Singularity at the demise of a black hole

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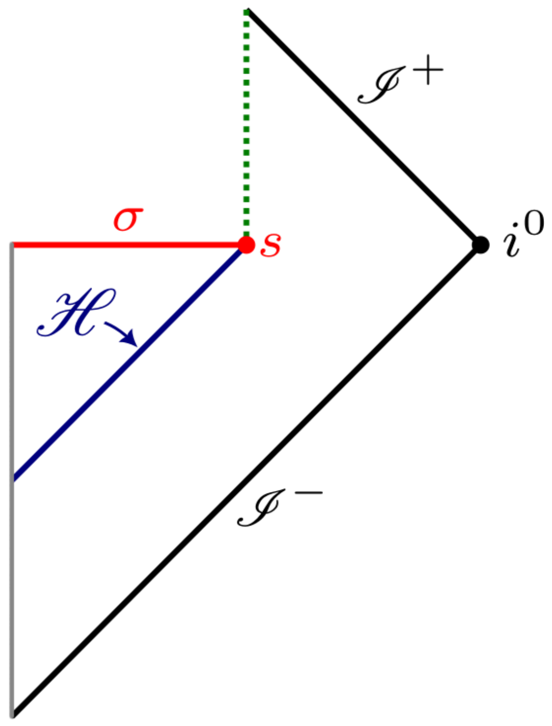
End state of a Black Hole



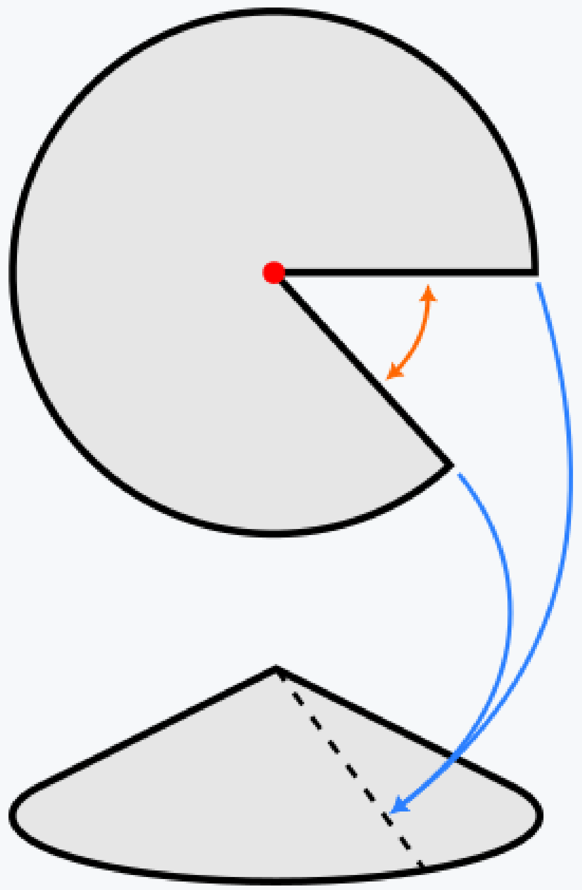
- What is the end state of an evaporating black hole?
  - Remnants, naked singularities, white holes, etc.[†]
- One possibility illustrated on left (horizon disappears)
- Can imagine regularizing singularity[†‡] σ, but what about s?
- In this talk, I claim that:

  i. If σ is regularized in the $1+1$ case, then s is a quasiregular singularity.

  ii. $\exists$ theories that can describe quasiregular singularities.

[†] S Hossenfelder, L Smolin, Phys.Rev.D 81 (2010) 064009; P Martin-Dussaud, C Rovelli, Class. Quantum Grav. 36, 245002 (2019)
[‡] A Simpson, M Visser, JCAP 02 (2019) 042 [arxiv:1812.07114]

