Dust Stars in the Minimal Exponential Measure (MEMe) Model

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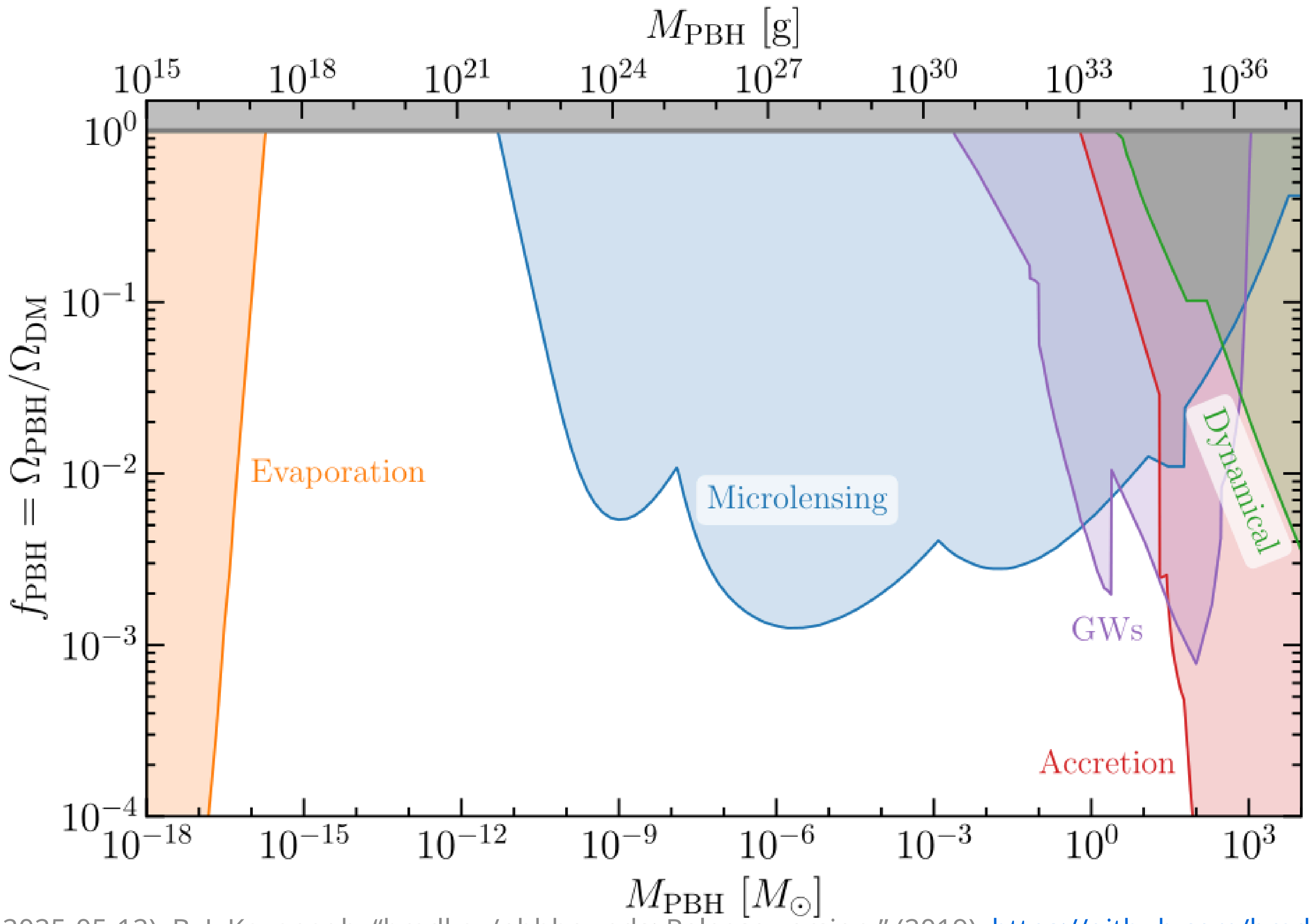


