Justin Bornais
110037759
bornais3

COMP2310 Midterm 1

1. $(\forall x)(Q(x) \Rightarrow (\exists y)(P(x,y) \Rightarrow (\forall x)P(y,x)))$

Let $x=a$ in $(\forall x)(Q(x) \Rightarrow (\exists y)(P(x,y) \Rightarrow (\forall x)P(y,x)))$

We obtain $Q(a) \Rightarrow (\exists y)(P(a,y) \Rightarrow (\forall x)P(y,x))$

Since $Q(a) \equiv T$, we must evaluate $(\exists y)(P(a,y) \Rightarrow (\forall x)P(y,x))$

let $y=a$ in $(\exists y)(P(a,y) \Rightarrow (\forall x)P(y,x))$
We obtain $P(a,a) \Rightarrow (\forall x)P(a,x)$
Since $P(a,a) \equiv T$, we must evaluate $(\forall x)P(a,x)$

Let $x=a$ in $(\forall x)P(a,x)$. We obtain $P(a,a) \equiv T$
Let $x=b$ in $(\forall x)P(a,x)$. We obtain $P(a,b) \equiv T$
Since $P(a,x)$ is evaluated to true for both values of x,
Hence $(\forall x)P(a,x) \equiv T$.

Since $(\forall x)P(a,x) \equiv T$, then $P(a,y) \Rightarrow (\forall x)P(a,x)$
$\therefore$ $(\exists y)(P(a,y) \Rightarrow (\forall x)P(y,x))$ is evaluated to true.
Hence, $Q(a) \Rightarrow (\exists y)(P(a,y) \Rightarrow (\forall x)P(y,x))$ is evaluated to true.

Now, let $x=b$ in $(\forall x)(Q(x) \Rightarrow (\exists y)(P(x,y) \Rightarrow (\forall x)P(y,x)))$
We obtain $Q(b) \Rightarrow (\exists y)(P(b,y) \Rightarrow (\forall x)P(y,x))$

Since $Q(b) \equiv F$, we have $F \Rightarrow (\exists y)(P(b,y) \Rightarrow (\forall x)P(y,x)) \equiv T$
Hence, $Q(b) \Rightarrow (\exists y)(P(b,y) \Rightarrow (\forall x)P(y,x))$ is evaluated to true.

Since $Q(x) \Rightarrow (\exists y)(P(x,y) \Rightarrow (\forall x)P(y,x))$ is evaluated to true for both values of x.
Hence, $(\forall x)(Q(x) \Rightarrow (\exists y)(P(x,y) \Rightarrow (\forall x)P(y,x)))$ is evaluated to true.

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2. P1: $(A \vee B) \Rightarrow (C \wedge D)$

P2: $(D \vee E) \Rightarrow F$

C: $A \Rightarrow F$

(Proof by Contradiction)

1. $\sim(A \Rightarrow F)$ Hypothesis
2. $(A \vee B) \Rightarrow (C \wedge D)$ from Γ
3. $(D \vee E) \Rightarrow F$ from Γ
4. $\sim(\sim A \vee F)$ 1, E18
5. $\sim\sim A \wedge \sim F$ 4, E17
6. $A \wedge \sim F$ 5, E15
7. A 6, I2
8. $\sim F \wedge A$ 6, E9
9. $\sim F$ 8, I2
10. $\sim(D \vee E)$ 9, 3, I4
11. $\sim D \wedge \sim E$ 10, E17
12. $\sim D$ 11, I2
13. $\sim D \vee \sim C$ 12, I1
14. $\sim C \vee \sim D$ 13, E10
15. $\sim(C \wedge D)$ 14, E16
16. $\sim(A \vee B)$ 15, 2, I4
17. $\sim A \wedge \sim B$ 16, E17
18. $\sim A$ 17, I2
19. $A \wedge \sim A$ 7, 18, I6
20. false 19, E1

