

PRIMORDIAL BLACK HOLES, CHARGE AND DARK MATTER:

RETHINKING EVAPORATION LIMITS

Justin C. Feng

CEICO, FZU-Institute of Physics of the Czech Academy of Sciences

In collaboration with Jessica Santiago, Sebastian Schuster, and Matt Visser [arXiv:2503.20696]

Workshop in Cosmology, Gravitation and Particle Physics — April 10, 2025, Prague, Czechia



Co-funded by
the European Union



PBHs as DM?

- Gap in PBH constraints below $M \sim 10^{-11} M_\odot$
- Can form large mass fraction $f(M)$ of DM in gap above $10^{-16} M_\odot$
- Very light PBHs can evaporate, but will emit high energy particles (via Hawking radiation)
- Either formation of very light PBHs suppressed, or radiation suppressed

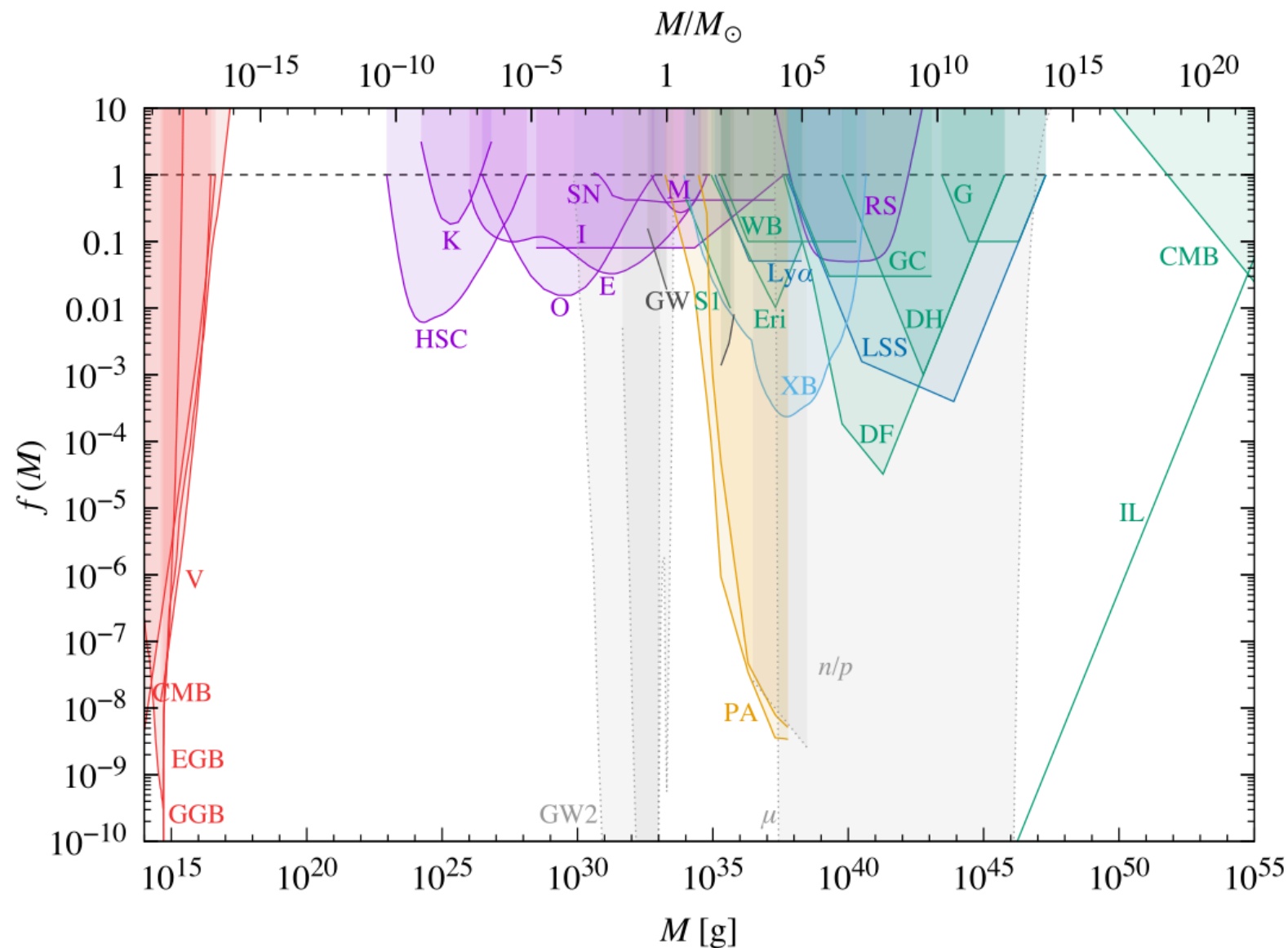


Figure from Carr, B., Kohri, K., Sendouda, Y. & Yokoyama, J. (2021)

Dark electromagnetism

- Idea: Extremally charged black holes have vanishing Hawking temperature
- Problem: charged black holes will be visible
- One solution: New degrees of freedom
 - Postulate dark sector contains a massive charged particle coupled to a dark $U(1)$ gauge field (dark electromagnetism)*
 - For instance, if fermion: $\mathcal{L} = \bar{\chi}(i\gamma^\mu D_\mu + m_\chi)\chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
 - Require no coupling to standard model particles
- Can varying mass and charge of dark "electron" permit long-lived low mass PBHs?

