Scaling Hidden Markov Language Models

April 30, 2021

Language Modeling

'How BERT and GPT models change the game for NLP'

Demi Ajayi (IBM)

Able to achieve strong performance in modern NLP tasks with

- Large, opaque models
- Pretrained via language modeling on large data
- Fine-tuned for downstream task

Language modeling either has useful information for or is a necessary component of downstream tasks

Language Modeling

How now, brown ____

- Given the words seen so far, predict the next word
- Requires encoding long-range context

Language Models

Modern language models are primarily

- Transformers
  - Encode context into a continuous vector with stacks of attention-based neural networks
- Recurrent neural networks
  - Encode context into a continuous vector with stacks of nonlinear dynamical systems

Both are difficult to interpret due to model complexity

Hidden Markov Language Models

- Interpretability as a design decision
- Context is encoded as a single integer
- Interpretable, but thought to be poor language models[1]

[1] Buys, Bisk, and Choi, 'Bridging HMMs and RNNs through Architectural Transformations'.

Research Question

To what extent is the performance of HMMs limited by scale and choices in parameterization?

This work: Scale HMMs on language modeling using techniques drawn from recent advances in neural networks

Background: Hidden Markov Models

Hidden Markov Models (HMMs)

- Classical models for unsupervised per-word tag induction
  - Part-of-speech induction[2]
  - Word alignment for translation[3]
- Admits tractable exact inference
  - Strong conditional independence assumptions
  - Simple transition dynamics
  - Finite set of discrete latent states

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[2] Merialdo, 'Tagging English Text with a Probabilistic Model'.

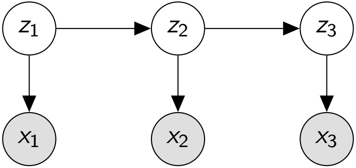
[3] Vogel, Ney, and Tillmann, 'HMM-Based Word Alignment in Statistical Translation'.

HMM State Sizes

Year	Data	Model	States
1989	Phoneme Segmentation	HMM	7
1994	POS	HMM	76
2005	Activity Monitoring	DMC HMM	1k
2006	2D Image Tracking	Convolutional HMM	100k
2009	LM	Split-POS HMM	450
2016	POS	Neural HMM	37
2016	Web Traffic Analysis	FFT HMM	81
2019	Char LM	Cloned HMM	30k

Hidden Markov Models (HMMs)

For times t, model states $z_t \in [Z]$, and tokens $x_t \in [X]$,



This yields the joint distribution

$$p(x, z) = \prod_t p(x_t \mid z_t) p(z_t \mid z_{t-1})$$

with

start state	$p(z_1)$,
transitions	$p(z_t \mid z_{t-1})$,
and emissions	$p(x_t \mid z_t)$

represented as vectors and matrices

Inference

Given observed $x = (x_1, \ldots, x_T)$ We wish to maximize

$$p(x) = \sum_{z_1} \cdots \sum_{z_T} p(x, z) = \alpha_1^\top \Lambda_2 \Lambda_3 \cdots \Lambda_T \mathbf{1},$$

where we have the

$$\begin{aligned} &\text{start,} && [\alpha_1]_{z_1} = p(x_1 \mid z_1) p(z_1), \\ &\text{and transition operators,} && [\Lambda_t]_{z_{t-1}, z_t} = p(x_t \mid z_t) p(z_t \mid z_{t-1}) \end{aligned}$$

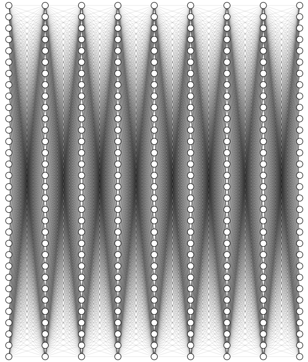
- Matrix representation of forward algorithm:

$$\alpha_t = \alpha_{t-1} \Lambda_t$$

- Requires $O(TZ^2)$ operations in total!

