Scaling Hidden Markov Models

March 12, 2021

Hidden Markov Models in NLP

- Simplest latent variable models for time series data
- Are thought to be very poor language models
- We show they are not!

Scaling HMMs with Emission Sparsity

Lessons from Large Neural Language Models

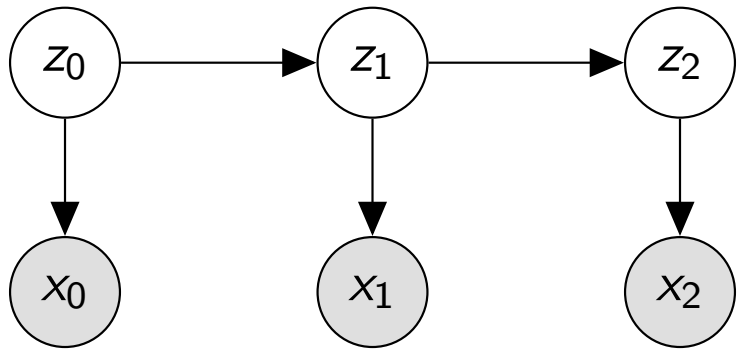
Large models perform better but are . . .

1. Slow to train
2. Prone to overfitting

We must overcome these issues when scaling HMMs

HMMs

For times t, model states $z_t \in [Z]$, and tokens $x_t \in [X]$,



We wish to optimize

$$p(x) = \sum_z p(x, z),$$

computed via the forward algorithm

3 Techniques for Training Large HMMs

- Block-sparse emission constraints

  ⬆ Speed

- Compact neural parameterization

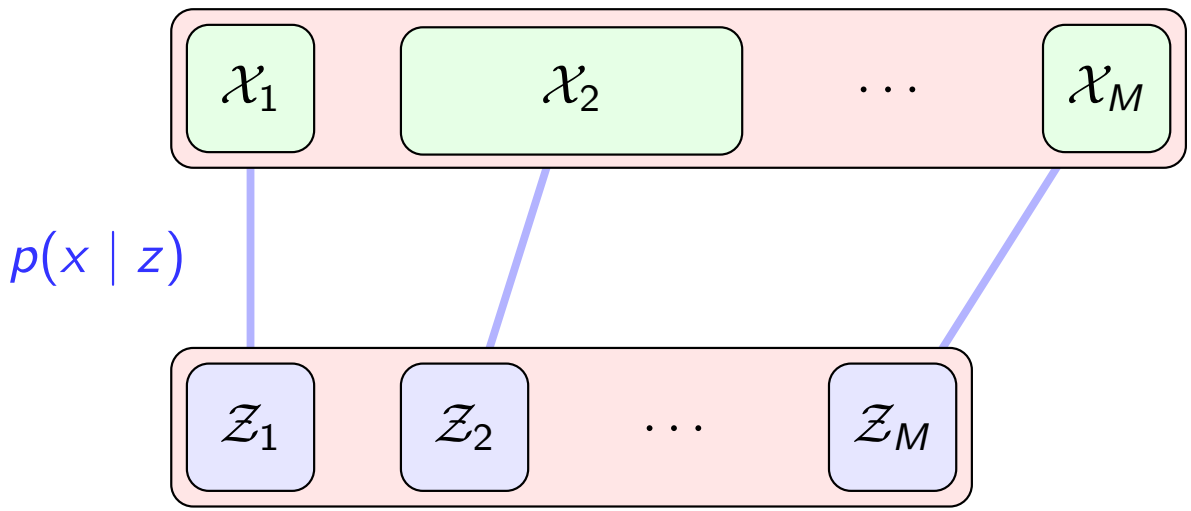
  ⬆ Generalization

- State dropout

  ⬆ Speed ⬆ Generalization

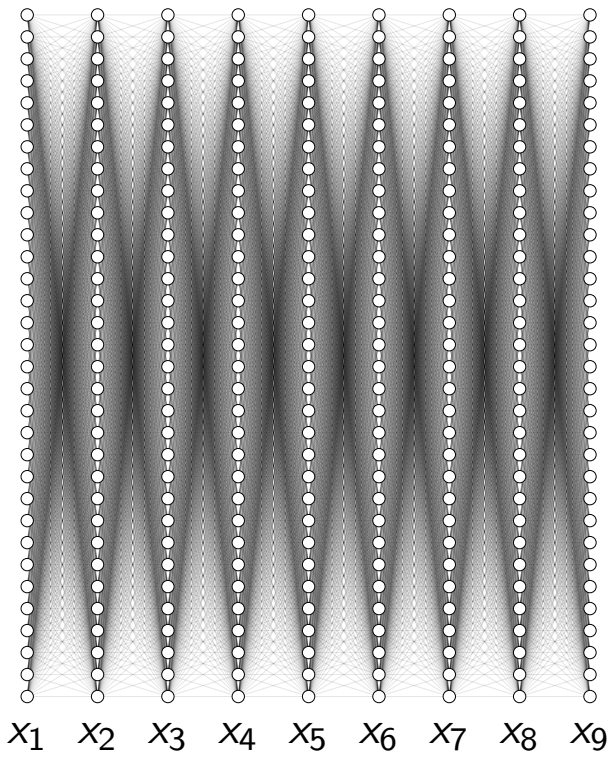
Technique 1: Block-Sparse Emission Constraints

- Reduce cost of marginalization by enforcing structure
- Partition words and states jointly
- Words can only be emit by states in same group

