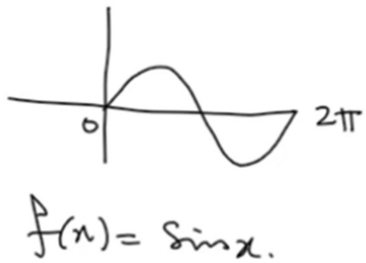
Trapezoidal rule is extremely accurate when periodic functions are integrated over the period.



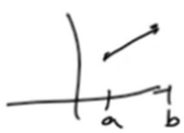
Trapezoidal→ $\int_0^{2\pi} sinx\ dx = 0$

Exact → $\int_0^{2\pi} sinx\ dx = -\cos x \Big|_0^{2\pi} = -\cos 2\pi + \cos 0$

$= -1 + 1$

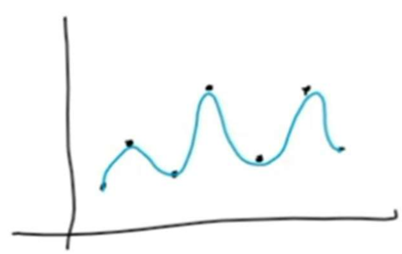
$= 0$

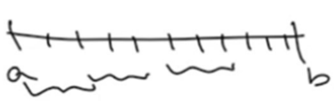
For function f such that $f, f', \& f''$ are cont. on [a, b], Simpson's rule converges faster than Trapezoidal rule. However, for various function which has weaker smoothness conditions, the trapezoidal rule works better than Simpson's rule. $(f''\ is\ precise??\ cont)$



The Newton cotes formulas are generally not suitable for use on large intervals. Higher degree formulas are generally not suitable for use on large intervals. Higher degree formulas are needed (to have small h) and the values of coefficient formulas are difficult to obtain. Also, the interpolation polynomial increases which means that it will oscillate more.

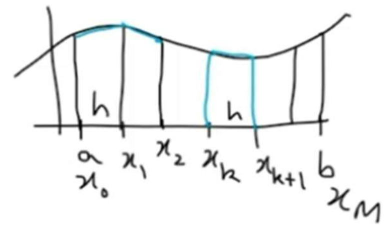
So, to avoid these issues a piecewise approach to numerical integration is used.





We divide [a, b] into smaller intervals and use low order Newton Cotes formulas for smaller intervals and then add them to get the value over the interval [a, b].

7.2 Composite Trapezoidal Rule



We have to integrate a function f over the interval [a, b]. We subdivide the interval [a, b] into M sub intervals $[x_k, x_{k+1}]$ of equal length $h = \frac{b-a}{M}$

So, we use equally spaced nodes $x_k = a + kh$, for $k = 0,1,2, \ldots , M$

$$or\ x_k = x_0 + kh$$

We apply trapezoidal rule to each subinterval $[x_k, x_{k+1}]$ and add them to get the approximation to definite integral of f over $[a, b]$.

$$\int_a^b f(x)\, dx = \frac{h}{2}[f_0 + f_1] + \frac{h}{2}[f_1 + f_2] + \cdots + \frac{h}{2}[f_{M-2} + f_{M-1}] + \frac{h}{2}[f_{M-1}, f_M]$$

$$f(x_0) \quad f(x_1) \quad f(x_1) \quad f(x_2) \qquad f(x_{M-2}) \quad f(x_{M-1}) \quad f(x_{M-1}) \quad f(x_M)$$

$\int_a^b f(x)\, dx = \frac{h}{2}[f_0 + 2(f_1 + f_2 + \cdots + f_{M-1}) + f_M]$ Where $h = \frac{(b-a)}{M}$ that is, there are $M + 1$ Points (nodes).

Or

$$\int_a^b f(x)\, dx = \frac{h}{2}\left[f_0 + 2\sum_{k=1}^{M-1} f_k + f_M\right]$$

The error for Composite trapezoidal rule is of $O(h^2)$

Error is $\frac{-(b-a)f''(c)h^2}{12}$