Introduction to Linear Regression

SML 312 – Fall 2022

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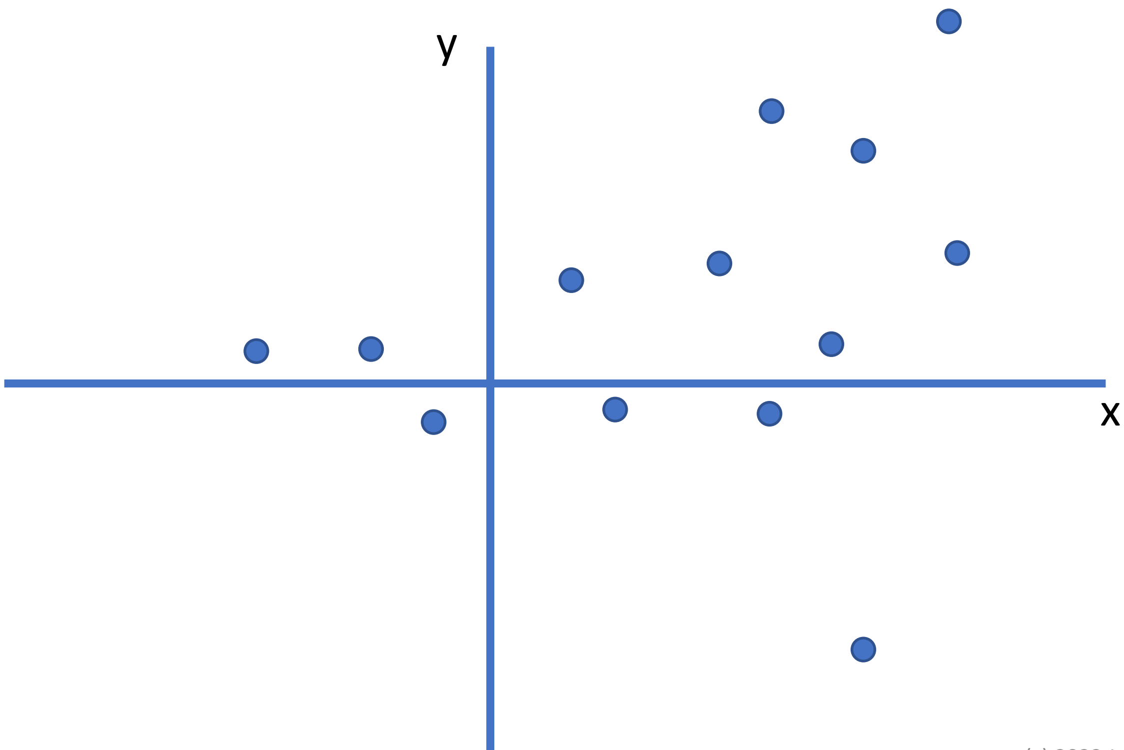

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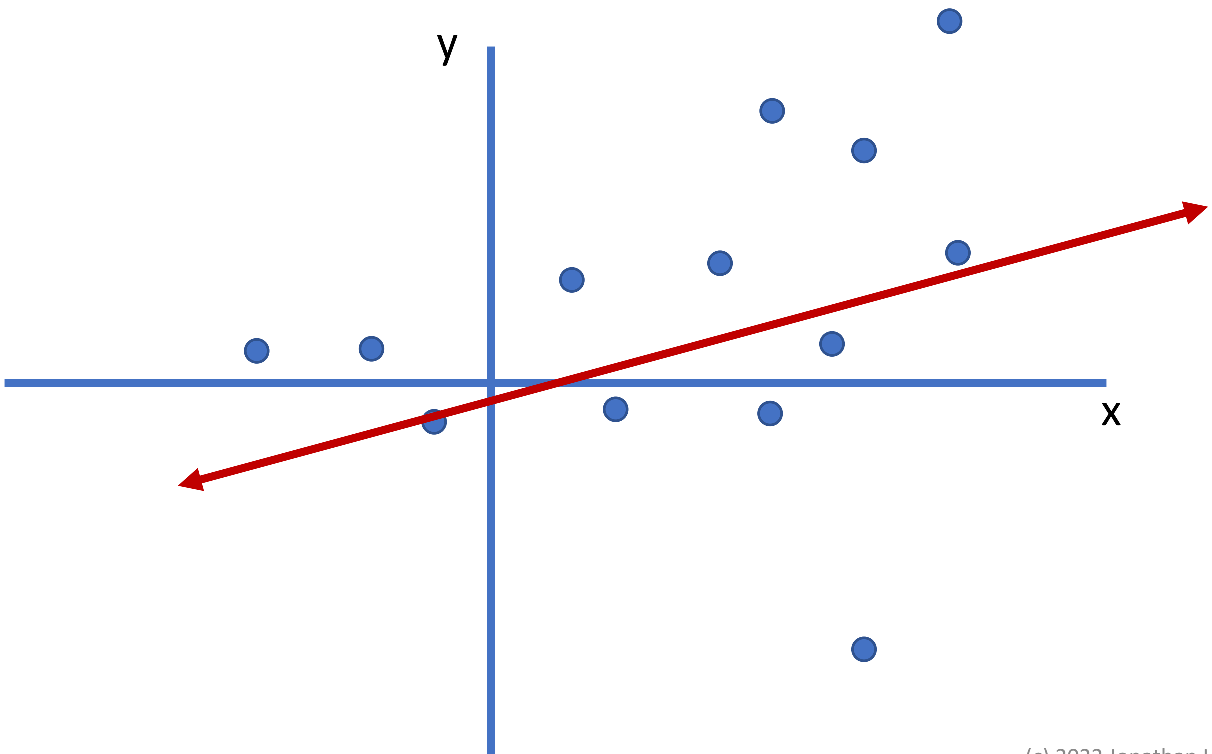

