Example 1:
Let $f(x) = x^3 + 3x^2 - 24x$ Perform 3 iterations of the golden ratio search method for finding local min. starting with the interval [1,3]

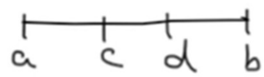
Solution:

r = 0.61803
$\Rightarrow 1 - r = 1 - 0.61803 = 0.3197$

f is unimodal on [1,3]

Let $a_0 = 1$ & $b_0 = 3$

$c_0 = a_0 = (1 - Y)(b_0 - a_0) = 1 + 0.38197(2) = 1.76394$
&
$d_0 = b_0 - (1 - Y)(b_0 - a_0) = 3 - 0.3897(2) = 2.23606$



$f(c_0) = f(1.76394) = -27.5116$
$f(d_0) = f(2.23606) = -27.4853$

$f(c_0) \leq f(d_0)$ So, the new interval will be $[\overset{a_0}{\underset{a_1}{1}}, \overset{d_0}{\underset{b_1}{2.33606}}]$

$|d_1 = c_0$

$c_1 = a_1 + (1 - r)(b_1 - a_1)$
$\quad = 1 + (0.38197)(1.23606)$
$\quad = 1.47214$

$d_1 = b_1 - (1 - r)(b_1 - a_1)$
$\quad = 2.23606 - (0.38197)(1.23606)$
$\quad = 1.76394$

$f(c_1) = f(1.47214) = -25.6393$

$f(d_1) we\ already\ have = f(1.76394) = -27.5116$

$f(d_1) \leq f(c_1)$ So, the new interval will be $[\overset{c_1}{\underset{a_2}{1.47214}}, \overset{b_1}{\underset{b_2}{2.23606}}]$

$c_2 = a_2 + (1 - r)(b_2 - a_2)$
$\quad = 1.47214 + (0.38197)(2.23606 - 1.47214)$
$\quad = 1.7639 \rightarrow$ Same as d_1

$$
\begin{aligned}
d_2 &= b_2 - (1 - r)(b_2 - a_2) \\
&= 2.23606 + (0.38197)(2.23606 - 1.47214) \\
&= 1.9443
\end{aligned}
$$

$f(c_2) = f(1.7639) = -27.5116$
$f(d_2) = f(1.9443) = -27.9723$

$f(d_2) < f(c_2)$ So, the new interval will be $[\overset{c_2}{1.7639}, \overset{b_2}{2.23606}]$ → 3rd iteration

Using Matlab method Converges to 2 in 27 iterations with

$\underbrace{\delta = 10^{-5}}_{|b - a| < \delta}$ & $\underbrace{\varepsilon = 10^{-7}}_{|f(b) - f(a)| < \varepsilon}$

Example 2:

Let $f(x) = \frac{x^2}{2} - 4x - x\cos x$, perform 2 iterations of the golden ratio search method for finding local min. starting with the interval $[0.5, 2.5]$

Solution:

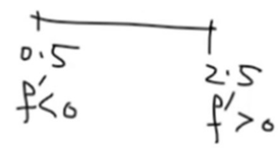
If you are asked that, do you expect golden ratio to converge starting with $[0.5, 2.5]$

f is cont. on $[0.5, 2.5]$

$f'(x) = \frac{1}{2}(2x) - 4 - \cos x + x\sin x$ $\quad\quad \rightarrow f'(x) = 0$ for critical numbers

use radians

$f'(0.5) = 0.5 - 4 - \cos 0.5 + 0.5\sin 0.5 = -3.638 < 0 \Rightarrow$ dec



$f'(2.5) = 2.5 - 4 - \cos 2.5 + 2.5\sin 2.5 = 3.927 > 0 \Rightarrow$ inc

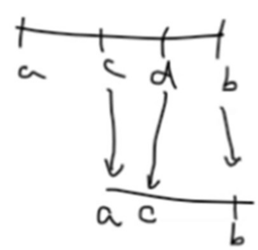
So, f is unimodal $\Rightarrow$ Golden ratio method will converge.

$[0.5, 2.5] \Rightarrow a_0 = 0.5$ & $b_0 = 2.5$

$c_0 = b_2 - (1 - r)(b_0 - a_0)$
$= 0.5 - (1 - 0.61803)(2.5 - 0.5)$
$= 0.5 + (\mathbf{0.76394})$
$= 1.26394 \approx 1.2639$

$d_0 = b_0 - (1 - r)(b_0 - a_0)$
$= 0.5 - (1 - 0.61803)(2.5 - 0.5)$
$= 2.5 - (\mathbf{0.76394})$
$= 1.73606 \approx 1.7361$

$f(c_0) = f(1.2639) = -4.6387$
$f(d_0) = f(1.7361) = -5.1517$



$f(d_0) < f(c_0)$ So, the new interval will be $[\underset{\downarrow\, a_1}{\overset{c_0}{1.2639}}, \underset{\downarrow\, b_1}{\overset{b_0}{2.5}}]$

$c_1 = a_1 + (1 - r)(b_1 - a_1)$
$= 1.2369 - (1 - 0.61803)(2.5 - 1.2639)$
$= 0.5 + (\mathbf{0.4722})$
$= 1.7361$ → Same as d_0

$d_1 = b_1 - (1 - r)(b_1 - a_1)$
$= 2.5 - (1 - 0.61803)(2.5 - 0.5)$
$= 2.5 - (\mathbf{0.4722})$
$= 2.0278$

$f(c_1) = f(1.7361) = -5.1517$ (already calculated)
$f(d_1) = f(2.0278) = -5.6104$

c_1 b_1

$f(d_1) < f(c_1)$ So, the new interval will be $[1.7361, 2.5]$ → 2nd iteration

Using Matlab it converges to 1.8907 in 26 iterations with
$\delta = 10^{-5}$ & $\varepsilon = 10^{-7}$