*Y. Bai, N. Orlofsky, Phys. Rev. D 101, 055006 (2020)

Reissner-Nordstrom (RN) metric

The Einstein-Maxwell equations admit a static, spherically symmetric solution of the form:

$$ds^2 = -F dt^2 + F^{-1} dr^2 + r^2 d\Omega^2, \quad F := 1 - \frac{2M}{r} + \frac{Q^2}{r^2},$$

with 4-potential $A_\mu = (Q/r, 0, 0, 0)$.

Has two event horizons at $r_\pm = M \pm \sqrt{M^2 - Q^2}$,

Hawking temperature is given by:

$$T = \frac{\hbar \kappa}{2\pi}, \quad \kappa = \frac{\sqrt{M^2 - Q^2}}{r_+^2}$$

In standard electrodynamics (in geometric units)

$$Q/M \ll m/e \simeq 10^{-21}$$

Assumptions of the Hiscock-Weems model*

Hawking radiation

$$\frac{dM}{dt} = -\frac{\pi^2}{15\hbar^3} T^2 \alpha \sigma_0$$

- Describes mass loss rate; assumes massless particles dominate spectrum
- Geometric optics scattering-cross section:

$$\sigma_0 := \pi \frac{(3M + \sqrt{9M^2 - 8Q^2})^4}{8(3M^2 - 2Q^2 + M\sqrt{9M^2 - 8Q^2})}$$

- Arises from quantum field theory in curved spacetime considerations

Schwinger effect

$$\Gamma = \frac{(eE)^2}{4\pi^3 \hbar^2 c} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{\pi m_e^2 c^3 n}{\hbar e E}\right)$$

- Describes electron positron pair creation rate per unit 4-volume
- Schwinger effect: strong electric field E create electron-positron pairs
- Gibbons:[†] for sufficiently massive black holes, discharge follows a Schwinger-type formula

*W. A. Hiscock and L. D. Weems, "Evolution of charged evaporating black holes,". Phys. Rev. D 41, 1142 (1990).

[†]G.W. Gibbons, "Vacuum polarization and spontaneous loss of charge by black holes,". Comm. Math. Phys. 44 (1975) 245.

Applicability of the Hiscock-Weems (HW) model

(i) Black hole mass larger than Compton wavelength of lightest charged particle[†]

$$M \gg \frac{\hbar}{m_e}$$

(ii) Weak field assumption (use to truncate series for Γ):

$$\frac{e^3 Q}{m_e^2 r^2} \ll 1$$

(iii) Another large mass assumption (use to get rid of an erfc term):

$$r_+^2 \gg QQ_0$$

[†]G.W. Gibbons, "Vacuum polarization and spontaneous loss of charge by black holes,". Comm. Math. Phys. 44 (1975) 245.

