MATH 3940-1 Numerical Analysis for Computer Scientists
Assignment 1

Due on Monday, September 28, 2020 at 1:00 pm

- You have to provide the inputs and outputs from Matlab/Octave. You will provide program if asked. Hand written programs will not be accepted.
- Show all your work to receive full credit.
- You can discuss assignments with each other but do not copy them. Identical or nearly identical assignments will not be accepted.

1. Consider the following system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= -3 \\ x_1 + x_2 - x_3 &= 0 \\ 3x_1 - x_2 - 9x_3 &= 2 \end{aligned}$$

(a) (5 marks) Use hand calculations to solve the system using Gaussian elimination method with no pivoting.

(b) (7 marks) Use hand calculations to solve the system using Gaussian elimination method with partial pivoting.

2. Consider the system of linear equations

$$\begin{aligned} 2x_1 + 3x_2 + x_3 + 2x_4 + x_5 &= 9 \\ x_1 - 4x_2 + x_3 - 2x_5 &= -4 \\ 5x_2 + 3x_3 + x_4 + x_5 &= 16 \\ 3x_1 - 6x_2 + 4x_4 + 3x_5 &= 14 \\ -x_1 + 2x_2 + 2x_3 + 5x_4 + x_5 &= 5 \end{aligned}$$

(a) (2 marks) Use Matlab to find the determinant and the inverse of the coefficient matrix A.

(b) (2 marks) Use Matlab built in command to solve the system $AX = B$.

3. Consider the system of linear equations

$$\begin{aligned} x_2 + 2x_3 - x_4 &= -1 \\ x_1 + x_2 - x_3 &= 5 \\ -x_1 - x_2 + x_3 + 3x_4 &= 1 \\ x_1 + 2x_2 + x_4 &= 9 \end{aligned}$$

(a) (10 marks) Use hand calculations to find the LU decomposition of the coefficient matrix A and then solve the resulting triangular system.

(b) (9 marks) Use Matlab to find the LU decomposition of the coefficient matrix A and then solve the resulting triangular system using forward and backward substitutions programs. (You need to provide program for forward substitution)

Question 2:

```
>> A=[2 3 1 2 1; 1 -4 1 0 -2; 0 5 3 1 1; 3 -6 0 4 3; -1 2 2 5 1];
>> det(A)

ans =  972

>> inv(A)

ans =   0.3549   0.1008  -0.0566   0.0154  -0.1430
        0.1543  -0.0576  -0.0391  -0.0802   0.0103
       -0.2253   0.1708   0.3837   0.0772  -0.0484
        0.1481   0.0247  -0.1975  -0.0370   0.2099
       -0.2438  -0.2490   0.2418   0.2068  -0.1163
```

(b)
```
>> B=[9 -4 16 14 5]';

>> x=A\B

x =  1.3868
    -0.0782
     4.2675
    -1.3951
     4.9835
```

Question 3: (b) Using the Matlab command lu, we find that

```
>> A=[0 1 2 -1; 1 1 -1 0; -1 -1 1 3; 1 2 0 1];

>> [L U P]=lu(A)

L =   1   0   0   0
      0   1   0   0
      1   1   1   0
     -1   0   0   1

U =   1   1  -1   0
      0   1   2  -1
      0   0  -1   2
      0   0   0   3
```

```
P =

   0   1   0   0

   1   0   0   0

   0   0   0   1

   0   0   1   0

>> B=[-1 5 1 9]';
```

We write this forward substitution program in M file

```
function X=forsub(A,B)
% A is an n x n lower triangular nonsingular matrix and B is an n x 1 matrix
% Find the dimension of B and initialize X
n=length(B);
X=zeros(n,1);
X(1)=B(1)/A(1,1);
for k=2:n
 X(k)=(B(k)-A(k,1:k-1)*X(1:k-1))/A(k,k);
end
```

We recall in Matlab as follows:

```
>> Y=forsub(L,P*B)

Y =    5

      -1

       5

       6
```

Then we use the backward substitution program in M file

```
>> X=backsub(U,Y)

X =   1

      3

     -1

      2
```