Low-Rank Constraints for Fast Inference in Structured Models

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Structured Models

- Explicitly model output associations
  - Directly or through latent variables
- Focus on combinatorially large latent discrete structures
  - Complementary to distributed representations

Scaling Structured Models

- Prior work demonstrated: Size ⬆ Performance ⬆
  - Hidden Markov Models (HMM)
  - Probabilistic Context-Free Grammars (PCFG)
- Prior work scaled via
  - Sparsity for HMMs[1]
  - Low-rank tensor decompositions for PCFGs[2]
- This work: low-rank matrix constraints
  - More general
  - Less speedup

[1] Chiu and Rush, *Scaling Hidden Markov Language Models*.

[2] Yang, Zhao, and Tu, 'PCFGs Can Do Better: Inducing Probabilistic Context-Free Grammars with Many Symbols'.

Inference as Matrix-Vector Products

- Inference: sequence of matrix-vector products
- Speed up via fast mat-vecs
- Applies to a large family of structured models

Fast Matrix-Vector Products

- Mat-vecs take $O(L^2)$ computation
- Various fast methods
  - Sparsity (nnz entries)
  - Fast Fourier Transform ($L \log L$)
  - Low-Rank factorization (LR)
- Connected to work in efficient attention and low-dimensional kernel approximations[3]

[3] Choromanski et al., *Rethinking Attention with Performers*; Peng et al., *Random Feature Attention*; Blanc and Rendle, *Adaptive Sampled Softmax with Kernel Based Sampling*.

Roadmap

- Speeding up HMM inference
- Speeding up PCFG inference
- Generalization to hypergraph inference
- Experiments

Two Examples

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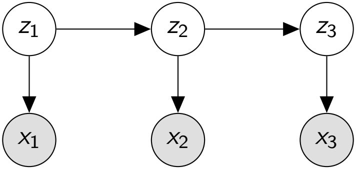
- Blah

Hidden Markov Models (HMMs)

For times t, model states $z_t \in [Z]$, and tokens $x_t \in [X]$,



with joint distribution

$$p(x, z) = \prod_t p(x_t \mid z_t) p(z_t \mid z_{t-1})$$

Inference

Given observed $x = (x_1, \ldots, x_T)$ We wish to maximize

$$p(x) = \sum_{z_1} \cdots \sum_{z_T} p(x, z) = \alpha_1^\top \Lambda_2 \Lambda_3 \cdots \Lambda_T \mathbf{1},$$

where we have the

$$\begin{aligned} &\text{start,} && [\alpha_1]_{z_1} = p(x_1 \mid z_1) p(z_1), \\ &\text{and transition operators,} && [\Lambda_t]_{z_{t-1}, z_t} = p(x_t \mid z_t) p(z_t \mid z_{t-1}) \end{aligned}$$

Inference: Backward Algorithm

- Performing multiplications from right to left

$$p(x) = \alpha_1^\top(\Lambda_2(\Lambda_3\mathbf{1}))$$

- Recursively

$$\beta_t = \Lambda_t\beta_{t+1}$$

- Requires $O(TZ^2)$ operations in total

Low-Rank Factorization

Factor matrices $\Lambda \in \mathbb{R}^{Z \times Z}$ into product of $U, V \in \mathbb{R}^{Z \times R}$

$$\boxed{\Lambda} \times \boxed{\beta} = \boxed{U} \times \left(\boxed{V^\top} \times \boxed{\beta} \right)$$

resulting in two matrix-vector products of cost $O(ZR)$ each

Hypergraph Marginalization

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Hypergraph Marginalization Algorithm

Experiments

Experiments

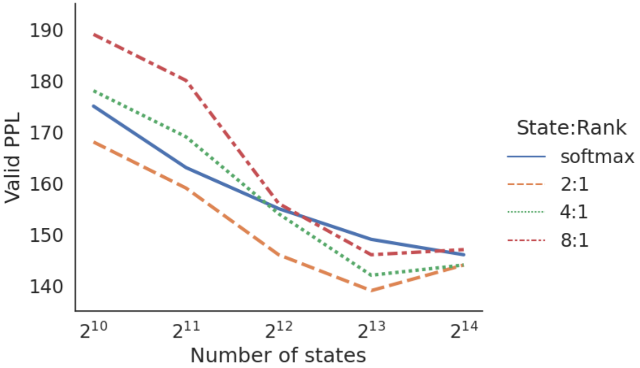
- Language modeling on PTB
- Feature map $\phi(U) = \exp(UW)$, with learned $W \in \mathbb{R}^{R \times R}$
- Baseline: Softmax HMM

HMM Accuracy

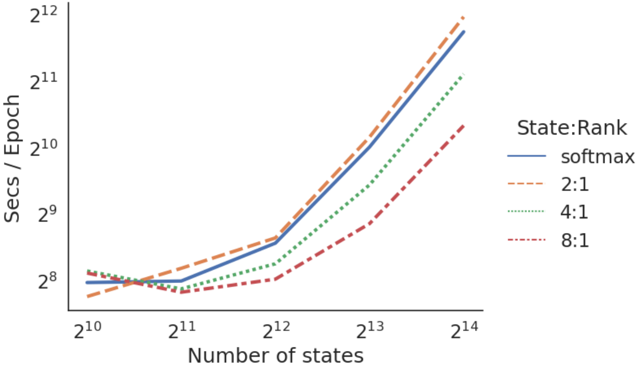
Model	Val	Test
AWD-LSTM	60.0	57.3
VL-HMM	128.6	119.5
HMM	144.3	136.8
LHMM	141.4	131.8

HMM Speed vs Accuracy Frontier

HMM Accuracy vs Rank



HMM Speed vs Rank



HMM Music Results

Model	Nott	Piano	Muse	JSB
RNN-NADE	2.31	**7.05**	**5.6**	5.19
R-Transformer	**2.24**	7.44	7.00	8.26
LSTM	3.43	7.77	7.23	8.17
LV-RNN	2.72	7.61	6.89	**3.99**
SRNN	2.94	8.20	6.28	4.74
TSBN	3.67	7.89	6.81	7.48
HMM	2.43	8.51	7.34	5.74
LHMM	2.60	8.89	7.60	5.80

HSMM Results

PCFG Results

Citations

- Blanc, Guy and Steffen Rendle. *Adaptive Sampled Softmax with Kernel Based Sampling*. 2018. arXiv: 1712.00527 [cs.LG].
- Chiu, Justin T. and Alexander M. Rush. *Scaling Hidden Markov Language Models*. 2020. arXiv: 2011.04640 [cs.CL].
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- Yang, Songlin, Yanpeng Zhao, and Kewei Tu. 'PCFGs Can Do Better: Inducing Probabilistic Context-Free Grammars with Many Symbols'. In: *CoRR* abs/2104.13727 (2021). arXiv: 2104.13727. URL: https://arxiv.org/abs/2104.13727.