MATH 3940-1 Numerical Analysis for Computer Scientists

Problem Set 3: Nonlinear Equations

Note: You can use Octave or Matlab for the questions that says to use Matlab.

1. Let $g(x) = \dfrac{x^2}{4} + \dfrac{5x}{4} - 3$.

   (a) Solve the equation $x = g(x)$.

   (b) Perform 3 iterations of the fixed point method starting with $p_0 = -3.5$.

   (c) Do you expect fixed point method to converge with an initial approximation $p_0 = -3.5$? Justify your answer using the condition of convergence.

   (d) Use Matlab to perform 40 iterations of fixed point method to solve $x = g(x)$, starting with $p_0 = -3.5$, and a tolerance of 10^{-5}. Do you get the expected convergence/divergence as your answer in part (c)?

   (e) Do you expect fixed point method to converge with an initial approximation $p_0 = -0.25$? Justify your answer using the condition of convergence.

2. Given the equation $x^3 + x^2 - 3x - 3 = 0$.

   (a) Use the Matlab built-in function to find all roots of the above equation.

   (b) Use Matlab to perform 25 iterations of the fixed point method for each of the following functions, starting with $p_0 = 1$ and a tolerance of 10^{-5}. In the case of convergence, mention the number of iterations when the convergence is achieved.

   (i) $g_1(x) = \sqrt{\dfrac{3 + 3x - x^2}{x}}$

   (ii) $g_2(x) = -1 + \dfrac{3x + 3}{x^2}$

   (iii) $g_3(x) = \dfrac{x^3 + x^2 - x - 3}{2}$.

3. Consider the equation: $x^3 + 2x = 1$.

   (a) Can we use bisection method to find a solution of the equation starting with the interval $[0, 1]$? Justify your answer using the conditions of convergence.

   (b) Using hand calculations, perform 3 iterations of the bisection method starting with the interval $[0, 1]$.

   (c) Using hand calculations, perform 2 iterations of the method of false position starting with the interval $[0, 1]$.

   (d) Using hand calculations, perform 3 iterations of the secant method starting with the initial values $p_0 = 0$ and $p_1 = 1$.

4. Consider the equation: $x - 2^{-x} = 0$

   (a) Use the Matlab built-in function to find the root near 0.

   (b) Use Matlab to perform 20 iterations of the bisection method with initial values $a = 0$, $b = 1$ and tolerance 10^{-5}.

   (c) Use Matlab to perform 20 iterations of the method of false position with initial values $a = 0$, $b = 1$, tolerance= 10^{-5} and epsilon= 10^{-10}.

   (d) Use Matlab to perform 20 iterations of the secant method starting with the initial values $p_0 = 0$, $p_1 = 1$, tolerance= 10^{-5} and epsilon= 10^{-10}.

   (e) Use Matlab to perform 20 iterations of Newton's method with the initial approximation $p_0 = 1$, tolerance= 10^{-5} and epsilon= 10^{-10}.

   (f) Based on your results from parts (b) - (e), which method is more successful. Explain your answer using the convergence rates.

5. Consider the equation: $x \cos x = x$

   (a) Using hand calculations find the exact solution(s) in the interval $[-\pi, \pi]$.

   (b) Using hand calculations, perform 2 iterations of Newton's method starting with the initial approximation $p_0 = 1$.

   (c) Use Matlab to perform 15 iterations of Newton's method with the initial approximation $p_0 = 1$, tolerance= 10^{-5} and epsilon= 10^{-7}.