Inference

$$p(x) = \alpha_1^\top \Lambda_2 \cdots \Lambda_T \mathbf{1}$$



- Each node corresponds to a state
- Each edge to an entry in the transition operator matrix

Scaling HMMs

Lessons from Large Neural Language Models

Large models perform better but are ...

1. Slow to train
2. Prone to overfitting

We must overcome these issues when scaling HMMs

3 Techniques for Training Large HMMs

- Compact neural parameterization
  - ⬆ Generalization
- State dropout
  - ⬆ Speed ⬆ Generalization
- Block-sparse emission constraints
  - ⬆ Speed
- Will cover a fourth in the second part of this talk

Technique 1: Neural Parameterization

- Transition and emission matrices have Z^2 and ZX entries
- More states lead to explosion in parameter count
- Solution: Low dimensional factorization

Neural Parameterization: Softmax Parameterization

The transition matrix A is factorized as follows:

$$A \propto \exp\left(U \times V^\top \right)$$

with state embeddings $U, V \in \mathbb{R}^{Z \times D}$

- Can further parameterize U or $V = \mathrm{MLP}(E_u)$
- Similar for emissions

Technique 2: State Dropout

- Dropout is a common technique for regularizing neural networks[4]
  - Reduces a network's reliance on any particular neuron by via random masking
- Extend dropout to the states of an HMM
  - Encourage broad utilization of all states

[4] Srivastava et al., 'Dropout: A Simple Way to Prevent Neural Networks from Overfitting'.

State Dropout

- At each batch, sample dropout mask $\mathbf{b} \in \{0, 1\}^Z$
- Compute distributional parameters by indexing into embeddings U, V

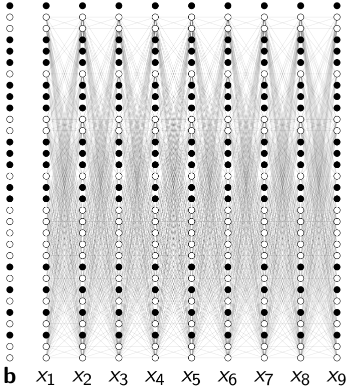
$$\left(\mathbf{b} \circ \boxed{U_{\text{trans}}}\right) \times \left(\mathbf{b} \circ \boxed{V_{\text{trans}}}\right)^\top$$

(a) Unnormalized transition logits

$$\left(\mathbf{b} \circ \boxed{U_{\text{emit}}}\right) \times \boxed{V_{\text{emit}}}^\top$$

(b) Unnormalized emission logits

State Dropout: Inference



- Shaded nodes depict dropped states
- Ignore dropped states during inference

Technique 3: Block-Sparse Emission Constraints

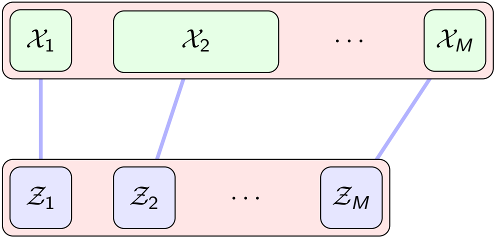
- Reduce cost of marginalization by enforcing structure
- Introduce emission constraints inspired by Cloned HMMs[5]
- Only allow each word to be emit by a subset of states
- Cost of inference is quadratic in the size of the largest subset due to sparsity

[5] Dedieu et al., *Learning higher-order sequential structure with cloned HMMs*.