PlotPBHbounds (as of 2025-05-12), B. J. Kavanagh, “bradkav/pbhbounds: Release version,” (2019). https://github.com/bradkav/pbhbounds

The Minimal Exponential Measure (MEMe) model

- Aims: maintain equivalence principle, no new dynamical dofs

$$\mathfrak{g}_{\mu\nu} = e^{(4-A^{\mu}{}_{\mu})/2}\, A_{\mu}{}^{\alpha}\, A_{\nu}{}^{\beta}\, g_{\alpha\beta}, \qquad \mathfrak{g} = \det(\mathfrak{g}_{\cdot\cdot})$$

$$S[g_{\cdot\cdot}, A_{\cdot}{}^{\cdot}] = \frac{1}{2\kappa}\int d^4x \left[\sqrt{|g|}(R - 2\,\tilde{\Lambda}) - \frac{2\kappa}{q}\sqrt{|\mathfrak{g}|}\right] + S_{\mathrm{m}}[\varphi, \mathfrak{g}_{\cdot\cdot}]$$

- Field equations ($\kappa := 8\pi G/c^4$, $\tilde{\Lambda} := \Lambda + \frac{2\kappa}{q}$):

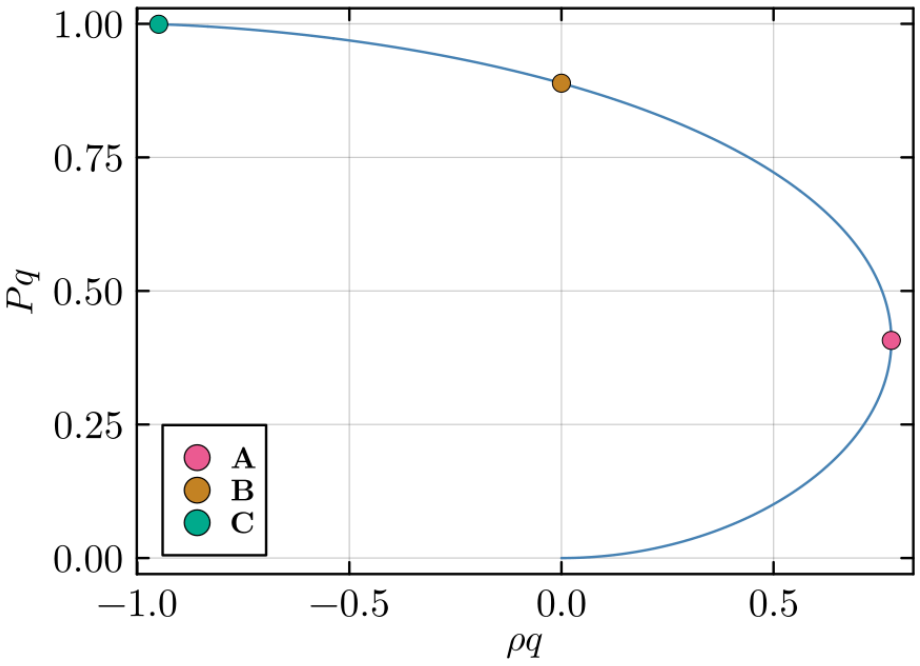
$$A_{\beta}{}^{\alpha} - \delta_{\beta}{}^{\alpha} = q\left[\tfrac{1}{4}\mathfrak{T}\, A_{\beta}{}^{\alpha} - \mathfrak{T}_{\beta\nu}\,\bar{\mathfrak{g}}^{\alpha\nu}\right]$$

$$G_{\mu\nu} + \left[\Lambda - \tfrac{\kappa}{q}(1 - |A|)\right] g_{\mu\nu} = \kappa\,|A|\,\bar{A}^{\alpha}{}_{\mu}\,\bar{A}^{\beta}{}_{\nu}\,\mathfrak{T}_{\alpha\beta}$$

- $A_{\beta}{}^{\alpha}$ nondynamical, coincides with GR in vacuum ($\mathfrak{T}_{\alpha\beta} = 0$) or vanishing q
- $1/q$ is characteristic energy density

[1]Generalized coupling theories: JCF and S Carloni, Phys. Rev. D 101, 064002 (2020)
[2]JCF, S Mukohyama, S Carloni, Phys.Rev.D 103 (2021) 8, 084055; Phys.Rev.D 105 (2022) 10, 104036

Perfect fluid



EoS for $\hat{p} = 0$, $q > 0$.

