

Title

March 20, 2018

- 1 Section Title
- 2 Language as a Latent Variable
- 3 Our Proposal: Attention as a Latent Variable

1 Section Title

2 Language as a Latent Variable

3 Our Proposal: Attention as a Latent Variable

- Attention Mechanism
- Attention as a Latent Variable

Frame Title

1 Section Title

2 Language as a Latent Variable

3 Our Proposal: Attention as a Latent Variable

- Attention Mechanism
- Attention as a Latent Variable

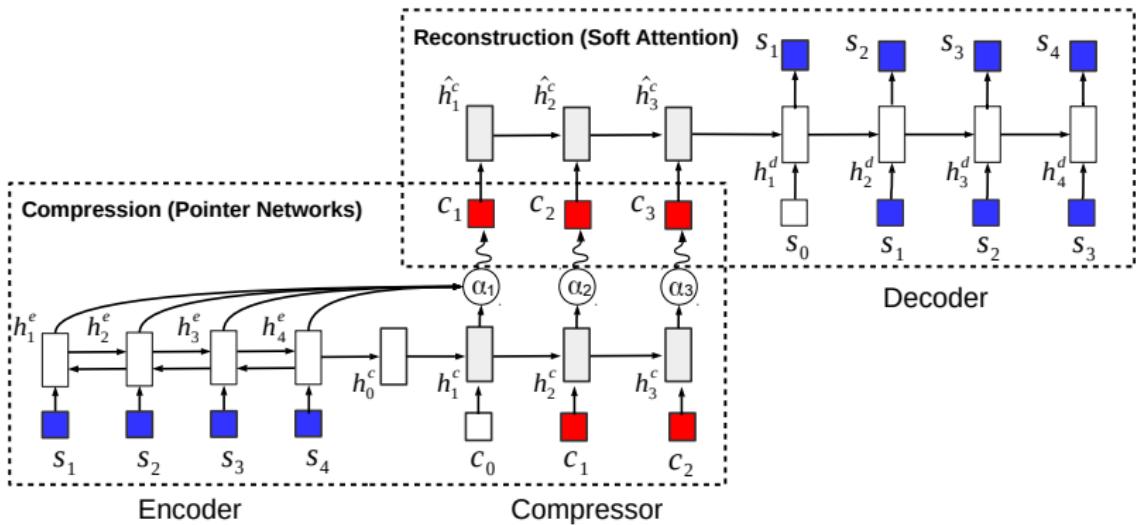
Overview of Paper

- Semi-supervised summarization
- Builds on Kingma et al. 2014's M2 (the semisupervised version)
- But uses a sequence of words as the latent representation

The Idea for Extractive Summarization

- Start with source sentence $x = \text{'I wish I could love dogs but I just hate them'}$
- Sample a summary $y = \text{'I love dogs'}$ by picking words from the source
- Reconstruct the source sentence given the summary using an attentive decoder
- Use the probability of the source sentence under the reconstruction decoder (and a couple other terms) as signal for how good the summary was

Auto-Encoding Sentence Compression



Note: Some of the parameters of the inference network are used as the decoder network's encoder, but the shared parameters are not updated using gradients from the decoder

Auto-Encoding Sentence Compression

- The inference network $q_\lambda(\mathbf{y} \mid \mathbf{x})$ uses hard attention at every timestep to select a source token
- Use a bidirectional source encoder on \mathbf{x} to get source embedding matrix $H^e = \text{BRNN}(\mathbf{x})$, whose i th element is the vector \mathbf{h}_i^e .
- Select source token \mathbf{x}_i with a pointer network ‘compressor’

$$\mathbf{h}_j^c = \text{RNN}(\mathbf{h}_{j-1}^c, \mathbf{y}_{j-1}) \quad (1)$$

$$\boldsymbol{\alpha}_j = \text{attention}(\mathbf{h}_j^c, H^e) \quad (2)$$

$$\mathbf{y}_j \sim \text{Cat}(\boldsymbol{\alpha}_j) \quad (3)$$

ASC Continued

- Let H^c be the concatenation of all the compressor hidden states
- The decoder $p_\theta(\mathbf{x} \mid \mathbf{y})$ is a conditional language model that attends over the hidden states H^c to reconstruct the original source sentence \mathbf{x}

$$\mathbf{h}_k^d = \text{RNN}(\mathbf{h}_{k-1}^d, \mathbf{x}_{k-1}) \quad (4)$$

$$\mathbf{v}_k = \text{attention}(\mathbf{h}_k^d, H^c) \quad (5)$$

$$\mathbf{d}_k = \mathbf{v}_k^T H^c \quad (6)$$

$$p_\theta(\mathbf{x}_k \mid \mathbf{x}_{<k}, \mathbf{y}) = \text{softmax}(W\mathbf{d}_k) \quad (7)$$

- (7) is quite strange, since it does not use the recurrent state. My guess is that it is a mistake from following the pointer network paper too closely (this is equation (10) in the paper).