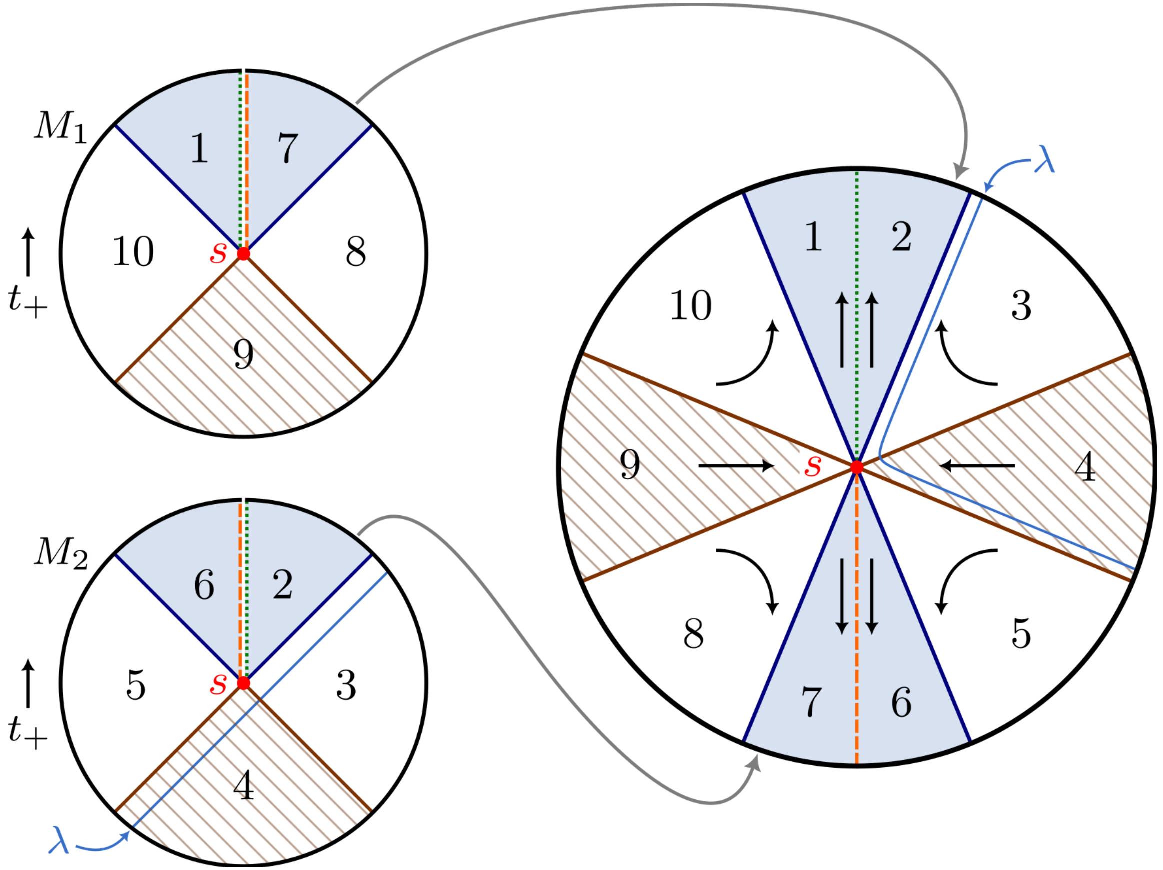
Quasiregular singularities



- Here, singularities defined as (boundary) points on which inextendible geodesics terminate
- Curvature singularities defined by diverging curvature in parallel frame along geodesic
- Quasiregular singularity† has well-behaved curvature (can even be zero) in its neigborhood
  - Can easily construct with cut-and-paste procedures
  - Conical singularity is an example

†G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

Saddlelike causally discontinuous singularity (SCDS)

