Singularity at the demise of a black hole

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Based on: *Phys. Rev. D 109, 024040 (2024)* and *arXiv:2505.00112v1 (to appear in Phys. Rev. D)*

17 July 2025 GR24 and Amaldi16, Glasgow, Scotland, UK

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Co-funded by the European Union

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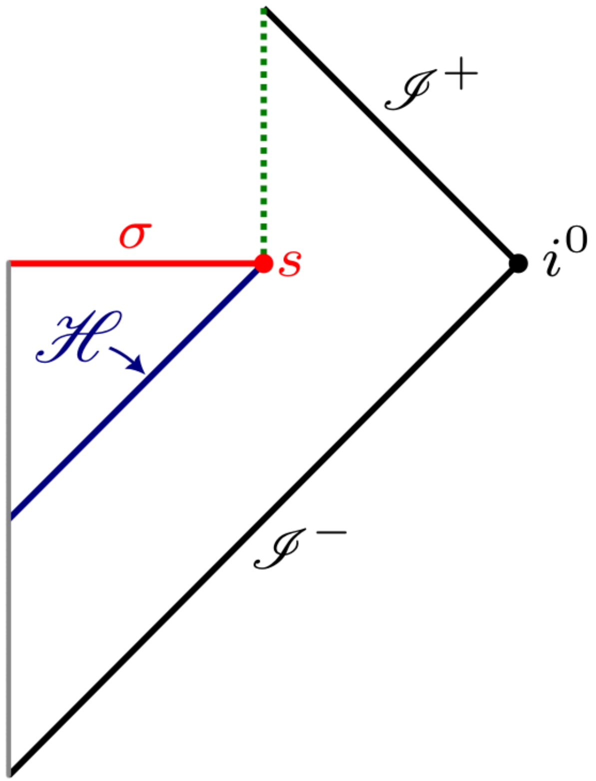
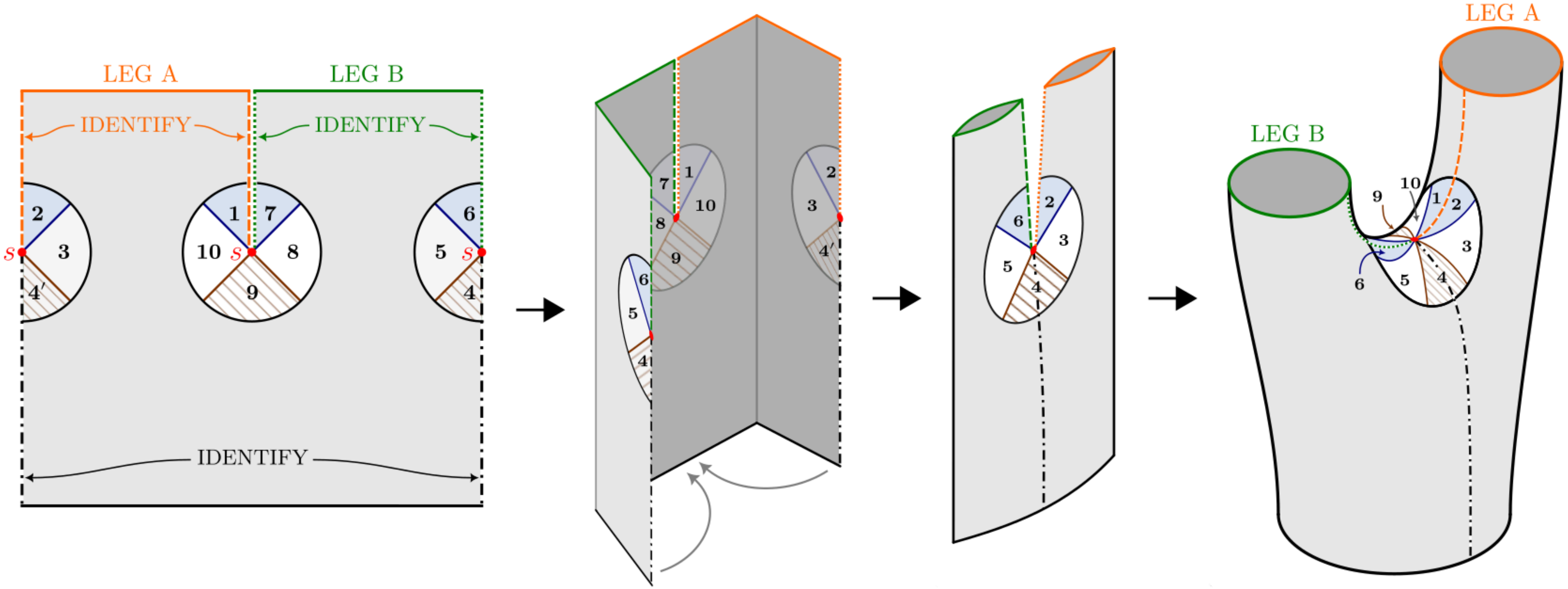
End state of a Black Hole



