Equivalences

In the following equivalences, α, β, γ are wffs in propositional logic.

E1.	$\alpha \wedge \sim \alpha \equiv false$	Law of Contradiction
E2.	$\alpha \vee \sim \alpha \equiv true$	Law of Excluded Middle
E3.	$\alpha \wedge \alpha \equiv \alpha$	
E4.	$\alpha \vee \alpha \equiv \alpha$	
E5.	$\alpha \wedge true \equiv \alpha$	
E6.	$\alpha \vee false \equiv \alpha$	
E7.	$\alpha \wedge false \equiv false$	
E8.	$\alpha \vee true \equiv true$	
E9.	$\alpha \wedge \beta \equiv \beta \wedge \alpha$	Commutative Law
E10.	$\alpha \vee \beta \equiv \beta \vee \alpha$	
E11.	$(\alpha \wedge \beta) \wedge \gamma \equiv \alpha \wedge (\beta \wedge \gamma)$	Associative Law
E12.	$(\alpha \vee \beta) \vee \gamma \equiv \alpha \vee (\beta \vee \gamma)$	
E13.	$\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$	
E14.	$\alpha \vee (\beta \wedge \gamma) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$	
E15.	$\sim\sim \alpha \equiv \alpha$	
E16.	$\sim (\alpha \wedge \beta) \equiv (\sim \alpha \vee \sim \beta)$	DeMorgan's Law
E17.	$\sim (\alpha \vee \beta) \equiv (\sim \alpha \wedge \sim \beta)$	
E18.	$\alpha \Rightarrow \beta \equiv \sim \alpha \vee \beta$	
E19.	$\alpha \Rightarrow \beta \equiv \sim \beta \Rightarrow \sim \alpha$	
E20.	$(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha) \equiv \alpha \Leftrightarrow \beta$	

Inference Rules

In the following inference rules, *p,q,r* are wffs in propositional logic.

I1. $\dfrac{p}{p \vee q}$ *addition*

I2. $\dfrac{p \wedge q}{p}$ *simplification*

I3. $\dfrac{p, p \Rightarrow q}{q}$ *modus ponens*

I4. $\dfrac{\sim q, p \Rightarrow q}{\sim p}$ *modus tollens*

I5. $\dfrac{p \Rightarrow q, q \Rightarrow r}{p \Rightarrow r}$ *hypotheical syllogism*

I6. $\dfrac{p, q}{p \wedge q}$ *conjunction*

First-order Logic Equivalences

Remark: In the following equivalences, $\alpha(x)$ represents a FOL-wff which may contain a free occurrence of x, and $\alpha(y)$ is the FOL-wff resulting from $\alpha(x)$ after every free occurrence of x in it is replaced by a y. Note that $\alpha(x)$ may contain other free variables.

FE1. $(\forall x)\alpha(x) \equiv (\forall y)\alpha(y)$ ($\alpha(x)$ does not contain y)
FE2. $(\exists x)\alpha(x) \equiv (\exists y)\alpha(y)$ ($\alpha(x)$ does not contain y)
FE3. $(\forall x)\alpha(x) \vee \beta \equiv (\forall x)(\alpha(x) \vee \beta)$ (β contains no free occurrence of x)
FE4. $(\exists x)\alpha(x) \vee \beta \equiv (\exists x)(\alpha(x) \vee \beta)$ (β contains no free occurrence of x)
FE5. $(\forall x)\alpha(x) \wedge \beta \equiv (\forall x)(\alpha(x) \wedge \beta)$ (β contains no free occurrence of x)
FE6. $(\exists x)\alpha(x) \wedge \beta \equiv (\exists x)(\alpha(x) \wedge \beta)$ (β contains no free occurrence of x)
FE7. $\sim (\forall x)\alpha(x) \equiv (\exists x) \sim \alpha(x)$
FE8. $\sim (\exists x)\alpha(x) \equiv (\forall x) \sim \alpha(x)$
FE9. $(\forall x)\alpha(x) \wedge (\forall x)\beta(x) \equiv (\forall x)(\alpha(x) \wedge \beta(x))$
FE10. $(\exists x)\alpha(x) \vee (\exists x)\beta(x) \equiv (\exists x)(\alpha(x) \vee \beta(x))$

First-order Logic Inference Rules

I1 — **I6** of Propositional logic with *p, q, r* being FOL-wffs.

Gen: *Generalization*

$$\frac{\alpha(x)}{(\forall x)\alpha(x)} \qquad x \textit{ is a variable}$$

UI: *Universal Instantiation*

$$\frac{(\forall x)\alpha(x)}{\alpha(t)} \qquad \alpha(x)\ is\ free\ for\ t\ \ (t \text{ is a variable or a constant})$$

where $\alpha(t)$ results from $\alpha(x)$ after *all* free occurrences of x in $\alpha(x)$ are replaced by t.

EQ: *Existential Quantification*

$$\frac{\alpha(t)}{(\exists x)\alpha(x)} \qquad \alpha(t)\ is\ free\ for\ x\ \ (t \text{ is a variable or a constant})$$

where $\alpha(x)$ results from $\alpha(t)$ after *some* (may be *all*) free occurrences of t in $\alpha(t)$ are replaced by x.

EI: *Existential Instantiation*

$$\frac{(\exists x)\alpha(x)}{\alpha(c)} \qquad c \textit{ is a constant}$$