

Best-First Rao-Blackwellization

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March 5, 2022

Gradient Estimation

- ▶ Goal: optimize

$$\max_{\theta} \sum_x p_{\theta}(x) f(x)$$

(optimizing wrt parameters of f is easy)

- ▶ Discrete x , expensive evaluation of $f(x)$
 - ▶ eg f is a giant neural network

Gradient Estimators

- ▶ Score-function estimator (SFE)

$$\nabla_{\theta} \sum_x p_{\theta}(x) f(x) = \mathbb{E}_{p_{\theta}(x)} [f(x) \nabla \log p_{\theta}(x)]$$

- ▶ High variance, consistent
 - ▶ Requires multiple evaluations, better with more compute
- ▶ Continuous relaxation, $x \approx g(\theta, \epsilon)$

$$\nabla_{\theta} \sum_x p_{\theta}(x) f(x) \approx \mathbb{E}_{p(\epsilon)} [\nabla_{\theta} f(g_{\theta}(\theta, \epsilon))]$$

- ▶ Biased, lower variance
 - ▶ Requires low number of evals, less benefit from more compute
 - ▶ Requires relaxable f
 - ▶ Stochastic softmax tricks (SST)
- ▶ Focus on improving SFE
 - ▶ SFE under-explored

Rao-Blackwellization

- ▶ Too expensive to enumerate over all x (f is expensive)
- ▶ Reduce effective width of distribution via sub-sampling (red)
- ▶ Enumerate over green portion

$p(1)$	$p(2)$	$p(3)$	$p(4)$
$f(1)$	$f(2)$	$f(3)$	$f(4)$

Rao-Blackwellization of score function estimator

- ▶ Condition on values of $f(x)$ for $x \in S$

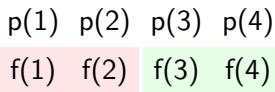
$$\begin{aligned} & \nabla_{\theta} \sum_x p_{\theta}(x) f(x) \\ &= \mathbb{E}_{p_{\theta}(x)} [f(x) \nabla \log p_{\theta}(x)] \\ &= \sum_{x \in S} p(x) f(x) \nabla \log p_{\theta}(x) + \mathbb{E}_{p_{\theta}(x \notin S)} [f(x) \nabla \log p_{\theta}(x)] \end{aligned}$$

- ▶ Conditioning on all values $f(x)$ = no variance
- ▶ Want S to have x with high $p(x)f(x)$
 - ▶ Need to compromise with high $p(x)$
 - ▶ Use heuristic estimate of $f(x)$?
 - ▶ What if x is structured?

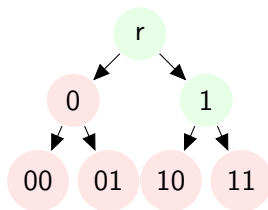
Structured Setting

- ▶ In the flat setting, only cared about width
- ▶ In structured setting, can also approximate depth

Flat



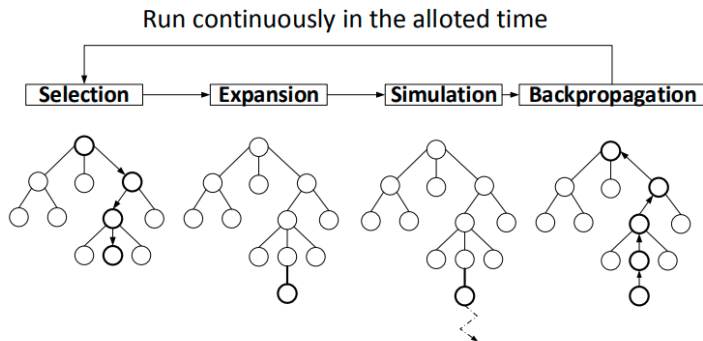
Structured



Best-first Rao-Blackwellization

- ▶ Modern MCTS
 - ▶ Prior $p(x)$ (limits width)
 - ▶ Cost-to-go estimate $\tilde{V}(x)$ (limits depth)
 - ▶ Search procedure that links the two
- ▶ Proposal
 - ▶ In many cases, already have the prior $p(x)$
 - ▶ Estimate cost-to-go with continuous relaxation or cheaper problem-dependent weaker model (learned value function approx)
 - ▶ Link the two by marginalization

BF RB



Minimal Experiment

Approximate DP when exact is tractable

- ▶ Depth approx: Learn cost-to-go estimate to bound depth in 32k state HMM
 - ▶ HMM with forward algo that doesn't go all the way to end
 - ▶ Bounded width = prior not important
 - ▶ Experiment with continuous relaxation + learned \tilde{V}
- ▶ Width approx: Utilize prior in large-state HMM
 - ▶ Limit num states considered at each time step
- ▶ Approximations for width + depth
 - ▶ Probably more useful in PCFGs or other more interesting models

Citations I