Block-Sparse Emission Constraints: Alignment

Start with a joint partitioning of both states and words

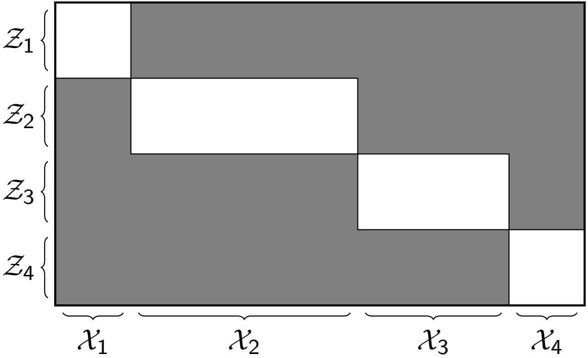
Indices $m \in [M]$ State partitions $\mathcal{Z}_m$ Word partitions $\mathcal{X}_m$



Block-Sparse Emission Constraints

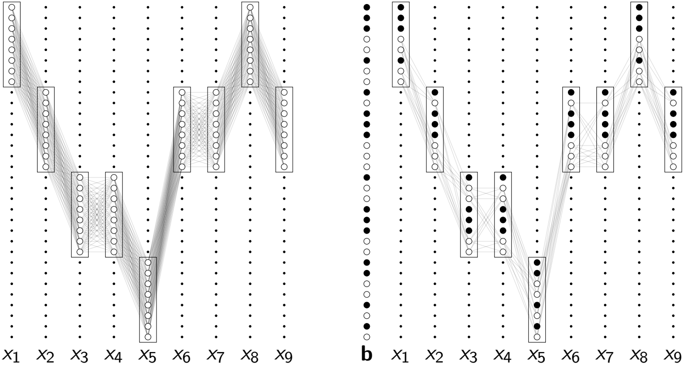
Given the unnormalized emission logits,

- Mask out unaligned state-word entries
- Normalize rows across words in aligned partition



Block-Sparse Emissions: Inference

Given each word x_t, only the states in the correct group can occur



(a) Block-sparse emission

(b) With state dropout

Method Recap

- Compact neural parameterization

  ⇧ Generalization

- State dropout

  ⇧ Speed ⇧ Generalization

- Block-sparse emission constraints

  ⇧ Speed

- A fourth after experiments

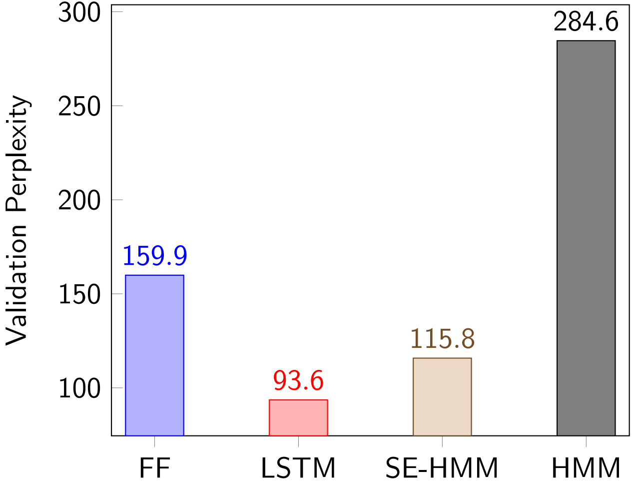
Experiments

Experiments

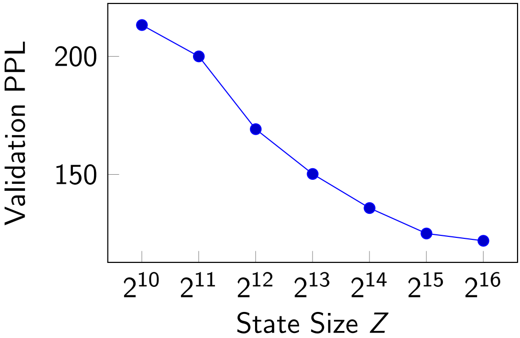
- Language modeling on Penn Treebank
- Evaluate perplexity
  - Function of $p(x)$
  - Lower is better
- Baselines
  - Feedforward 5-gram model
  - 2-layer LSTM
  - A 900 state HMM[6]
- Model
  - 2^{15} (32k) state spare emission HMM (SE-HMM)
  - $M = 128$ groups (256 states per group), obtained via Brown Clustering
  - Dropout rate of 0.5 during training

[6] Buys, Bisk, and Choi, 'Bridging HMMs and RNNs through Architectural Transformations'.

