Conditions of Convergence

Linear Systems of Equations

Jacobi Method

If A is Strictly Diagonally Dominant, then this method will converge to a unique solution of $A\vec{x} = \vec{b}$.

This is sufficient, but not necessary. That is, all strictly diagonally dominant matrices will converge, but not all convergent matrices are strictly diagonally dominant.

Gauss-Seidel Method

Same as the Jacobi Method.

Non-Linear Equations

Fixed Point Method

Let g be continuous in $[a, b]$ and $g(x) \in [a, b]$ for all $x \in [a, b]$. Also, have $g'(x)$ defined in (a, b) with there existing some $k \in (0, 1)$ such that $|g'(x)| \leq 1$ for all $x \in (a, b)$. If the initial guess $p_0 \in [a, b]$ then this method converges to a unique solution $p \in [a, b]$.

Note: for this course, we only need to find an interval $[a, b]$ such that:

1. $[a, b]$ contains the initial guess p_0 and the solution p
2. g and g' are continuous on $[a, b]$

Then, if:

- $|g'(p)| < 1$, the method converges
- $|g'(p)| > 1$ then method diverges
- $|g'(p)| = 1$ then the method may or may not converge.

Bisection Method

Let $[a, b]$ be an interval on which f is continuous with $f(a)$ and $f(b)$ having opposing signs. Then if the solution $p \in [a, b]$, this method converges.

Regula Falsi Method

Same as the Bisection Method.

Secant Method

Same as Bisection Method, minus the need for opposing signs.

Newton's Method

If f, f', and f'' are continuous on interval $[a, b]$, with $p_0 \in [a, b]$ and $f'(p_0) \neq 0$, then Newton's method converges.

Other

The Power Method

May converge when:

1. A has an eigenvalue that is strictly greater in magnitude than others. For example, may converge if the eigenvalues are $\{3, 1, 4\}$ but, not $\{-3, 1, 3\}$, as $|-3| = |3|$.
2. The initial guess x_0 is not an eigenvector of A.