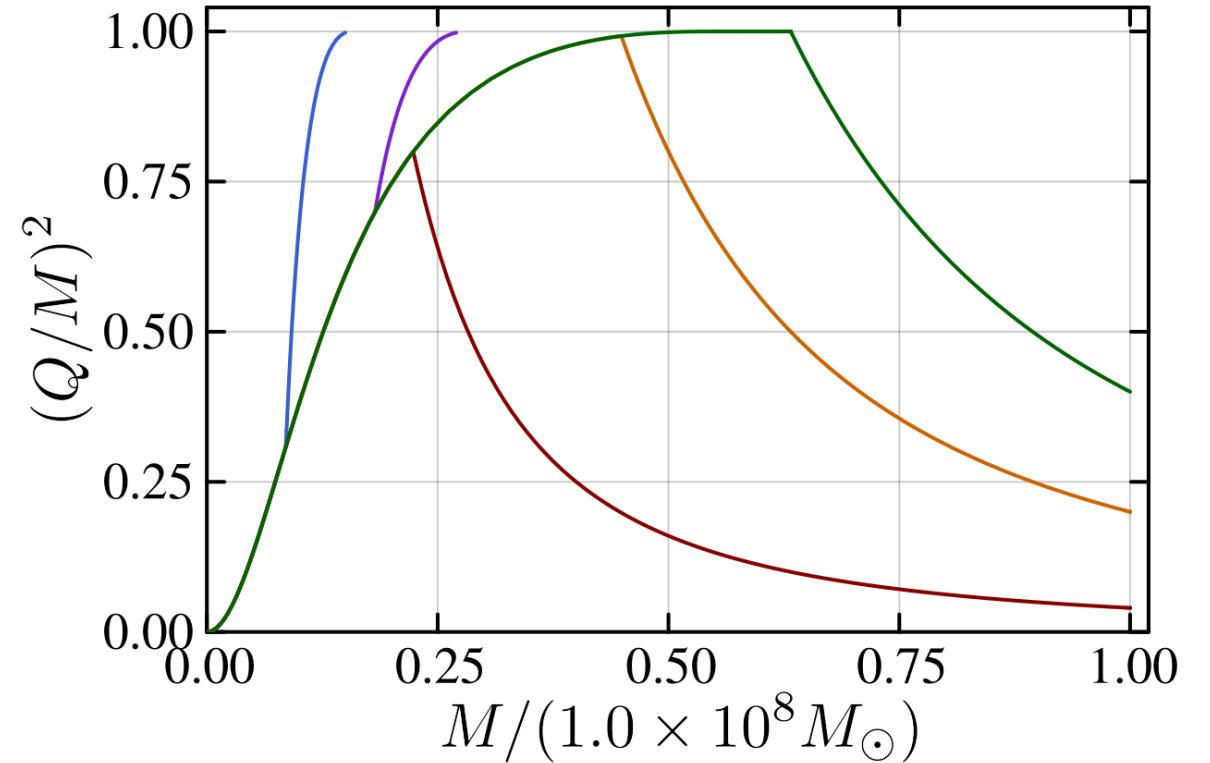
The Hiscock-Weems model

Evolution equations

$$\begin{aligned}\frac{dM}{dt} &= -\frac{\pi^2}{15\hbar^3}T^2\alpha\sigma_0 + \frac{Q}{r_+}\frac{dQ}{dt} \\ \frac{dQ}{dt} &= -\frac{e^2}{\pi^2\hbar^2r_+}\exp\left(-\frac{r_+^2}{QQ_0}\right) \\ &\quad -\frac{\pi}{\sqrt{QQ_0}}\operatorname{erfc}\left(-\frac{r_+^2}{QQ_0}\right)\end{aligned}$$

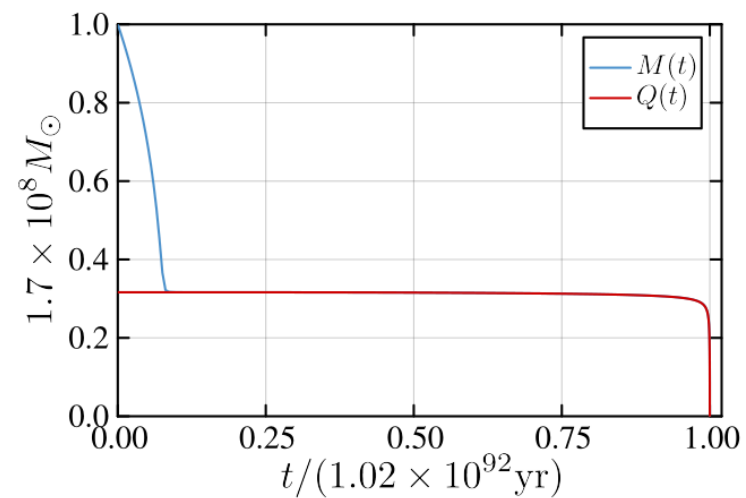
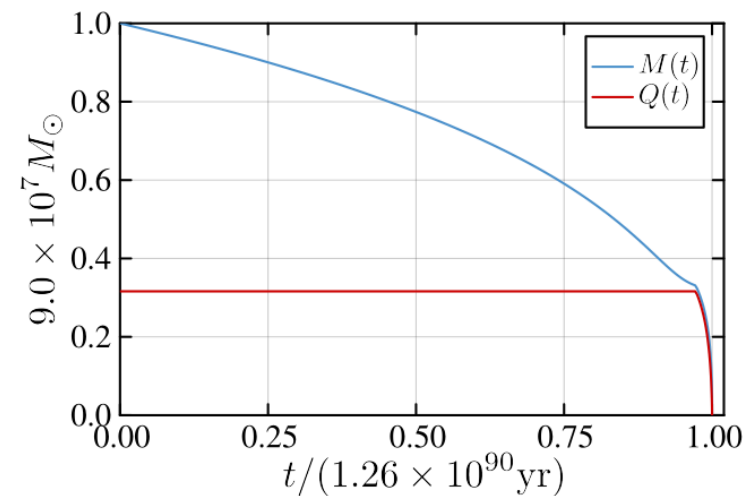
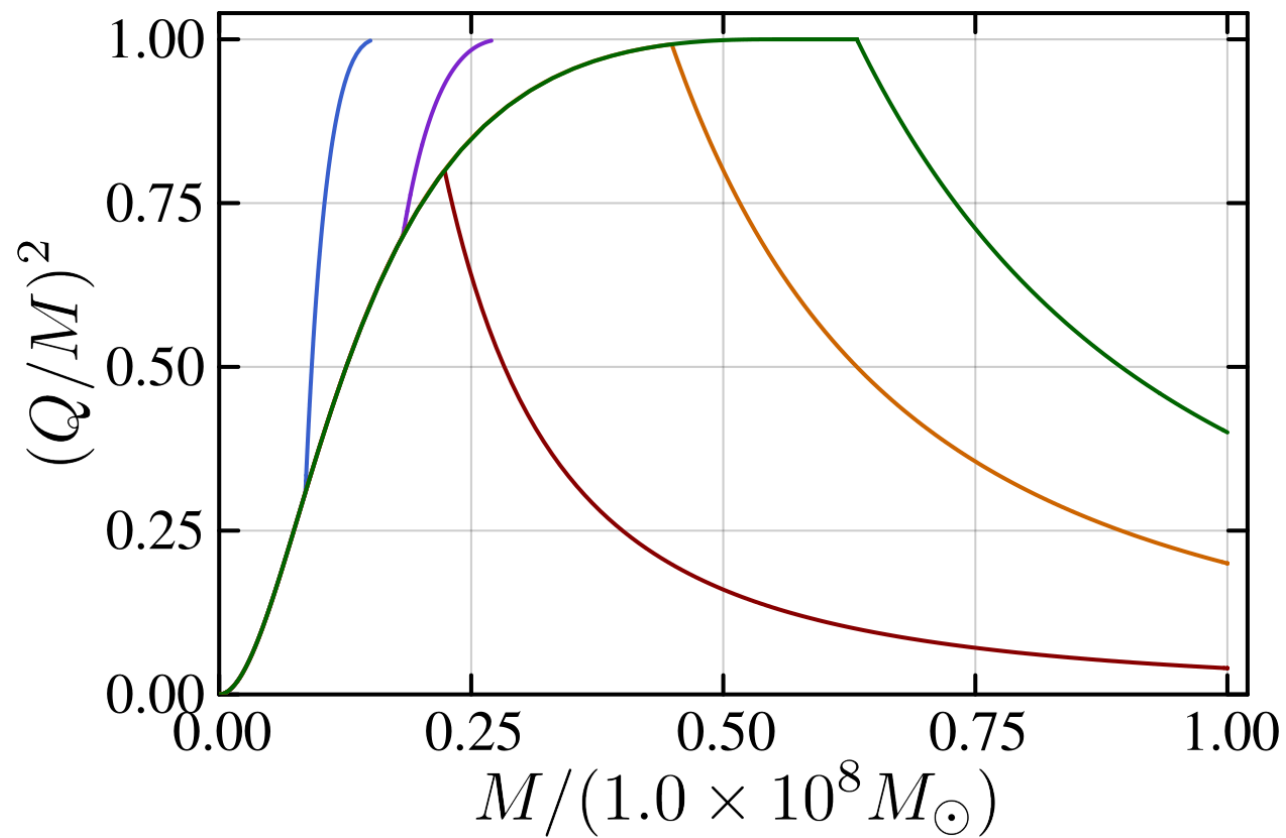
$$\sigma_0 := \pi \frac{(3M + \sqrt{9M^2 - 8Q^2})^4}{8(3M^2 - 2Q^2 + M\sqrt{9M^2 - 8Q^2})}$$

