MATH2790 Assignments

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1 Assignment 1

1.1 Practice Mobius Assignment

1.2 Marked Mobius Assignment

1.2.1 Question 1

Find the values of m so that $y = x^m, (x \neq 0)$ is a solution of the differential equation $x^2y'' - 28xy' + 210y = 0$

Solution $y = xm \Rightarrow y' = mx^{m-1} \Rightarrow y'' = m(m-1)x^{m-2}$
$x^2y'' - 28xy' + 210y = 0$
$x^2(m(m-1)x^{m-2}) - 28x(mx^{m-1}) + 210(x^m) = 0$
$\Rightarrow m(m-1)x^m - 28mx^m + 210x^m = 0$
$\Rightarrow x^m(m(m-1) - 28m + 210) = 0$
$\Rightarrow x^m(m^2 - m - 28m + 210) = 0$
$\Rightarrow x^m(m^2 - 29m + 210) = 0$

$m^2 - 29m + 210 = 0 \Rightarrow (m-14)(m-15) = 0 \Rightarrow m = 14, 15$

Similarly, $x^m = 0 \Rightarrow m = 0$ but $m \neq 0$
Thus $m = 14$ or $m = 15$

1.2.2 Question 2

Find the value of a such that the function $y = ax^2 - 7$ satisfies the differential equation $xy' - y + 4(x-1)^2 = 11 - 8x$

Solution $y = ax^2 - 7 \Rightarrow y' = 2ax$
$x(2ax) - ax^2 + 7 + 4x^2 - 8x + 4 - 11 + 8x = 0$
$\Rightarrow 2ax^2 - ax^2 + 7 + 4x^2 - 7 = 0$
$\Rightarrow ax^2 + 4x^2 = 0$
$\Rightarrow (a+4)x^2 = 0$
$\Rightarrow a + 4 = 0$
$\Rightarrow a = -4$