

Singularity at the demise of a black hole

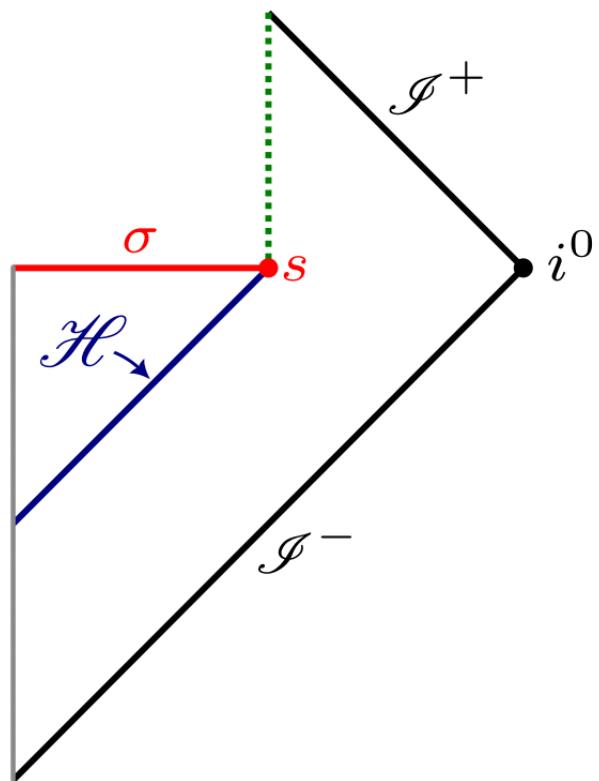
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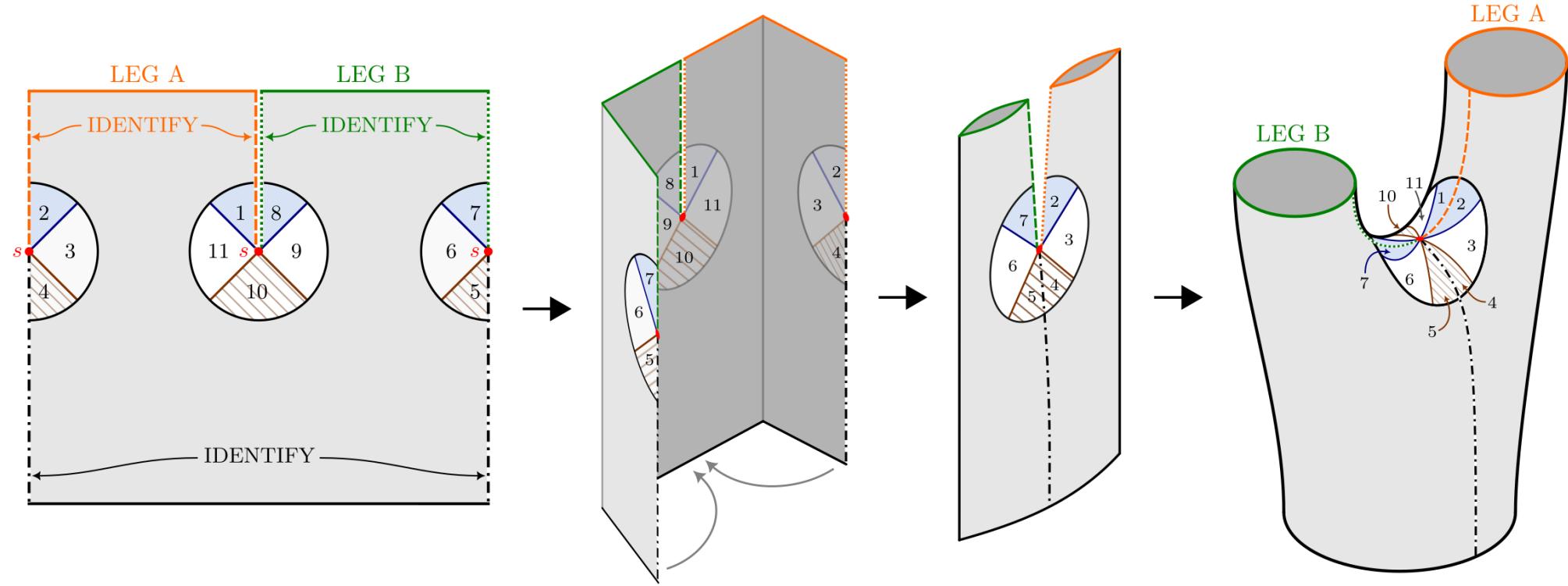
End state of a Black Hole



