Low-Rank Factorizations for Fast Inference in Structured Models

Justin Chiu* [1] Yuntian Deng* [2] Alexander Rush [1]

[1]Cornell Tech

[2]Harvard University

October 19, 2021

Structured Models

- Explicitly model output associations
  - Directly or through latent variables
- Focus on combinatorially large latent discrete structures
  - Complementary to continuous, deterministic representations
- More difficult to scale than alternative representations
  - Bottlenecked by time and space complexity of inference

Scaling Structured Models

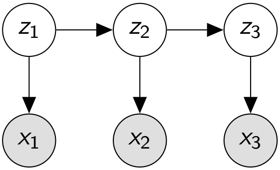
- Scaling (to the point of overparameterization) is key
- Target tractable models
  - Admit dynamic programs for exact inference
- Impose a low-rank model constraint
  - Trades off model expressivity for cheaper inference
- Only constrain parameters used in key steps of inference

Inference in Structured Models

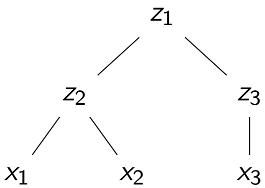
- Model an observation $x = (x_1, \ldots, x_T)$ via latent structure z
  - Latent nodes z_i
  - Nodes have discrete label set $[L]$

- Perform training and evaluation via marginalization

$$p(x) = \sum_z p(x, z)$$



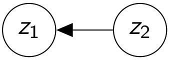
Hidden Markov models

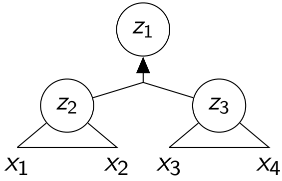


Probabilistic context-free grammars

Hypergraphs for Inference

- Represent dynamic programs for inference as hypergraphs
- Hypergraphs consist of nodes and hyperedges
  - Hyperedge consists of a head node and set of tail nodes
- Perform inference by traversing hypergraph
  - Aggregate marginals from tails to head via a matrix-vector product





Hyperedge representations for HMMs and PCFGs

Hypergraph Marginalization

For each hyperedge e in topological order,

- Combine tail marginals α_1, α_2 into joint tail marginal β_v
- Apply score matrix Ψ_e and aggregate in head marginal α_u
  - Multiple hyperedges may have the same head node

Algorithm 1 Hypergraph marginalization

---

for $u \leftarrow v$ hyperedge e topologically **do**

$\quad \beta_v \leftarrow \alpha_{v_1} \alpha_{v_2}^\top \qquad \triangleright\ O(L^{|e|})$

$\quad \alpha_u \xleftarrow{+} \Psi_e \beta_v \qquad \triangleright\ O(L^{|e|+1})$

return $\alpha_S^\top \mathbf{1}$

Scaling with Low-rank Factorizations

- Factor matrices $\Psi = UV^\top$, $U \in \mathbb{R}^{L \times R}$, $V \in \mathbb{R}^{L^{|e|} \times R}$

$$\boxed{\Psi} \times \boxed{\beta} = \boxed{U} \times \left(\boxed{V^\top} \times \boxed{\beta} \right)$$

- Two matrix-vector products of cost $O(LR)$ and $O(L^{|e|}R)$
  - Reduced from $O(L^{|e|+1})$
- Potentially large speedups for inference
  - HMM from $O(L^2)$ to $O(LR)$
  - PCFG from $O(L^3)$ to $O(L^2R)$

Low-rank Hypergraph Marginalization

Algorithm 2 Low-rank marginalization

for $u \leftarrow v_1, v_2$ hyperedge e topologically **do**

  $\beta_v \leftarrow \alpha_{v_1} \alpha_{v_2}^\top$ $\triangleright$ $O(L^{|e|})$

