Teleporters, time machines, and quasiregular singularities

Or, why care about quasiregular singularities?

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Motivation

- In this talk, I aim to:

    i. Describe some causally nontrivial spacetimes constructed from cut-and-paste procedures

    ii. Describe the quasiregular singularities present in these spacetimes

    iii. Present a possible microscopic description for said singularities

- Partly based on:

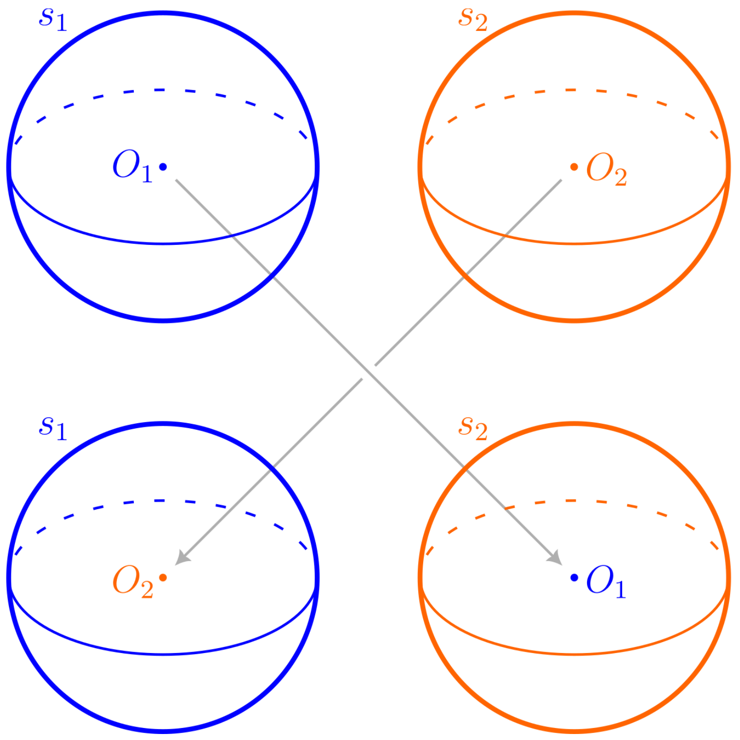
    JCF, S Mukohyama, S Carloni, **Singularity at the demise of a black hole**

    Phys. Rev. D 109, 024040 (2024) [arXiv:2310.17266]

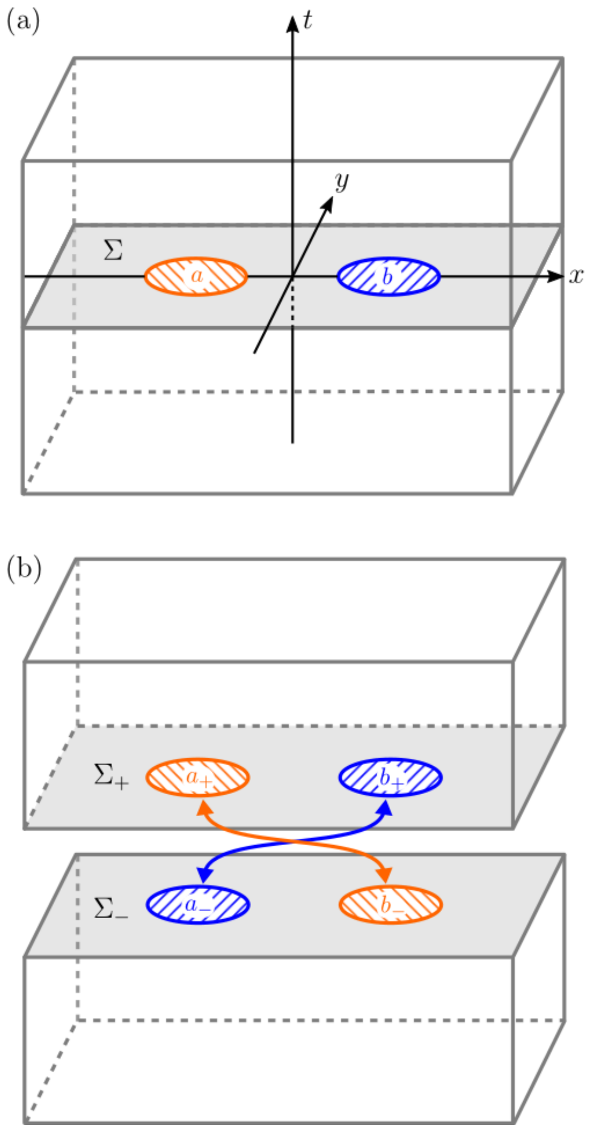
A thought experiment: "Teleporting space"