- What is end state of evaporating black hole?
- Penrose diagram for spherically symmetric case provides an illustration
 - $r-t$ spacetime diagram preserving angles
 - 45° null, \mathcal{I}^\pm and i^0 represent ∞
- Not too difficult to regularize singularity[†] σ
- But what about s ?

[†]A Simpson, M Visser, JCAP 02 (2019) 042 [arxiv:1812.07114]

Trousers^{†*} spacetime



Normally, spacetime points have one future and one past light cone

s is a quasiregular singularity, characterized by 2 future and 2 past light cones

[†]A Anderson, B S DeWitt, Found.Phys. 16 (1986) 91-105

*F. Dowker, S. Surya, PRD 58, 124019 (1998) [arXiv:gr-qc/9711070]; Buck et al., Class.Quant.Grav. 34 (2017) 5, 055002 [arXiv:1609.03573]

Quasiregular singularities

They generalize conical singularities, construct via cut and paste methods.

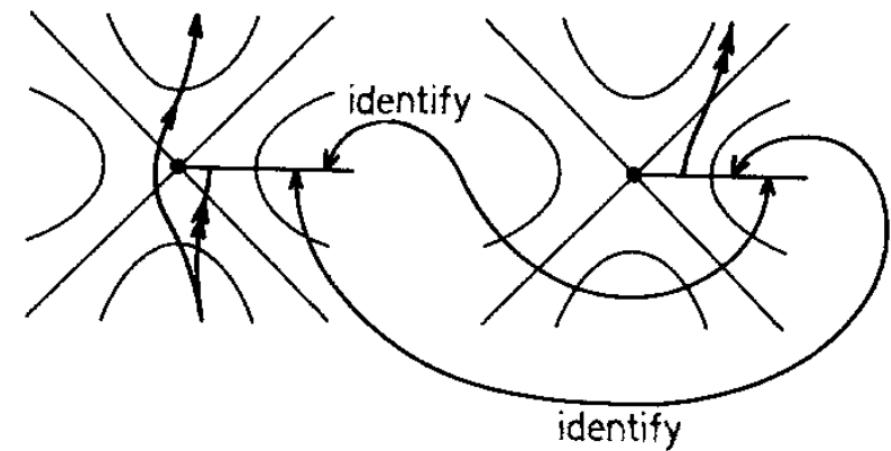
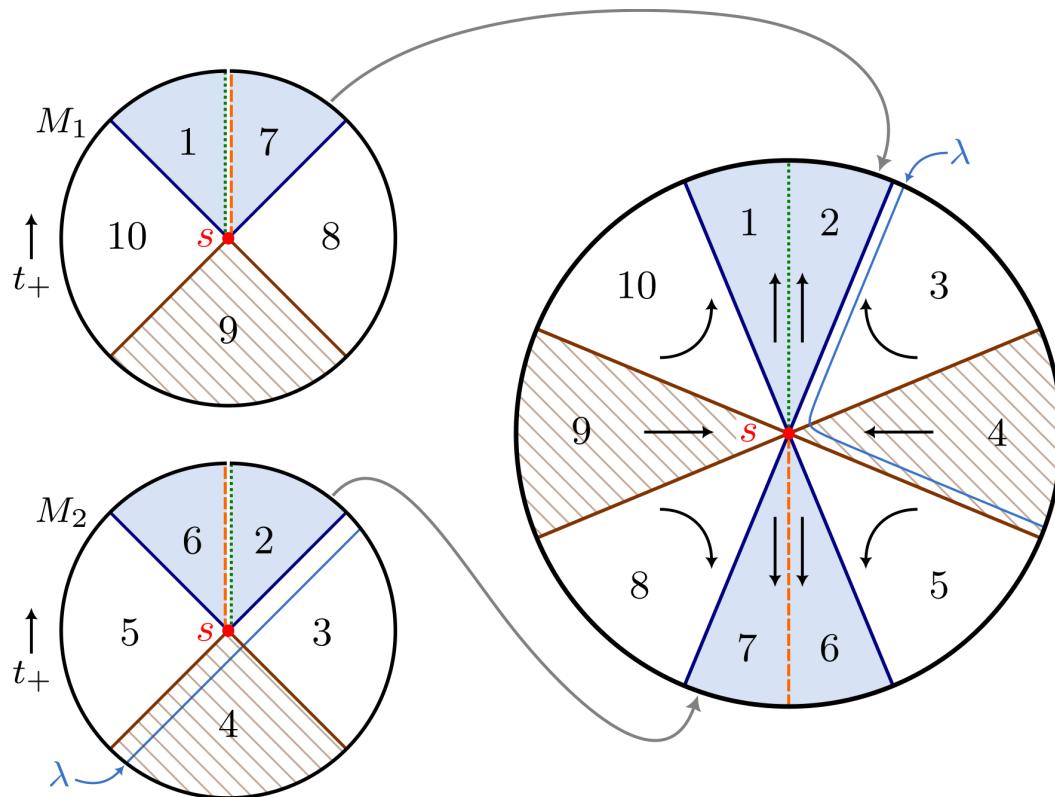
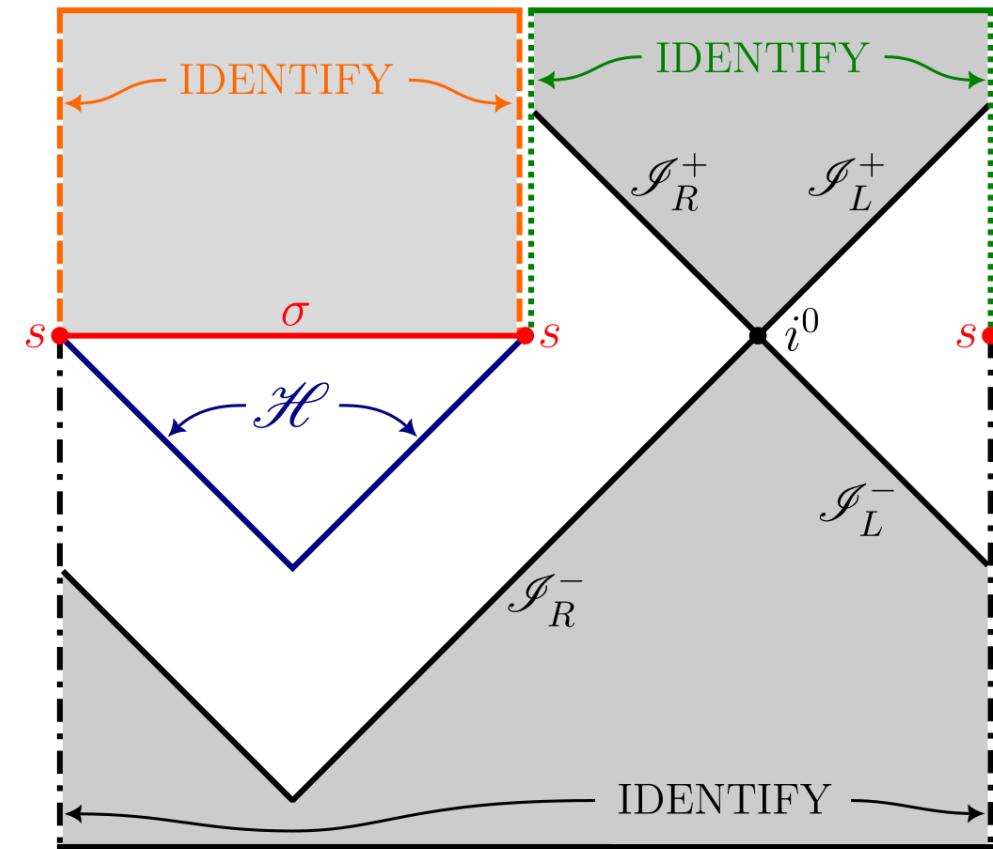
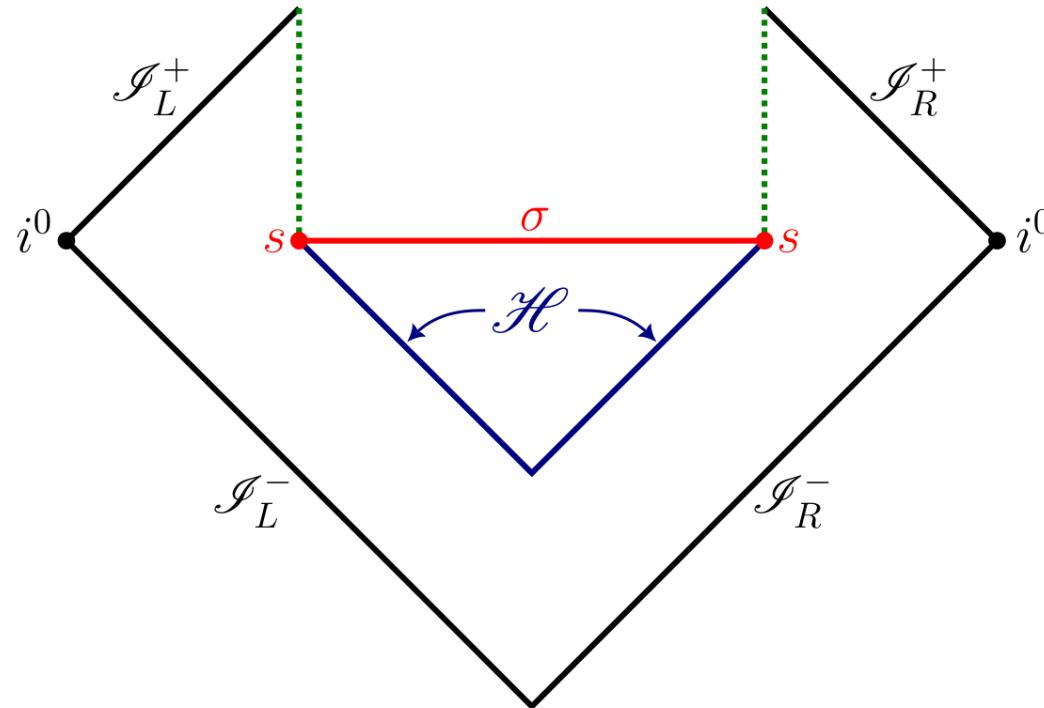


