

# Low-Rank Factorizations for Fast Inference in Structured Models

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# Structured Models

- ▶ Explicitly model output associations
  - ▶ Directly or through latent variables
- ▶ Focus on combinatorially large latent discrete structures
  - ▶ Complementary to continuous, deterministic representations
- ▶ More difficult to scale than alternative representations
  - ▶ Bottlenecked by time + space complexity of marginal inference

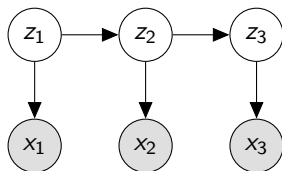
# Scaling Structured Models

- ▶ Scaling (to the point of overparameterization) is key
- ▶ Target tractable models
  - ▶ Admit dynamic programs for exact marginalization
- ▶ Impose a low-rank model constraint
  - ▶ Trades off model expressivity for cheaper marginalization
- ▶ Only constrain parameters used in key steps of marginalization

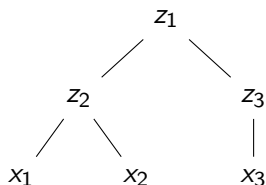
# Marginalization in Structured Models

- ▶ Model an observation  $x = (x_1, \dots, x_T)$  via latent structure  $z$ 
  - ▶ Latent nodes  $z_i$
  - ▶ Nodes have discrete label set  $[L]$
- ▶ Perform training and evaluation via marginalization

$$p(x) = \sum_z p(x, z)$$



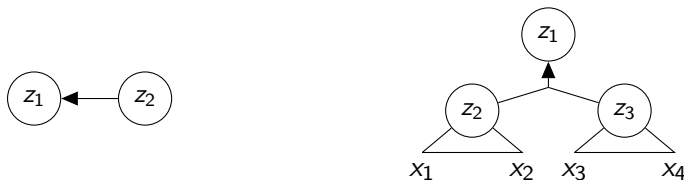
Hidden Markov models



Probabilistic context-free grammars

# Hypergraphs for Marginalization

- ▶ Represent marginalization dynamic programs as hypergraphs
- ▶ Hypergraphs consist of nodes and hyperedges
  - ▶ Hyperedge consists of a head node and set of tail nodes
- ▶ Perform marginalization by traversing hypergraph
  - ▶ Aggregate marginals from tails to head via a matrix-vector product



Hyperedge representations for HMMs and PCFGs

# Hypergraph Marginalization

For each hyperedge  $e$  in topological order,

- ▶ Combine tail marginals  $\alpha_1, \alpha_2$  into joint tail marginal  $\beta_v$
- ▶ Apply score matrix  $\Psi_e$  and aggregate in head marginal  $\alpha_u$ 
  - ▶ Matrix-vector product
  - ▶ Multiple hyperedges may have the same head node

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**Algorithm 1** Hypergraph marginalization / belief propagation

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**for**  $u \leftarrow v$  hyperedge  $e$  topologically **do**

$\beta_v \leftarrow \alpha_{v_1} \alpha_{v_2}^\top$   $\triangleright O(L^{|e|})$

$\alpha_u \stackrel{+}{\leftarrow} \Psi_e \beta_v$   $\triangleright O(L^{|e|+1})$

**return**  $\alpha_S^\top \mathbf{1}$

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# Our Method: Scaling with Low-Rank Factorizations

- ▶ Hypergraph marginalization bottlenecks
  - ▶ Number of hyperedges
  - ▶ Matrix-vector product
- ▶ Approach: Impose low-rank model constraint
- ▶ Improves time and space complexity of marginalization

# Low-Rank Factorizations

- ▶ Rank  $R < L$  factorization
- ▶ Factor matrices  $\Psi = UV^\top$ ,  $U \in \mathbb{R}^{L \times R}$ ,  $V \in \mathbb{R}^{L^{\text{el}} \times R}$

$$\boxed{\Psi} \times \boxed{\beta} = \boxed{U} \times \left( \boxed{V^\top} \times \boxed{\beta} \right)$$

- ▶ Two matrix-vector products of cost  $O(LR)$  and  $O(L^{\text{el}}R)$ 
  - ▶ Reduced from  $O(L^{\text{el}}+1)$



# Low-rank Hypergraph Marginalization

Applying the low-rank factorization,

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**Algorithm 2** Low-rank marginalization

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**for**  $u \leftarrow v_1, v_2$  hyperedge  $e$  topologically **do**

$$\beta_v \leftarrow \alpha_{v_1} \alpha_{v_2}^\top$$

$$\triangleright O(L^{|e|})$$

$$\gamma \leftarrow V_e^\top \beta_v$$

$$\triangleright O(L^{|e|}R)$$

$$\alpha_u \leftarrow^+ U_e \gamma$$

$$\triangleright O(LR)$$

**return**  $\alpha_S^\top \mathbf{1}$

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- ▶ Potentially large speedups for marginalization
  - ▶ HMM from  $O(L^2)$  to  $O(LR)$
  - ▶ PCFG from  $O(L^3)$  to  $O(L^2R)$