$$Q_0 = \frac{\hbar e}{\pi m^2} \sim 1.7 \times 10^5 M_\odot$$



Configuration space plot; mass dissipation zone on lower right, charge dissipation zone on upper left

Evolution profiles



Nondimensionalized Hiscock-Weems (NHW) model

Defining $\mu := M/M_s$, and $Y := (Q/M)^2$, where M_s is a mass scale, the HW equations are rewritten (after truncation to drop erfc):

$$\boxed{\frac{d\mu}{d\tau} = -\frac{(H(\mu, Y) + S(\mu, Y)Y^2)}{(\sqrt{1-Y} + 1)^4}, \quad \frac{dY}{d\tau} = \frac{2(H(\mu, Y) - S(\mu, Y)(1 - Y + \sqrt{1-Y})Y)Y}{\mu(\sqrt{1-Y} + 1)^4}}$$

where $\tau := ts_0/M_s$ and:

$$H(\mu, Y) := \frac{(\sqrt{9-8Y} + 3)^4(1-Y)^2}{\mu^2(\sqrt{1-Y} + 1)^4(3-2Y + \sqrt{9-8Y})}, \quad S(\mu, Y) := \exp \left\{ b_0 \left[z_0 - \mu(\sqrt{1-Y} + 1)^2 / \sqrt{Y} \right] \right\}$$

with the dimensionless constants:

$$s_0 = \frac{\alpha \hbar}{1920\pi M_s^2}, \quad z_0 = \frac{e\hbar}{\pi m_e^2 M_s} \ln \left(\frac{960e^4 M_s^2}{\pi^2 \alpha m_e^2 \hbar^2} \right), \quad b_0 = \frac{m_e^2 M_s \pi}{e\hbar}$$

Approximate solutions

Configuration space equation:

$$\frac{dY}{d\mu} = \frac{2S(\mu, Y) \left(\sqrt{1-Y} + 1 \right) Y^2}{\mu (H(\mu, Y) + S(\mu, Y) Y^2)} - \frac{2Y}{\mu}.$$

Hawking-dominated (mass dissipation)

