MATH 3940-1 Numerical Analysis for Computer Scientists
Assignment 1
Due in class on Wednesday, September 25, 2019

- You have to provide Matlab/Octave Sheets for any program used, inputs and the outputs. Hand written programs will not be accepted.
- Show all your work to receive full credit.
- You can discuss assignments with each other but do not copy them. Identical or nearly identical assignments will not be accepted.

1. Consider the following system

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & - & x_3 & = & 6 \\ 3x_1 & + & x_2 & & & = & 5 \\ 2x_1 & + & x_2 & + & x_3 & = & 3 \end{array}$$

(a) (5 marks) Use hand calculations to solve the system using Gaussian elimination method with no pivoting.

(b) (7 marks) Use hand calculations to solve the system using Gaussian elimination method with partial pivoting.

2. (a) (5 marks) Use Matlab to generate a 7×7 Hilbert matrix H, where $H(i,j) = \dfrac{1}{i+j-1}$. Use Matlab to find the determinant and the inverse of H. Also find the condition number of H, using the infinity (uniform) norm.

(b) (2 marks) Use Matlab built in function to solve the system $AX = B$, where A is the Hilbert matrix generated in part(a) and the matrix $B = [1\ 0\ 0\ 0\ 0\ 0\ 0]'$.

(c) (2 marks) Use Matlab built in function to solve the system $AX = B$, where A is the Hilbert matrix generated in part(a) and the matrix $B = [1\ 0\ 0\ 0.01\ 0\ 0\ 0]'$.

3. Consider the system of linear equations

$$\begin{array}{rcrcrcrcr} & & & - & x_3 & + & x_4 & = & -1 \\ x_1 & + & x_2 & - & x_3 & + & 2x_4 & = & -1 \\ x_1 & + & x_2 & & & + & 3x_4 & = & 2 \\ x_1 & + & 2x_2 & - & x_3 & + & 3x_4 & = & 1 \end{array}$$

(a) (10 marks) Use hand calculations to find the LU decomposition of the coefficient matrix A and then solve the resulting triangular system.
(b) (9 marks) Use Matlab to find the LU decomposition of the coefficient matrix A and then solve the resulting triangular system using forward and backward substitutions programs.

4. Consider the system of linear equations

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 0 \\ 2x_1 + 8x_2 - 4x_3 &= 6 \\ -x_1 - 4x_2 + 3x_3 &= -2 \end{aligned}$$

(a) (10 marks) Use hand calculations to find the Cholesky decomposition of the coefficient matrix A and then solve the resulting triangular system.

(b) (5 marks) Use Matlab to find the Cholesky decomposition of the coefficient matrix A and then solve the resulting triangular system using forward and backward substitutions programs.

Question 2: (a) You can use the command hilb(7) or generate a 7 X 7 Hilbert matrix as follows:

```
>> for i=1:7
   for j=1:7
    H(i,j)=1/(i+j-1);
   end
  end
>> H
H =
   1.0000   0.5000   0.3333   0.2500   0.2000   0.1667   0.1429
   0.5000   0.3333   0.2500   0.2000   0.1667   0.1429   0.1250
   0.3333   0.2500   0.2000   0.1667   0.1429   0.1250   0.1111
   0.2500   0.2000   0.1667   0.1429   0.1250   0.1111   0.1000
   0.2000   0.1667   0.1429   0.1250   0.1111   0.1000   0.0909
   0.1667   0.1429   0.1250   0.1111   0.1000   0.0909   0.0833
   0.1429   0.1250   0.1111   0.1000   0.0909   0.0833   0.0769

>> det(H)
ans =  4.8358e-025
>> inv(H)
ans =  1.0e+008 *
   0.0000  -0.0000   0.0001  -0.0003   0.0005  -0.0004   0.0001
  -0.0000   0.0004  -0.0032   0.0113  -0.0194   0.0160  -0.0050
   0.0001  -0.0032   0.0286  -0.1058   0.1871  -0.1572   0.0505
  -0.0003   0.0113  -0.1058   0.4032  -0.7277   0.6209  -0.2018
   0.0005  -0.0194   0.1871  -0.7277   1.3340  -1.1526   0.3784
  -0.0004   0.0160  -0.1572   0.6209  -1.1526   1.0059  -0.3330
   0.0001  -0.0050   0.0505  -0.2018   0.3784  -0.3330   0.1110
```

Using the infinity norm, the condition number of H is

```
>> condH=norm(H,inf)*norm(inv(H),inf)
condH = 9.8519e+008
```

If we use the condition number command in Matlab which uses norm 2, we obtain

```
>> cond(H)
  ans =  4.7537e+008
```

However we can use >> cond(H,inf)

```
ans =  9.8519e+008
```

(b) To solve HX=B, where B=[1 0 0 0 0 0 0]', we use the built in command in Matlab

```
>> B=[1 0 0 0 0 0 0]';
>> X=H\B
X =  1.0e+004 *
   0.0049
  -0.1176
   0.8820
  -2.9400
   4.8510
  -3.8808
   1.2012
```

(c) To solve HX=B, where B=[1 0 0 0.01 0 0]', we use the built in command Matlab

```
>> B=[1 0 0 0.01 0 0 0]';
>> X=H\B
X = 1.0e+005 *
  -0.0025
   0.1011
  -0.9702
   3.7380
  -6.7914
   5.8212
  -1.8979
```

Question 3: (b) Using the Matlab command lu, we find that

```
>> A=[0 0 -1 1; 1 1 -1 2; 1 1 0 3; 1 2 -1 3];
>> [L U P]=lu(A)
L =
    1    0    0    0
    1    1    0    0
    1    0    1    0
    0    0   -1    1
U =
    1    1   -1    2
    0    1    0    1
    0    0    1    1
    0    0    0    2
P =
    0    1    0    0
    0    0    0    1
    0    0    1    0
    1    0    0    0
```

After finding the L U and P, we have to find PB

```
>> B=[-1 -1 2 1]' ;
>> PB=P*B;
```

We write this forward substitution program in M file

```
function X=forsub(A,B)
% A is an n x n lower triangular nonsingular matrix and B is n x 1 matrix
n=length(B);
X=zeros(n,1);
X(1)=B(1)/A(1,1);
for k=2:n
  X(k)=(B(k)-A(k,1:k-1)*X(1:k-1))/A(k,k);
end
```

```
>> Y=forsub(L,PB)
Y =
        -1
         2
         3
         2
```

Then we use the backward substitution program in M file

```
function X=backsub(A,B)
% A is an n x n upper triangular nonsingular matrix and B is n x 1 matrix
% Find the dimension of B and initialize X
n=length(B);
X=zeros(n,1);
X(n)=B(n)/A(n,n);
for k=n-1:-1:1
  X(k)=(B(k)-A(k,k+1:n)*X(k+1:n))/A(k,k);
end
```

```
>> X=backsub(U,Y)
X =
    -2
     1
     2
     1
```

Question 4:

(b) >> A=[1 2 -1; 2 8 -4; -1 -4 3];

```
>> U=chol(A)
U =
   1   2  -1
   0   2  -1
   0   0   1
```

>> B=[-1 -10 7]';

Using forward substitutions and backward substitution program from previous question,

```
>> Y=forsub(U',B)
Y =
     0
     3
     1
```

```
>> X=backsub(U,Y)
X =
    -3
     2
     1
```