MATH 3940-1 Numerical Analysis for Computer Scientists

Problem Set 4: Nonlinear Systems and Interpolation Polynomials

Note: You can use Octave or Matlab for the questions that says to use Matlab.

1. Consider the system of nonlinear equations

$$\begin{array}{rcrcl} 2x & + & 2y & = & 3 \\ 3x^2 & + & 2y & = & 4 \end{array}$$

   (a) Find the exact solutions.

   (b) Perform 2 iterations of Jacobi method starting with the initial values $x_0 = 0$ and $y_0 = 0$.

   (c) Perform 2 iterations of Gauss-Seidel method starting with the initial values $x_0 = 0$ and $y_0 = 0$.

   (d) Use Matlab to perform 10 iterations of Jacobi method starting with $x_0 = 0$ and $y_0 = 0$, and tolerance 10^{-5}. Does it converge?

   (e) Use Matlab to perform 10 iterations of Gauss-Seidel method starting with $x_0 = 0$ and $y_0 = 0$, and tolerance 10^{-5}. Does it converge?

   (f) Do you expect Jacobi or Gauss Seidel method to converge starting with $x_0 = 0$ and $y_0 = 0$? Justify your answer using the conditions of convergence.

2. (a) Find the Taylor polynomial of degree 3 for $f(x) = \dfrac{1}{x+1}$ expanded about $x_0 = 0$.

   (b) Does $f(x) = \dfrac{1}{x+1}$ have a Taylor polynomial expansion about $x_0 = -1$? Justify your answer.

3. Consider the data

x_i	-2	-1	0	1	2	3
y_i	1	4	11	16	13	-4

   (a) Use Matlab built in functions to find an interpolation polynomial of degree 5 for the data and to approximate the value at $x = -1.5$.

   (b) Find Lagrange polynomial $P_2(x)$ using the nodes x_0, x_1, and x_2.

   (c) Use Matlab to find Lagrange polynomial $P_2(x)$ using the nodes x_0, x_1, and x_2.

   (d) Find divided difference table and Newton polynomial using all nodes in the above table.

   (e) Use Matlab to find divided difference table and Newton polynomial using all nodes in the above table.

4. Let $f(x) = x + e^{-x}$. The nodes are $x_0 = 0$, $x_1 = 0.5$, $x_2 = 1$, and $x_3 = 1.5$.

   (a) Use Matlab to find lagrange polynomial $P_3(x)$ using the above nodes.
   (b) Use Matlab to find Newton polynomial $P_3(x)$ using the above nodes.
   (c) Use the error formula to find a bound for the error for $P_3(1.2)$ and compare the bound to the actual error. (use the Lagrange or Newton polynomial $P_3(x)$ obtained from Matlab in part(a) or (b)).