$\mathbf{A}$: $\hat{\rho} = 2/q$ $\mathbf{B}$: $\hat{\rho} = 8/q$ $\mathbf{C}$: $\hat{\rho} \to +\infty$

- Can get ${A_\mu}^\alpha$ for: $\mathfrak{T}_{\mu\nu} = (\hat{\rho} + \hat{p})u_\mu u_\nu + \hat{p}\,\mathfrak{g}_{\mu\nu}$

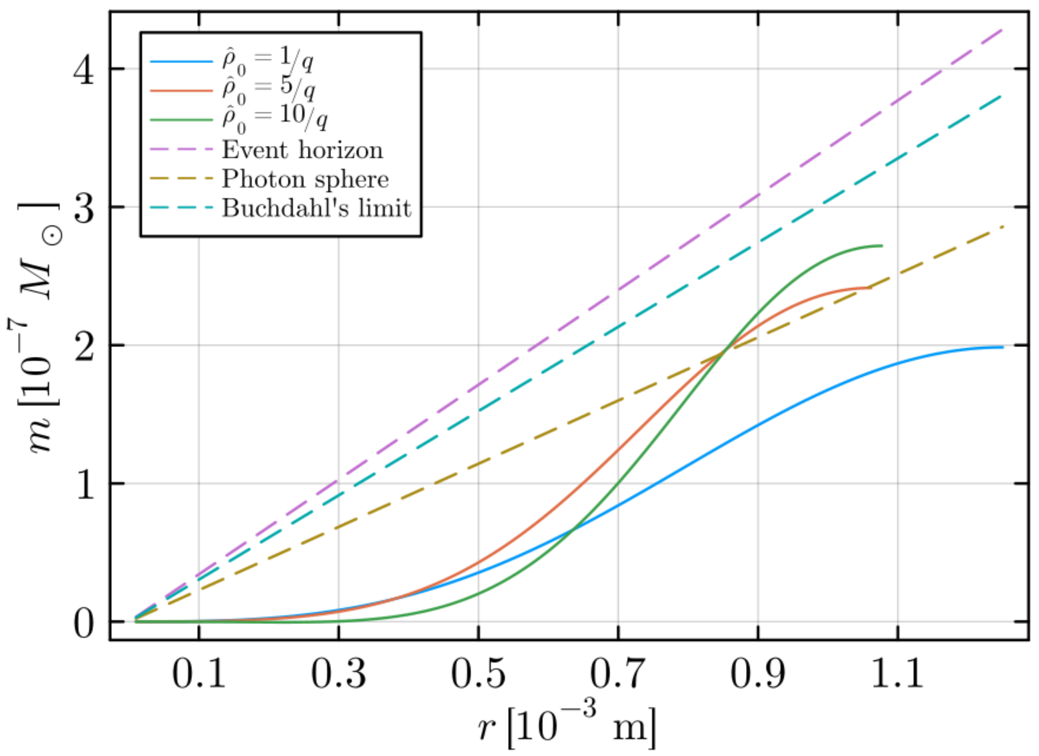
  $G_{\mu\nu} = \kappa\left[(\rho + p)U_\mu U_\nu + p\,g_{\mu\nu}\right]$, $|A| \propto (1 - \hat{p}\,q)^3$

- Effective p, ρ, modified EoS

$$p = \frac{|A|\,(q\,\hat{p} - 1) + 1}{q} - \frac{\Lambda}{\kappa}, \qquad \rho = |A|\,(\hat{p} + \hat{\rho}) - p$$

