

*Q	Method	Formula	Convergence/info	Rate
1	Gaussian Elimination	Swap, X, - rows of [A B] to get UTM Bckwd Sub to get values of [X]. → $X = A \setminus B$	Partial Pivot: $PAX = PB$ → pivot = largest magnitude	Both = $O(N^3)$ fwd/bwd Sub = $O(N^2)$
	L. U. Factorization	$U = UTM$, using Gauss. Elim. on [A]. $L = LTM$, 1 on diag., then multipliers. → $[L U P] = lu(A)$	$A = LU \Rightarrow LUX = B, UX = Y \wedge LY = B$ $PLUX = PB \Rightarrow UX = Y \wedge PLY = PB$	
1	Eigenvalues Eigenvectors Power Method	$\det(A - \lambda I) = 0 \Rightarrow [V D] = \text{eig}(A)$ $(A - \lambda I)\vec{V} = 0$ $Y_k = AY_k, X_{k+1} = \frac{1}{\ Y_k\ } Y_k$ → C_{k+1} is largest mag. coord. of Y_k .	* At least one a_{kk} will = 0 → let $V_k = 1$ dom. e-val is unique → conv. d. e-v. not unique (3, -3) → div.	N/A N/A $\left(\frac{\lambda_2}{\lambda_1}\right)^k$
2-3	Fixed Point $f(x) = 0 \rightarrow x = g(x)$ Bisection	$P_{k+1} = g(P_k)$ → roots(P) * $P = [\text{coeffs of poly}]$ $c = (a+b)/2$ & find $f(c)$ * $f(c) = 0 \rightarrow \text{stop}$ → use c to replace a or b to keep $[+, -]$ → $c = b - [f(b)(b-a) / (f(b) - f(a))]$	$g, g' \in C[a, b] \wedge p \in p_0 \in [a, b]$ $ g'(p) < 1 \rightarrow \text{conv.}$ " $> 1 \rightarrow \text{div.}$ $f \in C[a, b] \wedge f(a)f(b) < 0 \rightarrow \text{GC}$	N/A " = 1 → mag/not linear
	Regula Falsi; Secant {M} Newton	$P_{k+1} = P_k - [f(P_k)(P_k - P_{k-1}) / (f(P_k) - f(P_{k-1}))]$ $X_k = X_{k-1} - \{M\} [f(X_{k-1}) / f'(X_{k-1})]$	" \wedge " → GC $f \in C[a, b] \rightarrow \text{LC}$ " $\wedge f'(x_{k-1}) \neq 0 \rightarrow \text{LC}$	linear M=1 → quad. M>1 → linear
1	Jacobi	$X_{k+1} = f_1(y_k, z_k) \quad z_{k+1} = f_3(x_k, y_k)$ $y_{k+1} = f_2(x_k, z_k)$	A is strictly diagonally dom. → $ a_{kk} > \sum_{j=1, j \neq k}^n a_{kj} , \forall k$ → convergense → else, mag/not conv.	N/A
	Gauss-Seidel	$X_{k+1} = " \quad y_{k+1} = f_2(x_{k+1}, z_k)$ $z_{k+1} = f_3(x_{k+1}, y_{k+1})$		N/A
	Non-Linear	Both = same as linear, but conv. is new. → $g_1, g_2, \partial g \in [R] \wedge (P, q) \in (P_0, q_0) \in [R]$		N/A
	Lagrange	$P_N(x) = \sum_{k=0}^N y_k L_{N,k}(x) \wedge L_{N,k}(x) = \prod_{j=0, j \neq k}^N \frac{x - x_j}{x_k - x_j}$	$E_N = \frac{f^{(N+1)}(c)(x - x_0) \dots (x - x_N)}{(N+1)!}$ → $c \in [x_0, x_N]$	N/A
1	Newton	$P_1(x) = a_0 + a_1(x - x_0)$ $P_2(x) = " + a_2(x - x_0)(x - x_1)$ $P_n(x) = " + " + \dots + a_n(x - x_0) \dots (x - x_{n-1})$	$E_N = "$ $a_0 \dots a_N \rightarrow \text{use div. diff. table}$	N/A
1-2	Differentiation Taylor's Poly.	See Difference Formulas Sheet $f(x) + \frac{h}{1!}[f'(x)] + \frac{h^2}{2!}[f''(x)] + \frac{h^3}{3!}[f'''(x)] + \dots$ $f(x) - \frac{h}{1!}[f'(x)] + \frac{h^2}{2!}[f''(x)] - \frac{h^3}{3!}[f'''(x)] + \dots$	E-Err = exact - approx R-Err = $E\text{-Err}/\text{exact}$ $= f(x+h)$ $= f(x-h)$	N/A "
1	Line $y = Ax + B$ Power $y = Ax^M$ Change Var	$A \sum x_k^2 + B \sum x_k = \sum x_k y_k \wedge A \sum x_k + NB = \sum y_k$ $A = \frac{\sum x_k^M y_k}{\sum x_k^{2M}}$ ① $y = A/x + B \rightarrow X = 1/x$ ④ $y = (Ax+B)^{-2} \rightarrow Y = 1/y^2$ ② $y = 1/Ax + B \rightarrow Y = 1/y$ ⑤ $y = x/A + Bx$ ③ $y = A \ln x + B \rightarrow X = \ln x \rightarrow Y = 1/y \wedge X = 1/x$	⑥ $y = ce^{Ax} \rightarrow Y = \ln y \wedge B = \ln c$ ⑦ $y = cx^A \rightarrow Y = \ln y \wedge X = \ln x \wedge B = \ln c$	N/A " "
1	Trapezoidal Comp. Trap. Simpson Comp. Simp.	$h = b - a \wedge \int_a^b f(x) dx = \frac{h}{2} [f_0 + f_1]$ $\int_a^b f(x) dx = \frac{h}{2} [f_0 + 2(f_1 + \dots + f_{n-1}) + f_n]$ $\int_a^b f(x) dx = \frac{h}{3} [f_0 + 4f_1 + f_2]$ $\int_a^b f(x) dx = \frac{h}{3} [f_0 + 4(\sum f_{\text{odd}}) + 2(\sum f_{\text{even}}) + f_n]$	E-Err & R-Err are same. $h = (b-a)/M$ $h = (b-a)/2$ $h = (b-a)/2M$	$O(h^3)$, deg. 1 $O(h^2)$ $O(h^5)$, deg 3 $O(h^4)$
1	Optimization	$f'(x) = 0$ or DNE → x is possible max/min • f' changes from +ve to -ve @ $c \rightarrow \text{max} = c$ • f' changes from -ve to +ve @ $c \rightarrow \text{min} = c$	f' doesn't change sign @ c → c is neither max nor min → $\text{fminsearch}('f', [x_0, y_0])$ f is unimodal on $[a, b]$ → unique min point in $[a, b]$ AND $f \in C[a, b]$.	
	Golden Ratio	See formula sheet. • $f(c) \leq f(d) \rightarrow \text{new} = [a, d], d_{k+1} = c_k$ • $f(c) > f(d) \rightarrow \text{new} = [c, b], c_{k+1} = d_k$		