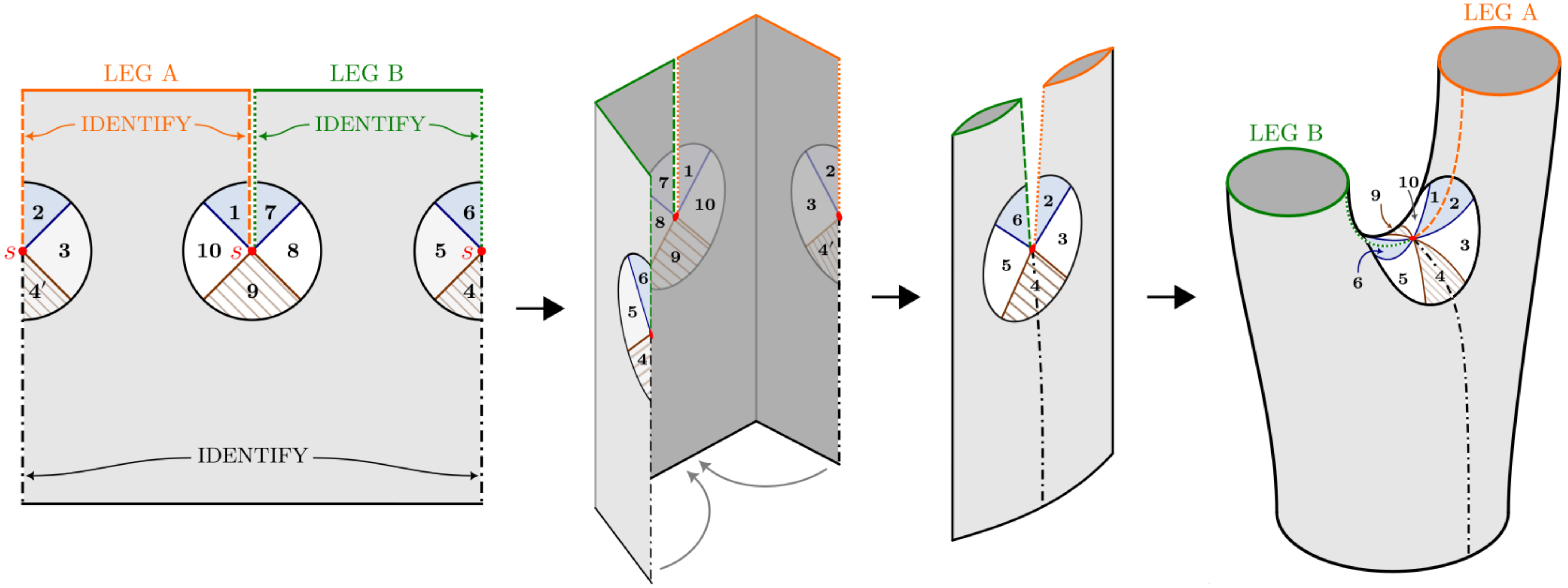


- Can construct by cut-and-paste procedure in $1+1$ flat spacetime
- Two regions of $1+1$ flat spacetime illustrated on left; nonconformal cartoon of result on right
- In $1+1$ flat spacetime, each point has one future light cone and one past light cone
- Point s (SCDS) characterized by *two* future and *two* past light cones

cf. Fig 4(e) of G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

There are further generalizations with more light cones---see G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

$1+1$ Trousers[†*] spacetime



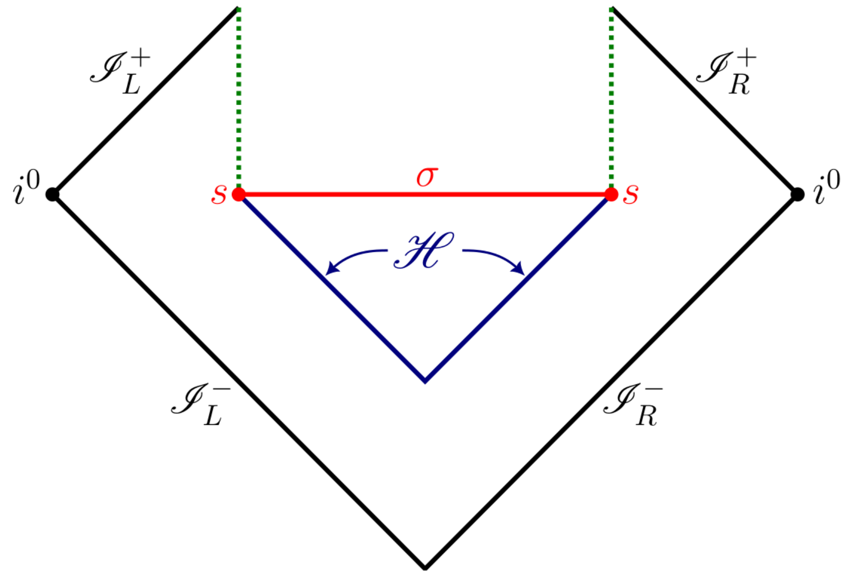
s is a SCDS, characterized by 2 future and 2 past light cones.

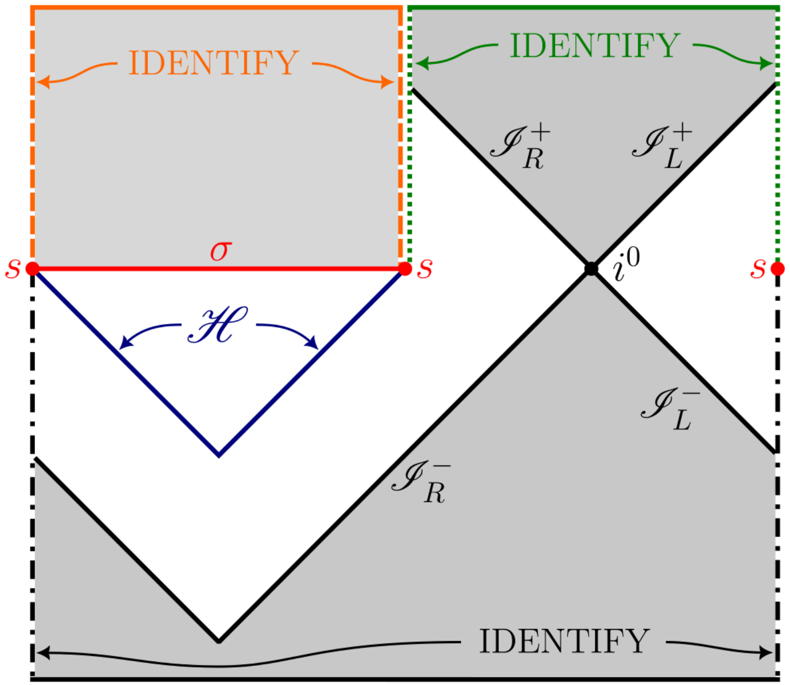
[†] A Anderson, B S DeWitt, Found.Phys. 16 (1986) 91-105