- What is end state of evaporating black hole?
- Penrose diagram for spherically symmetric case provides an illustration
  - r-t spacetime diagram preserving angles
  - 45° null, $\mathscr{I}^\pm$ and i^0 represent ∞
- Not too difficult to regularize singularity[†] σ
- But what about s?

[†] A Simpson, M Visser, JCAP 02 (2019) 042 [arxiv:1812.07114]

Trousers[†*] spacetime



Normally, spacetime points have one future and one past light cone

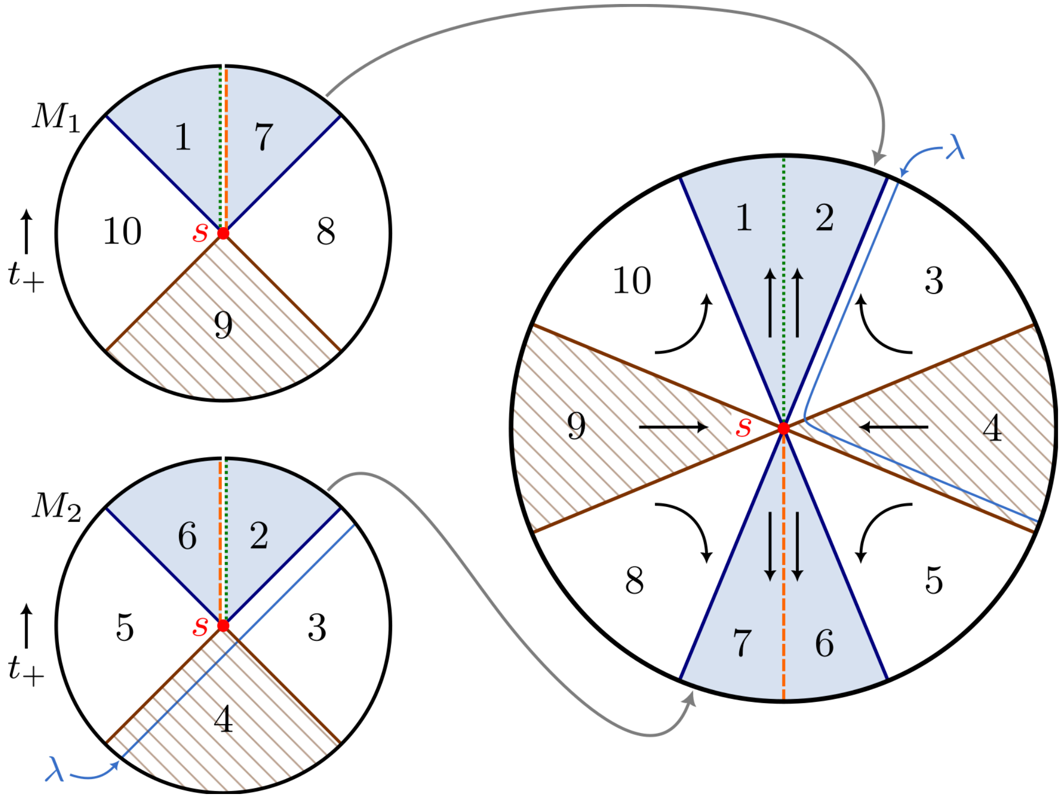
s is a quasiregular singularity, characterized by 2 future and 2 past light cones

[†] A Anderson, B S DeWitt, Found.Phys. 16 (1986) 91-105

[*] F. Dowker, S. Surya, PRD 58, 124019 (1998) [arXiv:gr-qc/9711070]; Buck et al., Class.Quant.Grav. 34 (2017) 5, 055002 [arXiv:1609.03573]

Quasiregular singularities

They generalize conical singularities, construct via cut and paste methods.