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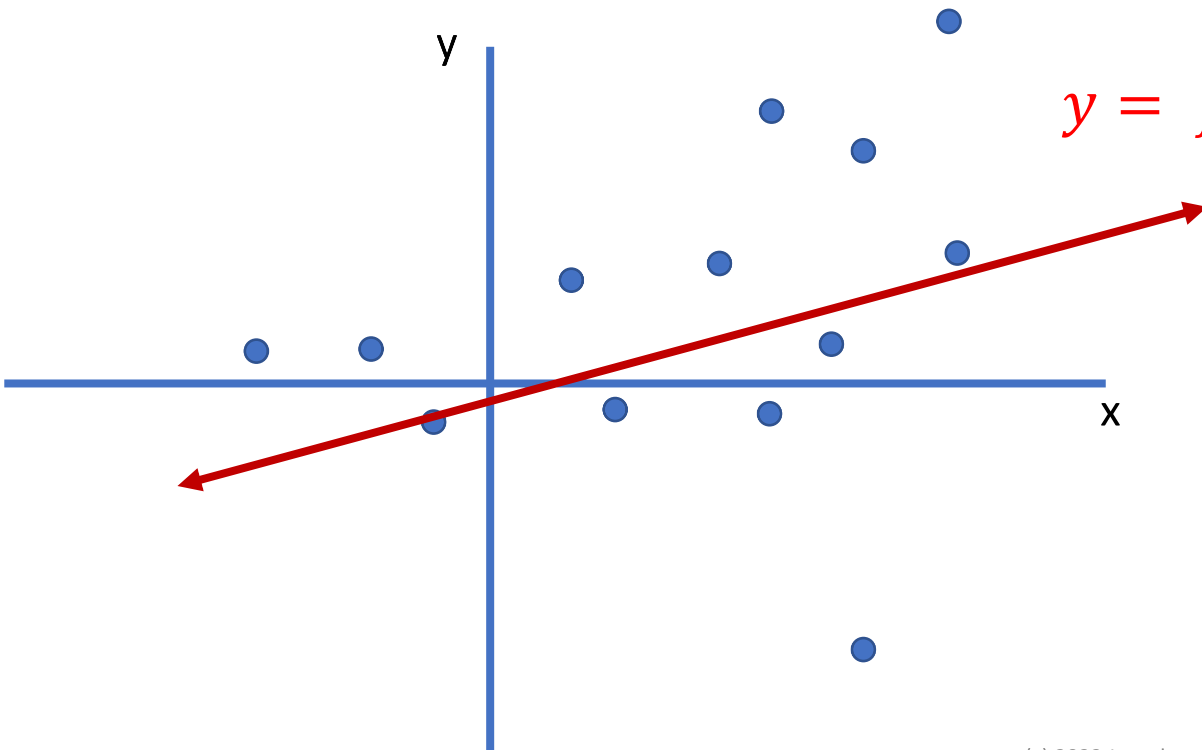

$$y = f(x) = b_0 + b_1 x$$

Ex:

