MATH 3940   Numerical Analysis for Computer Scientists
Final Exam

December 17, 2021   8:30 am to 11:00 am

Last Name: _____ First Name: _____ ID: _____

- Download this exam to avoid any problem with internet while you are solving it.

- You can print this exam and write your solutions on it, or you can work on your own paper and put just the question number next to your solutions. In both cases make sure you have written your name and ID on your papers. Try to keep the questions in the same order when you scan your solutions.

- This exam has 11 questions. You have 150 minutes (2 hours and 30 minutes).

- Read carefully and answer **all** questions.

- **Show all your work to receive full credit**.

- Answers in decimal form should be rounded to at least three decimal places.

- Only non-programmable and non-graphic calculators are allowed.

- Answers for polynomials should be written in simplified form.

- **You must stop writing at 11:00 am and must upload a single pdf file (not a zip file) with your solved exam by 11:30 am on Blackboard**. Make sure the pdf is clear and your solutions can be seen clearly to avoid any confusion during marking. If your exam cannot be read, it will not be marked. Better to use pen or dark pencil if possible. **You can submit your exam solutions once** (you can save draft and check that it is fine before you submit). Make sure to check your submission is submitted and not in progress (you should see a yellow icon in My Grades if it is submitted). You are responsible for your submission, this means **it is your responsibility to make sure your submission is on time and accurate**(the file is correct and includes all pages, pages on email will not be accepted). **Late submissions will not be accepted**.

- **Code of Conduct:** This open book examination must be written without collaboration. While writing these you must not communicate with any other person and you must not consult the internet or any material other than the course textbook, your course notes, assignments, and any material posted on the course Blackboard pages. If there is any evidence of academic misconduct, we will file an allegation of misconduct according to Bylaw 31.

1. (11 marks) Consider the system
$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & - & 2x_3 & = & -1 \\ 2x_1 & + & 4x_2 & - & 9x_3 & = & 3 \\ 3x_1 & + & x_2 & + & 2x_3 & = & -1 \end{array}$$

Find the LU decomposition of the coefficient matrix A with no pivoting and then solve the resulting triangular systems.

2. (4 marks) Let $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$ and $X_0 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

Perform one iteration of the power method starting with X_0.

3. (5 marks) Consider the equation: $x^2 + 2x - 4 = 2x$. Perform one iteration of Regula-Falsi method starting with the interval $[0, 3]$.

4. (6 marks) Consider the equation: $3e^x - 5x - 4 = 0$

(a) Can we use bisection method to solve the equation starting with the interval $[0, 2]$? Justify your answer using the conditions of convergence.

(b) Perform one iteration of Newton's method to solve the above equation starting with $p_0 = 0$.

5. (8 marks) Suppose Jacobi method is used to solve the system

$$
\begin{aligned}
x + \sqrt{y} &= 3 \\
x^4 + 2y &= 9
\end{aligned}
$$

starting with $(x_0, y_0) = (0.5, 3)$. Do you expect iterations to converge to the solution $(x, y) = (1, 4)$? Justify your answer using the conditions of convergence.

6. Let $f(x) = x^4 - 2x$. The nodes are $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$.

   (a) (4 marks) Find Lagrange polynomial $P_2(x)$ using the nodes x_0, x_1, and x_2.

   (b) (4 marks) Find divided difference table and Newton polynomial $P_2(x)$ using the nodes x_0, x_1, and x_2.

   (c) (4 marks) Calculate the approximate error for Lagrange polynomial $P_2(x)$ at $x = 0.8$. (use c=0.3)

7. (a) (6 marks) Consider the data

x_k	1	2	3
y_k	0.2	2	6

Find the least-squares power fit $y = Ax^2$ for the data and also calculate the root-mean-square error $E_2(f)$.

(b) (2 marks) Suppose you have to find the least-squares curve $y = (Ax + B)^{-2}$ by data linearization method, what would be the change of variable formulas?

8. (5 marks) Consider the data

x_k	2	2.06	2.12
$f(x_k)$	0.92	0.81	0.73

Find the approximations to $f'(2)$ and $f'(2.06)$ of order $\mathbf{O}(h^2)$.

9. (5 marks) Let $f(x) = 15x + 6x^2 - x^3$. Can Golden Ratio search method be used to find a local minimum of f starting with the interval $[0, 7]$? Justify your answer using the condition of convergence.

10. (9 marks) Consider the integral $\int_0^1 \tan x \, dx$

   (a) Approximate the above integral using Simpson's 3/8 rule.

   (b) Approximate the above integral using composite Trapezoidal rule with 4 points.

11. (7 marks) Find the order of error in the following approximation. (show your steps)

$$f'(x) = \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$$