Fig 4(e) from Ellis & Schmidt[†]

[†]G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

$1 + 1$ Black hole and trousers[†] spacetime

Schwarzschild in $1 + 1$ is conformal to trousers: s is quasiregular* singularity!



[†]A Anderson, B S DeWitt, Found.Phys. 16 (1986) 91-105

* Provided that σ is regularized and spacetime analytically extended.

Emergent Lorentz signature theory

- There is one theory that can describe a regularization of quasiregular singularities.
- Postulate* Euclidean-signature $g_{\mu\nu}$ with scalar-tensor action bounded below:

$$S = \int_M d^4x \sqrt{|g|} L, \quad \varphi_a := \nabla_a \varphi, \quad \varphi_{ab} := \nabla_a \nabla_b \varphi, \quad X := \varphi^a \varphi_a$$

$$\begin{aligned} L = & c_1 R^2 + c_2 R_{ab} R^{ab} + c_3 R_{abcd} R^{abcd} + c_4 X R + c_5 R^{ab} \varphi_a \varphi_b \\ & + c_6 X^2 + c_7 (\square^2 \varphi)^2 + c_8 \varphi_{ab} \varphi^{ab} + c_9 R + c_{10} X + c_{11} \end{aligned}$$

- At long distances, matter is coupled to $\mathbf{g}_{\mu\nu}$:

$$\mathbf{g}_{ab} = g_{ab} - \varphi_a \varphi_b / X_C$$

- Can show S reduces to Lorentzian scalar-tensor theory in long-distance limit.*‡

*S. Mukohyama, Phys. Rev. D 87, 085030 (2013)

†S Mukohyama, J Uzan, Phys. Rev D. 87:065020 (2013)

Regularized quasiregular singularity in quadratic ELST

Consider a saddle-like scalar field profile and flat metric:

$$\varphi = (u^2 - v^2)/(2L_0)$$

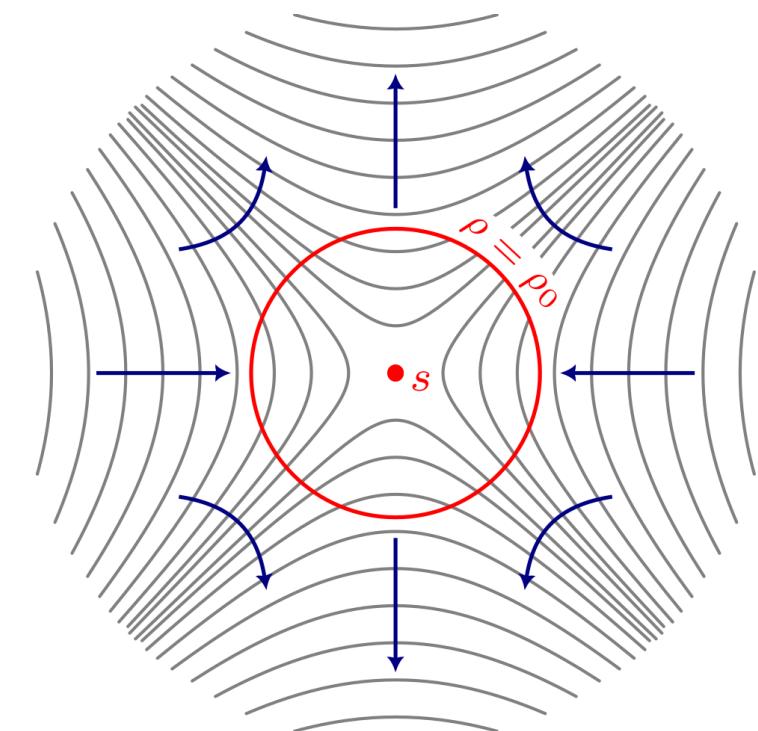
$$ds^2 = du^2 + dv^2 + dy^2 + dz^2$$

These form a soln. for the parameter choices
 $c_4 = c_6 = c_{10} = 0$ and $c_{11} = 8(c_5 - c_8)$.

Can get approximate soln. using Riemann normal coords:[†]

$$g_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{3} [R_{\mu\alpha\nu\beta}]_0 x^\alpha x^\beta - \frac{1}{6} [\nabla_\gamma R_{\mu\alpha\nu\beta}]_0 x^\alpha x^\beta x^\gamma$$
$$- \left[\frac{2}{45} R_{\mu\alpha\lambda\beta} R^\lambda{}_{\gamma\delta\nu} + \frac{1}{20} \nabla_\gamma \nabla_\delta R_{\mu\alpha\nu\beta} \right]_0 x^\alpha x^\beta x^\gamma x^\delta + O(x^5)$$

where $\textcolor{red}{s}$ is the origin.