$$b_0 = -0.2$$
$$b_1 = 0.43$$

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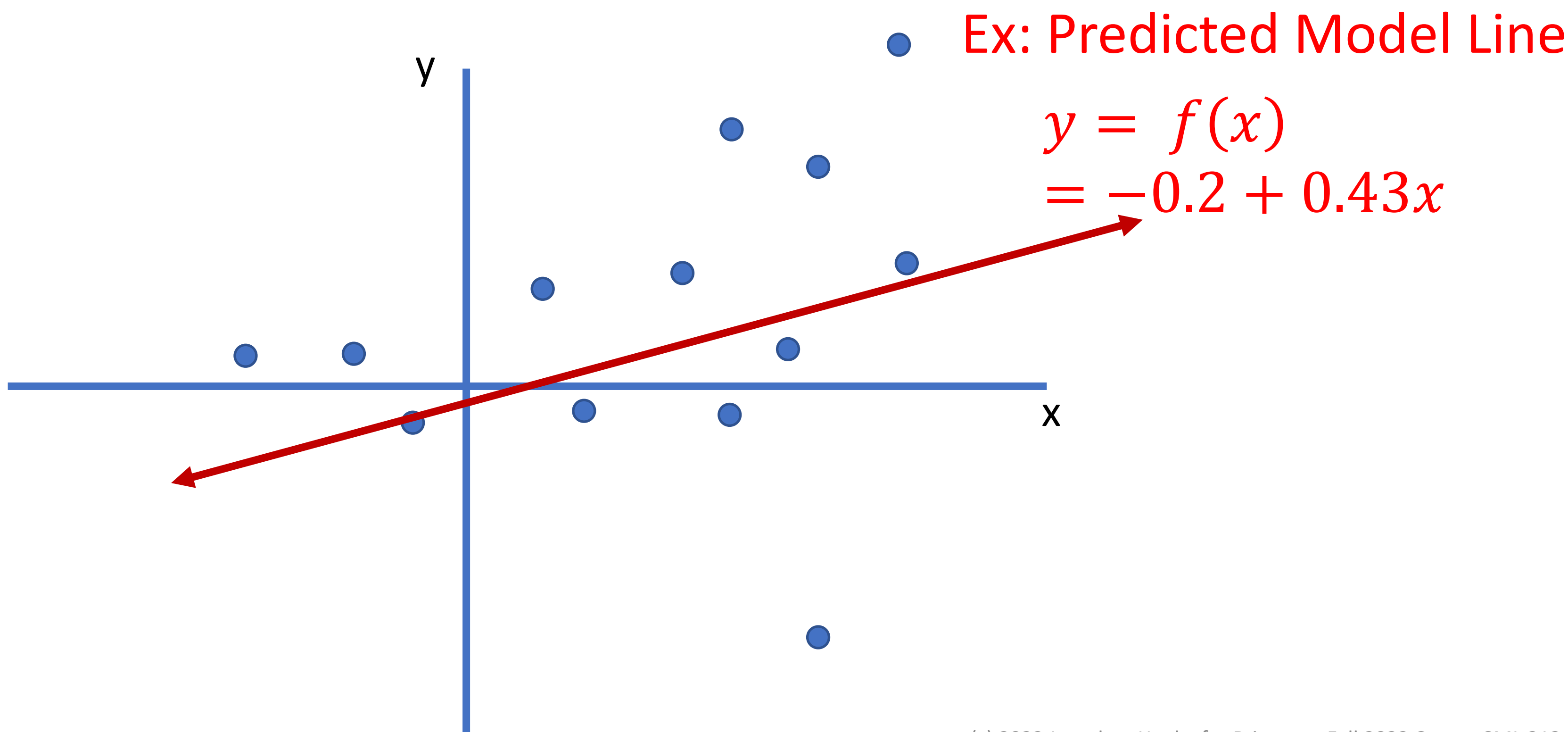
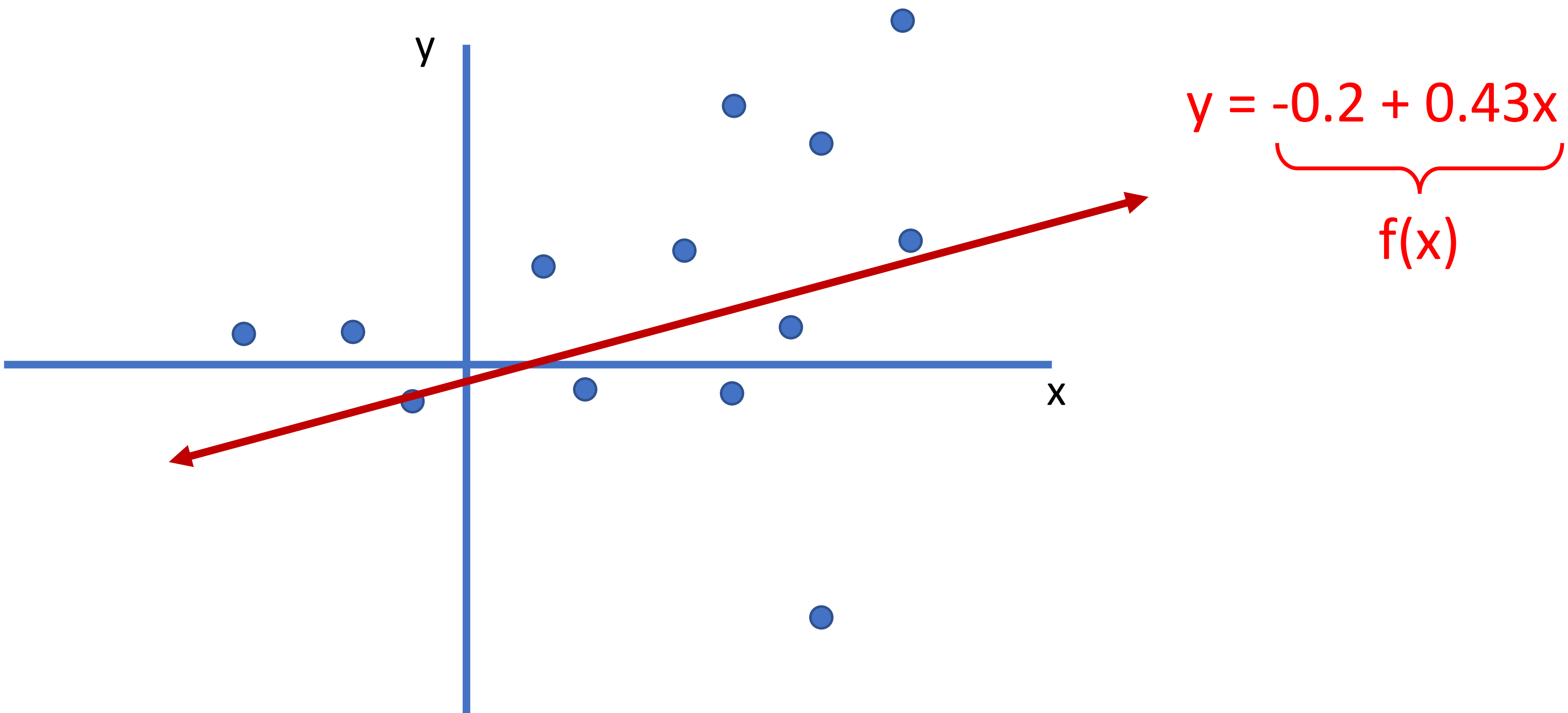