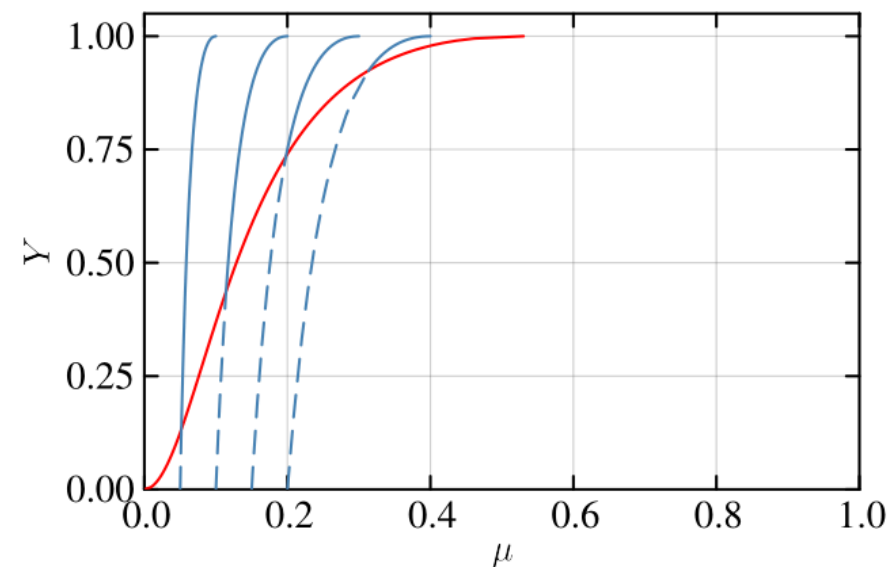
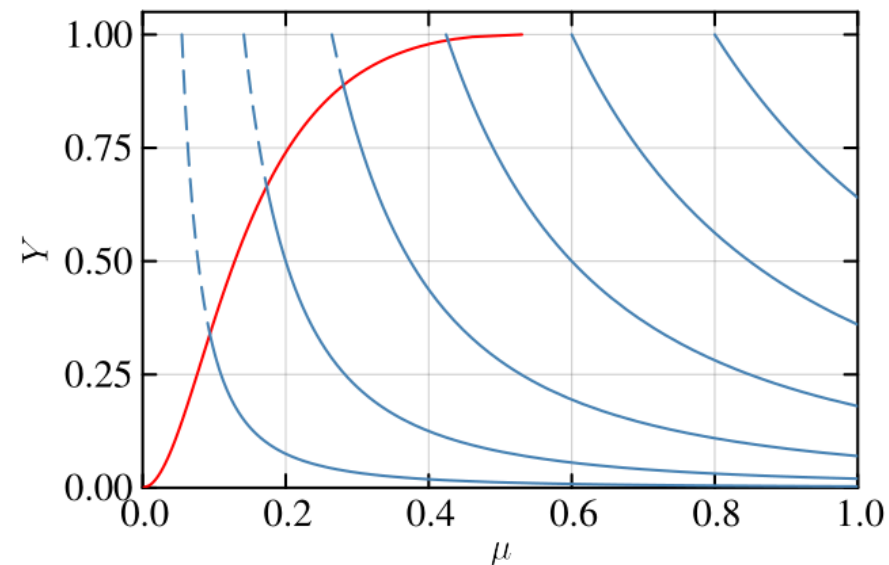
$$Y_H(\mu) = \left(\frac{\mu_h}{\mu} \right)^2$$

Schwinger-dominated (charge dissipation)

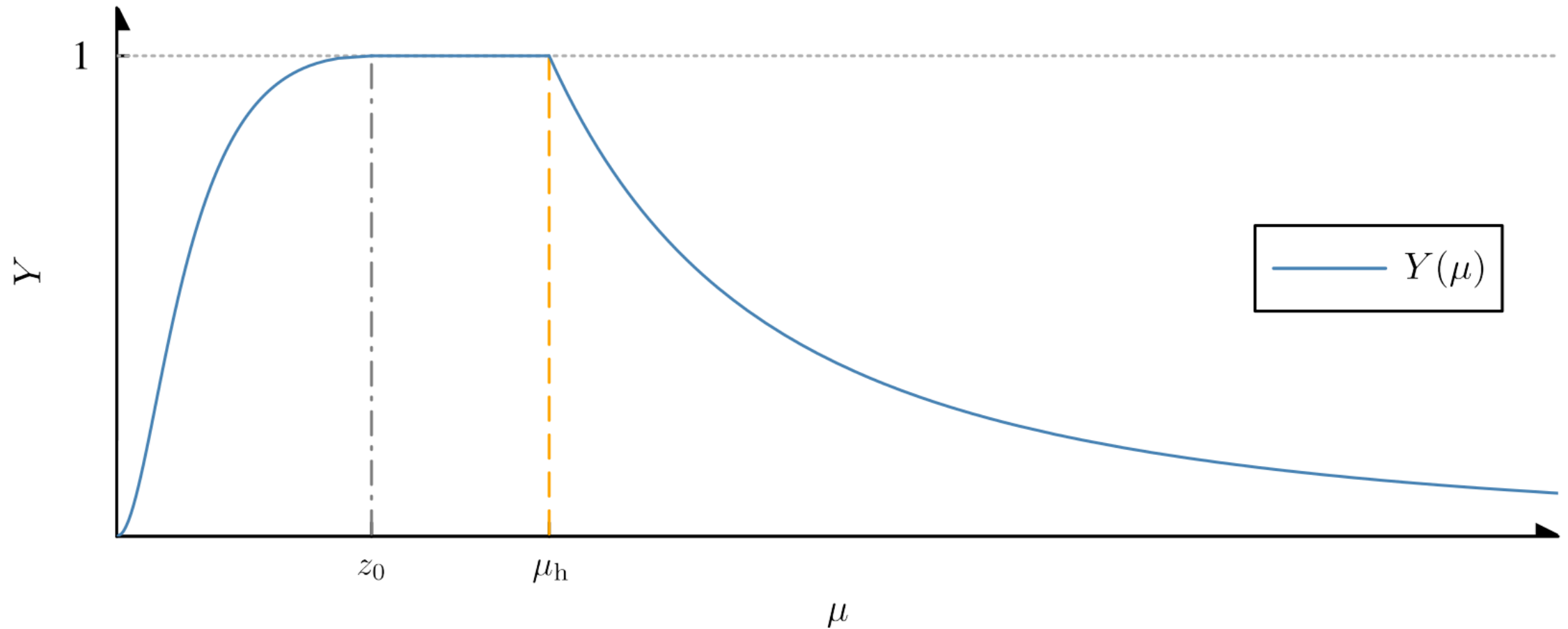
$$Y_S(\mu) = \frac{(2\mu - \mu_1)\mu_1}{\mu^2}$$

Attractor region curve: $\mu = z_0 \frac{\sqrt{Y}}{(\sqrt{1-Y}+1)^2}$

Near-extremal cond. is also soln.: $Y = 1$



Configuration space solution



z_0 determines beginning of attractor curve, μ_h begins near-extremal portion.