Details and Recap

- The attention formulation from Vinyals, Fortunato, and Jaitly 2015, which is pretty close to the ‘general’ attention in Luong, Pham, and Manning 2015

$$\text{attention}(q, C) = \text{softmax}(\mathbf{v}^T \tanh(Wq + VC)) \quad (8)$$

- $p_\theta(\mathbf{x} \mid \mathbf{y})$ is the reconstructive attention based decoder
- $q_\lambda(\mathbf{y} \mid \mathbf{x})$ is the summarizing pointer network, and we omit the conditioning on \mathbf{x} when convenient (randomly)
- $p(\mathbf{y})$ is a language model prior

Marginal Likelihood

- The inference network's parameters will be denoted by λ and the decoder network's by θ
- As usual, the marginal likelihood is intractable

$$\log p(\mathbf{x}) = \log \sum_{\mathbf{y}} p_{\theta}(\mathbf{x}, \mathbf{y}) \quad (9)$$

since we cannot enumerate all possible summaries, even if they are extractive

Objective

- So we lower bound it with Jensen's inequality and maximize the ELBO \mathcal{L}

$$\log \sum_{\mathbf{y}} p(\mathbf{x}, \mathbf{y}) = \log \sum_{\mathbf{y}} q_{\lambda}(\mathbf{y}) \frac{p_{\theta}(\mathbf{x}, \mathbf{y})}{q_{\lambda}(\mathbf{y})} \quad (10)$$

$$= \log \mathbb{E}_{\mathbf{y} \sim q_{\lambda}(\mathbf{y})} \left[\frac{p_{\theta}(\mathbf{x}, \mathbf{y})}{q_{\lambda}(\mathbf{y})} \right] \quad (11)$$

$$\geq \mathbb{E}_{\mathbf{y} \sim q_{\lambda}(\mathbf{y})} \left[\log \frac{p_{\theta}(\mathbf{x}, \mathbf{y})}{q_{\lambda}(\mathbf{y})} \right] \quad (12)$$

$$= \mathbb{E}_{\mathbf{y} \sim q_{\lambda}(\mathbf{y})} [\log p_{\theta}(\mathbf{x} \mid \mathbf{y})] \quad (13)$$

$$- D_{KL}(q_{\lambda}(\mathbf{y}) \| p(\mathbf{y}))$$

$$= \mathcal{L}$$

Training Details

- The gradient of the ELBO with respect to the reconstructive decoder only depends on $p_\theta(\mathbf{x} \mid \mathbf{y})$

$$\mathcal{L} = \underbrace{\mathbb{E}_{\mathbf{y} \sim q_\lambda(\mathbf{y})} [\log p_\theta(\mathbf{x} \mid \mathbf{y})]}_{\text{1. Reconstruction}} - \underbrace{KL[q_\lambda(\mathbf{z}) \parallel p(\mathbf{z})]}_{\text{2. Regularization towards prior}}$$

- It's given by term 1

$$\nabla_\theta \mathcal{L} = \nabla_\theta \mathbb{E}_{\mathbf{y} \sim q_\lambda(\mathbf{y})} [\log p_\theta(\mathbf{x} \mid \mathbf{y})] \quad (14)$$

$$\approx \frac{1}{M} \sum_m \nabla_\theta \log p_\theta(\mathbf{x} \mid \mathbf{y}^{(m)}) \quad (15)$$

where M sample summaries are generated through ancestral sampling

Training Details

- The gradient with respect to the inference network requires REINFORCE
- We rewrite the ELBO

$$\mathcal{L} = \mathbb{E}_{\mathbf{y} \sim q_\lambda(\mathbf{y})} \left[\log \frac{p_\theta(\mathbf{x}, \mathbf{y})}{q_\lambda(\mathbf{y})} \right] \quad (16)$$

$$= \mathbb{E}_{\mathbf{y} \sim q_\lambda(\mathbf{y})} [\log p_\theta(\mathbf{x} \mid \mathbf{y}) + \log p(\mathbf{y}) - \log q_\lambda(\mathbf{y})] \quad (17)$$

$$= \mathbb{E}_{\mathbf{y} \sim q_\lambda(\mathbf{y})} [l(\mathbf{x}, \mathbf{y})] \quad (18)$$