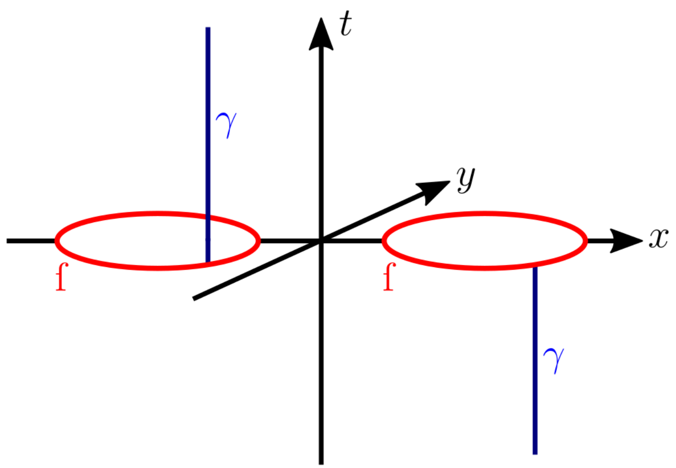
Consider pair of 2-spheres s_1 and s_2 in flat space

- Suppose BCs on s_1 and s_2 and areas are identical
- Consider observer O_1 enclosed within s_1 and O_2 enclosed within s_2
- If observers have no memories, they have no way of knowing which sphere encloses them
- If one can maintain identical BCs and areas (up to quantum uncerts.) on s_1 and s_2, could the interiors swap so that the respective enclosed observers O_1 and O_2 switch places?
- How can one describe such a process in spacetime?



Teleporter structures in spacetime (2+1 depiction)

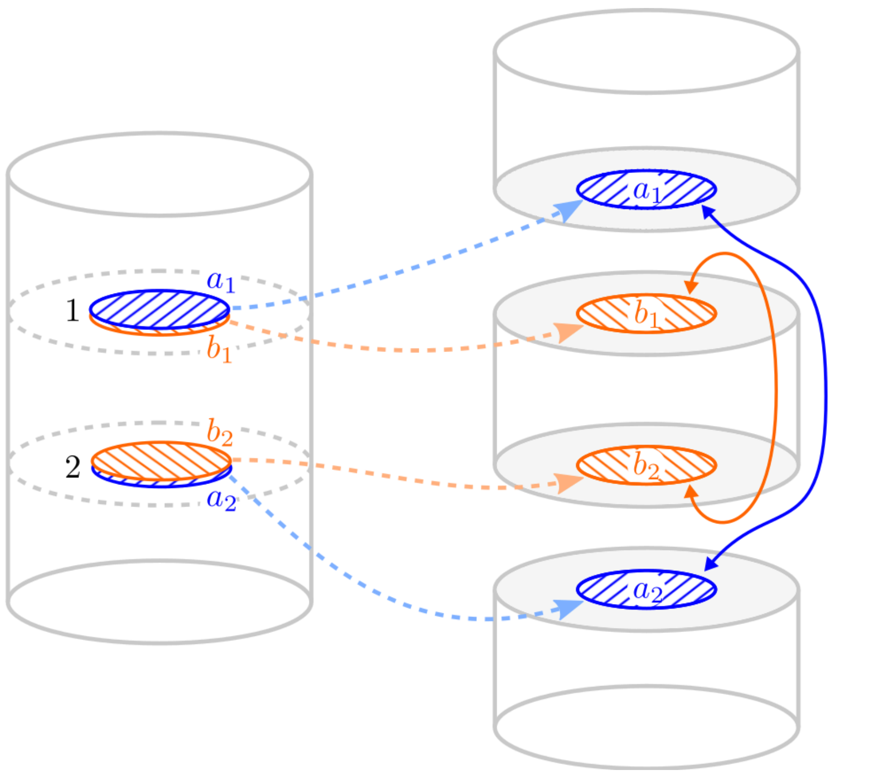




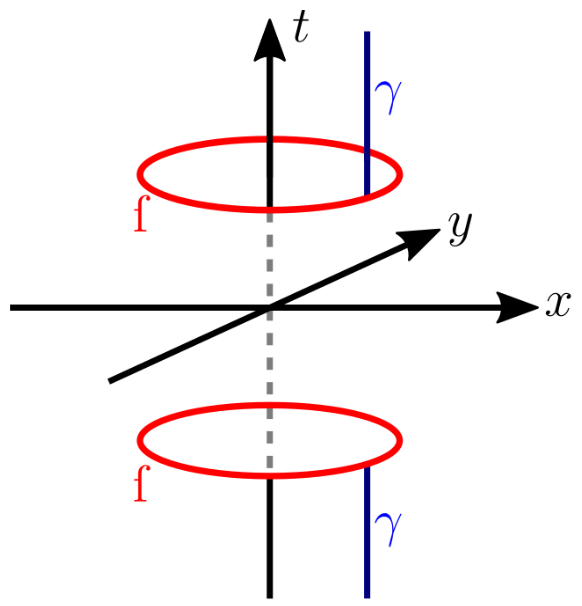
- Obtain a spacetime description of the aforementioned scenario by way of cut-and paste procedure
- Result illustrated above
  - Curve γ begins in $x > 0$ region when $t < 0$, continues in $x < 0$ region when $t > 0$
  - 2-surface $\mathfrak{f}$ is singular, characterized by *two* future and *two* past light cones (points in nbhd only have one future/past light cone)
- Can be regarded as a timehole for some definition of "one-way"