Comparison with numerics



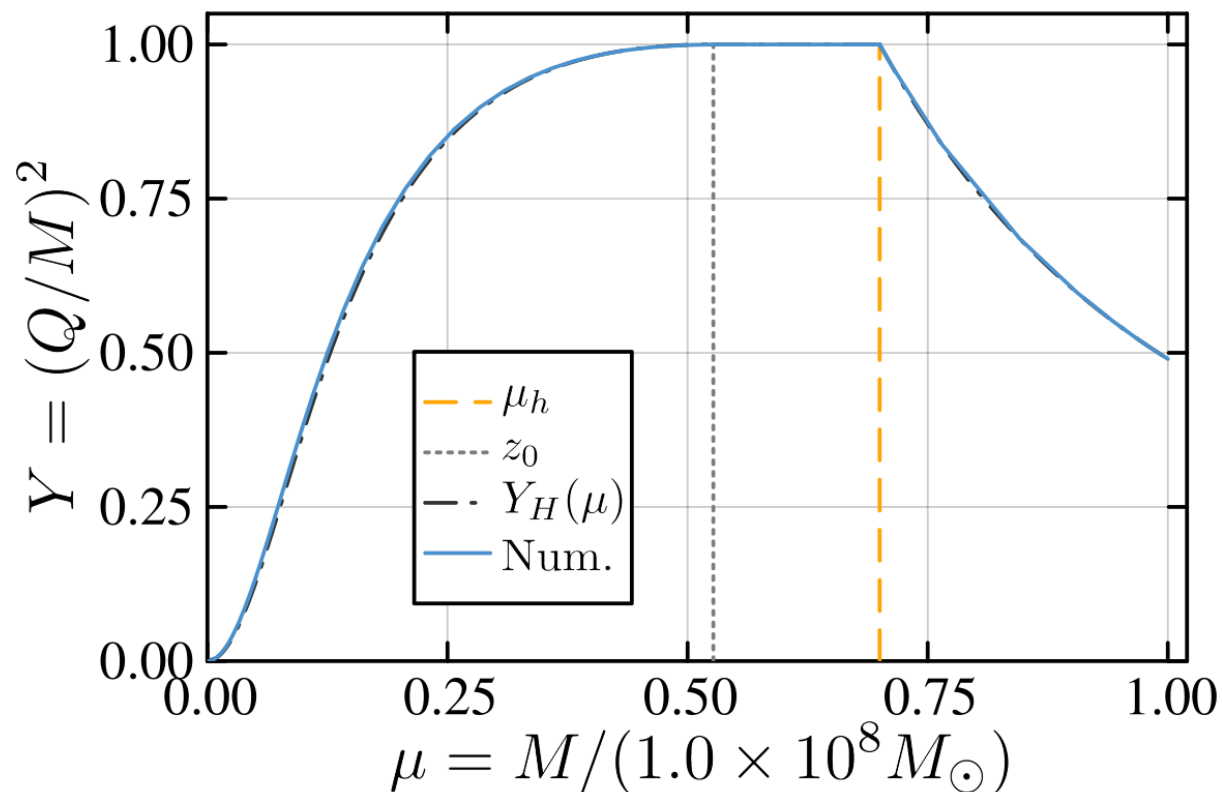
Numerical soln. well approximated by
piecewise soln. in config. space

- Residual is $\leq 10^{-2}$

For numerics, used OrdinaryDiffEq.jl in
the julia language

Advertisement for julia

- As easy as Python, as fast as C or FORTRAN
- Has state of the art ODE solvers
- Can autodifferentiate numerical ODE solutions!



Evaporation lifetime estimates

Near-extremal soln. $Y = 1$ implies:

$$\frac{d\mu}{d\tau} = -\exp[(z_0 - \mu)b_0]$$

which can be integrated to obtain:

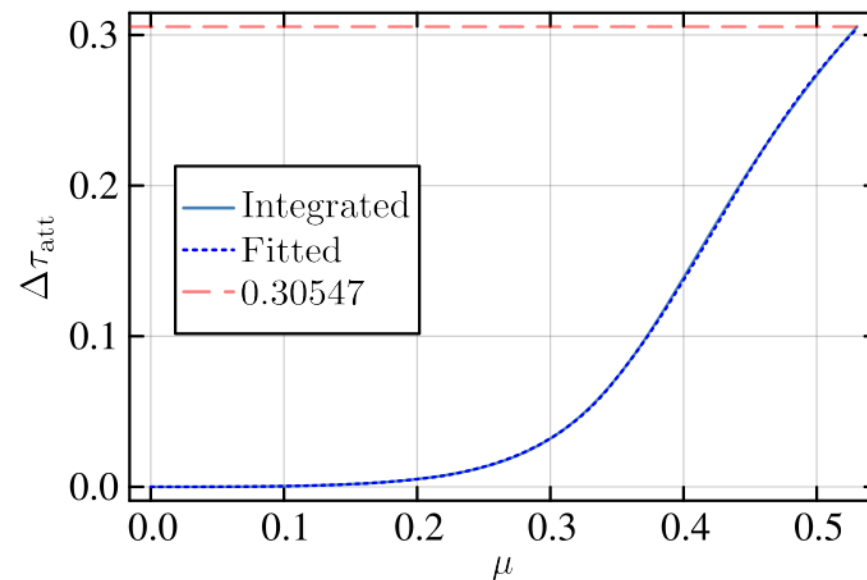
$$\Delta t \sim \frac{M_s}{s_0 b_0} \exp[(\mu_h - z_0) b_0]$$