Results on PTB Validation Data



State Size Ablation



Validation perplexity on PTB by state size ($\lambda = 0.5$ and $M = 128$)

Other Ablations

Model	Param	Train	Val
SE-HMM (2^{14})	7.2M	115	134
- neural param	423M	119	169
- state dropout	7.2M	88	157

Discussion

- Greatly scaled the state size of HMMs
- Performance improved with increasing state size
- Still a large gap between RNNs and HMMs
- Does the emission sparsity constraint improve computation complexity at the price of accuracy?

Speeding up HMMs with Low-Rank Factorizations

A work in progress

Fast Inference with Low-Rank Factorizations

- The previous approach relied a pre-specified emission sparsity constraint
- Can we scale inference with a weaker constraint?
- Exploit structure in the transition matrix to speed up inference

Inference

Start by unpacking inference to reveal the most expensive step

$$p(x) = \alpha_1^\top \Lambda_2 \Lambda_3 \cdots \Lambda_T \mathbf{1}$$

with

$$\begin{aligned} &\text{start,} & [\alpha_1]_{z_1} &= p(x_1 \mid z_1) p(z_1), \\ &\text{and transition operators,} & [\Lambda_t]_{z_{t-1}, z_t} &= p(x_t \mid z_t) p(z_t \mid z_{t-1}) \end{aligned}$$

Inference

Decompose transition operators into transition matrix A and emission matrix O

$$\begin{aligned} p(x) &= \alpha_1^\top \Lambda_2 \cdot \Lambda_T \mathbf{1} \\ &= \alpha_1^\top (A \operatorname{diag}([O]_{\cdot,x_2})) \cdots \Lambda_T \mathbf{1} \\ &= \alpha_1^\top A \operatorname{diag}([O]_{\cdot,x_2}) \cdots A \operatorname{diag}([O]_{\cdot,x_T}) \mathbf{1} \end{aligned}$$

where the most expensive steps are the matrix-vector products $\alpha_t^\top A$, which take $O(Z^2)$ computation

Fast Matrix-Vector Products

- Goal is to reduce the naive matvec complexity of $O(Z^2)$
- Various methods
  - Sparsity (nnz entries)
  - Fast Fourier Transform ($Z \log Z$)
  - Low-Rank factorization (ZR)
- We utilize low-rank factorizations
- Connected to work in efficient attention and kernel approximations[7]

[7] Choromanski et al., *Rethinking Attention with Performers*; Peng et al., *Random Feature Attention*; Blanc and Rendle, *Adaptive Sampled Softmax with Kernel Based Sampling*.

Low-Rank Factorization

Factor transitions $A \in [0,1]^{Z \times Z}$ into product of $U, V \in \mathbb{R}^{Z \times R}$

$$\boxed{\alpha^\top} \times \boxed{A} = \left(\boxed{\alpha^\top} \times \boxed{U} \right) \times \boxed{V^\top}$$

resulting in two matrix-vector products of cost $O(ZR)$ each

- Constraint: Entries of A must be nonnegative
- Solution: Use a nonnegative matrix factorization (NMF)

$$A = \phi(U)\phi(V)^\top,$$

with $\phi : \mathbb{R}^{Z \times R} \to \mathbb{R}_+^{Z \times R}$

Method Recap

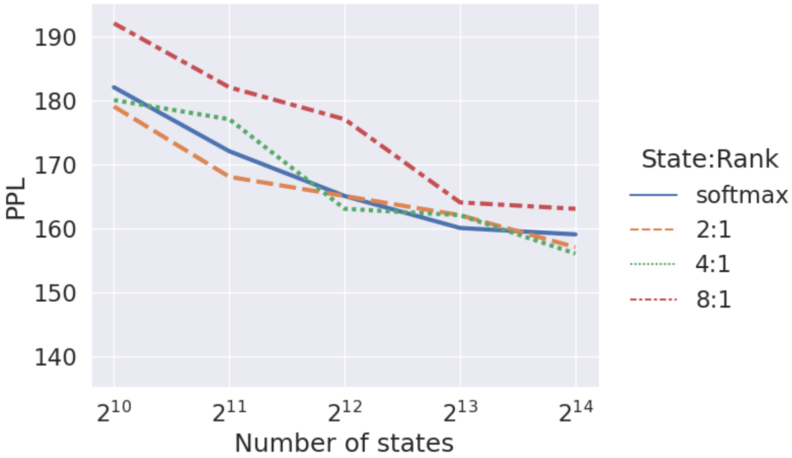
- Target key $O(Z^2)$ matvec step in inference
- Use NMF to reduce cost to $O(ZR)$
- How small can R be relative to Z without sacrificing accuracy?

