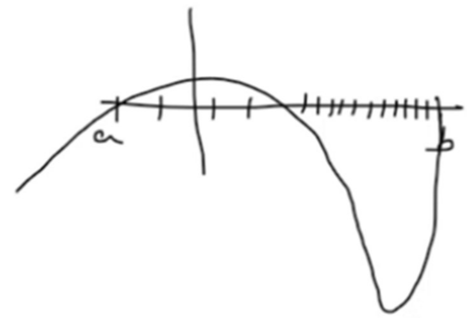
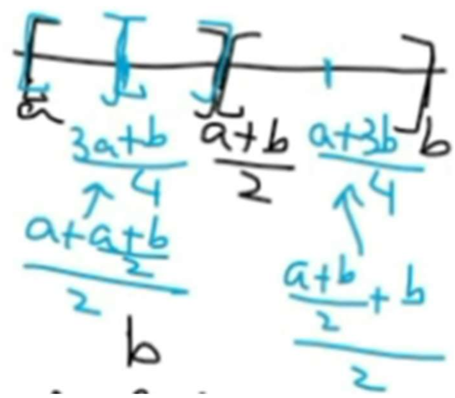
7.4 Adaptive Quadrature

Composite Simpson rule the nodes are equally spaced which is not good to use if a function varies a lot in certain interval compared to other intervals. So, in those cases it is good to divide some portion into smaller subintervals. Compared to subintervals in other portion, this is called adaptive quadrature. If we have to integrate $f(x)$ on the interval [a, b] with some desired accuracy (or tolerance) ε.



So, we apply Simpson rule to $[a,b]$, $\left[a,\frac{(a+b)}{2}\right]$ & $\left[\frac{(a+b)}{2},b\right]$



$$\text{If, } \frac{1}{10}\left|\overbrace{s(a,b)}^{\text{Simpson rule on [a, b]}}-\left\{s\left(a,\frac{a+b}{2}\right)+s\left(\frac{a+b}{2},b\right)\right\}\right|<\varepsilon$$

Then the integral $\int_a^b f(x)\,dx$ is $s\left(a,\frac{a+b}{2}\right)+s\left(\frac{a+b}{2},b\right)$

$$\text{If } \frac{1}{10}\left|\ s(a,b)-\left\{s\left(a,\frac{a+b}{2}\right)+s\left(\frac{a+b}{2},b\right)\right\}\right| \not< \varepsilon$$

Then we divide intervals further

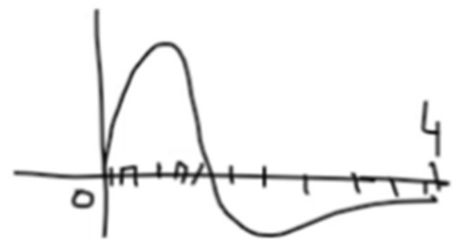
$$\text{If } \frac{1}{10}\left|\ s\left(a,\frac{a+b}{2}\right)-\left\{s\left(a,\frac{3a+b}{4}\right)+s\left(\frac{3a+b}{4},\frac{a+b}{2}\right)\right\}\right| \not< \frac{\varepsilon}{2}$$

Then we do not divide $\Big|\ \left[a,\frac{a+b}{2}\right]$ further, but if it is not less than $\frac{\varepsilon}{2}$ then we divide further.

Also, if $\frac{1}{10}\left| \; s(\frac{a+b}{2}, b) - \left\{ s\left(\frac{a+b}{2}, \frac{a+3b}{4}\right) + s\left(\frac{a+3b}{4}, b\right) \right\} \right| \nless \frac{\varepsilon}{2}$

Then we do not divide $\left[\frac{a+b}{2}, b\right]$ further but if it is not less than $\frac{\varepsilon}{2}$, then we divide it further.

14:00 to 18:00 Part 2 Lecture 20 watch



Textbook has example on page 395

$\int_0^4 13(x - x^2)\, e^{\frac{-3x}{2}}\, dx$ tolerance is $\varepsilon = 0.00001$

Book has table of values for Simpson rule for different subintervals. With adaptive quadrature [0, 4] is divided into 20 subintervals of vriable length & 81 function evaluations are needed to have accuracy of $\varepsilon = 10^{-5}$

For composite Simpson rule we can have accuracy of 10^{-5} by dividing [0, 4] into 128 subintervals and 256 function evaluations are needed.