† JCF, S Mukohyama, S Carloni, PRD 109, 024040 (2024) [arXiv:2310.17266]; JCF, JPS Lemos, RA Matzner, PRD 103, 124037 (2021) [arXiv:2105.03433]

Time machines: the Deutsch-Politzer† spacetime



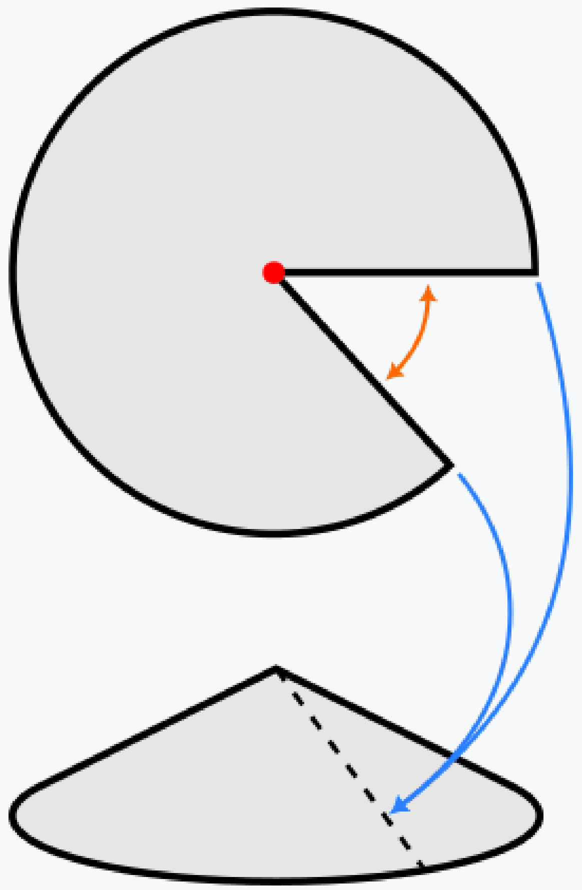
(Time direction is vertical in 2+1 illustration above)



- Deutsch-Politzer (DP) and Teleporter spacetimes are homeomorphic, ignoring causal structure‡
- Difference in causal structure: DP spacetime has closed timelike curves, (single) Teleporter does not.

†D. Deutsch, Phys. Rev. D 44, 3197–3217 (1991); H. D. Politzer, Phys. Rev. D 46, 4470–4476 (1992) [arXiv:hep-th/9207076]
‡Topology of DP discussed in Chamblin et al., PRD 50, R2353 (1994) [arXiv:gr-qc/9405001]; U Yurtsever, GRG 27, 691 (1995) [arXiv:gr-qc/9409040]

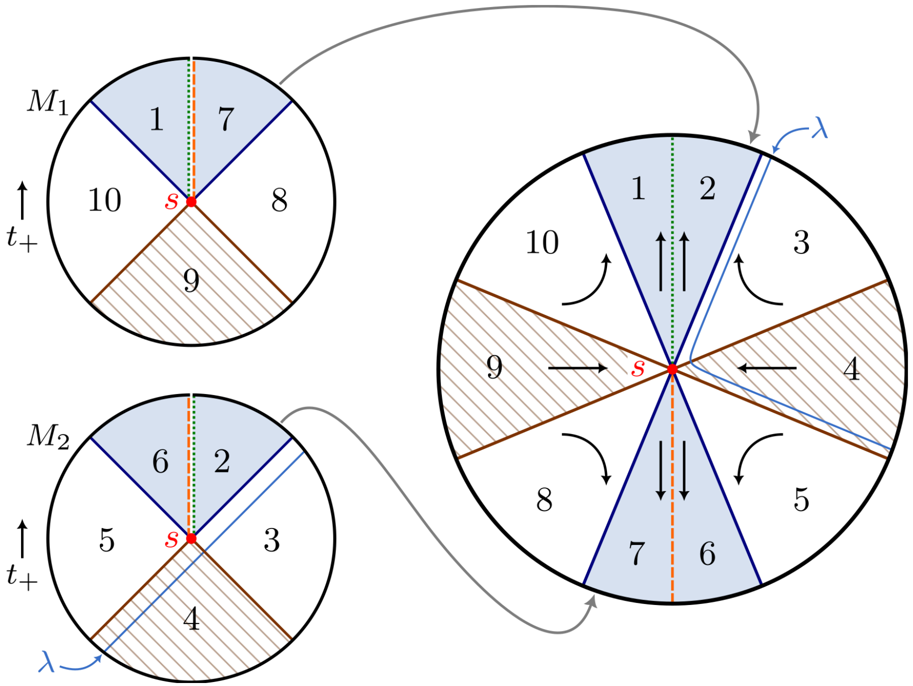
Quasiregular singularities



- Both Teleporters and DP spacetimes contain quasiregular singularities
- What's a quasiregular singularity?
  - Here, singularities defined as (boundary) points on which inextendible geodesics terminate
  - Curvature singularities identified by diverging curvature in parallel frame along geodesic
  - Quasiregular singularity[†] has well-behaved curvature (can even be zero) in its neigborhood
    - Can easily construct with cut-and-paste procedures
    - Conical singularity is an example