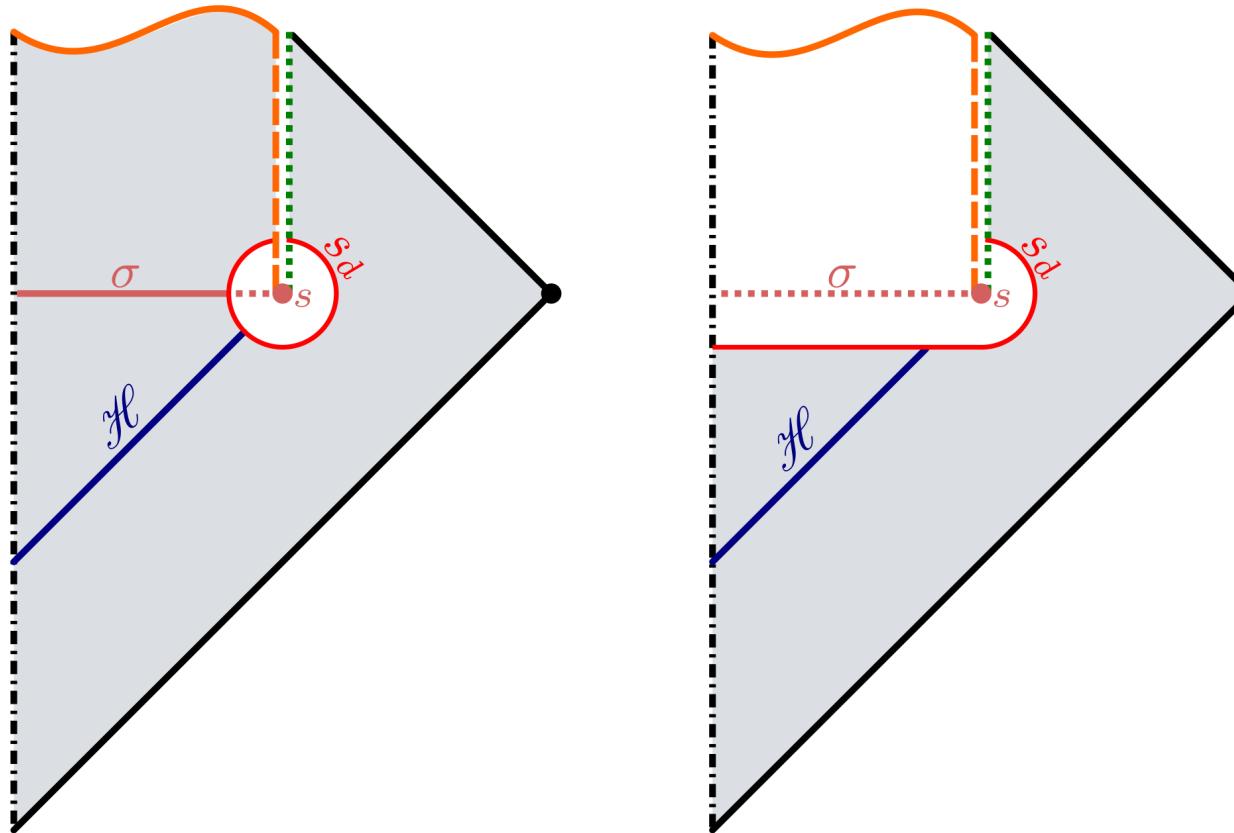
[†]U Muller, C Schubert, and A M E van de Ven, Gen. Rel. Grav. 31, 1759 (1999);

A Z Petrov, *Einstein Spaces*, Pergamon (1969); E Kreysig, *Intro. to Diff. Geom. and Riem. Geom.*, U Toronto Press (1968)