Hence $(A \vee B) \Rightarrow (C \wedge D), (D \vee E) \Rightarrow F \vdash A \Rightarrow F$

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3. P1: $(\forall x)(\forall y)((P(x) \wedge Q(y)) \Rightarrow R(x,y))$

P2: $(\exists x)(\forall y)((P(x) \wedge S(x,y)) \Rightarrow Q(y))$

P3: $(\forall x)(\exists y)(P(x) \wedge \sim R(x,y) \wedge T(x,y))$

C: $(\exists x)(\exists y)(P(x) \wedge \sim S(x,y) \wedge T(x,y))$

(proof by definition)

1. $(\forall x)(\forall y)((P(x) \wedge Q(y)) \Rightarrow R(x,y))$ from Γ
2. $(\exists x)(\forall y)((P(x) \wedge S(x,y)) \Rightarrow Q(y))$ from Γ
3. $(\forall x)(\exists y)(P(x) \wedge \sim R(x,y) \wedge T(x,y))$ from Γ
4. $(\forall y)((P(a) \wedge S(a,y)) \Rightarrow Q(y))$ 2, EI, a is a new constant
5. $(\exists y)(P(a) \wedge \sim R(a,y) \wedge T(a,y))$ 3, UI
6. $P(a) \wedge (\sim R(a,b) \wedge T(a,b))$ 5, EI, b is a new constant
7. $P(a)$ 6, I2
8. $(\sim R(a,b) \wedge T(a,b)) \wedge P(a)$ 6, E9
9. $\sim R(a,b) \wedge T(a,b)$ 8, I2
10. $\sim R(a,b)$ 9, I2
11. $T(a,b) \wedge \sim R(a,b)$ 9, E9
12. $T(a,b)$ 11, I2
13. $(\forall y)((P(a) \wedge Q(y)) \Rightarrow R(a,y))$ 1, UI
14. $(P(a) \wedge Q(b)) \Rightarrow R(a,b)$ 13, UI
15. $\sim(P(a) \wedge Q(b))$ 10, 14, I4
16. $\sim P(a) \vee \sim Q(b)$ 15, E16
17. $P(a) \wedge (\sim P(a) \vee \sim Q(b))$ 7, 16, I6
18. $(P(a) \wedge \sim P(a)) \vee (P(a) \wedge \sim Q(b))$ 17, E13
19. $\text{false} \vee (P(a) \wedge \sim Q(b))$ 18, E1
20. $(P(a) \wedge \sim Q(b)) \vee \text{false}$ 19, E10
21. $P(a) \wedge \sim Q(b)$ 20, E6
22. $\sim Q(b) \wedge P(a)$ 21, E9
23. $\sim Q(b)$ 22, I2
24. $((P(a) \wedge S(a,b)) \Rightarrow Q(b))$ 4, UI
25. $\sim(P(a) \wedge S(a,b))$ 23, 24, I4
26. $\sim P(a) \vee \sim S(a,b)$ 25, E16
27. $P(a) \wedge (\sim P(a) \vee \sim S(a,b))$ 7, 26, I6
28. $P(a) \wedge (\sim S(a,b) \vee \sim P(a))$ 27, E10
29. $(P(a) \wedge \sim S(a,b)) \vee (P(a) \wedge \sim P(a))$ 28, E13
30. $(P(a) \wedge \sim S(a,b)) \vee \text{false}$ 29, E1
31. $P(a) \wedge \sim S(a,b)$ 30, E6
32. $P(a) \wedge \sim S(a,b) \wedge T(a,b)$ 31, 12, I6
33. $(\exists y)(P(a) \wedge \sim S(a,y) \wedge T(a,y))$ 32, EQ
34. $(\exists x)(\exists y)(P(x) \wedge \sim S(x,y) \wedge T(x,y))$ 33, EQ