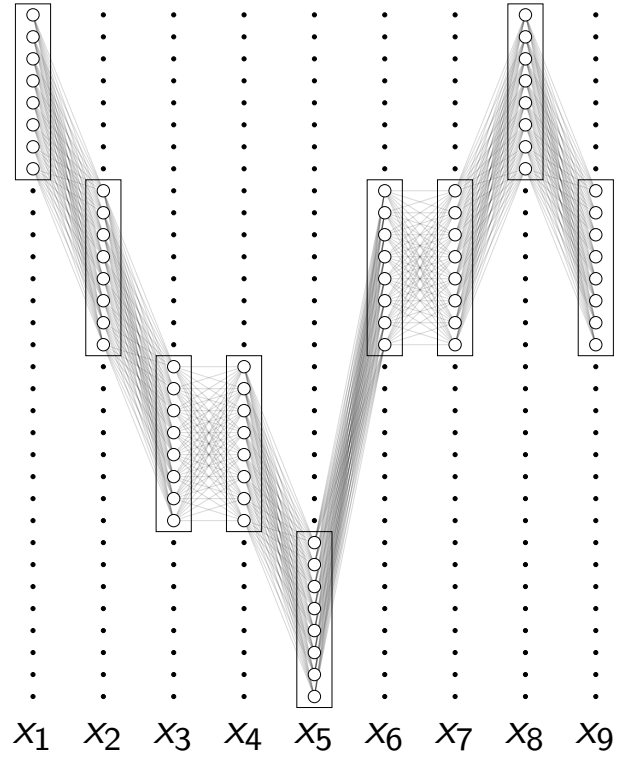


Block-Sparse Emissions: Effect on Inference

Given each word x_t, only the states in the correct group can occur



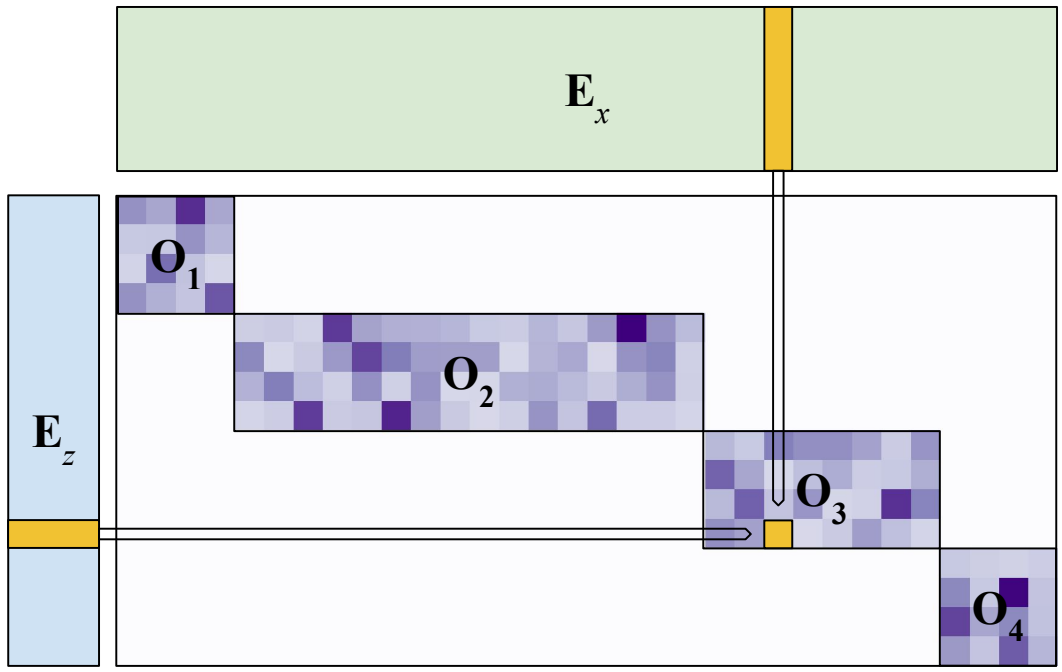
(a) No constraints



(b) Block-sparse emission

