

Low-Rank Factorizations for Fast Inference in Structured Models

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October 19, 2021

Structured Models

- ▶ Explicitly model output associations
 - ▶ Directly or through latent variables
- ▶ Focus on combinatorially large latent discrete structures
 - ▶ Complementary to continuous, deterministic representations
- ▶ More difficult to scale than alternative representations
 - ▶ Bottlenecked by time + space complexity of marginal inference

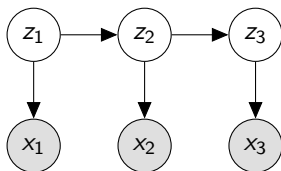
Scaling Structured Models

- ▶ Scaling (to the point of overparameterization) is key
- ▶ Target tractable models
 - ▶ Admit dynamic programs for exact marginalization
- ▶ Impose a low-rank model constraint
 - ▶ Trades off model expressivity for cheaper marginalization
- ▶ Only constrain parameters used in key steps of marginalization

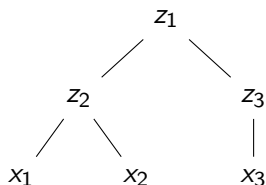
Marginalization in Structured Models

- ▶ Model an observation $x = (x_1, \dots, x_T)$ via latent structure z
 - ▶ Latent nodes z_i
 - ▶ Nodes have discrete label set $[L]$
- ▶ Perform training and evaluation via marginalization

$$p(x) = \sum_z p(x, z)$$



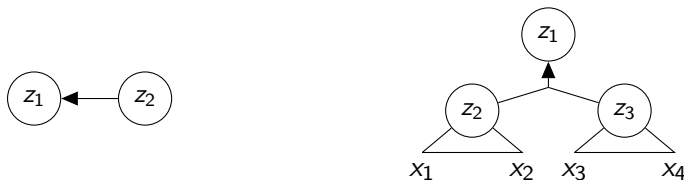
Hidden Markov models



Probabilistic context-free grammars

Hypergraphs for Marginalization

- ▶ Represent marginalization dynamic programs as hypergraphs
- ▶ Hypergraphs consist of nodes and hyperedges
 - ▶ Hyperedge consists of a head node and set of tail nodes
- ▶ Perform marginalization by traversing hypergraph
 - ▶ Aggregate marginals from tails to head via a matrix-vector product



Hyperedge representations for HMMs and PCFGs

Hypergraph Marginalization

For each hyperedge e in topological order,

- ▶ Combine tail marginals α_1, α_2 into joint tail marginal β_v
- ▶ Apply score matrix Ψ_e and aggregate in head marginal α_u
 - ▶ Matrix-vector product

Algorithm 1 Hypergraph marginalization / belief propagation

for $u \leftarrow v$ hyperedge e topologically **do**

$\beta_v \leftarrow \alpha_{v_1} \alpha_{v_2}^\top$ $\triangleright O(L^{|e|})$

$\alpha_u \stackrel{+}{\leftarrow} \Psi_e \beta_v$ $\triangleright O(L^{|e|+1})$

return $\alpha_S^\top \mathbf{1}$

Our Method: Scaling with Low-Rank Factorizations

- ▶ Hypergraph marginalization bottlenecks
 - ▶ Number of hyperedges
 - ▶ Matrix-vector product
- ▶ Approach: Impose low-rank model constraint
- ▶ Improves time and space complexity of marginalization

Low-Rank Factorizations

- ▶ Rank $R < L$ factorization
- ▶ Factor matrices $\Psi = UV^\top$, $U \in \mathbb{R}^{L \times R}$, $V \in \mathbb{R}^{L^{\text{el}} \times R}$

$$\boxed{\Psi} \times \boxed{\beta} = \boxed{U} \times \left(\boxed{V^\top} \times \boxed{\beta} \right)$$

- ▶ Two matrix-vector products of cost $O(LR)$ and $O(L^{\text{el}}R)$
 - ▶ Reduced from $O(L^{\text{el}}+1)$

Low-rank Hypergraph Marginalization

Applying the low-rank factorization,

Algorithm 2 Low-rank marginalization

for $u \leftarrow v_1, v_2$ hyperedge e topologically **do**

$$\beta_v \leftarrow \alpha_{v_1} \alpha_{v_2}^\top \quad \triangleright O(L^{|e|})$$

$$\gamma \leftarrow V_e^\top \beta_v \quad \triangleright O(L^{|e|}R)$$

$$\alpha_u \stackrel{+}{\leftarrow} U_e \gamma \quad \triangleright O(LR)$$

return $\alpha_S^\top \mathbf{1}$

- ▶ Potentially large speedups for marginalization
 - ▶ HMM from $O(L^2)$ to $O(LR)$
 - ▶ PCFG from $O(L^3)$ to $O(L^2R)$