[*] F. Dowker, S. Surya, PRD 58, 124019 (1998) [arXiv:gr-qc/9711070]; Buck et al., Class.Quant.Grav. 34 (2017) 5, 055002 [arXiv:1609.03573]

$1+1$ Black hole and trousers[†] spacetime

$1+1$ evaporating BH is conformal to trousers: $\boldsymbol{s}$ is quasiregular[*] singularity!





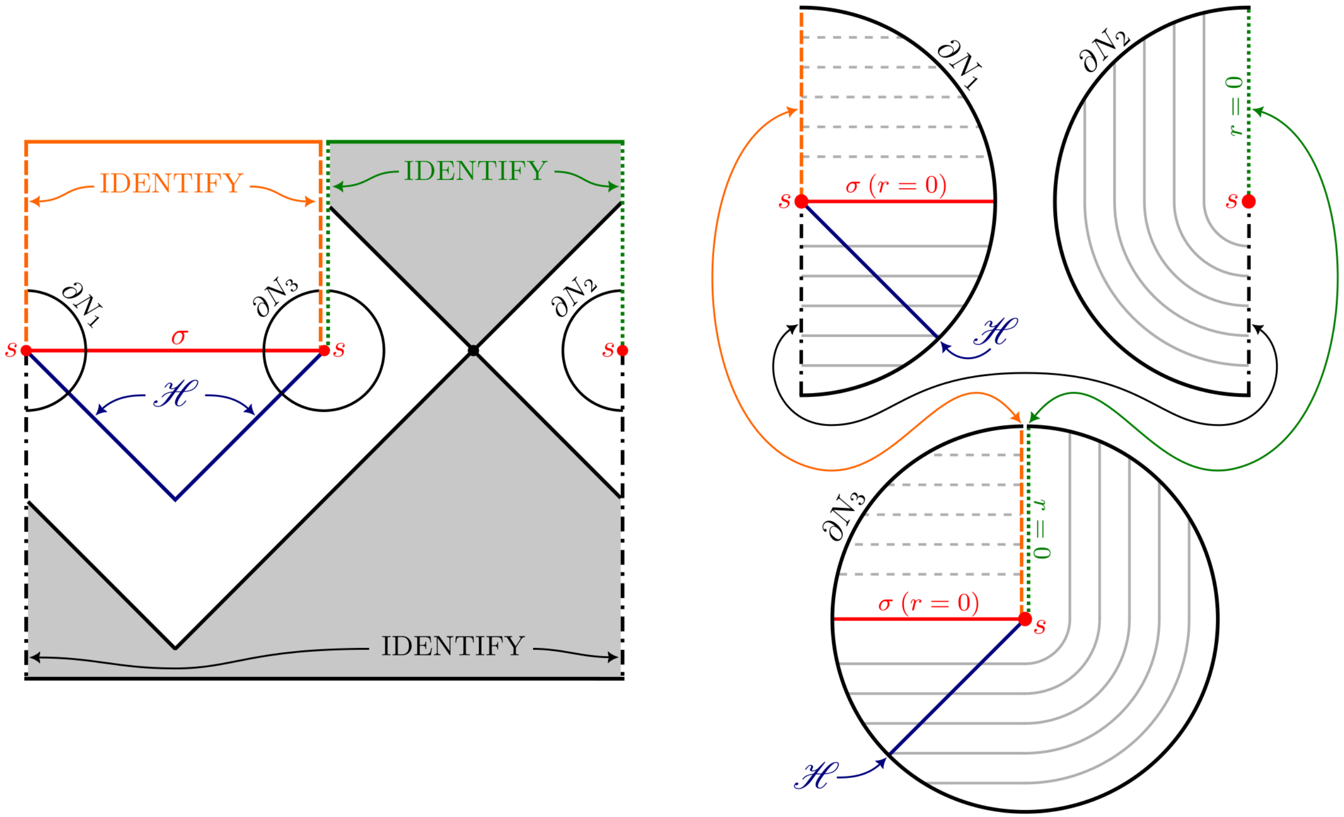
Planar slice of $d+1$ BH through origin can be regarded similarly.