Set $\Delta t = t_{\text{univ}} \sim 10^{10} \text{yr}$, get mass threshold:

$$M_{\text{ex}}^\chi \approx \frac{e_\chi}{m_\chi^2} \frac{\hbar}{\pi} \ln \left[\frac{e_\chi^3 t_{\text{univ}}}{2\pi^2 \hbar^2} \right]$$

Good if nearextremality reached, and $\bar{\mu} - z_0^\chi \gg 1/b_0^\chi$

Here, $\bar{\mu}$ is either μ_h or μ when soln. reaches attractor



More complete mass threshold includes time down attractor:

$$M_{\text{univ}} = \begin{cases} M_{\text{att}}^{\text{univ}}, & \bar{\mu} \leq z_0^\chi \\ M_{\text{ex}}^{\text{univ}} + M_{\text{att,max}}^{\text{univ}}, & \bar{\mu} > z_0^\chi \end{cases}$$

Long-lived PBH mass limits

M_{univ} is mass of a dark-charged BH that is near extremal or on attractor with lifetime t_{univ} .

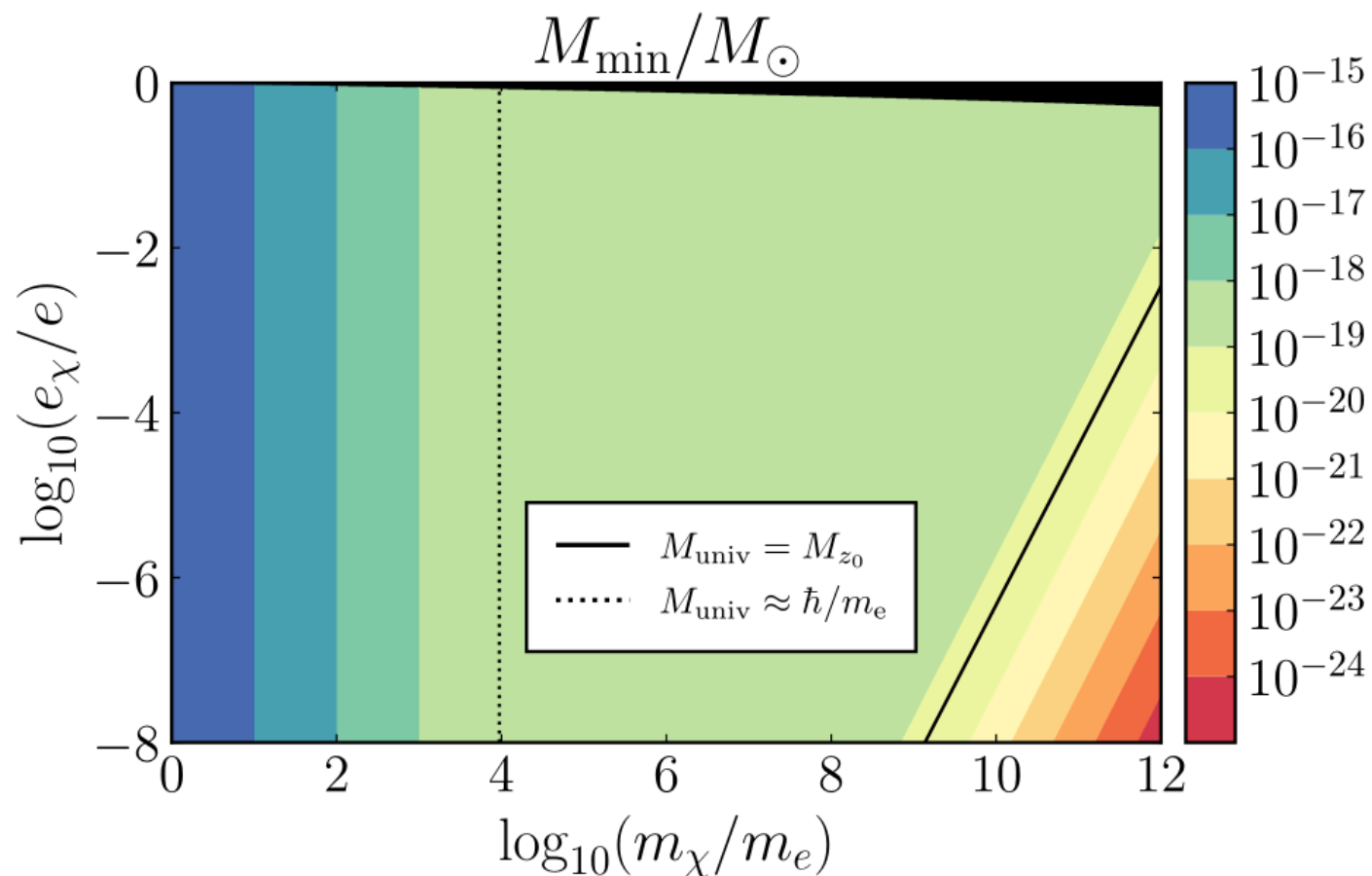
To right contours of following:

$$M_{\text{min}} = \max(M_{\text{univ}}, M_{(i)})$$

where $M_{(i)} := \hbar/m_e$ (cond. i)