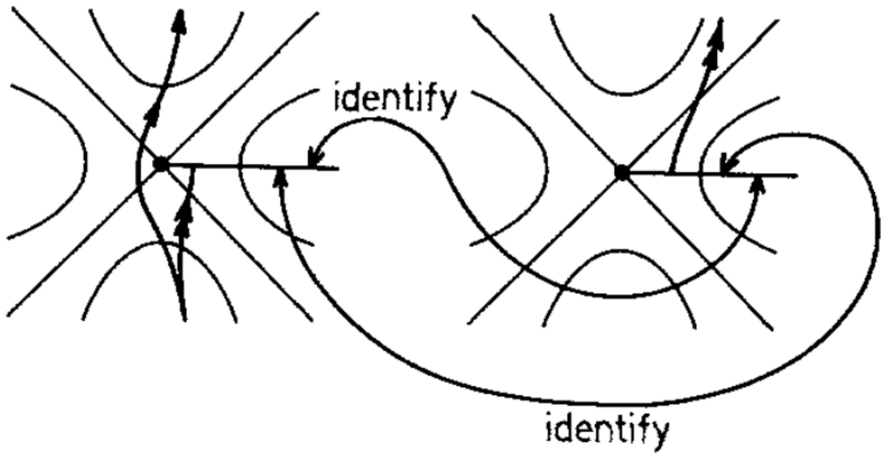
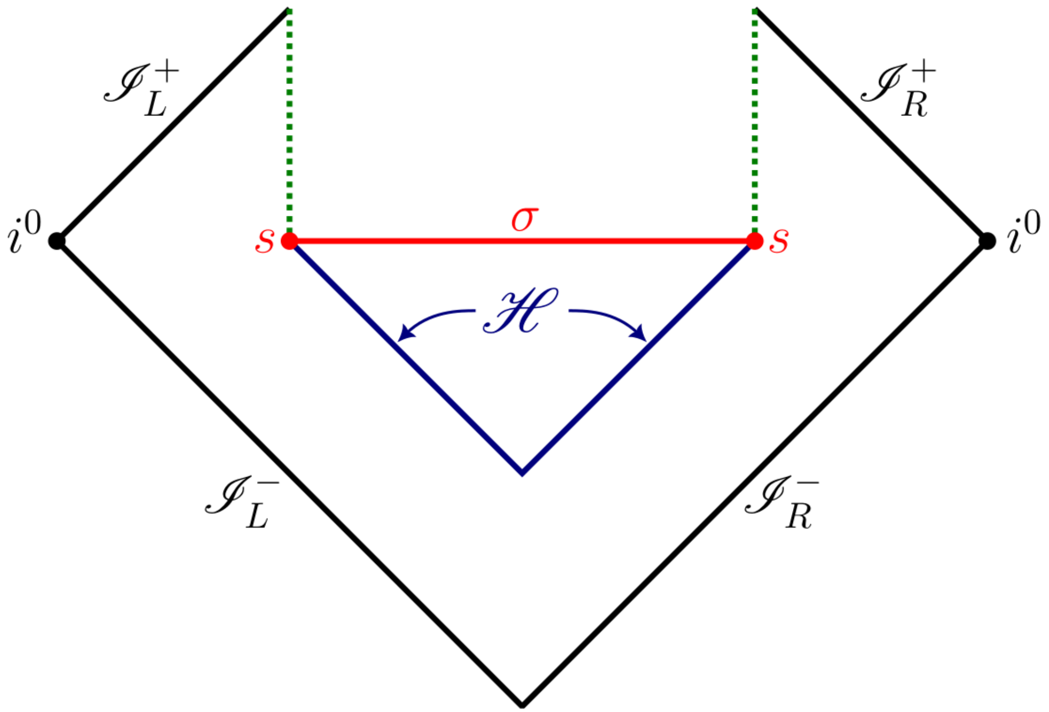


