MATH 3940 Numerical Analysis for Computer Scientists
Assignment 4

Due on Monday, November 16, 2020 at 1:00 pm

- You have to provide inputs and the outputs from Matlab/Octave for all questions. Hand written inputs and outputs will not be accepted.
- Show all your work to receive full credit.
- Answers for polynomial should be written in simplified form.
- You can discuss assignments with each other but do not copy them. Identical or nearly identical assignments will not be accepted.

1. (a) (6 marks) Find the Taylor polynomial of degree 3 for $f(x) = x^{3/2}$ expanded about $x_0 = 4$.
   (b) (2 marks) Does $f(x) = x^{3/2}$ have a Taylor polynomial expansion about $x_0 = 0$? Justify your answer.

2. Consider the data

x	-2	-1	0	1	2
y	4	-1	3	1	8

   (a) (5 marks) Find an interpolation polynomial of degree 4 by solving the system $AX = B$ (Use Matalb to solve the system $AX = B$).
   (b) (3 marks) Use Matlab built in functions to find an interpolation polynomial of degree 4 and then to approximate the value at $x = -1.4$.

3. Consider the data

x_k	0	1	2	3	4	5
y_k	5	5	3	5	17	45

   (a) (4 marks) Using hand calculations, find Lagrange polynomial $P_2(x)$ using the nodes x_0, x_1, x_2.

   (b) (2 marks) Use Matlab to find Lagrange polynomial $P_2(x)$ using the nodes x_0, x_1, and x_2.

   (c) (6 marks) Using hand calculations, find divided difference table and Newton polynomial using all nodes in the above table.

   (d) (2 marks) Use Matlab to find divided difference table and Newton polynomial using all nodes in the above table.

4. Let $f(x) = xe^x$. The nodes are $x_0 = -1$, $x_1 = -0.5$, $x_2 = 0$, $x_3 = 0.5$ and $x_4 = 1$.

   (a) (3 marks) Use Matlab built in functions to find an interpolation polynomial of degree 4 and to approximate the value at $x = 0.2$.

   (b) (2 marks) Use Matlab to find lagrange polynomial $P_4(x)$ using all nodes.

   (c) (2 marks) Use Matlab to find Newton polynomial $P_4(x)$ using all nodes.

   (d) (8 marks) Use hand calculations, calculate the exact error and the approximated error for Lagrange polynomial $P_4(x)$ at $x = -0.25$ (use $c = 0.1$).

MATH 3940 Assignment 4 Solutions - Matlab Fall 2020

Question 2: (a) To solve the system AX=B by Matlab

```
>> x5=[-2:1:2];
>> y5=[4 -1 3 1 8];
>> A=[ones(1,5);x5;x5.^2;x5.^3;x5.^4]';
>> B=y5';
>> X=A\B
X =  3.0000
     1.0000
    -4.2500
         0
     1.2500
```

(b) polyfit is the built in function to find interpolation polynomial

```
>> x5=[-2:1:2];
>> y5=[4 -1 3 1 8];
>> p=polyfit(x5,y5,4)
p =  1.2500  0.0000  -4.2500  1.0000  3.0000
```

The value at -1.4 is found by using the command polyval

```
>> polyval(p,-1.4)
ans =  -1.9280
```

Question 3: (b)

```
>> X=[ 0 1 2];
>> Y=[5 5 3];
>> [C L]=lagran(X,Y)
C =  -1   1   5
L =  0.5000  -1.5000   1.0000
    -1.0000   2.0000        0
     0.5000  -0.5000        0
```

(d) >> X=[0 1 2 3 4 5];

>> Y=[5 5 3 5 17 45];

>> [C D]=newtonpoly(X,Y)

C = 0 0 1 -4 3 5

D = 5 0 0 0 0 0

5 0 0 0 0 0

3 -2 -1 0 0 0

5 2 2 1 0 0

17 12 5 1 0 0

45 28 8 1 0 0

The Newton polynomial is x^3-4x^2+3x+5

Question 4: (a) >> X=[-1 -0.5 0 0.5 1];

>> Y=X.*(exp(X))

Y = -0.3679 -0.3033 0 0.8244 2.718

>> p=polyfit(X,Y,4)

p = 0.1773 0.5539 0.9979 0.9892 0.0000

The polynomial is 0.1773 x^4+ 0.5539 x^3+ 0.9979 x^2+ 0.9892 x

>> polyval(p,0.2)

ans = 0.2425

(b) >> X=[-1 -0.5 0 0.5 1];

>> Y=[-0.3679 -0.3033 0 0.8244 2.7183];

>> [C,L]=lagran(X,Y)

C = 0.1773 0.5539 0.9979 0.9892 0

L = 0.6667 -0.6667 -0.1667 0.1667 0

-2.6667 1.3333 2.6667 -1.3333 0

4.0000 0 -5.0000 0 1.0000

-2.6667 -1.3333 2.6667 1.3333 0

0.6667 0.6667 -0.1667 -0.1667 0

The Lagrange polynomial is 0.1773 x^4+ 0.5539 x^3+ 0.9979 x^2+ 0.9892 x

(c) >> [C D]=newtonpoly(X,Y)

C = 0.1773 0.5539 0.9979 0.9892 0

D = -0.3679 0 0 0 0

-0.3033 0.1292 0 0 0

0 0.6066 0.4774 0 0

0.8244 1.6488 1.0422 0.3765 0

2.7183 3.7878 2.1390 0.7312 0.1773

The Newton polynomial is 0.1773 x^4+ 0.5539 x^3+ 0.9979 x^2+ 0.9892 x