- So we have $l(\mathbf{x}, \mathbf{y}) = \log p_\theta(\mathbf{x} \mid \mathbf{y}) + \log p(\mathbf{y}) - \log q_\lambda(\mathbf{y})$

REINFORCE

- Recall the score function gradient estimator

$$p(\mathbf{x}) \nabla \log p(\mathbf{x}) = \nabla p(\mathbf{x}) \quad (19)$$

- We use this to find an approximation of

$$\nabla_{\lambda} \mathcal{L} = \nabla_{\lambda} \mathbb{E}_{\mathbf{y} \sim q_{\lambda}(\mathbf{y})} [l(\mathbf{x}, \mathbf{y})] \quad (20)$$

$$= \sum_{\mathbf{y}} l(\mathbf{x}, \mathbf{y}) \nabla_{\lambda} q_{\lambda}(\mathbf{y}) \quad (21)$$

$$= \sum_{\mathbf{y}} q_{\lambda}(\mathbf{y}) l(\mathbf{x}, \mathbf{y}) \nabla_{\lambda} \log q_{\lambda}(\mathbf{y}) \quad (22)$$

$$= \mathbb{E}_{\mathbf{y} \sim q_{\lambda}(\mathbf{y})} [l(\mathbf{x}, \mathbf{y}) \nabla_{\lambda} \log q_{\lambda}(\mathbf{y})] \quad (23)$$

Details

- They also train a baseline to predict $l(\mathbf{x}, \mathbf{y})$ for variance reduction (control variate)
- They use a variant of KL annealing and augment the loss as follows

$$l(\mathbf{x}, \mathbf{y}) = \log p_\theta(\mathbf{x} \mid \mathbf{y}) + \lambda(\log p(\mathbf{y}) - \log q_\lambda(\mathbf{y}))$$

with $\lambda = 0.1$ without justification, but note that increasing λ results in shorter summaries \mathbf{y} since the prior $p(\mathbf{y})$ prefers shorter sequences

Questions

- Why don't we use

$$\log \sum_{\mathbf{y}} p(\mathbf{y}) p_{\theta}(\mathbf{x} \mid \mathbf{y}) \geq \sum_{\mathbf{y}} \mathbb{E}_{\mathbf{y} \sim p(\mathbf{y})} [\log p_{\theta}(\mathbf{x} \mid \mathbf{y})]$$

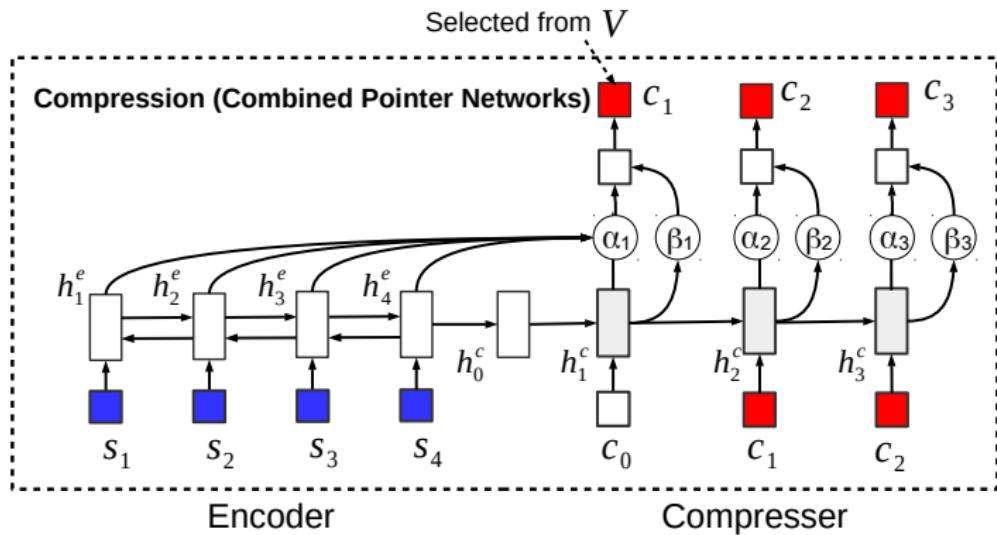
instead of introducing $q_{\lambda}(\mathbf{y})$ if we're using REINFORCE anyway?

