MATH2790 Assignments

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1 Assignment 1

1.1 Practice Mobius Assignment

1.2 Marked Mobius Assignment

1.2.1 Question 1

Find the values of m so that $y = x^m, (x \neq 0)$ is a solution of the differential equation $x^2y'' - 28xy' + 210y = 0$

Solution $y = xm \Rightarrow y' = mx^{m-1} \Rightarrow y'' = m(m-1)x^{m-2}$
$x^2y'' - 28xy' + 210y = 0$
$x^2(m(m-1)x^{m-2}) - 28x(mx^{m-1}) + 210(x^m) = 0$
$\Rightarrow m(m-1)x^m - 28mx^m + 210x^m = 0$
$\Rightarrow x^m(m(m-1) - 28m + 210) = 0$
$\Rightarrow x^m(m^2 - m - 28m + 210) = 0$
$\Rightarrow x^m(m^2 - 29m + 210) = 0$

$m^2 - 29m + 210 = 0 \Rightarrow (m-14)(m-15) = 0 \Rightarrow m = 14, 15$

Similarly, $x^m = 0 \Rightarrow m = 0$ but $m \neq 0$
Thus $m = 14$ or $m = 15$

1.2.2 Question 2

Find the value of a such that the function $y = ax^2 - 7$ satisfies the differential equation $xy' - y + 4(x-1)^2 = 11 - 8x$

Solution $y = ax^2 - 7 \Rightarrow y' = 2ax$
$x(2ax) - ax^2 + 7 + 4x^2 - 8x + 4 - 11 + 8x = 0$
$\Rightarrow 2ax^2 - ax^2 + 7 + 4x^2 - 7 = 0$
$\Rightarrow ax^2 + 4x^2 = 0$
$\Rightarrow (a+4)x^2 = 0$
$\Rightarrow a + 4 = 0$
$\Rightarrow a = -4$

1.2.3 Question 3

Find the value of c such that $y = \frac{1}{x^2+c}$ is a solution to the IVP $y' = -2xy^2;\ y(9) = \frac{1}{15}$

Solution The condition $y(9) = \frac{1}{15}$ means substituting $x = 9$ and $y = \frac{1}{15}$ into the solution $y = \frac{1}{x^2+c}$ gets us $\frac{1}{15} = \frac{1}{9^2+c} \Rightarrow \frac{1}{15} = \frac{1}{81+c}$
$\Rightarrow 81 + c = 15 \Rightarrow c = 15 - 81 = -66$

1.2.4 Question 4

Consider the IVP $(x-1)y' - 9y = -8;\ y(1) = 1$ Calculate $\frac{\partial f}{\partial y}$

Solution So $f(x,y) = \frac{-8+9y}{x-1}$ giving $\frac{\partial f}{\partial y} = \frac{9}{x-1}$
Both f and $\frac{\partial f}{\partial y}$ are continuous when $x \neq 1$.
But the existence theorem guarantees that the given IVP has a unique solution on an interval in a region.

1.2.5 Question 5

$x^4y' = e^{6x^2+5y^2}$ is or is not separable.

Solution Well, $\frac{dy}{dx} = \frac{e^{6x^2+5y^2}}{x^4} \Rightarrow \frac{dy}{dx} = \frac{e^{6x^2}}{x^4} \cdot e^{5y^2}$
Thus the equation is separable.

1.2.6 Question 6

Consider the IVP: $y' - 8x^9e^{-y} = 0;\ y(0) = 8$
Find the explicit solution of the IVP.

Solution We first use separation of variables to determine the solution to the DE:
$y' - 8x^9e^{-y} = 0 \Rightarrow \frac{dy}{dx} = 8x^9e^{-y}$
$\Rightarrow e^y dy = 8x^9 dx$
$\int e^y dy = \int 8x^9 dx$
$e^y = \frac{8x^{10}}{10} + C$
$y = \ln\left(\frac{8x^{10}}{10} + C\right)$

Imposing the initial condition $y(0) = 8 \Rightarrow \ln 0 + C = 8$
$C = e^8$

Thus the solution for the IVP is $y = \ln\left(\frac{8x^{10}}{10} + e^8\right)$

1.2.7 Question 8

What is the integrating factor of the linear differential equation? $xy' - 15y = x^{16}$ for $x \in (0, \infty)$

Solution First, in standard form $\Rightarrow y' - \frac{15}{x}y = x^{15}$
Thus $I.F. = e^{\int P(x)dx} = e^{\int -\frac{15}{x}dx} = e^{-15\ln|x|} = e^{(\ln|x|)^{-15}} = \frac{1}{|x|^{15}} = \frac{1}{x^{15}}$ for $x \in (0, \infty)$

1.2.8 Question 9

Which of the following intervals can be an interval of existence of the solution of the linear differential equation:
$(x - 13)y' - 12y = (x - 12)^3$

Solution To standard form: $y' - \frac{12}{x-13}y = \frac{(x-12)^3}{x-13}$ are continuous.
Thus the interval of existence is $(13, \infty)$

1.2.9 Question 10

Consider the IVP $y' - y = 2x;\ y(0) = 8$ Find the explicit solution of the IVP.

Solution The given equation is a linear equation.
$I.F. = e^{\int P(x)dx} = e^{\int -1dx} = e^{-x}$
Multiplying by the I.F. we get $e^{-x}y' - e^{-x}y = 2xe^{-x} \Rightarrow \frac{d}{dx}(e^{-x}y) = 2xe^{-x}$
Integrating both sides W.R.T. x: $e^{-x}y = -2xe^{-x} - 2e^{-x} + C \Rightarrow y = -2x - 2 + Ce^x$
Imposing the initial condition $y(0) = 8 \Rightarrow 8 = 0 - 2 + C \Rightarrow C = 10$
Thus the explicit solution to the IVP is $y = -2x - 2 + 10e^x$