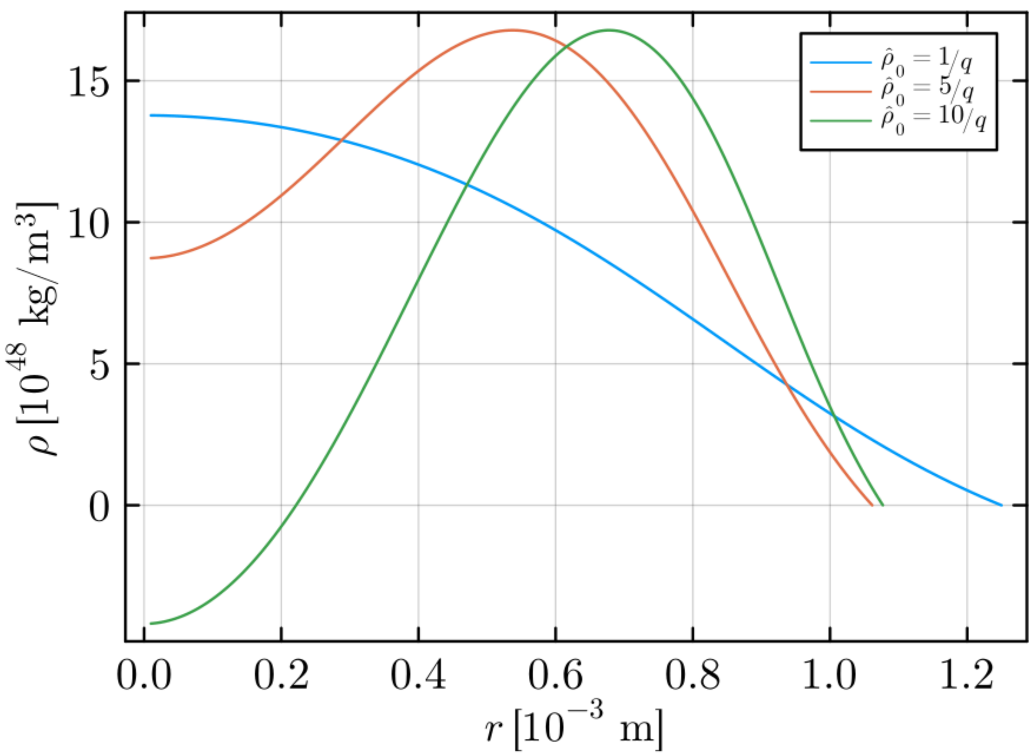
- Dust acquires pressure
- **EoS has maximum $\rho_{\max} = 7/(9q)$ for $q > 0$**
- For $q > 0$, can form compact objects from dust!
  - For $q < 0$, find no dust stars for TOV

Dust star TOV solutions for $q > 0$



Mass function

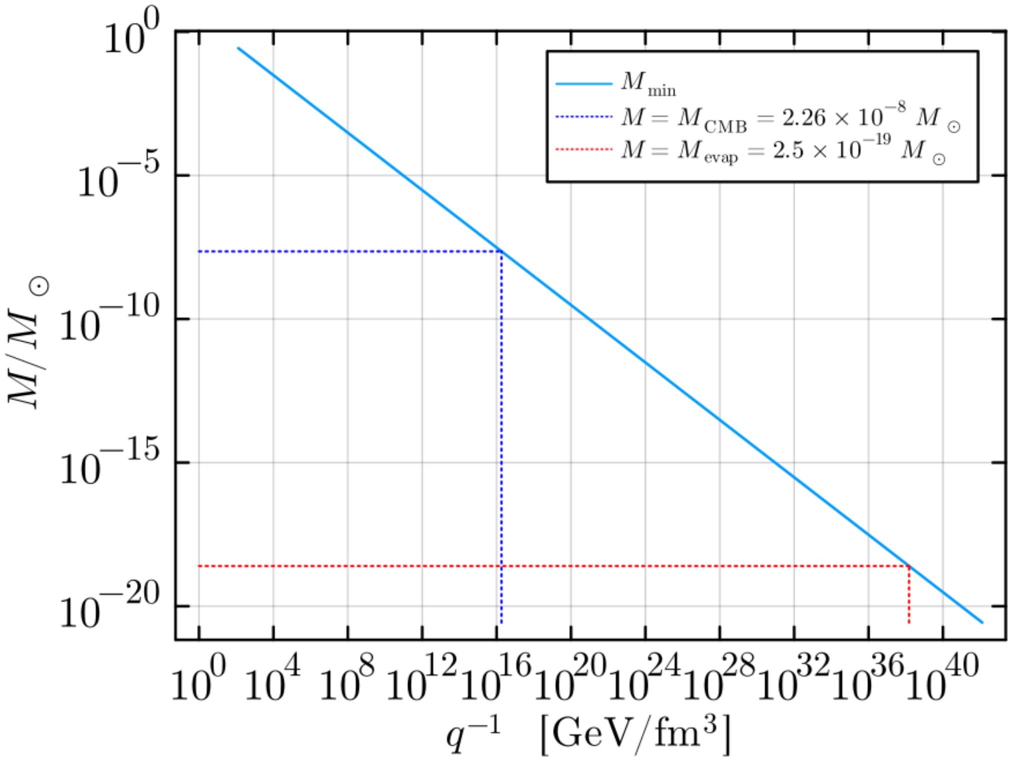
$[ds^2 = -e^{2\phi(r)}dt^2 + dr^2/\left(1 - 2m(r)/r\right) + r^2 d\Omega^2]$