- Which parts of the reward

$$l(\mathbf{x}, \mathbf{y}) = \log p_{\theta}(\mathbf{x} \mid \mathbf{y}) + \log p(\mathbf{y}) - \log q_{\lambda}(\mathbf{y} \mid \mathbf{x})$$

can we decompose to try to lower variance a bit more?

Forced Sentence Compression



Forced Sentence Compression

- The FSC generates from a mixture of the pointer network and a distribution over the full vocabulary
- Recall the pointer network attentions are α_j at time step j
- Let the distribution over the vocabulary be $\beta_j = \text{softmax}(W\mathbf{h}_j^c)$
- They use the weighted context $\eta_j = \alpha_j^T H^e$ as well as the current hidden state \mathbf{h}_j^c as a copy gate $\mathbf{t}_j = \sigma(\eta_j^T M \mathbf{h}_j^c)$
- The probability of a word in the summary is then

$$p(\mathbf{y}_j | \mathbf{y}_{<j}, \mathbf{x}) = \begin{cases} \mathbf{t}_j \alpha_j(i) + (1 - \mathbf{t}_j) \beta_j(\mathbf{y}_j), & \mathbf{y}_j = \mathbf{x}_i \\ (1 - \mathbf{t}_j) \beta_j(\mathbf{y}_j), & \mathbf{y}_j \notin \mathbf{x}_i \end{cases} \quad (24)$$

Connection to MIXER

- lol

Title

- lol

- 1 Section Title
- 2 Language as a Latent Variable
- 3 Our Proposal: Attention as a Latent Variable
 - Attention Mechanism
 - Attention as a Latent Variable

1 Section Title

2 Language as a Latent Variable

3 Our Proposal: Attention as a Latent Variable

- Attention Mechanism
- Attention as a Latent Variable

Attention Mechanism: Basics

- Source Input: $\mathbf{x} = \{x_1, x_2, \dots, x_S\}$
- Target Output: $\mathbf{y} = \{y_1, y_2, \dots, y_T\}$
- Encoded Source Features: $\text{enc}(\mathbf{x}) = \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\}$

Attention Mechanism: Basics

- Source Input: $\mathbf{x} = \{x_1, x_2, \dots, x_S\}$
- Target Output: $\mathbf{y} = \{y_1, y_2, \dots, y_T\}$
- Encoded Source Features: $\text{enc}(\mathbf{x}) = \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\}$

$$P(\mathbf{y}|\mathbf{x}) = \prod_{j=1}^T P(y_j|y_{<i}, \mathbf{x})$$

Attention Mechanism: Basics

- Source Input: $\mathbf{x} = \{x_1, x_2, \dots, x_S\}$
- Target Output: $\mathbf{y} = \{y_1, y_2, \dots, y_T\}$
- Encoded Source Features: $\text{enc}(\mathbf{x}) = \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\}$

$$\begin{aligned} P(\mathbf{y}|\mathbf{x}) &= \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{x}) \\ &= \prod_{j=1}^T P(y_j|y_{<j}, \text{enc}(\mathbf{x})) \\ &= \prod_{j=1}^T P(y_j|y_{<j}, \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\}) \end{aligned}$$

Attention Mechanism: Basics

- Recall that

$$P(\mathbf{y}|\mathbf{x}) = \prod_{j=1}^T P(y_j | y_{<j}, \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\})$$

Attention Mechanism: Basics

- Recall that

$$P(\mathbf{y}|\mathbf{x}) = \prod_{j=1}^T P(y_j|y_{<j}, \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\})$$

- $\{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\}$ is of varying lengths, in order to get fixed length vector, use weighted sum:

$$P(y_j|y_{<j}, \{\mathbf{h}_1, \dots, \mathbf{h}_S\}) = P(y_j|y_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i)$$

Attention Mechanism: Basics

- Recall that

$$P(\mathbf{y}|\mathbf{x}) = \prod_{j=1}^T P(y_j|y_{<j}, \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\})$$