What is Linear Regression?

- In particular, when we have a <u>single feature</u> (i.e. x), Linear regression answers the question: What is the "best" line through a set of data points $\{(x^{(i)}, y^{(i)})\}_{1 \le i \le n}$?



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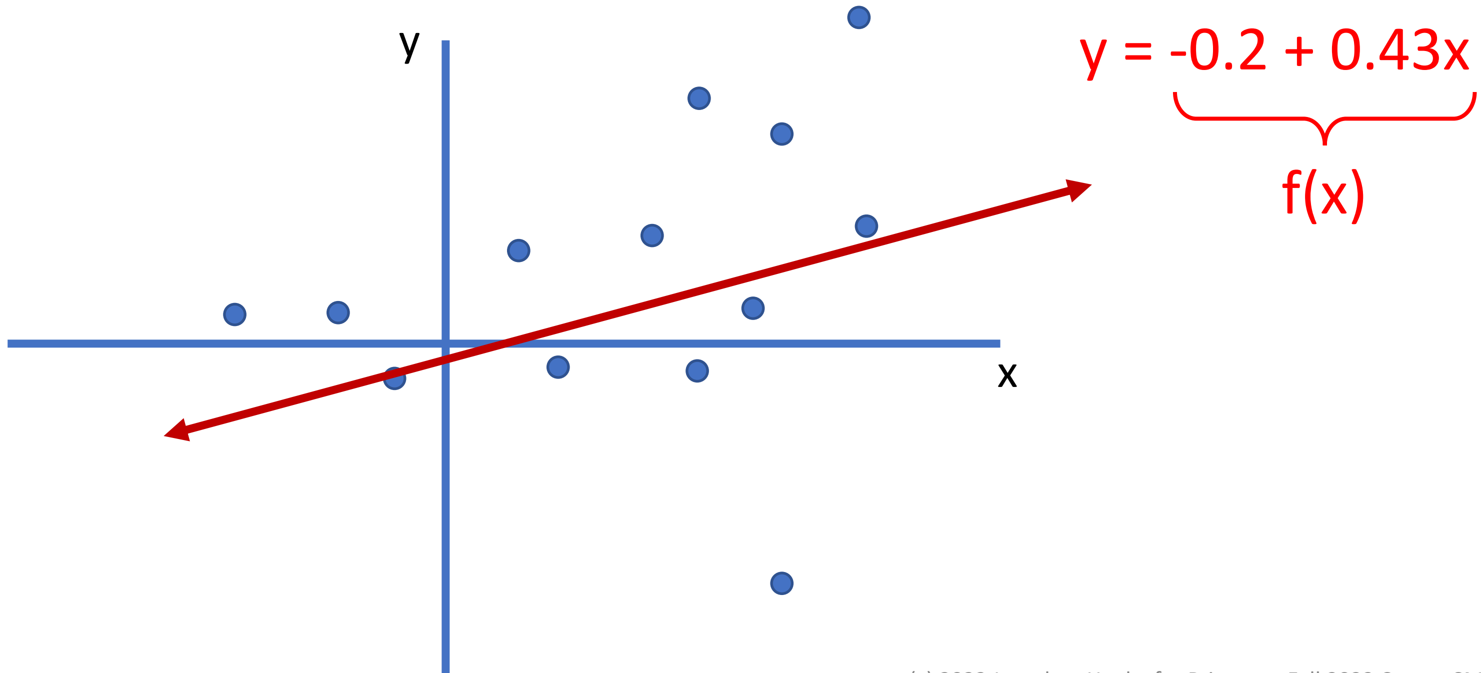