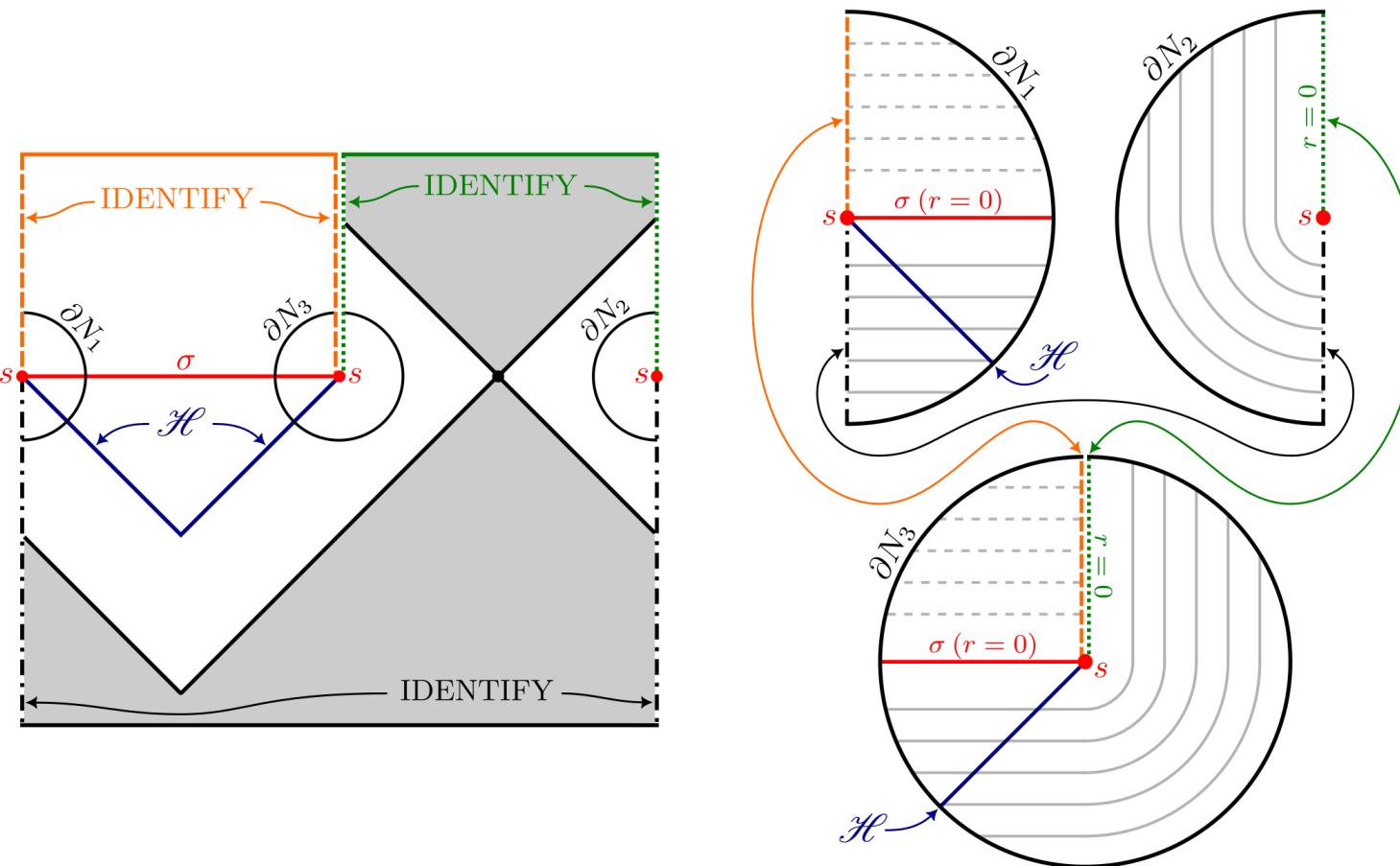
Issues/questions to think about

- Result compatible with a "baby universe" resolution to BH information paradox (but microscopic/quantum theory needs work)
- Singularity regularized only in fundamental metric and scalar field; effective metric still singular
- Analysis is very preliminary; a more comprehensive analysis is needed to determine whether the solutions are realized at the end of BH evaporation
- How might other theories handle quasiregular singularities?

Regularization possibilities



$d + 1$ spherically symmetric case



$2 + 1$ plane for $d + 1$ evap'ing BH on left, neighborhood ∂N of s on right.

To understand $d + 1$ case, treat areal radius r as a scalar function; contours of r in gray

ELST Field Equations

The field equations are as follows:

$$\nabla_a (J^a - \Phi^a) = 0$$

$$Q_{ab}^R + Q_{ab}^\varphi + g_{ab}Q = \frac{1}{2}T_{ab}$$

where Φ^a and T_{ab} are matter sources.