- $\{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\}$ is of varying lengths, in order to get fixed length vector, use weighted sum:

$$P(y_j|y_{<j}, \{\mathbf{h}_1, \dots, \mathbf{h}_S\}) = P(y_j|y_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i)$$

- The attention weights $\mathbf{a}_j = \{a_{j1}, a_{j2}, \dots, a_{jS}\}$ lay on a simplex

Attention Mechanism: Basics

- Recall that

$$P(\mathbf{y}|\mathbf{x}) = \prod_{j=1}^T P(y_j|y_{<j}, \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\})$$

- $\{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\}$ is of varying lengths, in order to get fixed length vector, use weighted sum:

$$P(y_j|y_{<j}, \{\mathbf{h}_1, \dots, \mathbf{h}_S\}) = P(y_j|y_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i)$$

- The attention weights $\mathbf{a}_j = \{a_{j1}, a_{j2}, \dots, a_{jS}\}$ lay on a simplex
- \mathbf{a}_j are modeled (discriminatively) via a neural network f :

$$\mathbf{a}_j = f(y_{<i}, \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\})$$

Attention Mechanism: Interpretation as Alignments

- Recall that

$$P(y_j|y_{<j}, \{\mathbf{h}_1, \dots, \mathbf{h}_S\}) = P(y_j|y_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i)$$

- $P(y_j|y_{<j}, \mathbf{x})$ is related to \mathbf{x} only through $\sum_{i=1}^S a_{ji} \mathbf{h}_i$
- The larger a_{ji} , the closer \mathbf{h}_i to $\sum_{i=1}^S a_{ji} \mathbf{h}_i$
- a_j can be interpreted as how target word j is aligned to source words $\{x_1, \dots, x_S\}$

1 Section Title

2 Language as a Latent Variable

3 Our Proposal: Attention as a Latent Variable

- Attention Mechanism
- Attention as a Latent Variable

Attention as a Latent Variable

- Formally treat attention (alignments) as a latent variable
- Introduce prior over attention

Attention as a Latent Variable

- Formally treat attention (alignments) as a latent variable
- Introduce prior over attention
- Still feed decoder with $\sum_{i=1}^S a_{ji} \mathbf{h}_i$

Attention as a Latent Variable

- Formally treat attention (alignments) as a latent variable
- Introduce prior over attention
- Still feed decoder with $\sum_{i=1}^S a_{ji} \mathbf{h}_i$

$$P(y_j, \mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

Attention as a Latent Variable

- Formally treat attention (alignments) as a latent variable
- Introduce prior over attention
- Still feed decoder with $\sum_{i=1}^S a_{ji} \mathbf{h}_i$

$$\begin{aligned} P(y_j, \mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ = P(y_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}, \mathbf{a}_j) P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \end{aligned}$$

Attention as a Latent Variable

- Formally treat attention (alignments) as a latent variable
- Introduce prior over attention
- Still feed decoder with $\sum_{i=1}^S a_{ji} \mathbf{h}_i$

$$\begin{aligned} P(y_j, \mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ = P(y_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}, \mathbf{a}_j) P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ = P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \end{aligned}$$

Attention as a Latent Variable

- Recall that

$$\begin{aligned} & P(y_j, \mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ &= P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \end{aligned}$$

Attention as a Latent Variable

- Recall that

$$\begin{aligned} P(y_j, \mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ = P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \end{aligned}$$

- But we do not observe attentions \mathbf{a}

Attention as a Latent Variable

- Recall that

$$\begin{aligned} P(y_j, \mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ = P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \end{aligned}$$

- But we do not observe attentions \mathbf{a}
- We need to marginalize \mathbf{a} :

$$\log P(\mathbf{y} | \mathbf{x}) = \log \int_{\mathbf{a}} \prod_{j=1}^T P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

Attention as a Latent Variable

- Recall that

$$\begin{aligned} P(y_j, \mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ = P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \end{aligned}$$

- But we do not observe attentions \mathbf{a}
- We need to marginalize \mathbf{a} :

$$\log P(\mathbf{y} | \mathbf{x}) = \log \int_{\mathbf{a}} \prod_{j=1}^T P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

- Normal attention is a special case if we set:

$$\begin{aligned} P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ = \delta(f(y_{<j}, \{\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_S\})) \end{aligned}$$

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x}) = \log \int_{\mathbf{a}} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x}) = \log \int_{\mathbf{a}} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

