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2310 Assignment 1

1.e) $(\sim r \vee q) \vee (p \wedge q) \vee (\sim p \wedge \sim r)$

p	q	r	$\sim p$	$\sim r$	$(\sim r \vee q)$	$(p \wedge q)$	$(\sim p \wedge \sim r)$	$(\sim r \vee q) \vee (p \wedge q) \vee (\sim p \wedge \sim r)$
F	F	F	T	T	T	F	T	T
F	F	T	T	F	F	F	F	F
F	T	F	T	T	T	F	T	T
F	T	T	T	F	T	F	F	T
T	F	F	F	T	T	F	F	T
T	F	T	F	F	F	F	F	F
T	T	F	F	T	T	T	F	T
T	T	T	F	F	T	T	F	T

3f) Prove $(a \Rightarrow \beta) \vee (\beta \Rightarrow a)$ is a theorem with a formal proof.

(Proof by contradiction)

1. $\sim((a \Rightarrow \beta) \vee (\beta \Rightarrow a))$ Hypothesis
2. $\sim((\sim a \vee \beta) \vee (\beta \Rightarrow a))$ 1, E18
3. $\sim((\sim a \vee \beta) \vee (\sim \beta \vee a))$ 2, E18
4. $\sim(((\sim a \vee \beta) \vee \sim \beta) \vee a)$ 3, E12
5. $\sim((\sim a \vee (\beta \vee \sim \beta)) \vee a)$ 4, E12
6. $\sim((\sim a \vee true) \vee a)$ 5, E2
7. $\sim(true \vee a)$ 6, E8
8. $\sim(a \vee true)$ 7, E10
9. $\sim(true)$ 8, E8
10. $\sim(a \vee \sim a)$ 9, E2
11. $\sim a \wedge \sim\sim a$ 10, E17
12. $\sim a \wedge a$ 11, E15
13. $a \wedge \sim a$ 12, E9
14. false 13, E1

Hence $\vdash (a \Rightarrow \beta) \vee (\beta \Rightarrow a)$ $\square$

3j) $\vdash (a \Rightarrow (\beta \Rightarrow (\beta \Rightarrow a)))$

(Indirect Proof)

1. $\sim(\beta \Rightarrow (\beta \Rightarrow a))$ assumption
2. $\sim(\beta \Rightarrow (\sim \beta \vee a))$ 1, E18
3. $\sim(\sim \beta \vee (\sim \beta \vee a))$ 2, E18
4. $\sim((\sim \beta \vee \sim \beta) \vee a)$ 3, E12
5. $\sim a$ 4, E4

Hence $\vdash (a \Rightarrow (\beta \Rightarrow (\beta \Rightarrow a)))$ $\square$

3g) $(a \Rightarrow \beta) \Rightarrow ((a \wedge \gamma) \Rightarrow (\beta \wedge \gamma))$

(Proof by contradiction)

1. $\sim((a \Rightarrow \beta) \Rightarrow ((a \wedge \gamma) \Rightarrow (\beta \wedge \gamma)))$ Hypothesis
2. $\sim(\sim(a \Rightarrow \beta) \vee ((a \wedge \gamma) \Rightarrow (\beta \wedge \gamma)))$ 1, E18
3. $\sim(\sim(a \Rightarrow \beta) \vee (\sim(a \wedge \gamma) \vee (\beta \wedge \gamma)))$ 2, E18
4. $\sim(\sim(\sim a \vee \beta) \vee (\sim(a \wedge \gamma) \vee (\beta \wedge \gamma)))$ 3, E18
5. $\sim(\sim(\sim a \vee \beta) \vee ((\sim a \vee \sim \gamma) \vee (\beta \wedge \gamma)))$ 4, E16
6. $\sim((\sim\sim a \wedge \sim \beta) \vee ((\sim a \vee \sim \gamma) \vee (\beta \wedge \gamma)))$ 5, E17
7. $\sim((a \wedge \sim \beta) \vee ((\sim a \vee \sim \gamma) \vee (\beta \wedge \gamma)))$ 6, E15
8. $\sim((a \wedge \sim \beta) \vee ((\sim a \vee \sim \gamma) \vee (\gamma \wedge \beta)))$ 7, E9
9. $\sim((a \wedge \sim \beta) \vee (\sim a \vee (\sim \gamma \vee (\gamma \wedge \beta))))$ 8, E12
10. $\sim(((a \wedge \sim \beta) \vee \sim a) \vee (\sim \gamma \vee (\gamma \wedge \beta)))$ 9, E12
11. $\sim((\sim a \vee (a \wedge \sim \beta)) \vee (\sim \gamma \vee (\gamma \wedge \beta)))$ 10, E10
12. $\sim(((\sim a \vee a) \wedge (\sim a \vee \sim \beta)) \vee (\sim \gamma \vee (\gamma \wedge \beta)))$ 11, E14
13. $\sim(((a \vee \sim a) \wedge (\sim a \vee \sim \beta)) \vee (\sim \gamma \vee (\gamma \wedge \beta)))$ 12, E10
14. $\sim((true \wedge (\sim a \vee \sim \beta)) \vee (\sim \gamma \vee (\gamma \wedge \beta)))$ 13, E2
15. $\sim(((\sim a \vee \sim \beta) \wedge true) \vee (\sim \gamma \vee (\gamma \wedge \beta)))$ 14, E9
16. $\sim((\sim a \vee \sim \beta) \vee (\sim \gamma \vee (\gamma \wedge \beta)))$ 15, E5
17. $\sim((\sim a \vee \sim \beta) \vee ((\sim \gamma \vee \gamma) \wedge (\sim \gamma \vee \beta)))$ 16, E14
18. $\sim((\sim a \vee \sim \beta) \vee ((\gamma \vee \sim \gamma) \wedge (\sim \gamma \vee \beta)))$ 17, E10
19. $\sim((\sim a \vee \sim \beta) \vee (true \wedge (\sim \gamma \vee \beta)))$ 18, E2
20. $\sim((\sim a \vee \sim \beta) \vee ((\sim \gamma \vee \beta) \wedge true))$ 19, E9
21. $\sim((\sim a \vee \sim \beta) \vee (\sim \gamma \vee \beta))$ 20, E5
22. $\sim((\sim a \vee \sim \beta) \vee (\beta \vee \sim \gamma))$ 21, E10
23. $\sim(((\sim a \vee \sim \beta) \vee \beta) \vee \sim \gamma)$ 22, E12
24. $\sim(\sim a \vee (\sim \beta \vee \beta) \vee \sim \gamma)$ 23, E12
25. $\sim(\sim a \vee (\beta \vee \sim \beta) \vee \sim \gamma)$ 24, E10
26. $\sim(\sim a \vee ((\beta \vee \sim \beta) \vee \sim \gamma))$ 25, E12
27. $\sim(\sim a \vee (true \vee \sim \gamma))$ 26, E2
28. $\sim(\sim a \vee (\sim \gamma \vee true))$ 27, E10
29. $\sim((\sim a \vee \sim \gamma) \vee true)$ 28, E12
30. $\sim true$ 29, E8
31. $\sim(a \vee \sim a)$ 30, E2
32. $\sim a \wedge \sim\sim a$ 31, E17
33. $\sim\sim a \wedge \sim a$ 33, E9
34. $a \wedge \sim a$ 33, E15
35. false 34, E1