[†]G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

Saddlelike causally discontinuous singularity (SCDS)



Left: Two regions of $1+1$ Minkowski spacetime

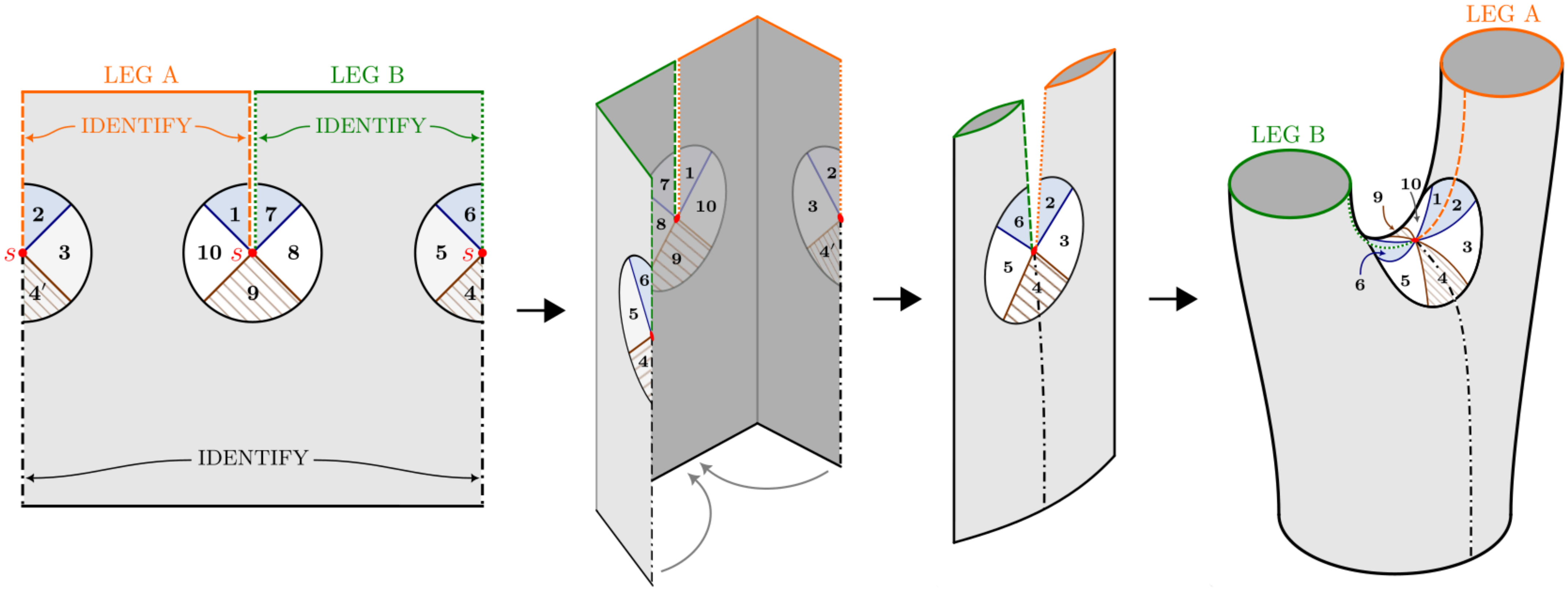
Right: Nonconformal illustration of result after gluing
Arrows indicate direction of increasing time

- Singularities ʃ in Deutsch-Politzer and Teleporter spacetimes characterized by *two* future and *two* past light cones
  - In Minkowski spacetime, each point has one future light cone and one past light cone
- Can construct an example by cut-and-paste procedure in $1+1$ flat spacetime
  - Direction of time near s resembles saddle point
  - Point s (SCDS) has *two* future and *two* past light cones
- There are further generalizations with more light cones--see Ellis and Schmidt[†]

cf. Fig 4(e) of Ellis and B G Schmidt[†]