Black region excluded by condition ii:

$$\frac{e^3 Q}{m_e^2 r^2} \ll 1$$



Summary

- In Hiscock & Weems model^{*} for evaporating BHs, discharge modeled by Schwinger effect,[†] mass mostly lost thru Hawking rad. of massless particles
- Relatively heavy nonextremal BHs initially dissipate mass faster than charge, can reach near-extremality
- If PBHs can form with dark $U(1)$ charge, there exists reasonable parameter space permitting cold, near-extremal PBHs with masses below $10^{-16} M_{\odot}$
- Such PBHs can comprise a significant fraction of DM

^{*}W. A. Hiscock and L. D. Weems, "Evolution of charged evaporating black holes,". Phys. Rev. D 41, 1142 (1990).

[†]G.W. Gibbons, "Vacuum polarization and spontaneous loss of charge by black holes,". Comm. Math. Phys. 44 (1975) 245.

After submitting to a journal, we discovered arXiv:2503.10755 (posted less than two weeks prior), which considers the HW equation for dark-charged PBHs, but with the very different aims of arguing that we *should* see BH explosions.

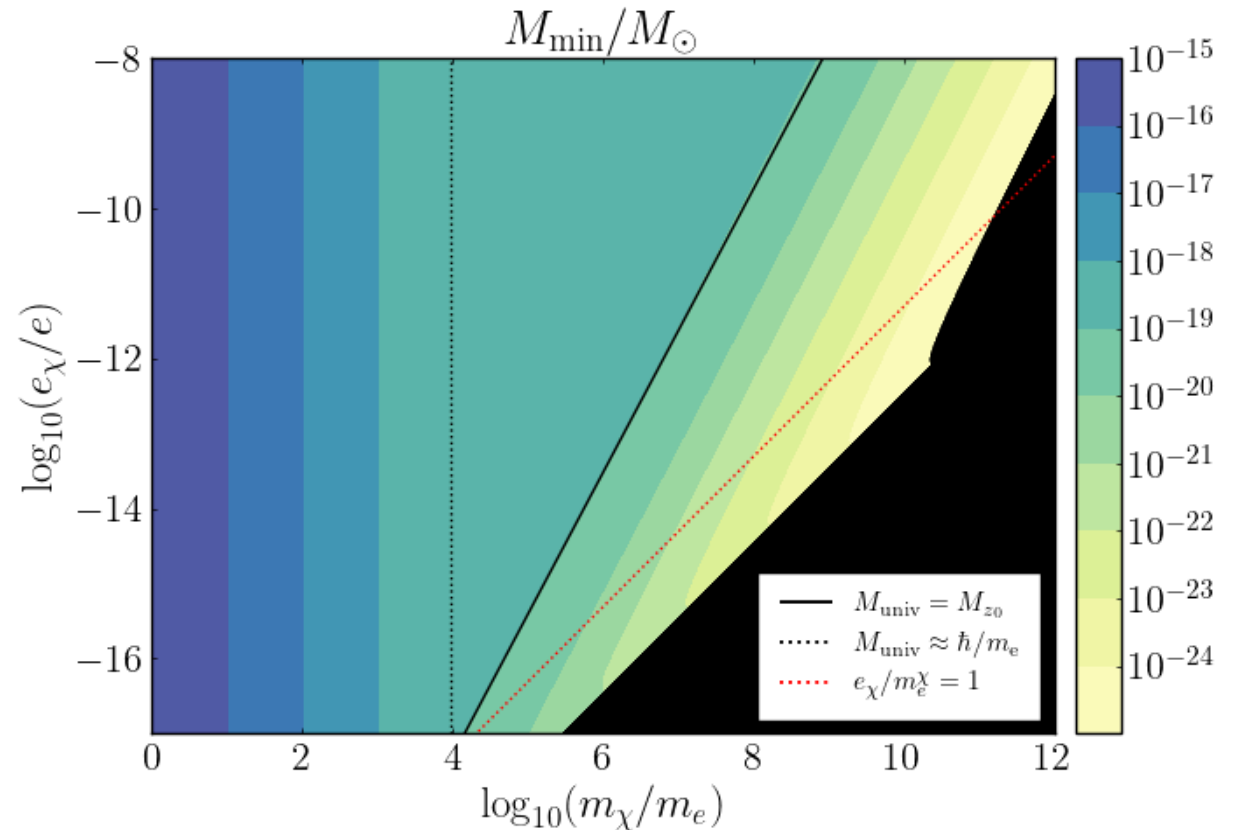
Superradiant discharge

Gibbons' derivation[†] indicates that for heavy, charged BHs, discharge follows Schwinger-type formula

Different approaches: superradiant discharge of $M \sim Q$ BHs when:

$$\frac{q}{m_e} > 1$$

Our analysis can accommodate above, but with reduced parameter space.



[†]G.W. Gibbons, "Vacuum polarization and spontaneous loss of charge by black holes,". Comm. Math. Phys. 44 (1975) 245.