Hence, $(\forall x)(\forall y)((P(x) \wedge Q(y)) \Rightarrow R(x,y))$, $(\exists x)(\forall y)((P(x) \wedge S(x,y)) \Rightarrow Q(y))$, $(\forall x)(\exists y)(P(x) \wedge \sim R(x,y) \wedge T(x,y))$ $\vdash (\exists x)(\exists y)(P(x) \wedge \sim S(x,y) \wedge T(x,y))$

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4. P1: The only type of movies that my son is allowed to watch are rated PG; (∀)
P2: No movie that I like is unsuitable for home entertainment; (∃)
P3: Movies with foul language are not rated PG; (∃)
P4: My son is allowed to watch any movie that is suitable for home entertainment; (∀)
C: I dislike movies with foul language; (∃)

Let U denote the set of all movies,
$S(x)$ denote x can be watched by my son,
$P(x)$ denote x is rated PG,
$L(x)$ denote x is liked by me,
$H(x)$ denote x is suitable for home entertainment,
$F(x)$ denote x contains foul language.

The premises are:

P1: $(\forall x)\,(P(x) \Leftrightarrow S(x))$
P2: $\sim(\exists x)\sim(L(x) \Rightarrow H(x))$
P3: $\sim(\exists x)\sim(F(x) \Rightarrow \sim P(x))$
P4: $(\forall x)\,(H(x) \Rightarrow S(x))$

C: $\sim(\exists x)\sim(F(x) \Rightarrow \sim L(x))$

We will prove P1, P2, P3, P4 $\vdash$ C
(proof by definition)

1. $(\forall x)\,(P(x) \Leftrightarrow S(x))$ from I
2. $\sim(\exists x)\sim(L(x) \Rightarrow H(x))$ from I
3. $\sim(\exists x)\sim(F(x) \Rightarrow \sim P(x))$ from I
4. $(\forall x)\,(H(x) \Rightarrow S(x))$ from I
5. $P(x) \Leftrightarrow S(x)$ 1, UI
6. $(\forall x)\sim\sim(L(x) \Rightarrow H(x))$ 2, FE8
7. $(\forall x)(L(x) \Rightarrow H(x))$ 6, E15
8. $L(x) \Rightarrow H(x)$ 7, UI
9. $(\forall x)\sim\sim(F(x) \Rightarrow \sim P(x))$ 3, FE8
10. $(\forall x)\,(F(x) \Rightarrow \sim P(x))$ 9, E15
11. $F(x) \Rightarrow \sim P(x)$ 10, UI
12. $H(x) \Rightarrow S(x)$ 4, UI
13. $(P(x) \Rightarrow S(x)) \wedge (S(x) \Rightarrow P(x))$ 5, E20
14. $(S(x) \Rightarrow P(x)) \wedge (P(x) \Rightarrow S(x))$ 13, E9
15. $S(x) \Rightarrow P(x)$ 14, I2
16. $H(x) \Rightarrow P(x)$ 12, 15, I5
17. $\sim\sim P(x) \Rightarrow \sim F(x)$ 11, E19
18. $P(x) \Rightarrow \sim F(x)$ 17, E15
19. $H(x) \Rightarrow \sim F(x)$ 16, 18, I5
20. $L(x) \Rightarrow \sim F(x)$ 8, 19, I5
21. $\sim\sim(L(x) \Rightarrow \sim F(x))$ 20, E15
22. $(\forall x)\sim\sim(L(x) \Rightarrow \sim F(x))$ 21, Gen
23. $\sim(\exists x)\sim(L(x) \Rightarrow \sim F(x))$ 22, FE8

Hence $(\forall x)\,(P(x) \Leftrightarrow S(x)),\ \sim(\exists x)\sim(L(x) \Rightarrow H(x)),\ \sim(\exists x)\sim(F(x) \Rightarrow \sim P(x)),\ (\forall x)\,(H(x) \Rightarrow S(x)) \vdash \sim(\exists x)\sim(F(x) \Rightarrow \sim L(x))$

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