[†] G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

A familiar example: $1+1$ Trousers[†*] spacetime

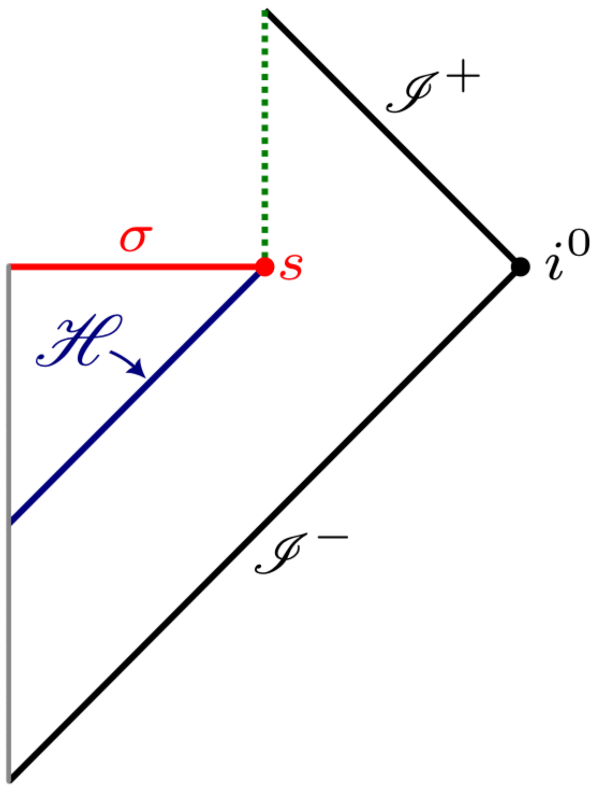


$\color{red}{s}$ is a SCDS, characterized by 2 future and 2 past light cones.

[†] A Anderson, B S DeWitt, Found.Phys. 16 (1986) 91-105

[*] F. Dowker, S. Surya, PRD 58, 124019 (1998) [arXiv:gr-qc/9711070]; Buck et al., Class.Quant.Grav. 34 (2017) 5, 055002 [arXiv:1609.03573]

End state of a Black Hole

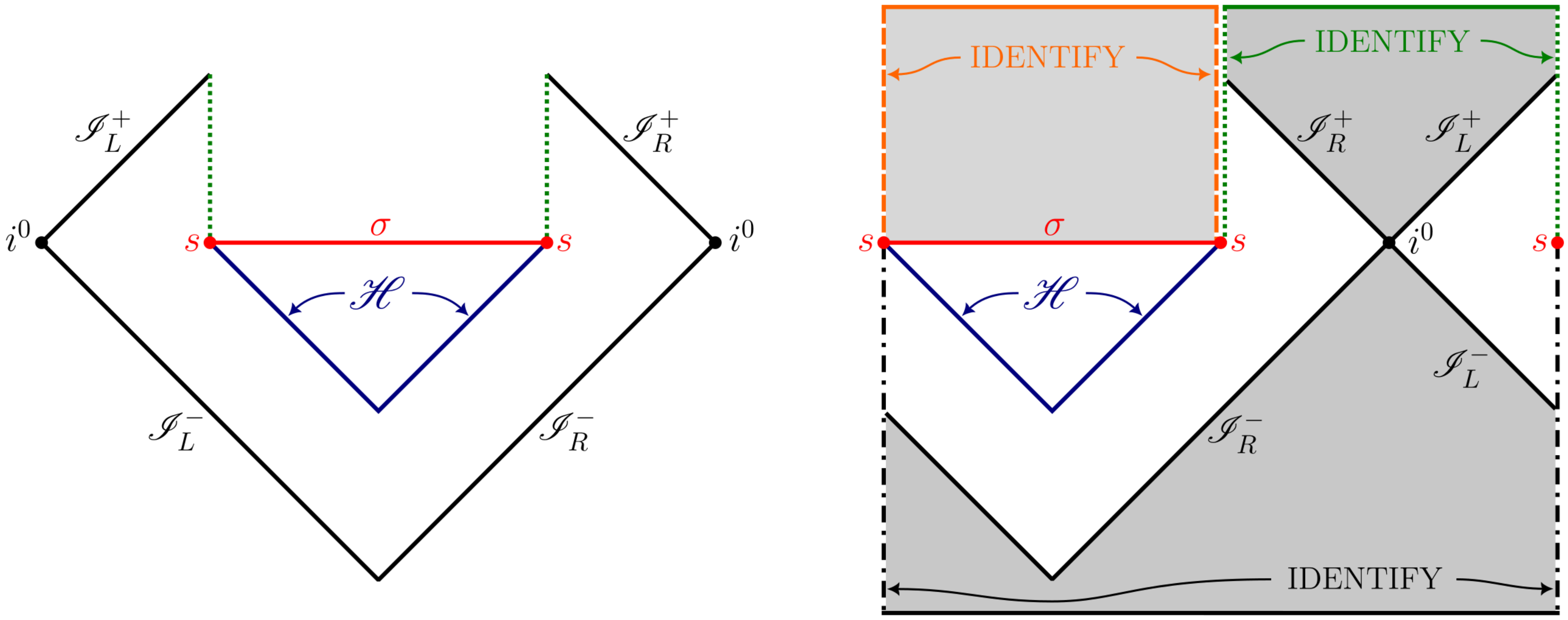


- What is the end state of an evaporating black hole?
  - Remnants, naked singularities, white holes, etc.[†]
- One possibility illustrated on left (horizon disappears)
- Can imagine regularizing singularity σ ("baby universes" for instance[†‡]), but what happens to s?
  - Claim: If one regularizes σ in a "baby universe" scenario (timelike curves continue through σ), then s must be a quasiregular singularity or some generalization of one

