## Scaling Hidden Markov Models

March 12, 2021

#### Hidden Markov Models in NLP

- ► Simplest latent variable models for time series data
- Are thought to be very poor language models
- We show they are not!

## Scaling HMMs with Emission Sparsity

## Lessons from Large Neural Language Models

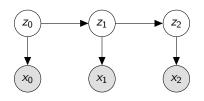
Large models perform better but are ...

- 1. Slow to train
- 2. Prone to overfitting

We must overcome these issues when scaling HMMs

#### **HMMs**

For times t, model states  $z_t \in [Z]$ , and tokens  $x_t \in [X]$ ,



We wish to optimize

$$p(x) = \sum_{z} p(x, z),$$

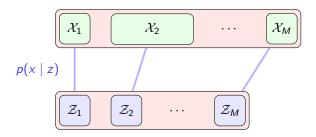
computed via the forward algorithm

## 3 Techniques for Training Large HMMs

- ▶ Block-sparse emission constraints
  - ♠ Speed
- Compact neural parameterization
  - **Generalization**
- State dropout
  - **↑** Speed **↑** Generalization

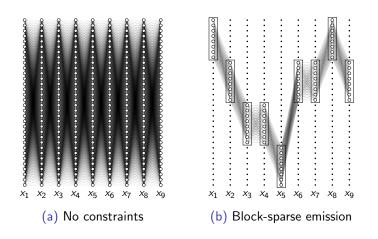
## Technique 1: Block-Sparse Emission Constraints

- Reduce cost of marginalization by enforcing structure
- Partition words and states jointly
- Words can only be emit by states in same group



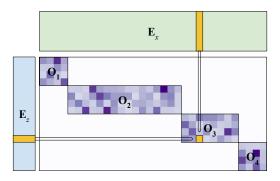
## Block-Sparse Emissions: Effect on Inference

Given each word  $x_t$ , only the states in the correct group can occur



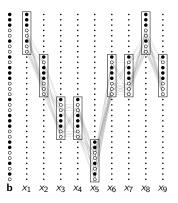
## Technique 2: Neural Parameterization

- A neural parameterization allows for parameter sharing
- Generate conditional distributions from state E<sub>z</sub> and token representations E<sub>x</sub>



## Technique 3: State Dropout

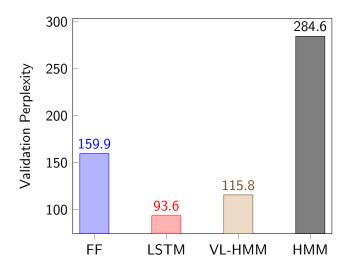
- State dropout encourages broad state usage
- ightharpoonup At each batch, sample dropout mask  $\mathbf{b} \in \{0,1\}^Z$



#### **Experiments**

- Language modeling on Penn Treebank
- Baselines
  - Feedforward 5-gram model
  - 2-layer LSTM
  - A 900 state HMM (Buys et al 2018)
- Model
  - ▶ 2<sup>15</sup> (32k) state very large HMM (VL-HMM)
  - M = 128 groups (256 states per type), obtained via Brown Clustering
  - Dropout rate of 0.5 during training

#### Results on PTB Validation Data



#### State Size Ablation



Validation perplexity on PTB by state size ( $\lambda = 0.5$  and M = 128)

## Other Ablations

Model	Param	Train	Val
VL-HMM (2 <sup>14</sup> )	7.2M	115	134
- neural param	423M	119	169
- state dropout	7.2M	88	157

# Scaling HMMs with Kernel Methods

#### **Embedded Structure Prediction**

▶ The previous work relied on emission sparsity constraints

- Can we scale with weaker assumptions?
- Reduce the quadratic dependence of inference on number of states to linear with kernel trick

#### Generalized Softmax

Focusing on the transition distribution,

Softmax

$$p(z_t \mid z_{t-1}) = \frac{\exp(\mathbf{u}_{z_{t-1}}^{\top} \mathbf{v}_{z_t})}{\sum_{z} \exp(\mathbf{u}_{z_{t-1}}^{\top} \mathbf{v}_{z})}$$

Generalized Softmax

$$p(z_t \mid z_{t-1}) = \frac{K(\mathbf{u}, \mathbf{v})}{\sum_z K(\mathbf{u}, \mathbf{v}_z)} = \frac{\phi(\mathbf{u})^\top \phi(\mathbf{v})}{\sum_z \phi(\mathbf{u})^\top \phi(\mathbf{v}_z)},$$

for positive kernel  $K: \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}_+$  and feature map  $\phi: \mathbb{R}^d \to \mathbb{R}^f$ 

#### Inference

▶ The key  $O(Z^2)$  step in the forward algorithm:

$$p(z_t \mid x_{< t}) = \sum_{z_{t-1}} p(z_t \mid z_{t-1}) p(z_t \mid x_{< t})$$

In matrix form,

$$oldsymbol{\gamma}_t = \underbrace{lpha_{t-1}}_{\mathbb{R}^Z} \underbrace{igwedge_{\mathbb{R}^{Z imes Z}}}_{\mathbb{R}^{Z imes Z}},$$

where

$$egin{aligned} [\gamma_t]_{z_t} &= p(z_t \mid x_{< t}) \ [lpha_{t-1}]_{z_{t-1}} &= p(z_{t-1} \mid x_{< t}) \ [\Lambda]_{z_{t-1}, z_t} &= p(z_t \mid z_{t-1}) \end{aligned}$$

#### **Embedded Inference**

Use generalized softmax in transition distribution

$$[\Lambda]_{z_{t-1},z_t} = p(z_t \mid z_{t-1}) \propto \phi(\mathbf{u}_{z_{t-1}})^{\top} \phi(\mathbf{v}_{z_t})$$

▶ In matrix form,

$$\gamma_t = \alpha_{t-1} \Lambda = \underbrace{\alpha_{t-1}}_{\mathbb{R}^Z} \underbrace{\operatorname{diag}(d)}_{\mathbb{R}^{Z \times Z}} \underbrace{\phi(U)}_{\mathbb{R}^{Z \times f}} \underbrace{\phi(V)^{\top}}_{\mathbb{R}^{f \times Z}},$$

with stacked embeddings  $\phi(U), \phi(V)$  and normalizing constants d

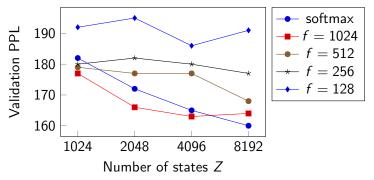
► Takes *O*(*Zf* ) time from left to right!

## **Experiments**

- Language modeling on PTB
- ▶ Work directly with feature map  $\phi(\mathbf{x}) = \exp\left(W\mathbf{x} \frac{\|\mathbf{x}\|^2}{2}\right)$ , with learned  $W \in \mathbb{R}^{d \times f}$

No dropout or sparsity constraints

#### Results on PTB Validation



- ► Holding number of features fixed, perplexity mostly improves or remains the same with an increasing number of states
- ► Achieve similar performance as softmax with around 4:1 state to feature ratio
- Results are a bit noisy, hoping dropout will clean things up
- ► Currently running that plus larger state sizes

#### Conclusion

- ► HMMs are competitive language models
- Introduced 4 techniques for tackling speed and overfitting
- A great time to revisit other discrete latent variable models

## EOS

#### Citations