Density profile

Nonmonotonic for central $\hat{\rho}_0 > 2/q$

Suppression of low mass BH formation



Minimum mass for BHs formed from collapse

- Recall EoS for $q > 0$ has max $\rho_{\rm max} = 7/(9q)$
- From Buchdahl limit,* $R > \frac{9}{4}M$, get collapse threshold density:

$$\rho_{\rm c} = \frac{16}{243\pi M^2}$$

Avg. densitites $\rho > \rho_{\rm c}$ collapse to BHs.

- Maximum density in EoS places lower limit on mass that can be formed from collapse:

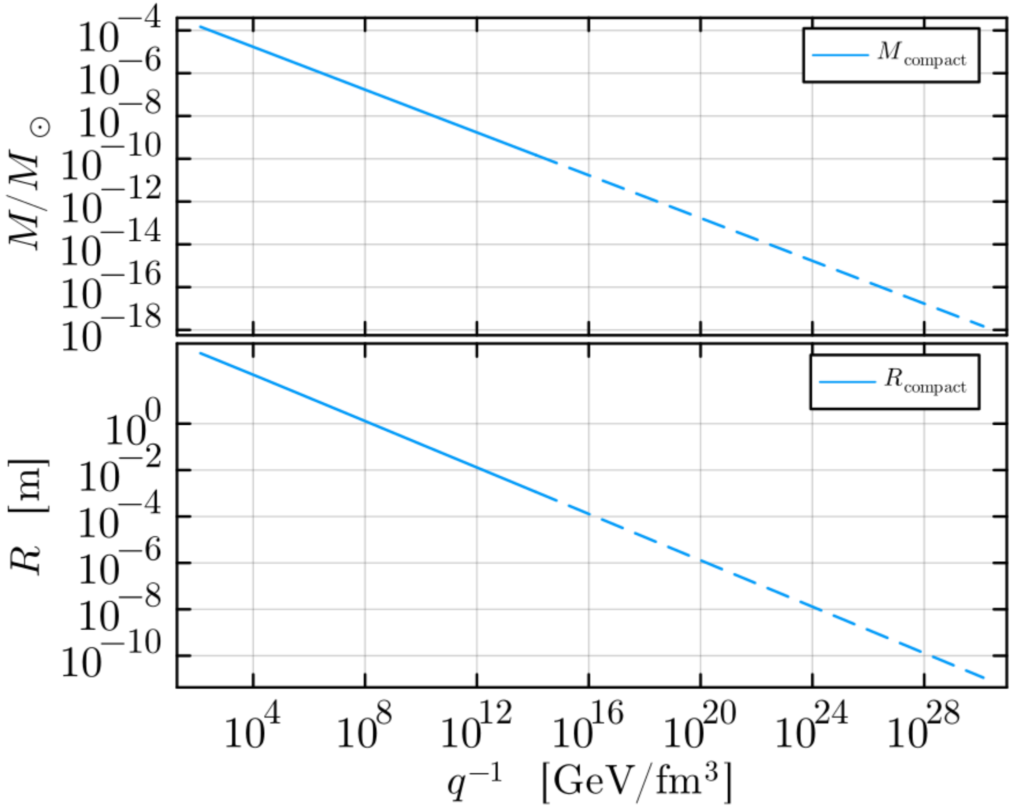
$$M \gtrsim M_{\rm min} = \frac{4}{9}\sqrt{\frac{3q}{7\pi}}$$