Hence $\vdash (a \Rightarrow \beta) \Rightarrow ((a \wedge \gamma) \Rightarrow (\beta \wedge \gamma))$ $\square$

6b) $P1: (p \vee q) \Rightarrow (r \wedge s)$
$P2: (s \vee t) \Rightarrow u$

$C: p \Rightarrow u$

(Proof by Contradiction)

1. $\sim(p \Rightarrow u)$	Hypothesis
2. $(p \vee q) \Rightarrow (r \wedge s)$	from Σ
3. $(s \vee t) \Rightarrow u$	from Σ
4. $\sim(\sim p \vee u)$	1, E18
5. $\sim\sim p \wedge \sim u$	4, E17
6. $p \wedge \sim u$	5, E15
7. $\sim u \wedge p$	6, E9
8. $\sim u$	7, I2
9. $\sim(s \vee t)$	8, 3, I4
10. $\sim s \wedge \sim t$	9, E17
11. $\sim s$	10, I2
12. p	6, I2
13. $p \vee q$	12, I1
14. $r \wedge s$	13, 2, I3
15. $(r \wedge s) \wedge \sim s$	14, 11, I6
16. $r \wedge (s \wedge \sim s)$	15, E11
17. $r \wedge false$	16, E1
18. $false$	17, E7

$\therefore (p \vee q) \Rightarrow (r \wedge s), (s \vee t) \Rightarrow u \vdash p \Rightarrow u$ □

6(c) $P1: (r \vee s) \Rightarrow (p \vee q)$
$P2: (t \Rightarrow u) \Rightarrow r$
$P3: t \Rightarrow (u \wedge s)$

$C: \sim p \Rightarrow q$

(Direct Proof)

1. $(r \vee s) \Rightarrow (p \vee q)$	from Σ
2. $(t \Rightarrow u) \Rightarrow r$	from Σ
3. $t \Rightarrow (u \wedge s)$	from Σ
4. $\sim(t \Rightarrow u) \vee r$	2, E18
5. $\sim t \vee (u \wedge s)$	3, E18
6. $(\sim t \vee u) \wedge (\sim t \vee s)$	5, E14
7. $\sim t \vee u$	6, I2
8. $t \Rightarrow u$	7, E18
9. r	8, 2, I3
10. $r \vee s$	9, I1
11. $p \vee q$	10, 1, I3
12. $\sim\sim p \vee q$	11, E15
13. $\sim p \Rightarrow q$	12, E18

Hence $(r \vee s) \Rightarrow (p \vee q), (t \Rightarrow u) \Rightarrow r, t \Rightarrow (u \wedge s) \vdash \sim p \Rightarrow q$
□

7d) Let O denote the casino is open,
G denote a gambling tax is imposed,
T denote that tourism will decline,
S denote the city will suffer,
P denote the city is a safe place to live.

The sentence "If the casino is shut down or a gambling tax is imposed then tourism will decline and the city will suffer." translates to:
$(\sim O \vee G) \Rightarrow (T \wedge S)$

The sentence "If tourism declines, then the city will be a safer place to live." translates to:
$T \Rightarrow P$

The sentence "The city is not a safe place to live." translates to: $\sim P$

The concluding sentence "Therefore, the casino is not closed." translates to: O

We thus have:
$P1: (\sim O \vee G) \Rightarrow (T \wedge S)$
$P2: T \Rightarrow P$
$P3: \sim P$
$C: O$

and we are to prove $P1, P2, P3 \vdash C$

(Proof by Contradiction)

1. $\sim O$	Hypothesis
2. $(\sim O \vee G) \Rightarrow (T \wedge S)$	From Σ
3. $T \Rightarrow P$	From Σ
4. $\sim P$	From Σ
5. $\sim O \vee G$	1, I1
6. $T \wedge S$	5, 2, I3
7. T	6, I2
8. P	7, 3, I3
9. $P \wedge \sim P$	8, 4, I6
10. $false$	9, E1

Hence, $(\sim O \vee G) \Rightarrow (T \wedge S), T \Rightarrow P, \sim P \vdash O$.

□