

MATH 3940 Numerical Analysis for Computer Scientists
Problem Set 6: Numerical Differentiation and Integration

1. Consider the data

x	0.9	0.97	1.04
$f(x)$	-0.17101	-0.05733	0.08486

Find the approximations to $f'(0.9)$, $f'(0.97)$, $f'(1.04)$, and $f''(0.97)$ of order $\mathbf{O}(h^2)$.

2. Let $f(x) = xe^x$ and $h = 0.06$.

(a) Find $f'(2)$ using the formula $f'(x) = \frac{f(x+h) - f(x)}{h}$

(b) Find $f''(2)$ using the formula $f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

(c) Find the exact error for the approximation obtained in part (b).

3. Find the order of error in the following approximation (show your steps)

$$f''(x) = \frac{2f(x) - 5f(x-h) + 4f(x-2h) - f(x-3h)}{h^2}$$

4. Consider the integral $\int_0^1 \sin \pi x \, dx$

(a) Approximate the above integral using trapezoidal rule.

(b) Approximate the above integral using Simpson's rule.

(c) Approximate the above integral by Simpson's $\frac{3}{8}$ rule.

(d) Using the error formula find a bound for the error of Trapezoidal rule and compare this to the actual error.

5. Consider the integral $\int_0^4 x^2 e^{-x} \, dx$

(a) Approximate the above integral using composite Trapezoidal rule with $n = 8$.

(b) Approximate the above integral using composite Simpson's rule with $n = 4$.