5 \\ x_1 - 4x_2 + x_3 - 2x_5 &= 9 \\ 5x_2 + 3x_3 + x_4 + x_5 &= 2 \\ 3x_1 - 6x_2 + 4x_4 + 3x_5 &= -1 \end{aligned}$$

   (a) Use Matlab to find the determinant and the inverse of the coefficient matrix A.

   (b) Use Matlab built in command (mentioned during lectures) to solve the linear system $AX = B$

3. Consider the system of linear equations

$$\begin{aligned} x_2 + 2x_3 - x_4 &= -1 \\ x_1 + x_2 - x_3 &= 5 \\ -x_1 - x_2 + x_3 + 3x_4 &= 1 \\ x_1 + 2x_2 + x_4 &= 9 \end{aligned}$$

   (a) Find the LU factorization of the coefficient matrix A and then solve the resulting triangular system.
   (b) Use Matlab built in command to find the LU factorization of the coefficient matrix A and then solve the resulting triangular system using forward and backward substitutions programs.

4. Consider the system of linear equations

$$\begin{aligned} x_1 + 2x_2 - x_3 &= -1 \\ 2x_1 + 8x_2 - 4x_3 &= -10 \\ -x_1 - 4x_2 + 3x_3 &= 7 \end{aligned}$$

   (a) Find the Cholesky factorization of the coefficient matrix A and then solve the resulting triangular system.

(b) Use Matlab built in command to find the Cholesky factorization of the coefficient matrix A and then solve the resulting triangular system using forward and backward substitutions programs.

5. Consider the linear system

$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 0 \\ 2x & + & 8y & - & 4z & = & 6 \\ -x & - & 4y & + & 3z & = & -2 \end{array}$$

(a) Perform two iterations of Jacobi method starting with the zero vector.
(b) Use Matlab to perform a maximum of 35 iterations of Jacobi method starting with the zero vector and tolerance of 10^{-6}. Does it converge? If yes, how many iterations does it take to converge?
(c) Perform two iterations of Gauss-Seidel method starting with the zero vector.
(d) Use Matlab to perform a maximum of 35 iterations of Gauss-Seidel method starting with the zero vector and tolerance of 10^{-6}. Does it converge? If yes, how many iterations does it take to converge?