Experiments

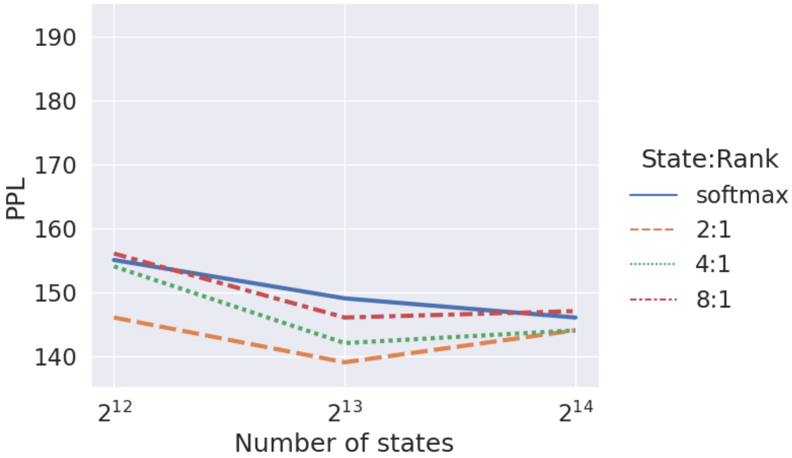
Experiments

- Language modeling on PTB
- Feature map $\phi(U) = \exp(UW)$, with learned $W \in \mathbb{R}^{R \times R}$
- Baseline: Softmax HMM

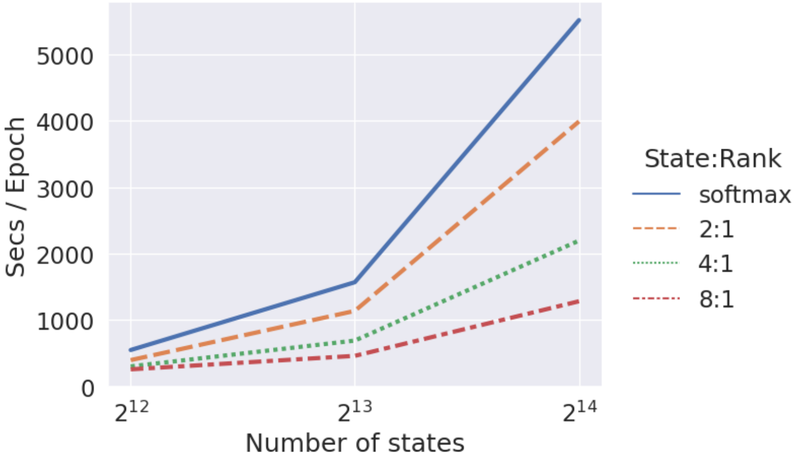
Scaling on PTB (Validation)



Further Scaling on PTB with Dropout (Validation)



Speed Comparison[8]



[8] 2^{14} (16k) state SE-HMM takes 506 s/epoch on the same data

Discussion

- Reduced computation complexity of inference by 4x with NMF vs softmax HMM
- Scaling factor not as large as SE-HMM
- Validation PPL worse than SE-HMM

Conclusion

- Demonstrated improvements in perplexity with larger state spaces
- Extended techniques from neural networks to HMMs

Future Work

- Explore the performance of more complex interpretable models
  - Probabilistic context-free grammars
  - Switching linear dynamical systems[9]
  - Latent vector grammars[10]
- Explore other structure for fast matrix-vector products and tensor generalizations
  - FFT-inspired algorithms[11]
- Learn sparsity constraints in SE-HMM
- Apply sparsity constraints to embedded HMMs[12] for use in approximate inference

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[9] Foerster et al., 'Intelligible Language Modeling with Input Switched Affine Networks'.