[†] S Hossenfelder, L Smolin, Phys.Rev.D 81 (2010) 064009; P Martin-Dussaud, C Rovelli, Class. Quantum Grav. 36, 245002 (2019)
[‡] A Simpson, M Visser, JCAP 02 (2019) 042 [arxiv:1812.07114]

Evaporating $1+1$ Black hole and trousers spacetime

Evaporating BH in $1+1$ is conformal to trousers: s is quasiregular[*] singularity!



Planar slice of $d+1$ BH through origin can be regarded similarly.

[†]JCF, S Mukohyama, S Carloni, Phys. Rev. D 109, 024040 (2024) [arXiv:2310.17266]

[*]Provided that σ is regularized and spacetime analytically extended. If future topology of $\mathscr{H}$ differs, this is still true but s may not be SCDS.

Emergent Lorentz signature theory

- There is one theory that can describe a regularization of a SCDS.
- Postulate Euclidean-signature g_{ab} with shift-symmetric scalar-tensor action:[†]

$$S = \int_M d^4x\sqrt{|g|}L, \quad \varphi_a := \nabla_a\varphi, \quad \varphi_{ab} := \nabla_a\nabla_b\varphi, \quad X := \varphi^a\varphi_a$$

$$L = c_1R^2 + c_2R_{ab}R^{ab} + c_3R_{abcd}R^{abcd} + c_4XR + c_5R^{ab}\varphi_a\varphi_b \\ + c_6\mathrm{X}^2 + c_7(\Box\varphi)^2 + c_8\varphi_{ab}\varphi^{ab} + c_9R + c_{10}X + c_{11}$$

- At long distance scales, matter coupled to $\mathbf{g}_{ab}$:

$$\mathbf{g}_{ab} = g_{ab} - \varphi_a\varphi_b/X_C$$

- Can show S reduces to Lorentzian scalar-tensor theory in long-distance limit.[†‡]
- Can avoid Ostrogradsky instability for S bounded below, theory is renormalizable[*]

[†] S. Mukohyama, Phys. Rev. D 87, 085030 (2013)

[‡] S Mukohyama, J Uzan, Phys. Rev D. 87:065020 (2013)

[*] K Muneyuki, N Ohta, Phys. Lett. B 725 (2013) 495-499

Regularized SCDS in quadratic ELST

Consider a saddle-like scalar field profile and flat metric:

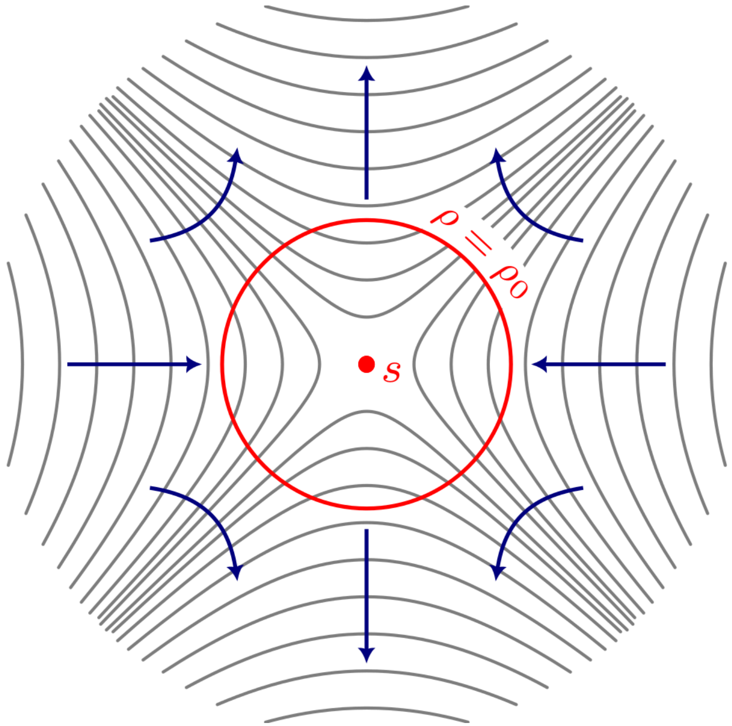
$$\varphi = (u^2 - v^2)/l_0^2 \quad , \quad g = \mathrm{diag}(1,1,1,1).$$

These form an exact soln. for the parameter choices $c_4 = c_6 = c_{10} = 0$ and $c_{11} = 8(c_5 - c_8)/l_0^4$.