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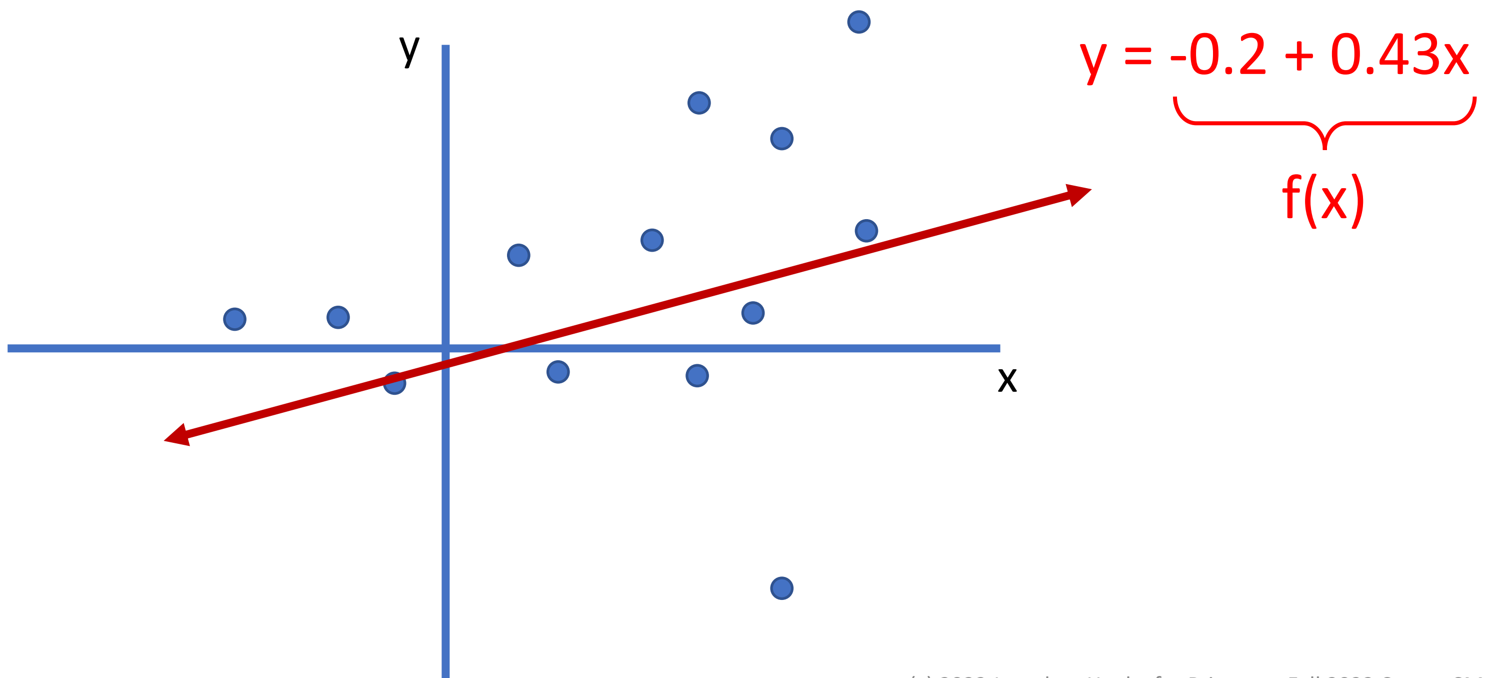