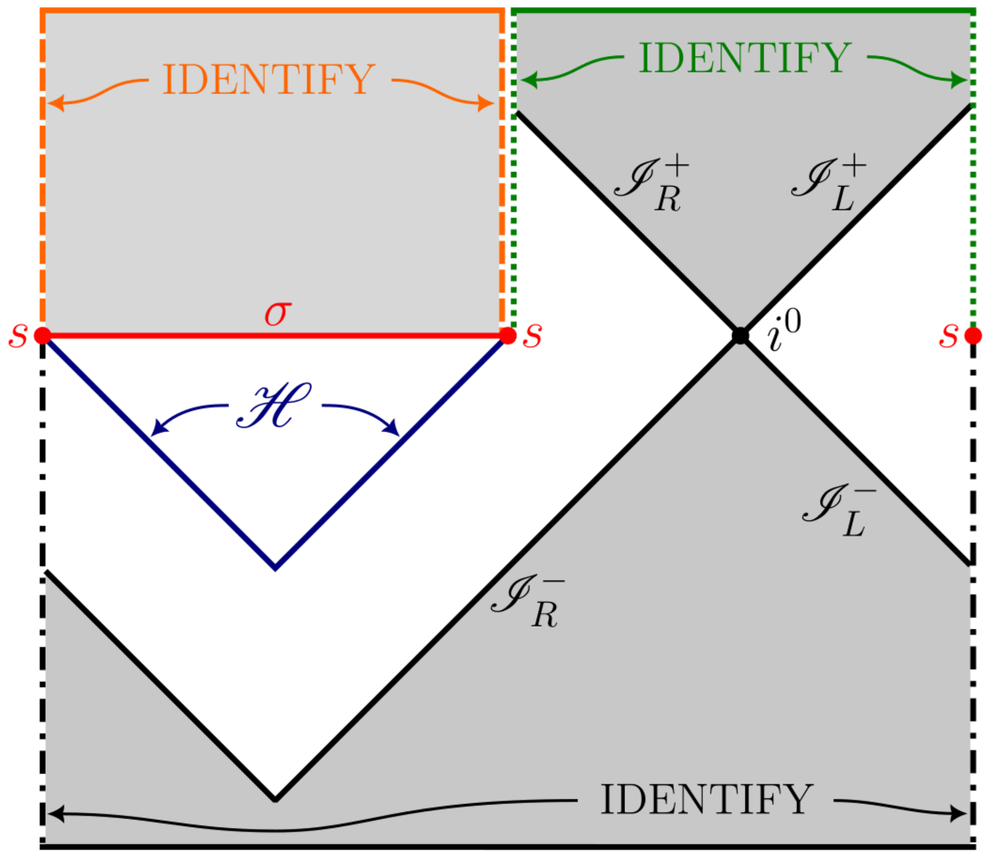
Fig 4(e) from Ellis & Schmidt†

†G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

$1+1$ Black hole and trousers† spacetime

Evaporating $1+1$ BH is conformal to trousers: s is quasiregular* singularity!





†A Anderson, B S DeWitt, Found.Phys. 16 (1986) 91-105

* Provided that σ is regularized and spacetime analytically extended.

Emergent Lorentz signature theory

- Postulate[1] Euclidean-signature $g_{\mu\nu}$ with scalar-tensor action bounded below:

$$S = \int_M d^4x \sqrt{|g|} L, \quad \varphi_a := \nabla_a \varphi, \quad \varphi_{ab} := \nabla_a \nabla_b \varphi, \quad X := \varphi^a \varphi_a$$

$$L = c_1 R^2 + c_2 R_{ab} R^{ab} + c_3 R_{abcd} R^{abcd} + c_4 X R + c_5 R^{ab} \varphi_a \varphi_b \\ + c_6 \mathrm{X}^2 + c_7 (\Box \varphi)^2 + c_8 \varphi_{ab} \varphi^{ab} + c_9 R + c_{10} X + c_{11}$$

- At long distances, matter is coupled to $\mathbf{g}_{\mu\nu}$:

$$\mathbf{g}_{ab} = g_{ab} - \varphi_a \varphi_b / X_C$$

- Renormalizable[2] (expected, since this is the case for quadratic gravity[3])
- Can argue S reduces to Lorentzian scalar-tensor theory in long-distance limit.[1,4]

[1] S. Mukohyama, Phys. Rev. D 87, 085030 (2013)
[2] K Muneyuki, N Ohta, Phys. Lett. B 725 (2013) 495-499
[3] K. S. Stelle, Phys. Rev. D 16, 953 (1977)
[4] S Mukohyama, J Uzan, Phys. Rev D. 87:065020 (2013)

A quadratic ELST can 'regularize' a quasiregular singularity[†]

Consider a saddle-like scalar field profile and flat metric:

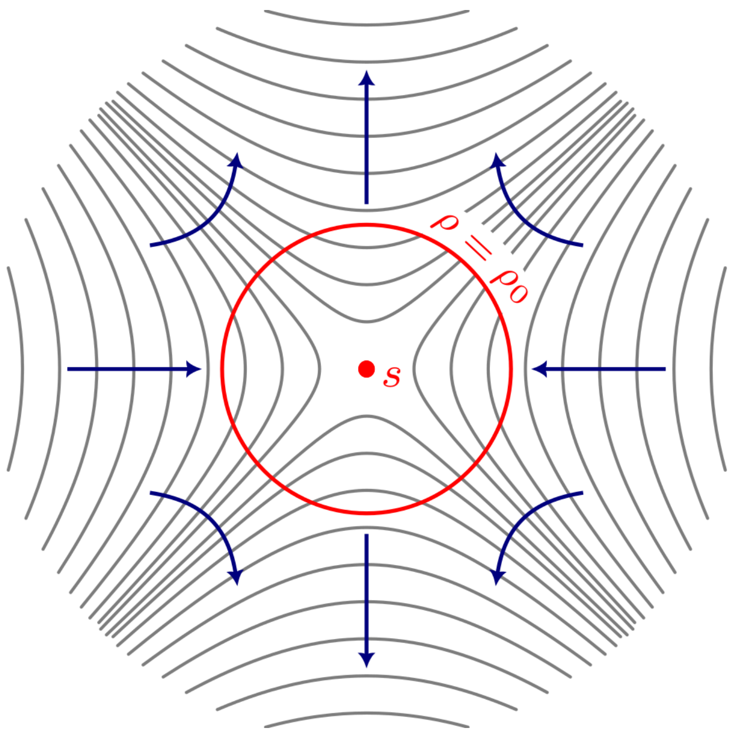
$$\varphi = (u^2 - v^2)/(2L_0)$$
$$ds^2 = du^2 + dv^2 + dy^2 + dz^2$$

These form a soln. for the parameter choices[†]
$c_4 = c_6 = c_{10} = 0$ and $c_{11} = 8(c_5 - c_8)$.

Can get approximate soln. using Riemann normal coords:[‡]

$$g_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{3}\left[R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta - \frac{1}{6}\left[\nabla_\gamma R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma$$
$$-\left[\frac{2}{45}R_{\mu\alpha\lambda\beta}R^\lambda{}_{\gamma\delta\nu} + \frac{1}{20}\nabla_\gamma\nabla_\delta R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma x^\delta + O(x^{\cdot 5})$$

where s is the origin.



[†]JCF, S. Mukohyama, S. Carloni, Phys. Rev. D 109, 024040 (2024)
[‡]U Muller, C Schubert, and A M E van de Ven, Gen. Rel. Grav. 31, 1759 (1999); A Z Petrov, *Einstein Spaces*, Pergamon (1969)

A Lorentzian dispersion relation emerges[1]

- Theory is fundamentally Euclidean---can one be fooled into thinking that it is Lorentzian?

- Rewrite L (C_{abcd} is Weyl, E is Gauss-Bonnet):

$$\tilde{L} := P_0 + (X - X_0)^2 - Zr + \eta(\chi^2 - \chi R) + \tfrac{1}{2\lambda} C_{abcd} C^{abcd} + \gamma_0 G^{ab} \varphi_a \varphi_b + \alpha_0 {\varphi_a}^a {\varphi_b}^b + \beta_0 \varphi_{ab} \varphi^{ab} + \sigma E$$

- Set $P_0 = 0$, have flat background soln:

$$\bar{g}_{ab} = \mathrm{diag}(1,1,1,1), \qquad \bar{\varphi} = t\sqrt{X_0}, \qquad \bar{\chi} = 0$$

- Perturb, restrict to modes in x-direction:

$$\delta g_{ab} = s \begin{pmatrix} U & 0 & B_y & B_z \\ 0 & \psi & 0 & 0 \\ B_y & 0 & \psi + h_+ & h_\times \\ B_z & 0 & h_\times & \psi - h_+ \end{pmatrix}, \qquad \delta\varphi = s\phi, \qquad \delta\chi = s\xi.$$

tensor: $(h_+, h_\times)$, vec.: (B_y, B_z), scalar: (ϕ, ξ, U, ψ)

- In long distance limit $\dot{h}, k \ll M_{\mathrm{Pl}}$, $\ddot{h} \ll M_{\mathrm{Pl}}^2$, tensor sector action has form:

$$\underline{S}_h \approx \frac{s^2 v_0}{8} \int dt \left[\mu_3\, \dot{h}^2 - \mu_2\, k^2\, h^2 \right], \qquad \begin{aligned} \mu_3 &:= X_0(\beta_0 + \gamma_0) + Z \\ \mu_2 &:= \gamma_0 X_0 - Z \end{aligned}$$

If $\mu_2 \mu_3 > 0$, tensor modes satisfy massless Lorentzian dispersion relation.

