1-Solve the system using Gaussian elimination method with partial pivoting.

2-Find all the eigenvalues of A and find the eigenvector corresponding to the dominant eigenvalue.

(b) (3 marks) Perform one iteration of the power method starting with X0.

(c) (1 mark) What can you say about the convergence of the power method for the matrix A? Justify your answer.

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1. (6 marks) Solve the system using Gaussian elimination method with partial pivoting.

$$
\begin{aligned}
2x_1 + 6x_2 - 8x_3 &= 14 \\
4x_2 + 3x_3 &= -10 \\
x_1 - 2x_2 + x_3 &= 2
\end{aligned}
$$

2. Consider the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 2 & -4 \\ 0 & 1 & -3 \end{bmatrix}$ and $X_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(a) (5 marks) Find all the eigenvalues of A and find the eigenvector corresponding to the dominant eigenvalue.

(b) (3 marks) Perform one iteration of the power method starting with X_0.

(c) (1 mark) What can you say about the convergence of the power method for the matrix A? Justify your answer.

4

3. (6 marks) Consider the equation: $2\sin x + 3x = 7$

   (a) Perform one iteration of Newton's method starting with $x_0 = 4$.

   (b) Can we use secant method to solve the equation starting with the interval $[0, 1]$? Justify your answer using the condition of convergence.

5

4. (4 marks) Let $g(x) = x^3 - x + 1$ and fixed point method is used to solve the equation $x = g(x)$. Do you expect that the fixed point method will converge to the solution $x = 1$ starting with $x_0 = 0$? Justify your answer using the condition of convergence.

5. (5 marks) Consider the linear system
$$\begin{array}{rcrcrcr} 3x & - & y & + & z & = & 9 \\ 2x & + & 6y & + & z & = & 12 \\ x & + & 4y & - & 7z & = & 3 \end{array}$$

   (a) Starting with $X_0 = (0, 0, 0)$, perform one iteration of Gauss-Seidel method.

   (b) Do you expect that the iterations of Gauss-Seidel method for the above system will converge? Justify your answer using the condition of convergence.

6. Let $f(x) = x^3 + 2x^2 - 1$. The nodes are $x_0 = -1$, $x_1 = 0$, and $x_2 = 1$.
(a) (4 marks) Find Lagrange polynomial $P_2(x)$ using the nodes x_0, x_1, and x_2.
(b) (4 marks) Find divided difference table and Newton polynomial $P_2(x)$ using the nodes x_0, x_1, and x_2.
(c) (3 marks) Calculate the approximate error for Lagrange polynomial $P_2(x)$ at $x = 0.5$.

7. (a) (6 marks) Find the least-squares line $y = f(x) = Ax + B$ for the data

x_k	1	2	3
y_k	4	7	12

(b) (2 marks) Suppose you have to find the least-squares curve $y = (Ax + B)^{-1}$ by data linearization method, what would be the change of variable formulas?

8. (5 marks) Consider the data

x_k	1	1.04	1.08
$f(x_k)$	0.841	0.862	0.882

Find the approximations to $f'(1)$ and $f''(1.04)$ of order $\mathbf{O}(h^2)$.

9. (5 marks) Find the order of error in the approximation (show your steps)
$f'(x) = \dfrac{f(x+h) - f(x-h)}{2h}$

10. (11 marks) Consider the integral $\displaystyle\int_1^4 \ln x \, dx$

(a) Approximate the above integral using trapezoidal rule. Also find the exact error for the approximation.

(b) Approximate the above integral using composite Simpson's rule with 5 points.

11. (5 marks) Let $f(x) = 2x^3 - 6x + 1$. Can Golden Ratio search method be used to find a local minimum of f starting with the interval $[0, 3]$? Justify your answer using the condition of convergence.