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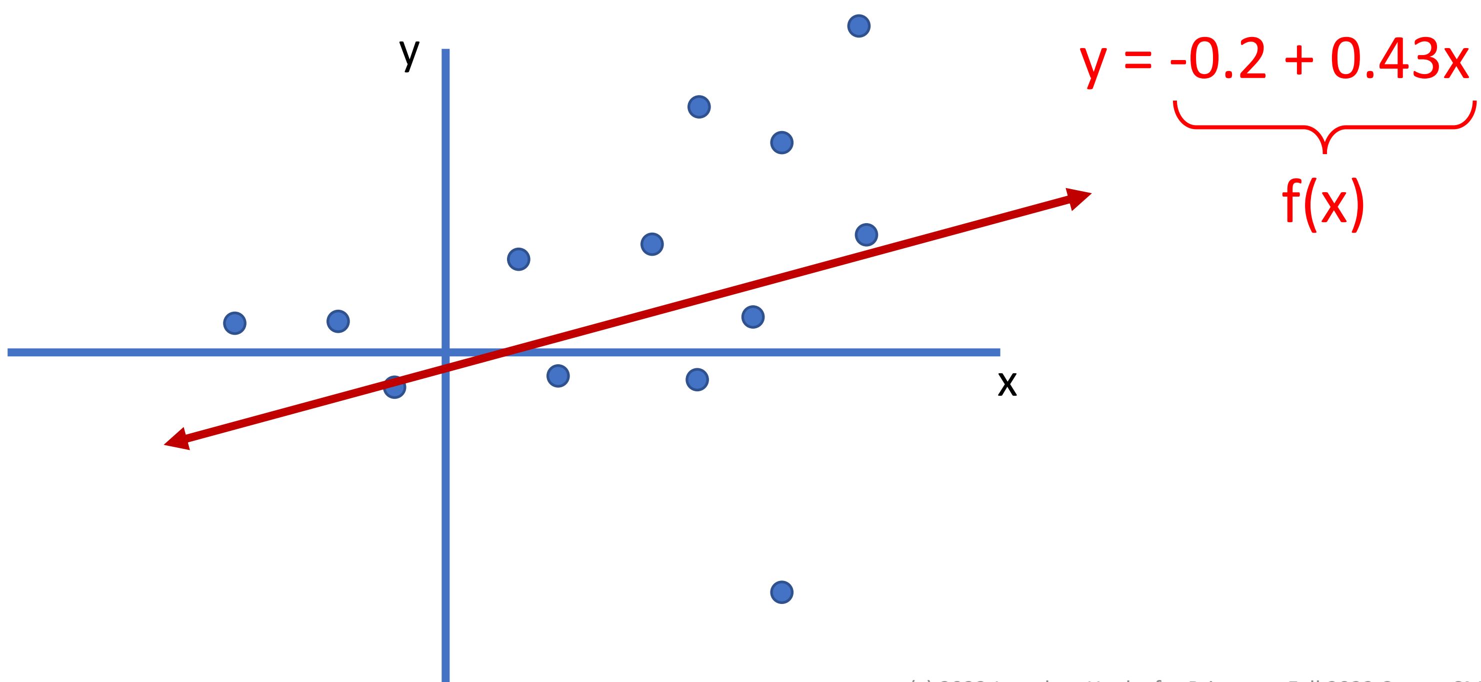

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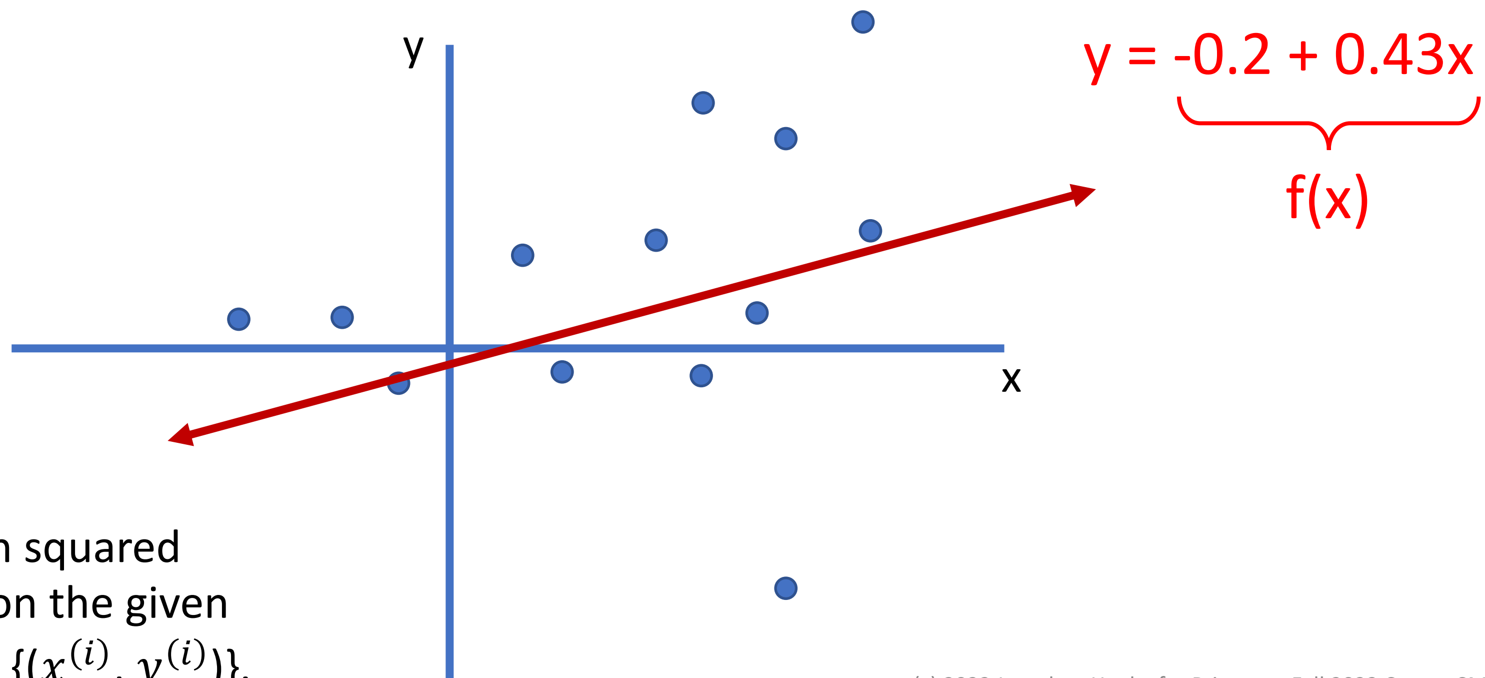

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Summary:

- $b_1 = \frac{\mathrm{Cov}[X,Y]}{\mathrm{Var}[X]}$.
- b_0 ensures that the point $(\mathbb{E}[X], \mathbb{E}[Y])$ is on the line.

