COMP-2310
Theoretical Fundamentals of Computer Science

Fall 2021

Assignment 4

Due Date: November 30 9 (before 12:59p.m.)

Section 5.8

3. Let R and S be relations in a set A. Prove or disprove the following:

(b) If R and S are antisysmmetric, then (i) $R \cup S$ is an antisymmetric relation. (ii) $R \cap S$ is an antisymmetric relation.

17. Let R be a relation from X to Y. Let $X_=$ and $Y_=$ be the equality relation in X and Y, respectively. Prove that $R \circ Y_= = R$ and $X_= \circ R = R$.

Section 6.6

14. Let $f : X \longrightarrow Y$. If there exist $g : Y \longrightarrow X$ and $h : Y \longrightarrow X$ such that $f \circ g = I_X$ and $h \circ f = I_Y$. Prove that:

(a) the function f is a one-to-one correspondence.

(b) $g = h = f^{-1}$.

Section 7.8

Figure 2

	1	2	3	. . .	i	. . .
F_1	$F_1(1)$	. . .	. . .	. . .	. . .	. . .
F_2	. . .	$F_2(2)$	. . .	. . .	. . .	. . .
F_3	. . .	. . .	$F_3(3)$	. . .	. . .	. . .
$\vdots$	. . .	. . .	. . .	$\ddots$	. . .	. . .
$\vdots$	. . .	. . .	. . .		. . .	. . .
F_i	. . .	. . .	. . .	. . .	$F_i(i)$	. . .
$\vdots$	. . .	. . .	. . .	. . .	. . .	$\ddots$
d	$F_1(1)$+1	$F_2(2)$+1	$F_3(3)$+1	. . .	$F_i(i)$+1	. . .

7.8 Exercises

1. Prove that if R is a symmetric relation, then so is $R^k, \forall k > 0$.
2. Let A be a nonempty set, and 2^A be the set of all functions from A to $\{0,1\}$. Prove that $\mathcal{P}(A) \sim 2^A$, where $\mathcal{P}(A)$ is the power set of A.
3. Let A and B be any two sets. Prove that $A \times B \sim B \times A$.
4. Let A and B be any two sets. Prove that $A \times (B \cup C) \sim (A \times B) \cup (A \times C)$.
5. Prove that if $(X - Y) \sim (Y - X)$, then $X \sim Y$.
6. Prove that if $A \sim C$ and $B \sim D$, then $A \times B \sim C \times D$.
7. Prove that if $A \sim B$, then $A^C \sim B^C$ for any set C.
8. Let A, B, C be sets such that $B \cap C = \emptyset$. Prove that $A^{B \cup C} \sim A^B \times A^C$.
9. Let A, B, C be sets. Prove that $(A \times B)^C \sim A^C \times B^C$.
10. Prove that the open interval $(0,1) = \{x \in \mathbf{R} \mid 0 < x < 1\}$ is an infinite set.
11. Let $\mathfrak{B} = \{\{b_i\}_{i=1}^{\infty} \mid b_i \in \{0,1\}\}$. i.e. $\mathfrak{B}$ is the set of all infinite sequences of 0's and 1's. Prove that $\mathfrak{B}$ is uncountable.
12. Determine $F_i(x)$ and $d(i)$ for each of the following $g(i)$ where g is defined in Lemma 7.7.14.
    (a) $g(i)$ = "**program** *h*(input,output); **var** x:**integer**; **begin** read(*x*); write(2**x*) **end**."
    (b) $g(i)$ = "**program** *h*(input,output); **var** x:**integer**; **begin** read(*x*); **repeat** *x*:=*x* **until** (false) **end**."
    (c) $g(i)$ = "**program** *h*(input,output); **var** x:**integer**; **begin** anjgishgragjskklsjjbumijk **end**."
    (d) $g(i)$ = "**program** *h*(input,output); **var** x:**integer**; **begin** read(*x*); **if** (*x*<50) **then** write(*x*-30) **elseif** (*x*<1000) **then** write(*x*/2) **else** write("too large") **end**."
13. Use the diagonalization method to prove that there exists a one-variable number theoretic function which cannot be defined mathematically using a finite set of symbols.
14. Let A and B be two finites sets. Prove that $|A| \leq |B|$ if and only if $\exists f : A \xrightarrow{1-1} B$.