- Large parameter space in which all remaining modes satisfy Euclidean dispersion relations.
  - All but one have a large tachyonic mass, can be set to zero with appropriate BCs.
  - Last mode is approximately harmonic (with corrections suppressed by $1/M_{\mathrm{Pl}}^2$)

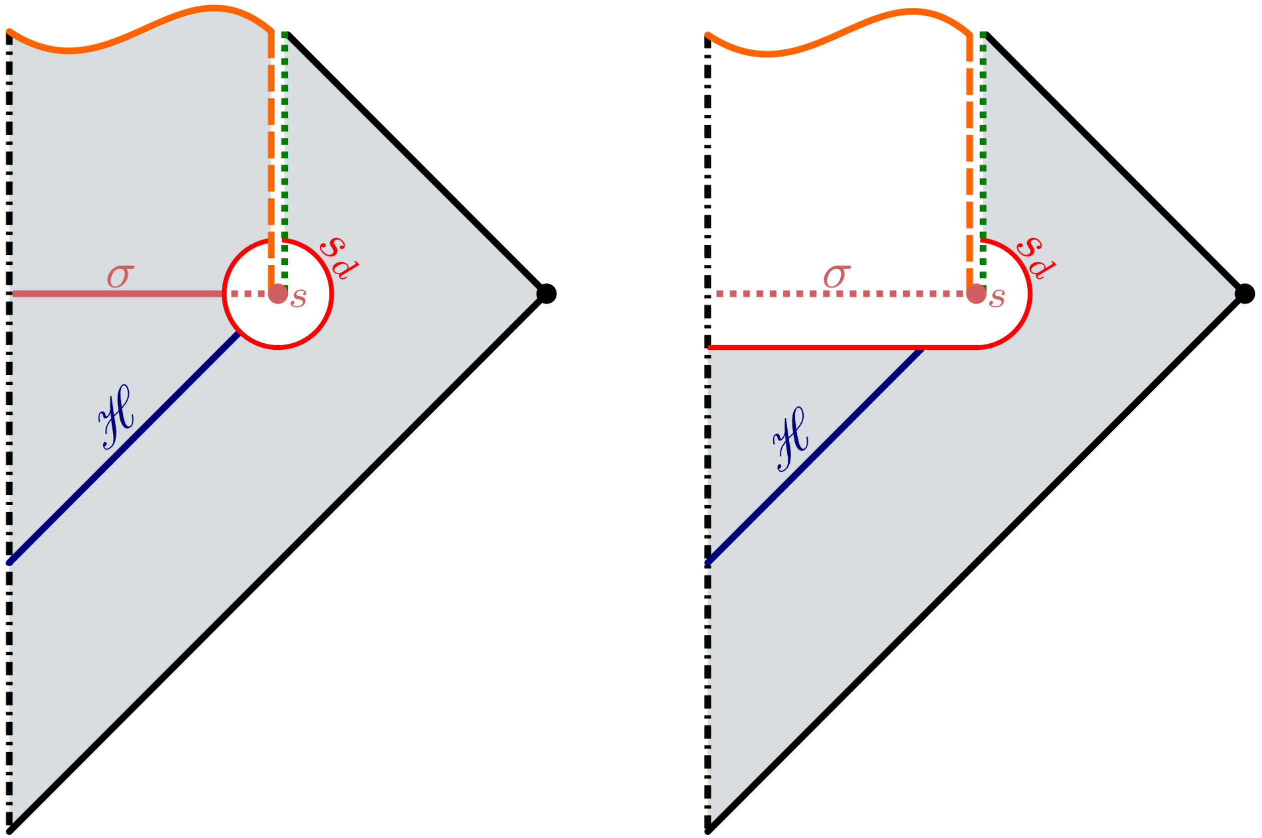
[1] JCF, S. Mukohyama, S. Carloni, arXiv:2505.00112v1