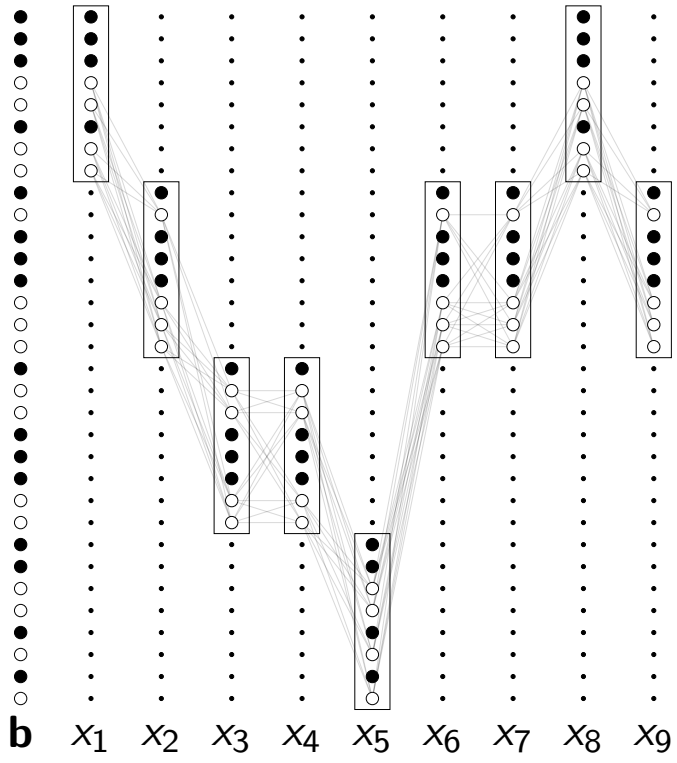
Technique 2: Neural Parameterization

- A neural parameterization allows for parameter sharing
- Generate conditional distributions from state $\mathbf{E}_z$ and token representations $\mathbf{E}_x$