[†] A Anderson, B S DeWitt, Found.Phys. 16 (1986) 91-105

[*] Provided that σ is regularized and spacetime analytically extended. If future topology of $\mathscr{H}$ differs, this is still true but s may not be SCDS.
Cf. Fig. 4 of S Hossenfelder, L Smolin, Phys.Rev.D81:064009,2010 [arXiv:0901.3156]

Generalization to $d+1$ spherically symmetric case

- Consider a planar $(1+1)$ slice of spherically symmetric $d+1$ evaporating BH through origin
  - Can construct similar conformal diagram in slice
- Can treat areal radius r as a scalar function (contours of r in gray)
- Causal past of s is not a light cone; has topology $(\mathbb{S}^2 \times \mathbb{R}) \cup (\mathbb{S}^2 \times \mathbb{R})$
  - One class of radial null curves corresponds to horizon
  - Another distinct class corresponds to incoming null curves from $\mathscr{I}_-$
- s is a quasiregular singularity, not a SCDS, maybe a generalization.



Left: $1+1$ planar slice of $d+1$ evaporating BH thru origin
Right: Neighborhood ∂N of s

Emergent Lorentz signature theory

- There is at least one gravity theory that can describe a regularization of a SCDS.
- Postulate Euclidean-signature g_{ab} with shift-symmetric scalar-tensor action:[†]

$$S = \int_M d^4x\sqrt{|g|}L, \quad \varphi_a := \nabla_a\varphi, \quad \varphi_{ab} := \nabla_a\nabla_b\varphi, \quad X := \varphi^a\varphi_a$$

$$L = c_1 R^2 + c_2 R_{ab}R^{ab} + c_3 R_{abcd}R^{abcd} + c_4 XR + c_5 R^{ab}\varphi_a\varphi_b$$
$$+ c_6 \mathrm{X}^2 + c_7(\Box\varphi)^2 + c_8\varphi_{ab}\varphi^{ab} + c_9 R + c_{10}X + c_{11}$$

- Theory is renormalizable,[*] can avoid Ostrogradsky instability if S bounded
- Postulate that at long distance scales, matter coupled to effective metric: $\mathbf{g}_{ab} = g_{ab} - \varphi_a\varphi_b/X_c$
- Eff. metric $\mathbf{g}_{ab}$ is a disformal transformation; can show S reduces to Lorentzian scalar-tensor theory for $\mathbf{g}_{ab}$ in long-distance limit.[†‡]

[†] S. Mukohyama, Phys. Rev. D 87, 085030 (2013)

[‡] S Mukohyama, J Uzan, Phys. Rev D. 87:065020 (2013)

[*] K Muneyuki, N Ohta, Phys. Lett. B 725 (2013) 495-499

ELST Field Equations

The field equations are as follows:

$$\nabla_a(J^a - \Phi^a) = 0$$

$$Q^R_{ab} + Q^\varphi_{ab} + g_{ab}Q = \tfrac{1}{2}T_{ab}$$

where Φ^a and T_{ab} are matter sources.

The quantities J^a, Q^R_{ab}, Q^φ_{ab}, and Q are defined to the right, where as before:

$$\varphi_a := \nabla_a\varphi \qquad \varphi_{ab} := \nabla_a\nabla_b\varphi.$$

Note that the quantities on the right simplify under the harmonic condition $\Box\varphi = 0$ and vanishing curvature.

$$J^a = (c_7 + c_8)\Box\varphi^a - (c_5 + c_7)R^{ab}\varphi_b - \varphi^a(c_{10} + c_4 R + 2c_6 X)$$

$$\begin{aligned}
Q^R_{ab} = &-2c_9 R_{ab} - 4c_2 R_{ac}R_b{}^c - 4c_1 R_{ab}R - 4c_3 R_a{}^{cde}R_{bcde} \\
&+ 4(c_2 + 2c_3)(R_{ac}R_b{}^c - R^{cd}R_{acbd}) \\
&+ 2(2c_1 + c_2 + 2c_3)\nabla_a\nabla_b R - 2(c_2 + 4c_3)\Box R_{ab},
\end{aligned}$$