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Residuals and the “best” linear model

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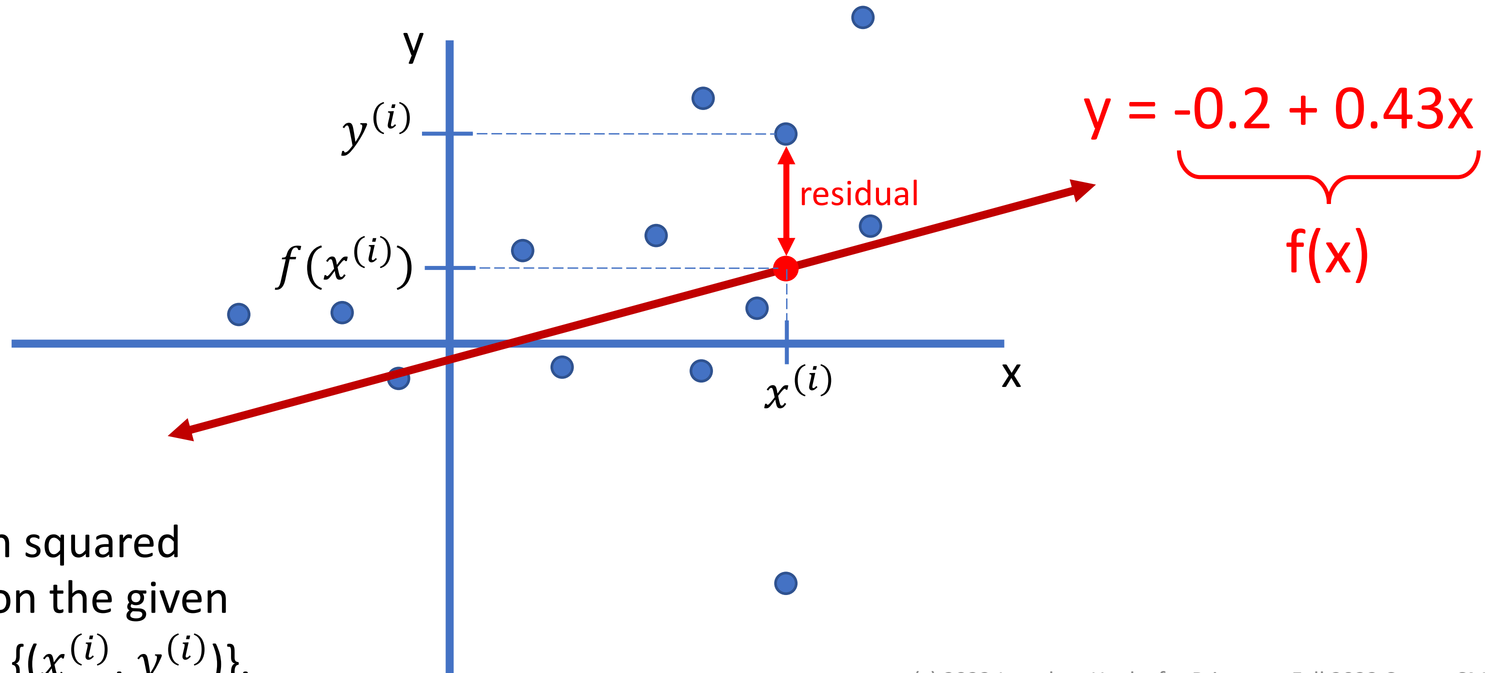