Can extend to general param. choices via Riemann normal coordinates.†

Gradient $\varphi_a := \nabla_a \varphi$, yields a timelike direction for effective metric when $\varphi_a \varphi_a > X_C$:

$$\mathbf{g}_{ab} = g_{ab} - \varphi_a \varphi_b / X_C$$



Solution has vanishing curvature at ∞ and looks like a SCDS at long distances (two future directed light cones and two past)

Direction of $\nabla_a \varphi$ indicated by arrows, and $\mathbf{g}_{ab}$ changes signature at red curve

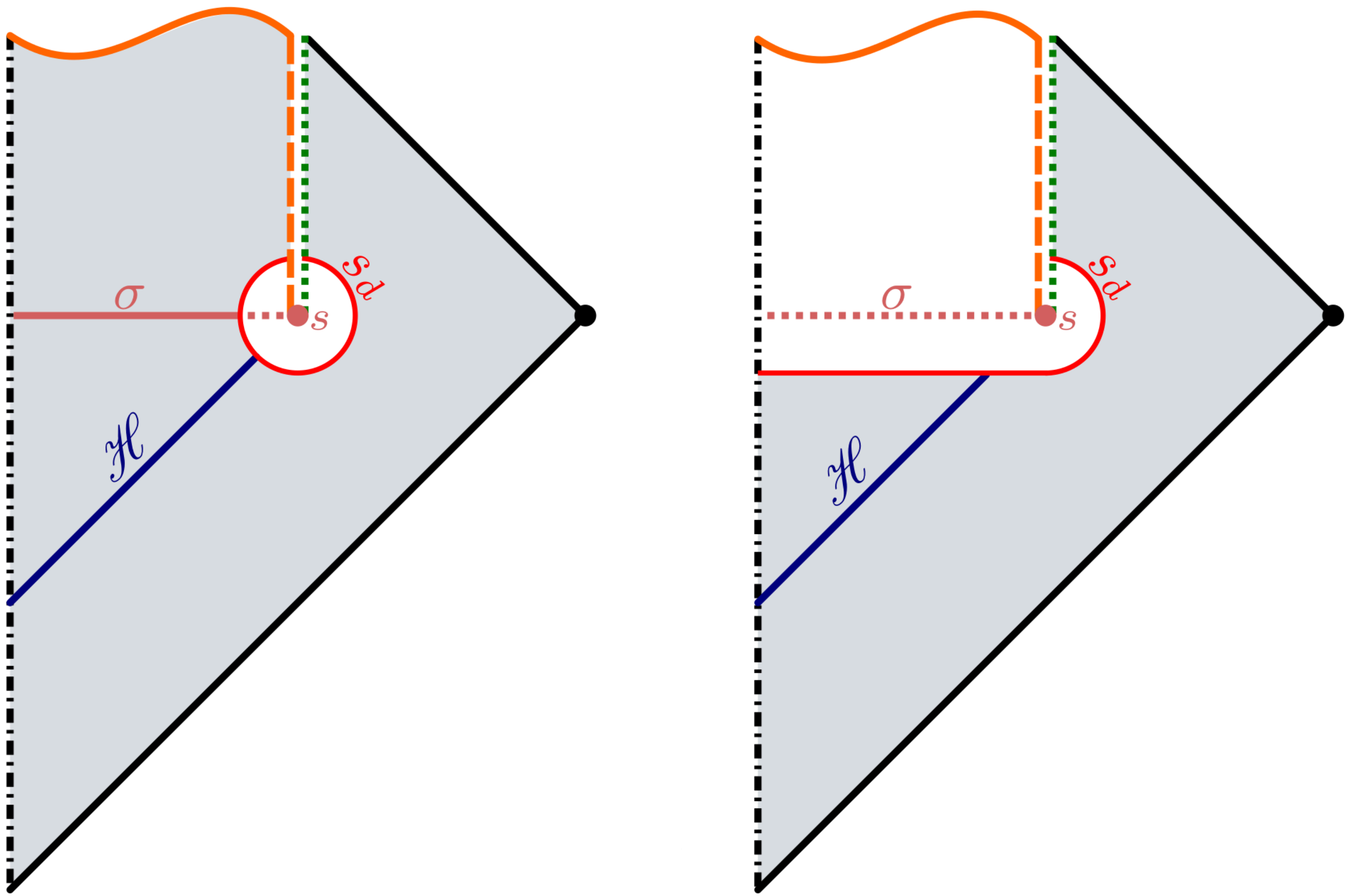
†U Muller, C Schubert, and A M E van de Ven, Gen. Rel. Grav. 31, 1759 (1999); A Z Petrov, *Einstein Spaces*, Pergamon (1969); E Kreysig, *Intro. to Diff. Geom. and Riem. Geom.*, U Toronto Press (1968).