- Generally intractable due to the integral

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x}) = \log \int_{\mathbf{a}} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

- Generally intractable due to the integral
- Use VAE by introducing an approximate posterior $Q(\mathbf{a}|\mathbf{y}, \mathbf{x})$:

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x}) = \log \int_{\mathbf{a}} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

- Generally intractable due to the integral
- Use VAE by introducing an approximate posterior $Q(\mathbf{a}|\mathbf{y}, \mathbf{x})$:

$$\log P(\mathbf{y}|\mathbf{x})$$

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x}) = \log \int_{\mathbf{a}} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

- Generally intractable due to the integral
- Use VAE by introducing an approximate posterior $Q(\mathbf{a}|\mathbf{y}, \mathbf{x})$:

$$\log P(\mathbf{y}|\mathbf{x})$$

$$= \log \int_{\mathbf{a}} Q(\mathbf{a}) \frac{1}{Q(\mathbf{a})} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x}) = \log \int_{\mathbf{a}} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

- Generally intractable due to the integral
- Use VAE by introducing an approximate posterior $Q(\mathbf{a}|\mathbf{y}, \mathbf{x})$:

$$\begin{aligned} & \log P(\mathbf{y}|\mathbf{x}) \\ &= \log \int_{\mathbf{a}} Q(\mathbf{a}) \frac{1}{Q(\mathbf{a})} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ &\geq \int_{\mathbf{a}} Q(\mathbf{a}) \log \frac{1}{Q(\mathbf{a})} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \end{aligned}$$

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x}) = \log \int_{\mathbf{a}} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$$

- Generally intractable due to the integral
- Use VAE by introducing an approximate posterior $Q(\mathbf{a}|\mathbf{y}, \mathbf{x})$:

$$\begin{aligned} & \log P(\mathbf{y}|\mathbf{x}) \\ &= \log \int_{\mathbf{a}} Q(\mathbf{a}) \frac{1}{Q(\mathbf{a})} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ &\geq \int_{\mathbf{a}} Q(\mathbf{a}) \log \frac{1}{Q(\mathbf{a})} \prod_{j=1}^T P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) \\ &= \mathbb{E}_{\mathbf{a} \sim Q} [-\log Q(\mathbf{a}) + \sum_{j=1}^T \log P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) + \log P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x})] \end{aligned}$$

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x})$$

$$\begin{aligned}&= \mathbb{E}_{\mathbf{a} \sim Q} [-\log Q(\mathbf{a}) + \sum_{j=1}^T \log P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) + \log P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x})] \\&= \underbrace{\mathbb{E}_{\mathbf{a} \sim Q} [\sum_{j=1}^T \log P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i)]}_{\text{Fit data}} + \underbrace{KL(Q(\mathbf{a}) || P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}))}_{Q(a) \text{ be close to prior } P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x})}\end{aligned}$$

- Apply reparameterization trick by specifying reparamterizable $Q(\mathbf{a}|\mathbf{y}, \mathbf{x})$: Dirichlet or Logistic Normal

Attention as a Latent Variable: Training

- Dirichlet:

$$\mathbf{a}_j \sim \text{Dir}(\alpha_1, \dots, \alpha_S)$$

- Mean: $\frac{\alpha_1}{\alpha_0}, \dots, \frac{\alpha_S}{\sum_i \alpha_0}$ where $\alpha_0 = \sum_i \alpha_i$
- Variance: $\frac{\alpha_1(\alpha_0 - \alpha_1)}{\alpha_0^2(\alpha_0 + 1)}, \dots, \frac{\alpha_S(\alpha_0 - \alpha_S)}{\alpha_0^2(\alpha_0 + 1)}$
- Use inference network to generate the mean $\frac{\alpha_1}{\alpha_0}, \dots, \frac{\alpha_S}{\sum_i \alpha_0}$ and α_0 .