Issues/questions to think about

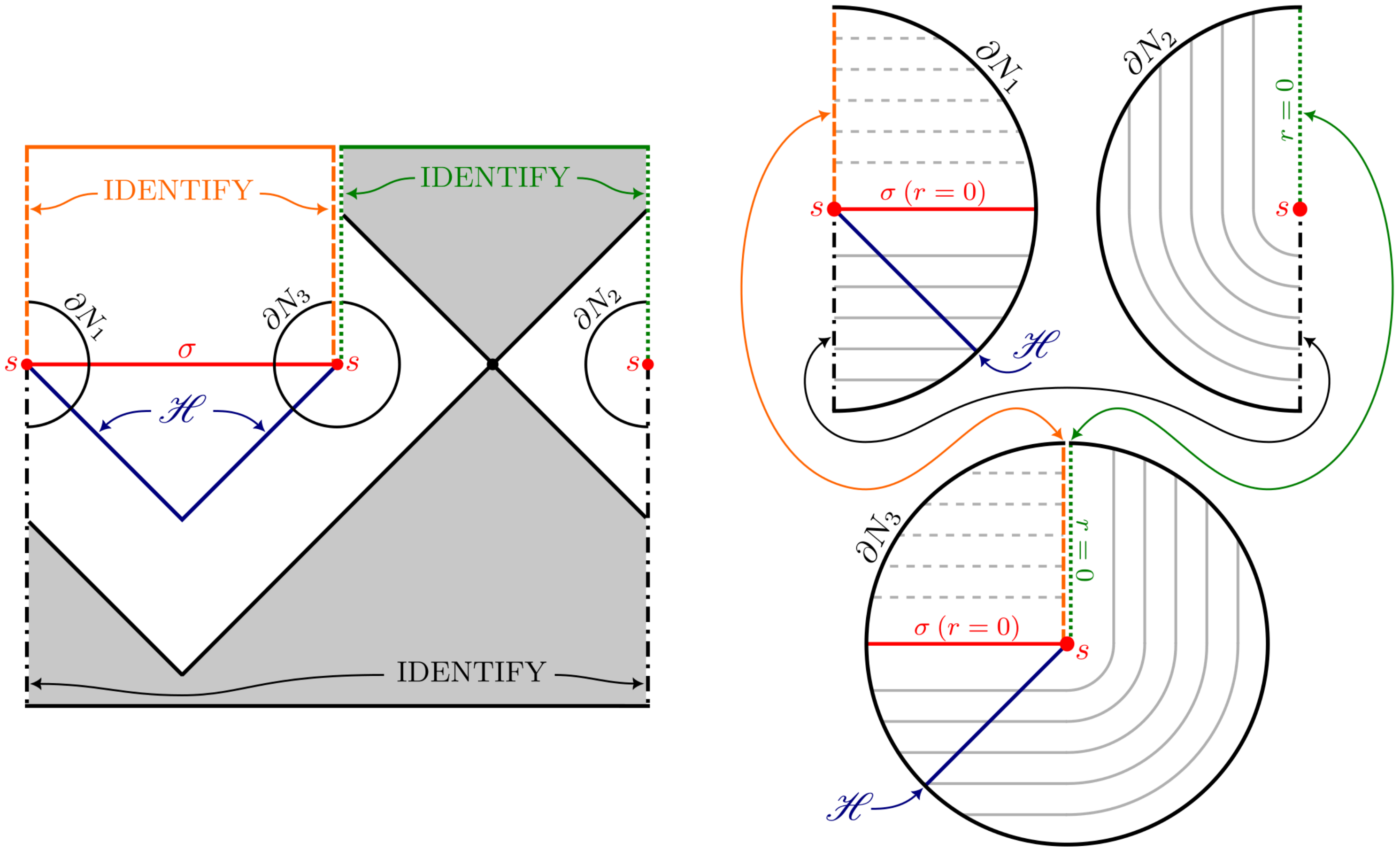
- Result compatible with "baby universe" resolution to BH information paradox, but:
    - microscopic/quantum theory needs work (how to define time evolution?)
    - unclear whether such a solution is realized at the end of BH evaporation
    - singularity regularized only in fundamental metric and scalar field; effective metric still singular.
- Emergence of Lorentzian dispersion relation:
    - only shown for a flat background
    - effective metric has sound speed independent of tensor modes; either need fine tuning or some other mechanism to recover Lorentz symmetry in IR[1]

[1] See S. Chadha, H. B. Nielsen, Nucl. Phys. B217 125 (1983)

Regularization possibilities



$d + 1$ spherically symmetric case



$2+1$ plane for $d+1$ evap'ing BH on left, neighborhood ∂N of s on right.

