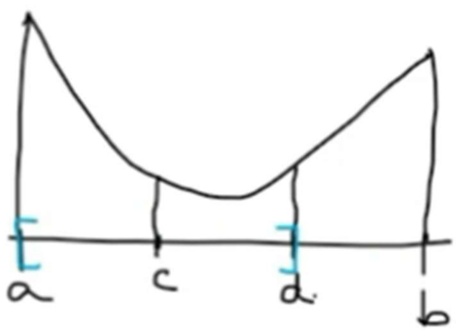
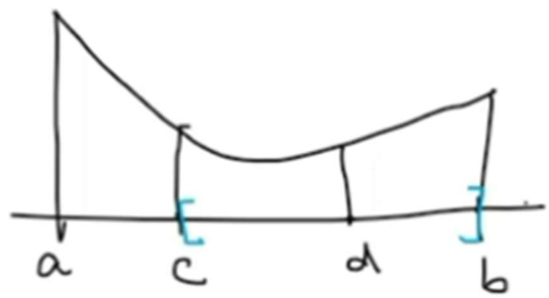
Golden Ratio Method

f is unimodal on [a, b] then we can make the interval smaller with the following procedure which is golden ratio search method for finding local min. value of f.
We find two interior points c and d





If $f(c) \leq f(d)$ then we squeeze from the right i.e., the new interval will be [a, d]	If $f(D) < f(c)$ then we squeeze from the left i.e., the new interval will be [c, b]

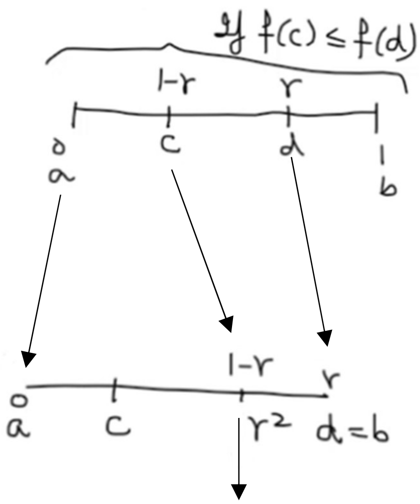
The interior points c and d are selected such that the resulting interval [a, c] & [d, b] are symmetrical. That is b-d= c-a, where

$c = a + (1 - r)(b - a)$

$d = b - (1 - r)(b - a)$

and $\frac{1}{2} < r < 1$ to ensure that $c < d$ always

We want the value of r to remain constant on each subinterval. Also, we want to have r such that the one of the interior points will remain the interior point of the new interval and one interior point will become the exterior point. So, only one new function evaluation is needed at each iteration.



new d

$$d = b - (1-r)(b-a)$$
$$= r - (1-r)(r-0)$$
$$= r - r + r^2$$

$b - a = 1$

$c = 0 + (1-r)(b-a)$

$= 1 - r$

$d = 1 - (1-r)(1)$

$= 1 - 1 + r$

$= r$

⇒We need

$r^2 = 1 - r$

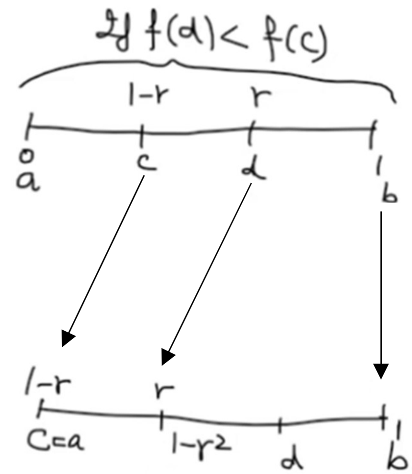
$r^2 + r - 1 = 0$

We need ⇐

$r = 1 - r^2$

$r^2 + r - 1 = 0$

same



new c

$$c = a + (1-r)(b-a)$$
$$= 1 - r + (1-r)(1-1+r)$$
$$= 1 - r + r - r^2$$
$$= 1 - r^2$$

r will have to satisfy $r^2 + r - 1 = 0$

$$r = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

$$= 0.61803, \underbrace{-1.61803}_{\text{reject as } \frac{1}{2} < r < 1}$$

So, $r = 0.61803$ or $r = \frac{\sqrt{5}-1}{2}$

Textbook has program on page 422