Attention as a Latent Variable: Training

- Dirichlet:

$$\mathbf{a}_j \sim \text{Dir}(\alpha_1, \dots, \alpha_S)$$

- Mean: $\frac{\alpha_1}{\alpha_0}, \dots, \frac{\alpha_S}{\sum_i \alpha_0}$ where $\alpha_0 = \sum_i \alpha_i$
- Variance: $\frac{\alpha_1(\alpha_0 - \alpha_1)}{\alpha_0^2(\alpha_0 + 1)}, \dots, \frac{\alpha_S(\alpha_0 - \alpha_S)}{\alpha_0^2(\alpha_0 + 1)}$
- Use inference network to generate the mean $\frac{\alpha_1}{\alpha_0}, \dots, \frac{\alpha_S}{\sum_i \alpha_0}$ and α_0 .
- Logistic Normal

$$\mathbf{r}_j \sim \mathcal{N}(\mu, \Sigma)$$

$$\mathbf{a}_j = \text{softmax}(\mathbf{r}_j)$$

- Use inference network to generate μ and Σ

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x})$$

$$= \mathbb{E}_{\mathbf{a} \sim Q} [-\log Q(\mathbf{a}) + \sum_{j=1}^T \log P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) + \log P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x})]$$

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x})$$

$$= \mathbb{E}_{\mathbf{a} \sim Q} [-\log Q(\mathbf{a}) + \sum_{j=1}^T \log P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) + \log P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x})]$$

- Sampling from Q is equivalent to sampling from a simple distribution \mathcal{U} followed by transformation g_ϕ :

$$\epsilon_j \sim \mathcal{U}$$

$$\mathbf{a}_j = g_\phi(\epsilon_j)$$

Attention as a Latent Variable: Training

- Recall that

$$\log P(\mathbf{y}|\mathbf{x})$$

$$= \mathbb{E}_{\mathbf{a} \sim Q} [-\log Q(\mathbf{a}) + \sum_{j=1}^T \log P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i) + \log P(\mathbf{a}_j | y_{<j}, \mathbf{a}_{<j}, \mathbf{x})]$$

- Sampling from Q is equivalent to sampling from a simple distribution \mathcal{U} followed by transformation g_ϕ :

$$\epsilon_j \sim \mathcal{U}$$

$$\mathbf{a}_j = g_\phi(\epsilon_j)$$

- The objective can be written as

$$\mathbb{E}_{\epsilon_1, \dots, \epsilon_T \sim \mathcal{U}} \sum_{j=1}^T \log P(y_j | y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S (g_\phi(\epsilon_j))_i \mathbf{h}_i) + \log P(g_\phi(\epsilon_j) | y_{<j}, \mathbf{a}_{<j}, \mathbf{x}) + \mathcal{H}(Q)$$

Attention as a Latent Variable: Inference

- Intractable to decode directly due to the integral w.r.t. \mathbf{a}
- First, we can find the joint argmax of \mathbf{a} and \mathbf{y} using beam search (stepwise) and back-propagation for optimizing w.r.t. \mathbf{a}
- Alternatively, we can use the model for rescoring

Attention as a Latent Variable: Inference

- Intractable to decode directly due to the integral w.r.t. \mathbf{a}
- First, we can find the joint argmax of \mathbf{a} and \mathbf{y} using beam search (stepwise) and back-propagation for optimizing w.r.t. \mathbf{a}
- Alternatively, we can use the model for rescoring

Attention as a Latent Variable: Network Structure

- $P(y_j|y_{<j}, \mathbf{a}_{<j}, \sum_{i=1}^S a_{ji} \mathbf{h}_i)$: vanilla decoder
- $P(\mathbf{a}_j|y_{<j}, \mathbf{a}_{<j}, \mathbf{x})$: vanilla attention network: the mean of \mathbf{a}_j , MLP with pooling over time: parameter controlling variance (σ^2 in logistic normal or α_0 for Dirichlet).
- Inference network Q : normalized dot-products of embeddings: mean, MLP with pooling: variance

Attention as a Latent Variable: Conclusion

The proposed framework enjoys three-fold benefits:

- alleviate decoder's burden to learn alignment model with a proper approximate attention posterior
- the attention network gets better supervision with the KL term
- more flexible compared to vanilla attention

References I

-  Kingma, Diederik P. et al. (2014). "Semi-Supervised Learning with Deep Generative Models". In: *CoRR* abs/1406.5298. arXiv: 1406.5298. URL: <http://arxiv.org/abs/1406.5298>.
-  Luong, Minh-Thang, Hieu Pham, and Christopher D. Manning (2015). "Effective Approaches to Attention-based Neural Machine Translation". In: *Proceedings of EMNLP*.
-  Vinyals, Oriol, Meire Fortunato, and Navdeep Jaitly (2015). "Pointer Networks". In: *Proceedings of NIPS*.