  $\gamma \leftarrow V_e^\top \beta_v$ $\triangleright$ $O(L^{|e|} R)$

  $\alpha_u \stackrel{+}{\leftarrow} U_e \gamma$ $\triangleright$ $O(LR)$

return $\alpha_S^\top \mathbf{1}$

Expressivity of Rank-constrained Models

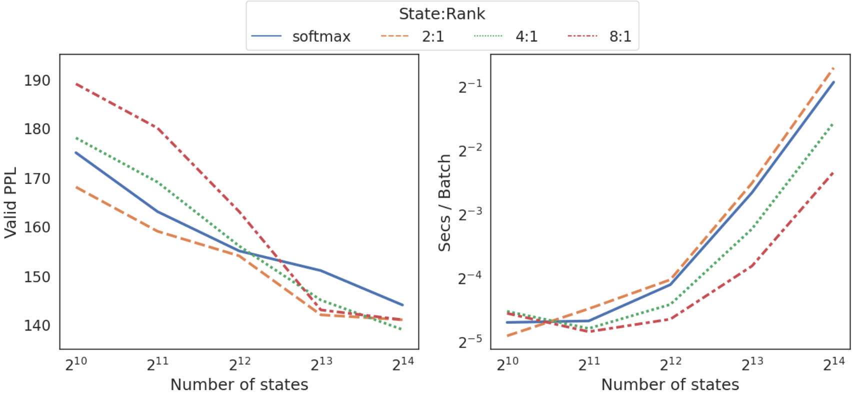
- Rank constraints limit expressivity
- Only apply to a subset of parameters
  - Transition matrix for HMMs
  - Subset of the transition matrix for PCFGs
- Is it more expressive than a smaller model?
  - An L-state HMM with rank R ($< L$) is more expressive than an unconstrained R-state HMM

Experiments

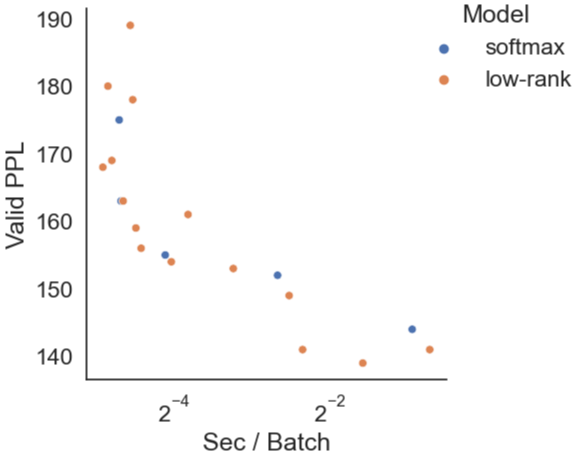
- Language modeling on PENN TREEBANK[1]
- Compare size vs speed and accuracy
  - Size = 1k to 16k state HMM, 90 to 300 state PCFG
  - Speed = Sec/Batch
  - Accuracy = Perplexity (function of likelihood)
- Unconstrained softmax HMM, PCFG vs low-rank versions

[1] Marcus, Santorini, and Marcinkiewicz, 'Building a Large Annotated Corpus of English: The Penn Treebank'.

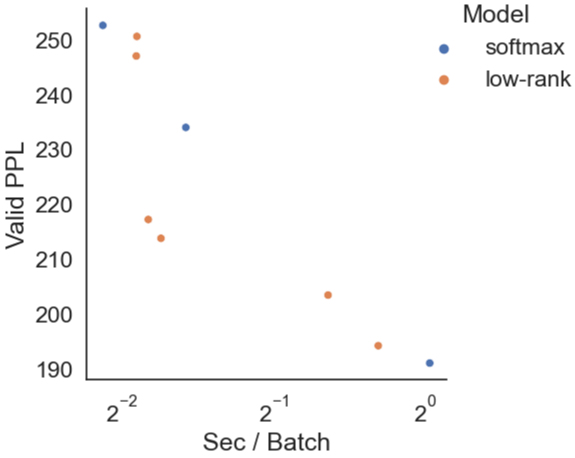
HMM Results


State:Rank
softmax
2:1
4:1
8:1
Valid PPL
190
180
170
160
150
140
Number of states
2^{10}
2^{11}
2^{12}
2^{13}
2^{14}
Secs / Batch
2^{-1}
2^{-2}
2^{-3}
2^{-4}
2^{-5}
Number of states

HMM Speed vs Accuracy Frontier



PCFG Speed vs Accuracy Frontier



Conclusion

- Introduce a low-rank factorization to speed up inference in hypergraph models
- Constrain only parameters used in inference bottlenecks
- Most effective with large models
- Please see the paper for more experiments and analysis!

Citations I

Marcus, Mitchell P., Beatrice Santorini, and Mary Ann Marcinkiewicz. 'Building a Large Annotated Corpus of English: The Penn Treebank'. In: *Computational Linguistics* 19.2 (1993), pp. 313–330. URL: https://www.aclweb.org/anthology/J93-2004.