$$\begin{aligned}
Q^\varphi_{ab} = &-2c_{10}\varphi_{ab} + 4(c_5 - c_8)\varphi_{ab}\Box\varphi + 4c_4\varphi_{cb}\varphi^c{}_a \\
&+ 4(c_7 + c_8)(\varphi_{(a}\Box\varphi_{b)}) + 2(2c_4 + c_5 - c_8)\varphi^c\nabla_c\varphi_{ab} \\
&- 2c_4(R\varphi_a\varphi_b + R_{ab}X - 2R_{acbd}\varphi^c\varphi^d) - 4c_6\varphi_a\varphi_b X \\
&- 4\varphi^c(c_5 + c_7)R_{c(a}\varphi_{b)},
\end{aligned}$$

$$\begin{aligned}
Q = L &- (c_5 + 2c_7)(\Box\varphi)^2 + (c_5 + 2c_7)R^{cd}\varphi_c\varphi_d \\
&- (4c_1 + c_2)\Box R - 2(2c_4 + c_5 + c_7)\varphi^c\Box\varphi_c \\
&- (4c_4 + c_5)\varphi_{cd}\varphi^{cd}.
\end{aligned}$$

Regularized SCDS in quadratic ELST

Consider a saddle-like scalar field profile and flat metric:

$$\varphi = (u^2 - x^2)/(2l_0), \qquad ds^2 = g_{\mu\nu}x^\mu x^\nu = du^2 + dx^2 + dy^2 + dz^2$$

$$\text{Eff. Metric:} \qquad \mathbf{g}_{ab} = g_{ab} - \varphi_a \varphi_b / X_c$$

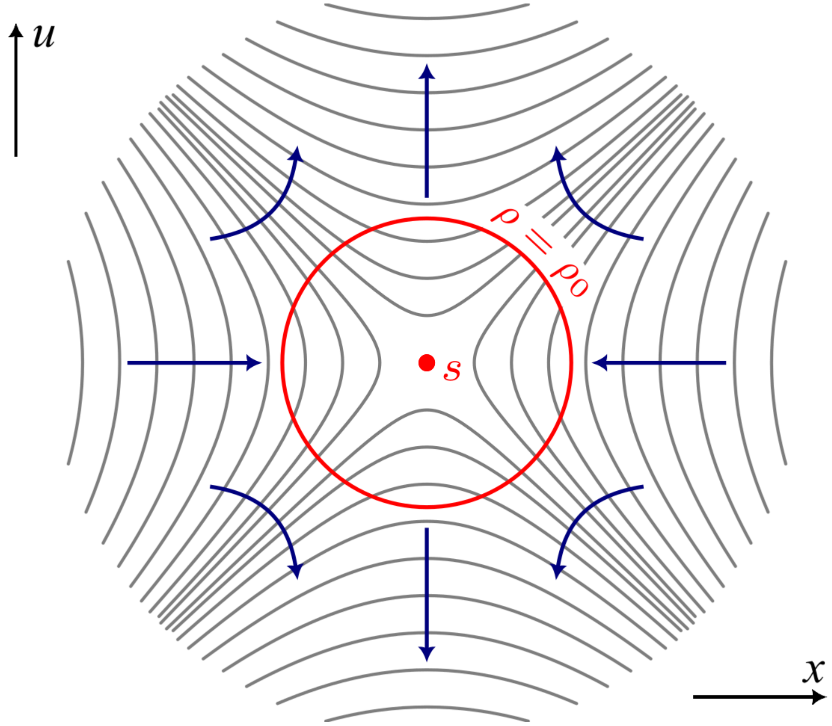
This is exact soln. for the parameters $c_4 = c_6 = c_{10} = 0$ and $l_0^2 = 2(c_5 - c_8)/c_{11}$. Can also have exact solution for:

$$\varphi = (3u^2 - x^2 - y^2 - z^2)/(2L_0)$$

Can get approximate soln. using Riemann normal coords:†

$$g_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{3}\left[R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta - \frac{1}{6}\left[\nabla_\gamma R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma$$
$$-\left[\frac{2}{45}R_{\mu\alpha\lambda\beta}R^\lambda{}_{\gamma\delta\nu} + \frac{1}{20}\nabla_\gamma\nabla_\delta R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma x^\delta + O(x^{\cdot 5})$$