The quantities J^a , Q_{ab}^R , Q_{ab}^φ , and Q are defined to the right, where as before:

$$\varphi_a := \nabla_a \varphi \quad \varphi_{ab} := \nabla_a \nabla_b \varphi.$$

$$J^a = (c_7 + c_8)\square\varphi^a - (c_5 + c_7)R^{ab}\varphi_b - \varphi^a(c_{10} + c_4R + 2c_6X)$$

$$\begin{aligned} Q_{ab}^R = & -2c_9R_{ab} - 4c_2R_{ac}R_b{}^c - 4c_1R_{ab}R - 4c_3R_a{}^{cde}R_{bcde} \\ & + 4(c_2 + 2c_3)(R_{ac}R_b{}^c - R^{cd}R_{acbd}) \\ & + 2(2c_1 + c_2 + 2c_3)\nabla_a\nabla_bR - 2(c_2 + 4c_3)\square R_{ab}, \end{aligned}$$

$$\begin{aligned} Q_{ab}^\varphi = & -2c_{10}\varphi_{ab} + 4(c_5 - c_8)\varphi_{ab}\square\varphi + 4c_4\varphi_{cb}\varphi^c{}_a \\ & + 4(c_7 + c_8)(\varphi_{(a}\square\varphi_{b)}) + 2(2c_4 + c_5 - c_8)\varphi^c\nabla_c\varphi_{ab} \\ & - 2c_4(R\varphi_a\varphi_b + R_{ab}X - 2R_{acbd}\varphi^c\varphi^d) - 4c_6\varphi_a\varphi_bX \\ & - 4\varphi^c(c_5 + c_7)R_{c(a}\varphi_{b)}, \end{aligned}$$

$$\begin{aligned} Q = & L - (c_5 + 2c_7)(\square\varphi)^2 + (c_5 + 2c_7)R^{cd}\varphi_c\varphi_d \\ & - (4c_1 + c_2)\square R - 2(2c_4 + c_5 + c_7)\varphi^c\square\varphi_c \\ & - (4c_4 + c_5)\varphi_{cd}\varphi^{cd}. \end{aligned}$$

Approximate solution in Riemann normal coordinates

Explicitly, the approximate solution near the origin has the form:

$$\varphi = (u^2 - v^2)/(2L_0) + \varphi_{\mu\nu}x^\mu x^\nu + O(x^3)$$

$$g_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{3} [R_{\mu\alpha\nu\beta}]_0 x^\alpha x^\beta - \frac{1}{6} [\nabla_\gamma R_{\mu\alpha\nu\beta}]_0 x^\alpha x^\beta x^\gamma$$
$$- \left[\frac{2}{45} R_{\mu\alpha\lambda\beta} R^\lambda{}_{\gamma\delta\nu} + \frac{1}{20} \nabla_\gamma \nabla_\delta R_{\mu\alpha\nu\beta} \right]_0 x^\alpha x^\beta x^\gamma x^\delta + O(x^5)$$

with coefficients satisfying:

$$\begin{aligned} & \left[4\{(c_2 + 4c_3)\square R_{\mu\nu} - (2c_1 + c_2 + 2c_3)\nabla_\mu \nabla_\nu R\} + 4c_{10}\varphi_{\mu\nu} \right. \\ & + 8\{c_3 R_\mu{}^{\alpha\beta\gamma} R_{\nu\alpha\beta\gamma} - (c_2 + 2c_3)(R_{\mu\alpha} R_\nu{}^\alpha - R^{\alpha\beta} R_{\mu\alpha\nu\beta})\} \\ & + 8\{c_2 R_{\mu\alpha} R_\nu{}^\alpha - c_4 \varphi_{\alpha\mu} \varphi^\alpha{}_\nu\} + 4(c_9 + 2c_1 R) R_{\mu\nu} \\ & + 2g_{\mu\nu} \left\{ (4c_1 + c_2)\square R - c_{11} - c_9 R - c_1 R^2 - c_2 R_{\alpha\beta} R^{\alpha\beta} \right. \\ & \left. - c_3 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - (c_8 - 4c_4 - c_5) \varphi_{\alpha\beta} \varphi^{\alpha\beta} \right\} + T_{\mu\nu} \Big]_0 = 0 \end{aligned}$$