# Expressivity of Rank-constrained Models

- ▶ Rank constraints limit expressivity
- ▶ Only apply to a subset of parameters
  - ▶ Transition matrix for HMMs
  - ▶ Subset of the transition matrix for PCFGs
- ▶ Is it more expressive than a smaller model?
  - ▶ An  $L$ -state HMM with rank  $R$  ( $< L$ ) is more expressive than an unconstrained  $R$ -state HMM

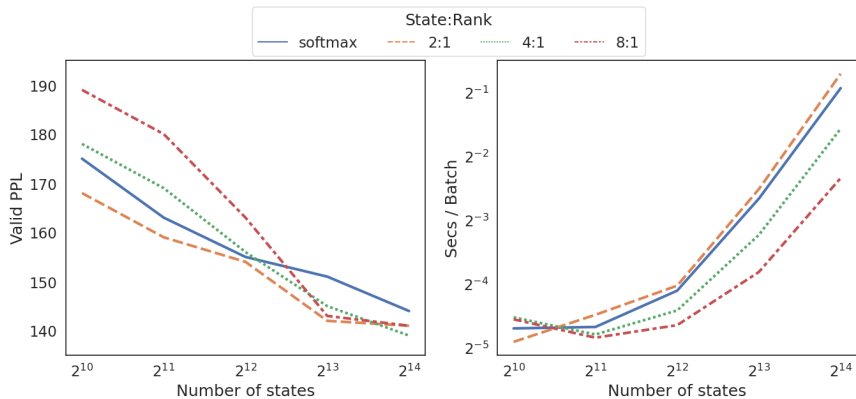
# Experiments

- ▶ Language modeling on PENN TREEBANK<sup>1</sup>
- ▶ Compare size vs speed and accuracy
  - ▶ Size = 1k to 16k state HMM, 90 to 300 state PCFG
  - ▶ Speed = Sec/Batch
  - ▶ Accuracy = Perplexity (function of likelihood)
- ▶ Unconstrained softmax HMM, PCFG vs low-rank versions
- ▶ Further experiments in paper
  - ▶ Polyphonic music modeling with HMMs
  - ▶ Video modeling with HSMMs

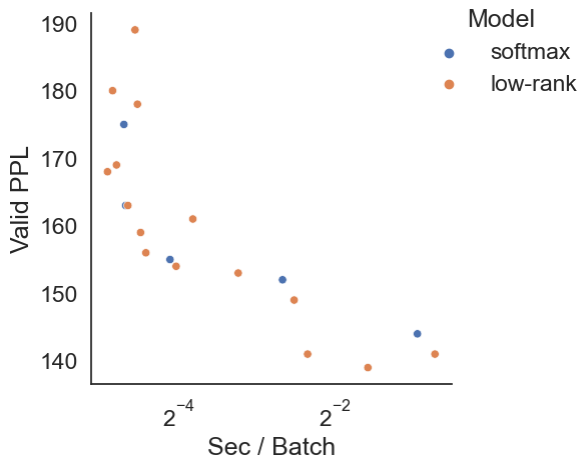
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<sup>1</sup>Marcus, Santorini, and Marcinkiewicz, 'Building a Large Annotated Corpus of English: The Penn Treebank'.

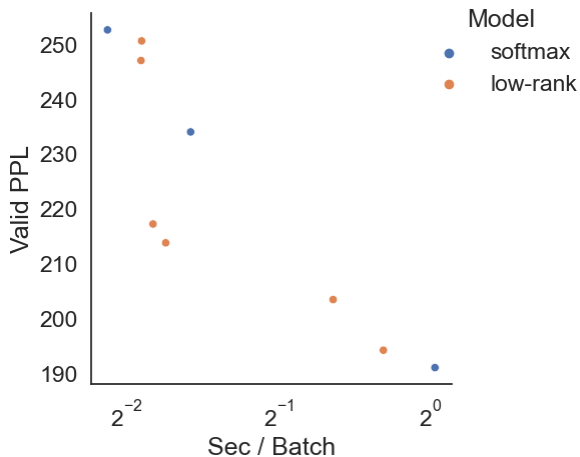
# HMM Results



# HMM Speed vs Accuracy Frontier



# PCFG Speed vs Accuracy Frontier



# Conclusion

- ▶ Low-rank factorization sped up marginalization
  - ▶ Constrain only parameters used in bottlenecks
  - ▶ Most effective with large models
- ▶ Performance gap with neural models still large
  - ▶ Scale further with more aggressive constraints
  - ▶ Compose with different representations
- ▶ Please see the paper for more experiments and analysis!

# Citations I



Marcus, Mitchell P., Beatrice Santorini, and Mary Ann Marcinkiewicz. 'Building a Large Annotated Corpus of English: The Penn Treebank'. In: *Computational Linguistics* 19.2 (1993), pp. 313–330. URL: <https://www.aclweb.org/anthology/J93-2004>.