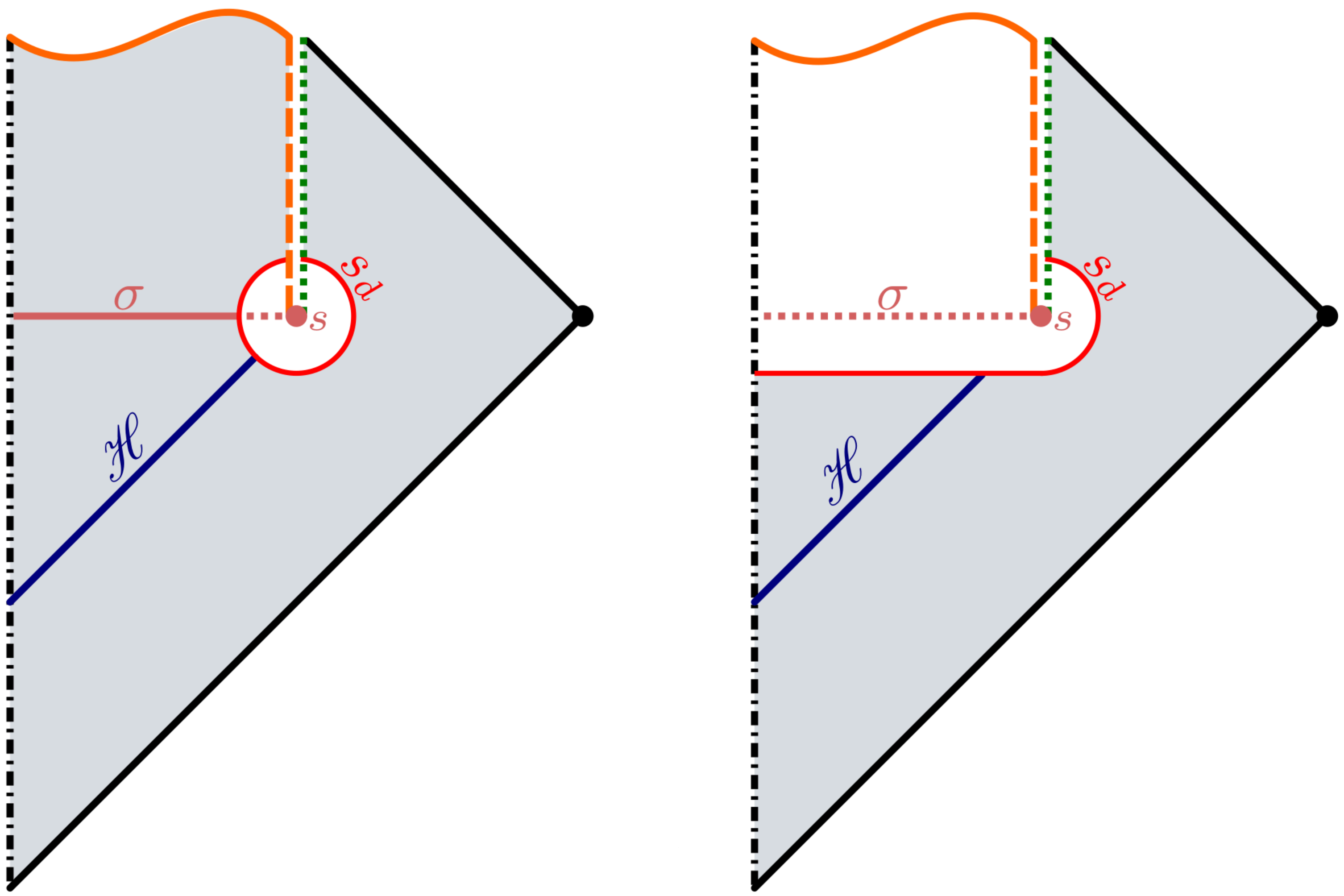
where s is the origin.*



Where: $\rho = \sqrt{u^2 + x^2}, \quad \rho_0 = \sqrt{X_c}\, l_0$

† U Muller, C Schubert, and A M E van de Ven, Gen. Rel. Grav. 31, 1759 (1999);
A Z Petrov, *Einstein Spaces*, Pergamon (1969); E Kreysig, *Intro. to Diff. Geom. and Riem. Geom.*, U Toronto Press (1968)

Regularization possibilities



Remarks on BH Information paradox

- A "baby universe" resolution to BH information paradox?
- There was some debate whether pinch off point relevant for unitarity[†,‡]
- Clear that pinch off point is singular; expect nontrivial consequences for unitarity
- Whether ELST can resolve this depends on its quantization