Expressivity of Rank-constrained Models

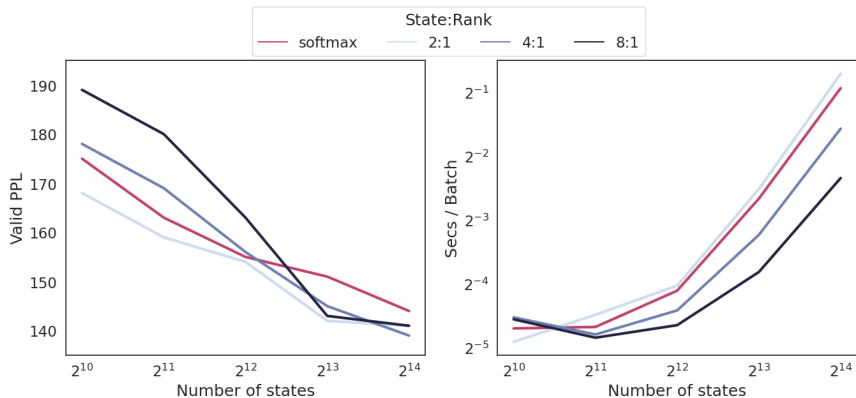
- ▶ Rank constraints limit expressivity
- ▶ Only need to constrain bottleneck parameters
 - ▶ Transition matrix for HMMs
 - ▶ Subset of the transition matrix for PCFGs
- ▶ Is it more expressive than a smaller model?
 - ▶ An L -state HMM with rank R ($< L$) is more expressive than an unconstrained R -state HMM

Experiments

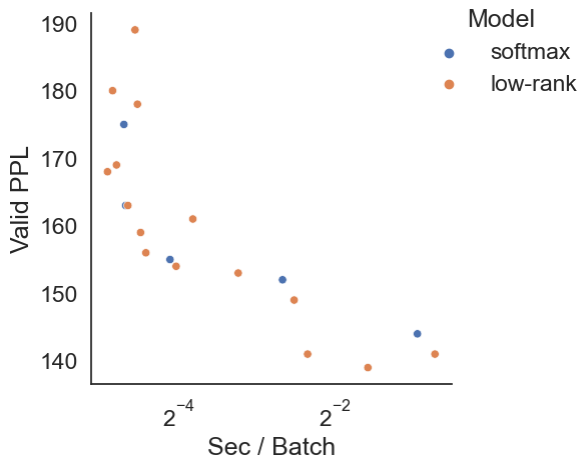
- ▶ Language modeling on PENN TREEBANK¹
- ▶ Compare size vs speed and accuracy
 - ▶ Size = 1k to 16k state HMM, 90 to 300 state PCFG
 - ▶ Speed = Sec/Batch
 - ▶ Accuracy = Perplexity (function of likelihood)
- ▶ Unconstrained softmax HMM, PCFG vs low-rank versions
- ▶ Further experiments in paper
 - ▶ Polyphonic music modeling with HMMs
 - ▶ Video modeling with HSMMs

¹Marcus, Santorini, and Marcinkiewicz, 'Building a Large Annotated Corpus of English: The Penn Treebank'.

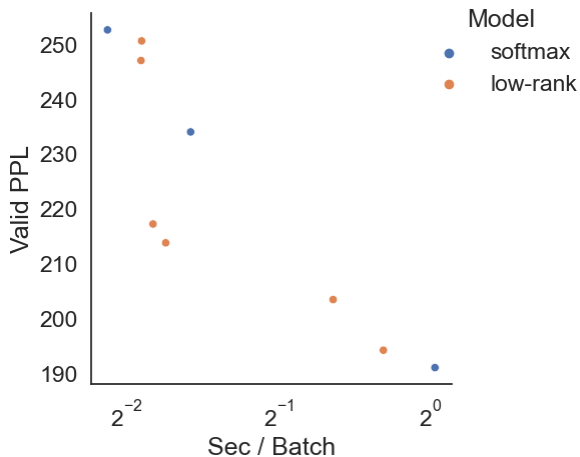
HMM Results



HMM Speed vs Accuracy Frontier



PCFG Speed vs Accuracy Frontier



Conclusion

- ▶ Low-rank factorization speeds up marginalization
 - ▶ Constrain only bottleneck parameters
 - ▶ Most effective with large models
- ▶ Scaling improves accuracy
 - ▶ Gap with neural models still large
 - ▶ Scale further with more aggressive constraints
 - ▶ Compose with different representations
- ▶ Please see the paper for more experiments and analysis!

Citations I



Marcus, Mitchell P., Beatrice Santorini, and Mary Ann Marcinkiewicz. 'Building a Large Annotated Corpus of English: The Penn Treebank'. In: *Computational Linguistics* 19.2 (1993), pp. 313–330. URL: <https://www.aclweb.org/anthology/J93-2004>.