Last thoughts

- Deutsch-Politzer and teleporter spacetimes require quasiregular singularities (SCDS)
- "Baby universe" resolution[†] to BH information paradox seems to require a theory that can describe a quasiregular singularity (or something like it)
- There exists an emergent Lorentz signature theory[‡] (with nice properties for quantum gravity) that can provide a microscopic description for SCDS
  - Singularity regularized only in g_{ab} and φ; effective metric $\mathbf{g}_{ab}$ still singular
  - Meaning of time needs to be carefully considered in the quantum theory
  - Provides toy model useful for studying Deutsch-Politzer and teleporter structures (which may be suppressed by topological terms in action, Gauss-Bonnet for instance)

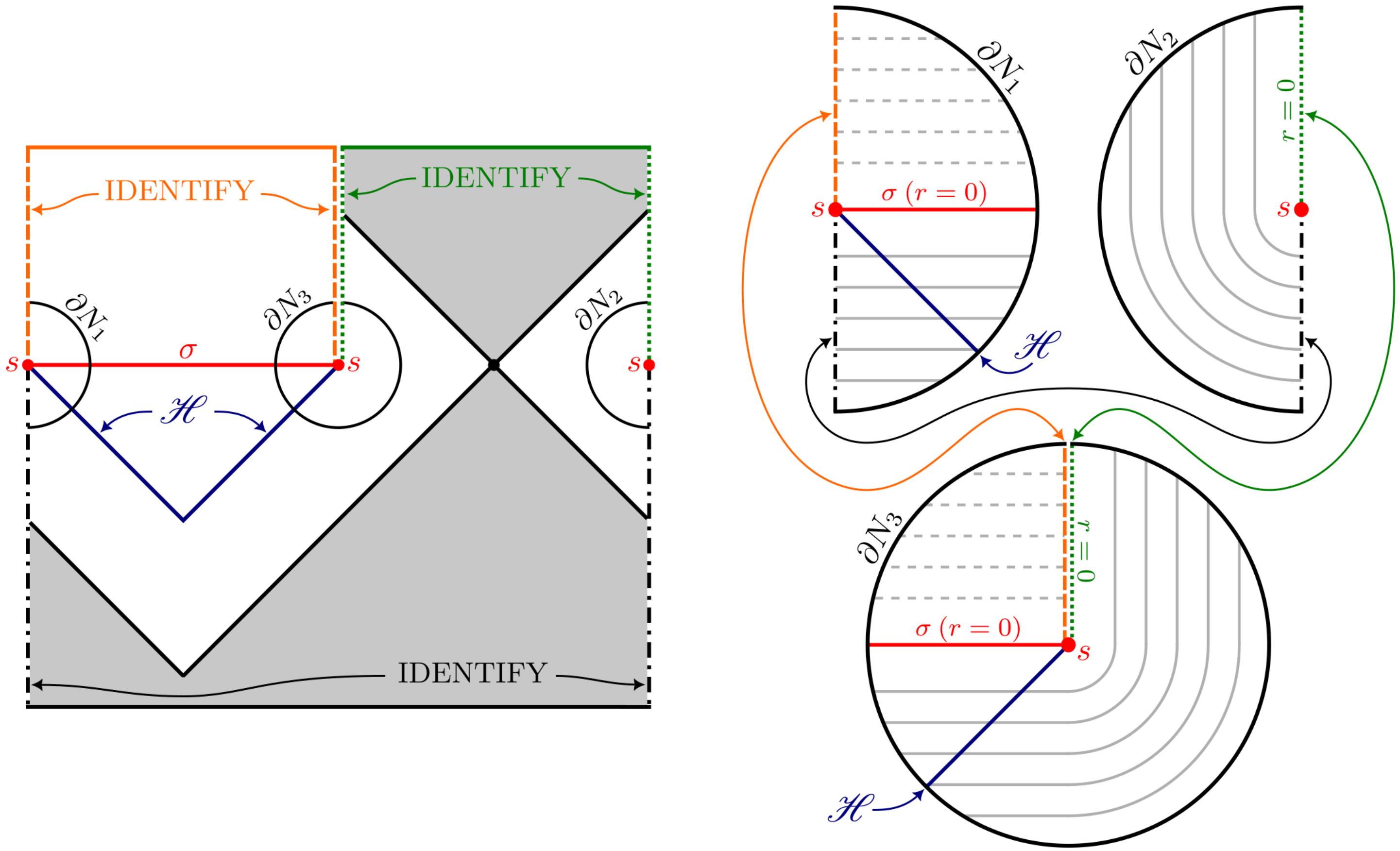
[†] S Hossenfelder, L Smolin, Phys.Rev.D81:064009,2010 [arXiv:0901.3156]
[‡] S. Mukohyama, Phys. Rev. D 87, 085030 (2013)

Regularization possibilities



Cf. Fig. 4 of S Hossenfelder, L Smolin, Phys.Rev.D81:064009,2010 [arXiv:0901.3156], but they did not consider microscopic description for s

Generalization to $d + 1$ spherically symmetric case



- $1 + 1$ plane for $d + 1$ evaporating BH on left, neighborhood ∂N of $\boldsymbol{s}$ on right.
- One way to understand $d + 1$: treat areal radius r as a scalar function (contours of r in gray)