MATH 3940-1 Numerical Analysis for Computer Scientists

Problem Set 2: Eigenvalues and Eigenvectors

Note: You can use Octave or Matlab for the questions that says to use Matlab.

1. Let
$$A = \begin{bmatrix} 2 & -7 & 0 \\ 5 & 10 & 4 \\ 0 & 5 & 2 \end{bmatrix}$$
 and the initial approximation is $X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

- (a) Find all the eigenvalues and eigenvectors of matrix A.
- (b) Use the Matlab built-in function to find all the eigenvalues and eigenvectors of matrix A.
- (c) Perform two iterations of the power method for matrix A starting with X_0 .
- (d) Use Matlab to find the dominant eigenvalue of A and the associated eigenvector using the power method with a tolerance of 10^{-5} , starting with X_0 .

2. Let
$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$
 and the initial approximation is $X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

- (a) Find all the eigenvalues of A and find the eigenvector associated with the dominant eigenvalue of the matrix A.
- (b) Perform two iterations of the power method starting with X_0 .
- (c) Use Matlab to find the dominant eigenvalue of B and the associated eigenvector using the power method with a tolerance of 10^{-5} , starting with X_0 .

3. Let
$$A = \begin{bmatrix} 4 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & -4 \end{bmatrix}$$
 and the initial approximation is $X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

- (a) Find all the eigenvalues and eigenvectors of A.
- (b) Use the Matlab built-in function to find all the eigenvalues and eigenvectors of matrix A.
- (c) Use Matlab to find the dominant eigenvalue and the associated eigenvector of A using the power method with a tolerance of 10^{-5} , starting with X_0 . Does it converge or diverge? Explain the reason for its convergence/divergence.