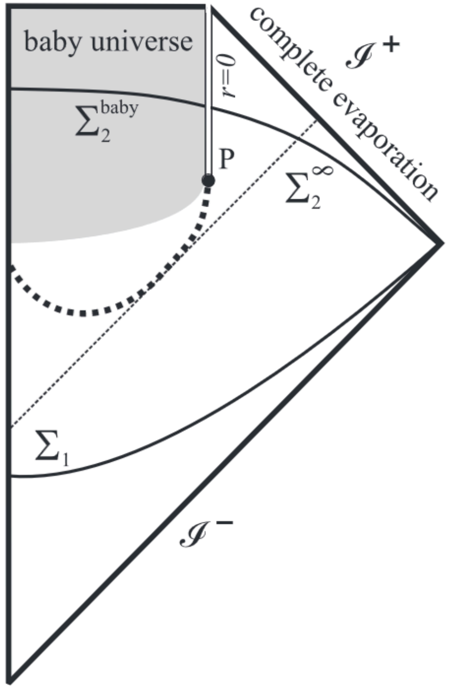


Figure 4: Option 4A: Example of a baby universe scenario. The pinch-off point is marked with P, the thick dotted line indicates the apparent horizon. The double thin line is a boundary between two disconnected regions. The grey shaded region is potentially subject to non-negligible quantum gravitational corrections. The thin dashed line represents the lightlike surface where the event horizon of the collapsing matter had been without evaporation. †

†S Hossenfelder, L Smolin, Phys.Rev.D81:064009,2010 [arXiv:0901.3156]

‡Bianchi et al. Class.Quant.Grav. 35 (2018) 22, 225003

Issues/questions to think about

- A "baby universe" resolution[†] to BH information paradox seems to require a study of quasiregular singularities and their regularization
  - In the $1+1$ case, you have SCDS, in the $d+1$ case, have a generalization
- ELSTs can describe SCDS (the analysis here is classical!)
  - Singularity regularized only in fundamental metric and scalar field; $\mathbf{g}_{ab}$ still singular
  - SCDS might work in $d+1$ case, but need to better understand structure of $d+1$ singularity
  - More comprehensive analysis needed to see whether scenario is indeed realized at end of BH evap.
  - Regarding ELSTs: Questions about quantization, time evolution, long-distance behavior remain
    - Preliminary results suggest ELST requires strong constraints on parameters if massless DoFs in pert. theory propagate with Lorentzian dispersion relation
- How might other theories handle quasiregular singularities?

[†] S Hossenfelder, L Smolin, Phys.Rev.D81:064009,2010 [arXiv:0901.3156]

Finite areal radius at the origin

Consider line element for $R \times S^2$ manifold:

$$ds^2 = du^2 + dv^2 + r(u,v)^2(d\theta^2 + \sin^2\phi d\phi^2)$$

For a the $v = 0$ surface, extrinsic curvature is:

$$K^{\cdot}{}_{\cdot} = \mathrm{diag}\left(0, \frac{\partial_v r(u,v)|_{v=0}}{r(u,0)}, \frac{\partial_v r(u,v)|_{v=0}}{r(u,0)}\right)$$

If left side is reflection of right side, can have finite areal radius r at $v = 0$ and smooth (C^1 at least) geometry provided that $\partial_v r(u,v)|_{v=0} = 0$.

Approximate solution in Riemann normal coordinates

Explicitly, the approximate solution near the origin has the form:

$$\varphi = (u^2 - v^2)/(2L_0) + \varphi_{\mu\nu} x^\mu x^\nu + O(x^{\cdot 3})$$

$$g_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{3}\left[R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta - \frac{1}{6}\left[\nabla_\gamma R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma$$
$$-\left[\frac{2}{45} R_{\mu\alpha\lambda\beta} R^\lambda{}_{\gamma\delta\nu} + \frac{1}{20}\nabla_\gamma \nabla_\delta R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma x^\delta + O(x^{\cdot 5})$$

with coefficients satisfying:

$$\Big[4\{(c_2 + 4c_3)\Box R_{\mu\nu} - (2c_1 + c_2 + 2c_3)\nabla_\mu \nabla_\nu R\} + 4c_{10}\varphi_{\mu\nu}$$
$$+ 8\{c_3 R_\mu{}^{\alpha\beta\gamma} R_{\nu\alpha\beta\gamma} - (c_2 + 2c_3)(R_{\mu\alpha} R_\nu{}^\alpha - R^{\alpha\beta} R_{\mu\alpha\nu\beta})\}$$
$$+ 8\{c_2 R_{\mu\alpha} R_\nu{}^\alpha - c_4 \varphi_{\alpha\mu}\varphi^\alpha{}_\nu\} + 4(c_9 + 2c_1 R)R_{\mu\nu}$$
$$+ 2g_{\mu\nu}\Big\{(4c_1 + c_2)\Box R - c_{11} - c_9 R - c_1 R^2 - c_2 R_{\alpha\beta} R^{\alpha\beta}$$
$$- c_3 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - (c_8 - 4c_4 - c_5)\varphi_{\alpha\beta}\varphi^{\alpha\beta}\Big\} + T_{\mu\nu}\Big]_0 = 0$$