*H. A. Buchdahl, Phys. Rev. 116, 1027 (1959)

Summary

- MEMe model is simple modification of couplings between metric and matter DoFs
  - Equivalent to GR in vacuum
  - For a perfect fluid: GR with a modified equation of state
- For $q > 0$, can find compact dust star solutions
  - Can have solutions slightly larger than photon radius, potential BH mimickers
- Equation of state for $q > 0$ has a maximum density $\rho_{\max} = 7/(9q)$
  - Maximum density places lower limit on mass that can be formed from collapse

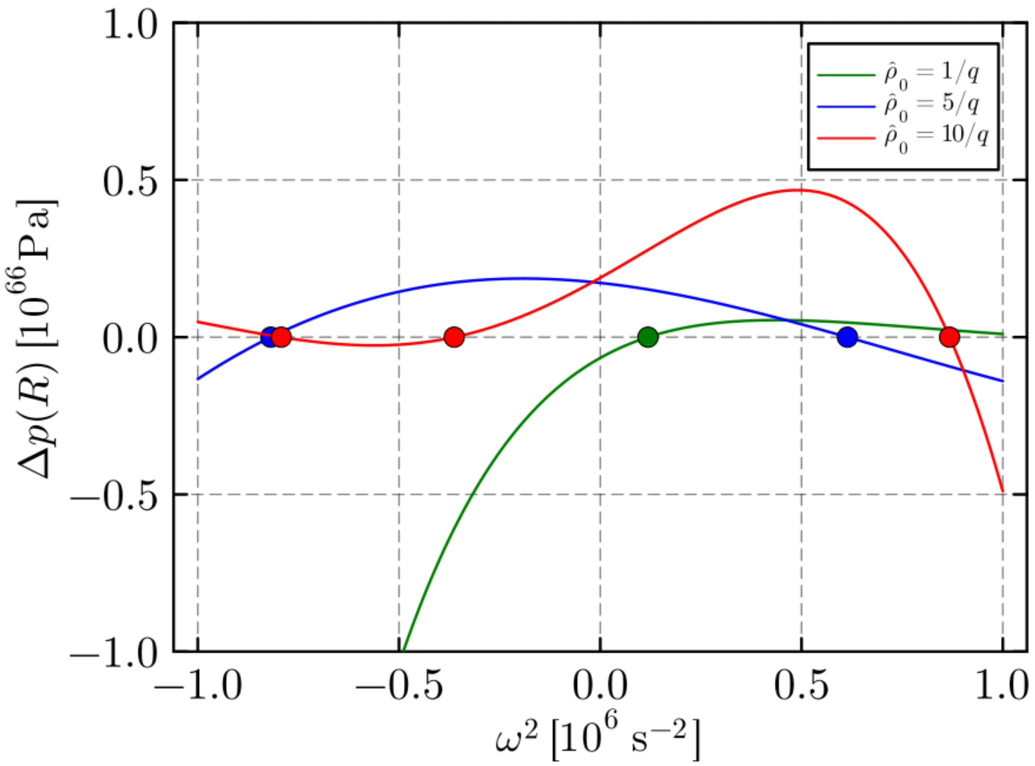
Dust star properties



Scaling behavior for mass and radius of dust stars with a central density of $\hat{\rho}_0 = 1/q$