To understand $d+1$ case, treat areal radius r as a scalar function; contours of r in gray

ELST Field Equations

The field equations are as follows:

$$\nabla_a(J^a - \Phi^a) = 0$$

$$Q^R_{ab} + Q^\varphi_{ab} + g_{ab}Q = \tfrac{1}{2}T_{ab}$$

where Φ^a and T_{ab} are matter sources.

The quantities J^a, Q^R_{ab}, Q^φ_{ab}, and Q are defined to the right, where as before:

$$\varphi_a := \nabla_a\varphi \qquad \varphi_{ab} := \nabla_a\nabla_b\varphi.$$

$$J^a = (c_7 + c_8)\Box\varphi^a - (c_5 + c_7)R^{ab}\varphi_b - \varphi^a(c_{10} + c_4 R + 2c_6 X)$$

$$\begin{aligned}
Q^R_{ab} = &-2c_9 R_{ab} - 4c_2 R_{ac}R_b{}^c - 4c_1 R_{ab}R - 4c_3 R_a{}^{cde}R_{bcde} \\
&+ 4(c_2 + 2c_3)(R_{ac}R_b{}^c - R^{cd}R_{acbd}) \\
&+ 2(2c_1 + c_2 + 2c_3)\nabla_a\nabla_b R - 2(c_2 + 4c_3)\Box R_{ab}, \\
Q^\varphi_{ab} = &-2c_{10}\varphi_{ab} + 4(c_5 - c_8)\varphi_{ab}\Box\varphi + 4c_4\varphi_{cb}\varphi^c{}_a \\
&+ 4(c_7 + c_8)(\varphi_{(a}\Box\varphi_{b)}) + 2(2c_4 + c_5 - c_8)\varphi^c\nabla_c\varphi_{ab} \\
&- 2c_4(R\varphi_a\varphi_b + R_{ab}X - 2R_{acbd}\varphi^c\varphi^d) - 4c_6\varphi_a\varphi_b X \\
&- 4\varphi^c(c_5 + c_7)R_{c(a}\varphi_{b)}, \\
Q = &\; L - (c_5 + 2c_7)(\Box\varphi)^2 + (c_5 + 2c_7)R^{cd}\varphi_c\varphi_d \\
&- (4c_1 + c_2)\Box R - 2(2c_4 + c_5 + c_7)\varphi^c\Box\varphi_c \\
&- (4c_4 + c_5)\varphi_{cd}\varphi^{cd}.
\end{aligned}$$

Approximate solution in Riemann normal coordinates

Explicitly, the approximate solution near the origin has the form:

$$\varphi = (u^2 - v^2)/(2L_0) + \varphi_{\mu\nu} x^\mu x^\nu + O(x^{\cdot 3})$$

$$g_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{3}\left[R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta - \frac{1}{6}\left[\nabla_\gamma R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma$$
$$-\left[\frac{2}{45} R_{\mu\alpha\lambda\beta} R^\lambda{}_{\gamma\delta\nu} + \frac{1}{20}\nabla_\gamma \nabla_\delta R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma x^\delta + O(x^{\cdot 5})$$

with coefficients satisfying:

$$\Big[4\{(c_2 + 4c_3)\Box R_{\mu\nu} - (2c_1 + c_2 + 2c_3)\nabla_\mu \nabla_\nu R\} + 4c_{10}\varphi_{\mu\nu}$$
$$+ 8\{c_3 R_\mu{}^{\alpha\beta\gamma} R_{\nu\alpha\beta\gamma} - (c_2 + 2c_3)(R_{\mu\alpha} R_\nu{}^\alpha - R^{\alpha\beta} R_{\mu\alpha\nu\beta})\}$$
$$+ 8\{c_2 R_{\mu\alpha} R_\nu{}^\alpha - c_4 \varphi_{\alpha\mu} \varphi^\alpha{}_\nu\} + 4(c_9 + 2c_1 R) R_{\mu\nu}$$
$$+ 2g_{\mu\nu}\Big\{(4c_1 + c_2)\Box R - c_{11} - c_9 R - c_1 R^2 - c_2 R_{\alpha\beta} R^{\alpha\beta}$$
$$- c_3 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - (c_8 - 4c_4 - c_5)\varphi_{\alpha\beta}\varphi^{\alpha\beta}\Big\} + T_{\mu\nu}\Big]_0 = 0$$