[10] Zhao, Zhang, and Tu, 'Gaussian Mixture Latent Vector Grammars'.

[11] Dao et al., 'Kaleidoscope: An Efficient, Learnable Representation For All Structured Linear Maps'.

[12] Neal, Beal, and Roweis, 'Inferring State Sequences for Non-Linear Systems with Embedded Hidden Markov Models'.

EOS

Citations

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Merialdo, Bernard. 'Tagging English Text with a Probabilistic Model'. In: *Computational Linguistics* 20.2 (1994), pp. 155–171.

Generalized Softmax

- Softmax

$$p(z_t \mid z_{t-1}) = \frac{\exp(\mathbf{u}_{z_{t-1}}^\top \mathbf{v}_{z_t})}{\sum_z \exp(\mathbf{u}_{z_{t-1}}^\top \mathbf{v}_z)}$$

- Generalized Softmax

$$p(z_t \mid z_{t-1}) = \frac{K(\mathbf{u}, \mathbf{v})}{\sum_z K(\mathbf{u}, \mathbf{v}_z)} = \frac{\phi(\mathbf{u})^\top \phi(\mathbf{v})}{\sum_z \phi(\mathbf{u})^\top \phi(\mathbf{v}_z)},$$

for positive kernel $K : \mathbb{R}^D \times \mathbb{R}^D \to \mathbb{R}_+$ and feature map $\phi : \mathbb{R}^D \to \mathbb{R}^R$

Generalized Softmax: Inference

- The key $O(Z^2)$ step in the forward algorithm:

$$p(z_t \mid x_{<t}) = \sum_{z_{t-1}} p(z_t \mid z_{t-1}) p(z_{t-1} \mid x_{<t})$$

- In matrix form,

$$\boldsymbol{\gamma}_t = \underbrace{\boldsymbol{\alpha}_{t-1}}_{\mathbb{R}^Z} \underbrace{\Lambda}_{\mathbb{R}^{Z \times Z}},$$

  where we have the probability of the

current state,	$[\boldsymbol{\gamma}_t]_{z_t} = p(z_t \mid x_{<t})$,
last state,	$[\boldsymbol{\alpha}_{t-1}]_{z_{t-1}} = p(z_{t-1} \mid x_{<t})$,
transition probability,	$[\Lambda]_{z_{t-1}, z_t} = p(z_t \mid z_{t-1})$

Generalized Softmax: Inference

- Use generalized softmax in transition distribution

$$[\Lambda]_{z_{t-1},z_t} = p(z_t \mid z_{t-1}) \propto \phi(\mathbf{u}_{z_{t-1}})^\top \phi(\mathbf{v}_{z_t})$$

- Allows us to apply associative property of matrix multiplication

$$\begin{aligned}
\boldsymbol{\gamma}_t &= \boldsymbol{\alpha}_{t-1}\Lambda \\
&= \boldsymbol{\alpha}_{t-1}(\mathrm{diag}(d)\phi(U)\phi(V)^\top) \\
&= \underbrace{(\boldsymbol{\alpha}_{t-1} \circ d)}_{\mathbb{R}^Z}\underbrace{\phi(U)}_{\mathbb{R}^{Z\times f}}\underbrace{\phi(V)^\top}_{\mathbb{R}^{f\times Z}},
\end{aligned}$$

  with stacked embeddings $\phi(U), \phi(V) = [\phi(\mathbf{v}_1), \dots, \phi(\mathbf{v}_Z)]$ and normalizing constants d

- Takes $O(Zf)$ time from left to right!