- For $\hat{\rho}_0 > 2/q$, density peaks at some $r > 0$
  - Expect a Rayleigh–Taylor instability
  - Radially unstable for $\hat{\rho}_0 > 2/q$
- For $\hat{\rho} q \ll 1$, dust EoS: $p \approx 3q\rho^2/8$
- For small $\hat{\rho}_0 \ll 1/q$, radius tied to scale $R \approx \pi\sqrt{3q/2\kappa}$ (since $p \approx 3q\rho^2/8$,)
  - Scaling holds for even for $\hat{\rho}_0 \sim 1/q$
  - Dust star size strongly tied to scale
- Dust stars can be compact; if formed from DM, can mimic low mass black holes.

Radial stability analysis



Lagrangian perturbation of pressure at surface

- We follow procedure of Chandrasekhar and Chanmugam, with radial pert. Eqs in the comoving gauge

$$\frac{d\zeta}{dr} = -\left(\frac{3}{r} + \frac{dp}{dr}\frac{1}{p+\rho}\right)\zeta - \frac{1}{r}\frac{\Delta p}{\Gamma p},$$

$$\begin{aligned}\frac{d(\Delta p)}{dr} &= \zeta\left\{\frac{(p+\rho)r\omega^2}{e^{2\phi}(1-2m/r)} - 4\frac{dp}{dr}\right\} \\ &+ \zeta\left\{\left(\frac{dp}{dr}\right)^2\frac{r}{(p+\rho)} - \frac{8\pi(p+\rho)pr}{(1-2m/r)}\right\} \\ &+ \Delta p\left\{\frac{dp}{dr}\frac{1}{p+\rho} - \frac{4\pi(p+\rho)r}{(1-2m/r)}\right\}.\end{aligned}$$

- Assuming regularity at $r = 0$, we solve perturbation equations for trial freqs. ω
- Solns. that satisfy $\Delta p = 0$ when $p = 0$ are eigenmodes.

S. Chandrasekhar, Phys. Rev. Lett. 12, 114 (1964); Ap. J. 140, 417 (1964), [Err: Ap. J. 140, 1342 (1964)]. G. Chanmugam, Ap. J. 217, 799 (1977).

Perfect fluids, high densities

- Perfect fluid: $\mathfrak{T}_{\mu\nu} = (\hat{\rho} + \hat{p}) u_\mu u_\nu + \hat{p}\, \mathfrak{g}_{\mu\nu}$

  can solve for ${A_\mu}^\alpha$:

$$ {A_\mu}^\alpha = Y\, {\delta_\mu}^\alpha - 4(1 - Y)\, U_\mu\, U^\alpha $$

$$ Y := \frac{4(1 - \hat{p}\, q)}{4 - q\,(3\,\hat{p} - \hat{\rho})} $$

- Determinant $|A|$

$$ |A| = Y^3(4 - 3Y) = \frac{256\,(1 - \hat{p}\, q)^3 (q\,\hat{\rho} + 1)}{[4 - q\,(3\hat{p} - \hat{\rho})]^4} $$

- If $q < 0$ then there is a crit. density $q\hat{\rho} = -1$,
  If $q > 0$ then there is a crit. pressure $\hat{p}q = 1$,

$$ G_{\mu\nu} \approx (\kappa/q) g_{\mu\nu}, \qquad |\kappa/q| \gg |\Lambda| $$

- Can rewrite gravitational eqn:

$$ G_{\mu\nu} = \kappa\,[(\rho + p) U_\mu\, U_\nu + p\, g_{\mu\nu}] $$

  Effective fluid with pressure and density:

$$ p = \frac{|A|\,(q\,\hat{p} - 1) + 1}{q} - \frac{\Lambda}{\kappa} $$

$$ \rho = |A|\,(\hat{p} + \hat{\rho}) - p $$

- Tolman-Oppenheimer-Volkoff (TOV) equation:

$$ p'(r) = -[p(r) + \rho(r)] \frac{m(r) + 4\pi r^3 p(r)}{[r - 2m(r)] r} $$

$$ m'(r) = 4\pi \rho(r) r^2 $$