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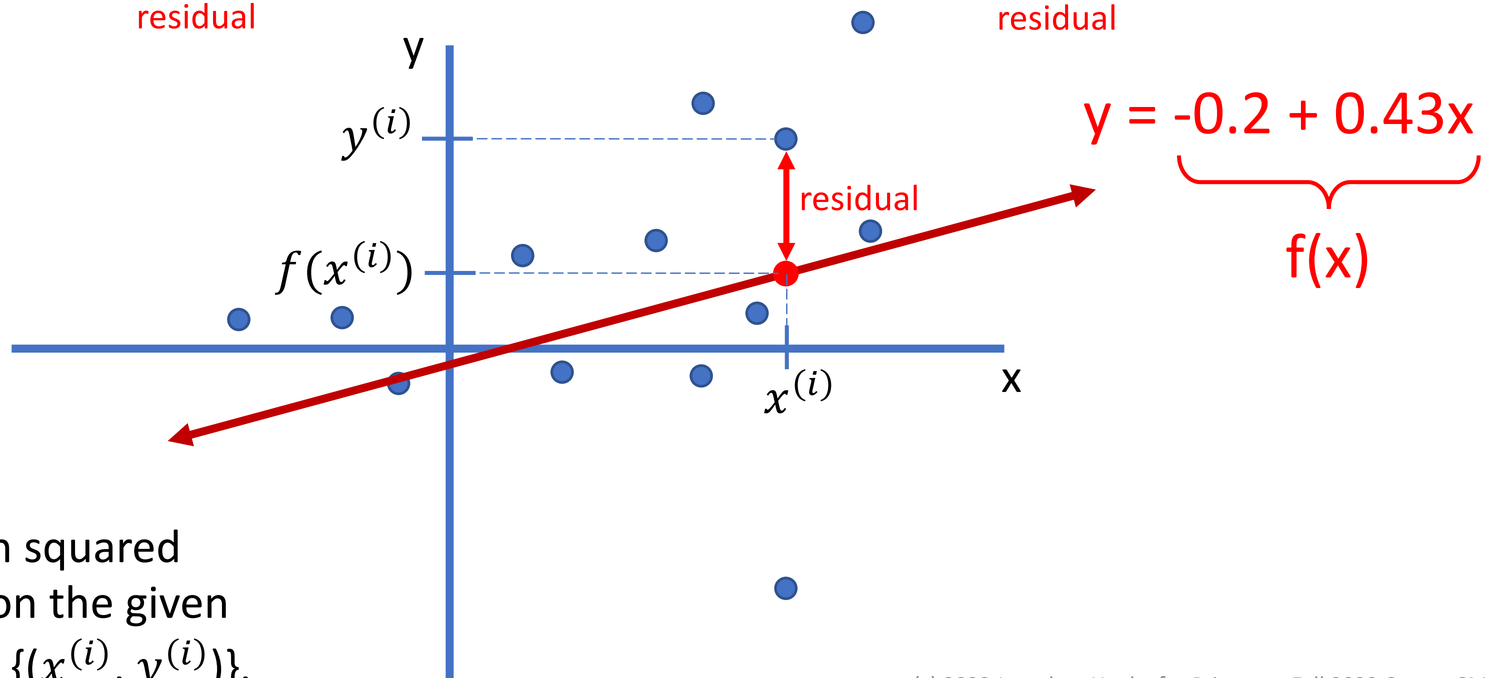

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Understanding Residuals

- **Residuals** are the differences between the actual values and the predicted values. The MSE is the sum of the squares of the residuals.

- The (MSE) "best" linear regression line will have residuals with mean zero. This is true for the residual random variable, but also for the statistic measuring the residual on actual data (i.e. the computed residuals for any given x, and over all x.)

- I.e. $Y = f(X) + r(X)$

$$\Rightarrow \mathbb{E}[Y] = \mathbb{E}[f(X)] + \mathbb{E}[r(X)],$$

but $(\mathbb{E}[X], \mathbb{E}[Y])$ is already on the linear regression line,

so $\mathbb{E}[r(X)] = 0$.

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$$\rho = \frac{\mathrm{Cov}[X,Y]}{\mathrm{StdDev}[X] \cdot \mathrm{StdDev}[Y]}.$$

- This is closely related to the slope of the (MSE) best line

$$b_1 = \frac{\mathrm{Cov}[X,Y]}{\mathrm{Var}[X]}.$$

Mid-class Break!

- Stretch your legs / get a snack break!

Special Case: Linearly related RVs

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$$b_1 = \frac{\text{Cov}[X,Y]}{\text{Var}[X]} = \frac{\text{Cov}[X, a_0 + a_1 X + \varepsilon]}{\text{Var}[X]} = \frac{\text{Cov}[X, a_0] + \text{Cov}[X, a_1 X] + \text{Cov}[X, \varepsilon]}{\text{Var}[X]}$$

$$= \frac{0 + a_1 \text{Var}[X] + 0]}{\text{Var}[X]} = a_1$$

so linear regression correctly recovers b_1 as the slope a_1 of the underlying relationship!

Also we can understand b_0 by computing the expected value

$$\mathbb{E}[Y] = \mathbb{E}[a_0 + a_1 X + \varepsilon] = \mathbb{E}[a_0] + \mathbb{E}[a_1 X] + \mathbb{E}[\varepsilon] = a_0 + a_1 \mathbb{E}[X] + 0,$$

so since $b_1 = a_1$, we see that the given coefficient a_0 also agrees with the coefficient b_0 appearing in the linear regression!

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5. In the special case where Y is linearly related to X (plus some mean zero noise not correlated to X), we see that Linear Regression recovers the coefficients a_0 and a_1 of linear relationship. In this case the underlying probability distribution of X does not play a role.