Technique 3: State Dropout

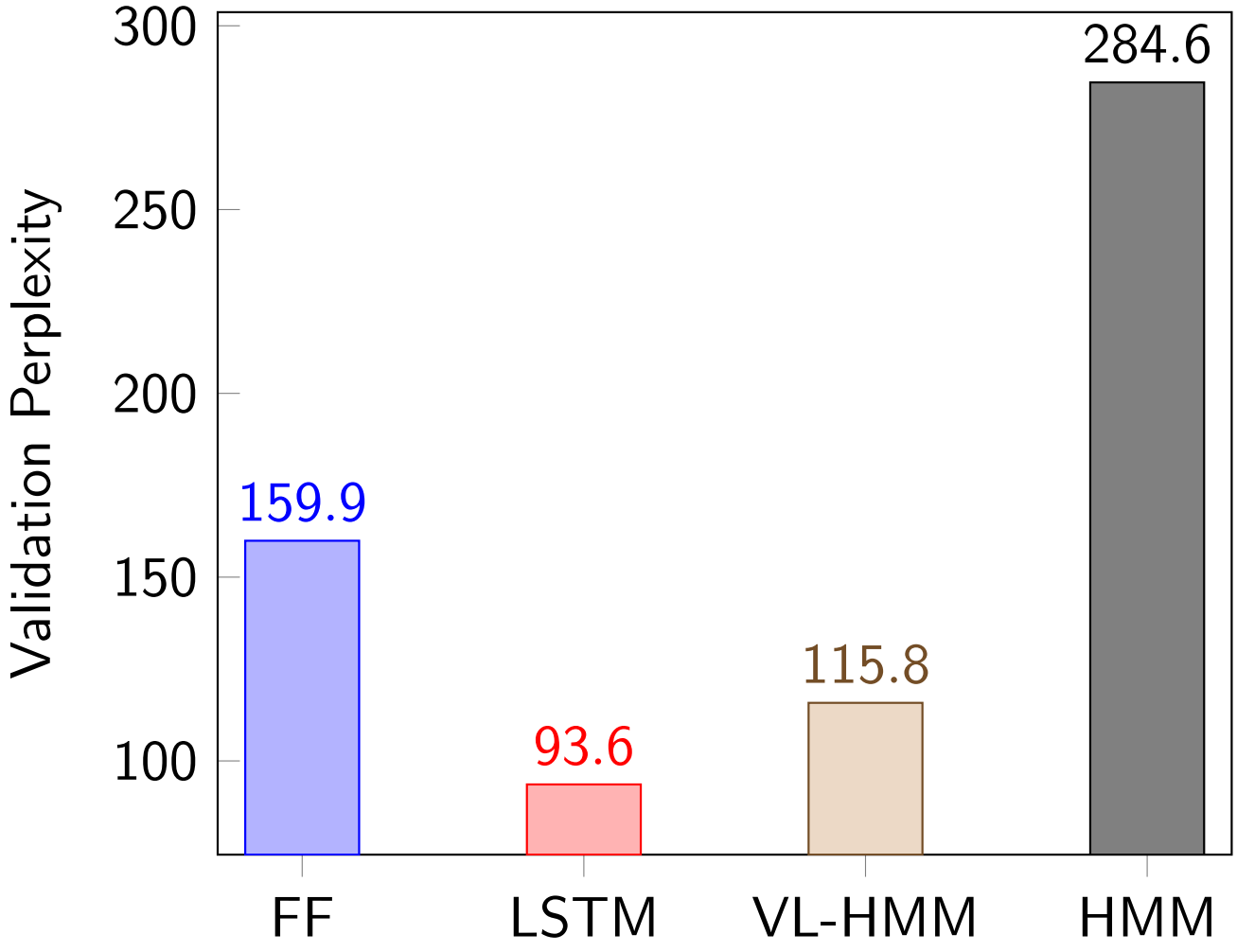
- State dropout encourages broad state usage
- At each batch, sample dropout mask $\mathbf{b} \in \{0, 1\}^z$



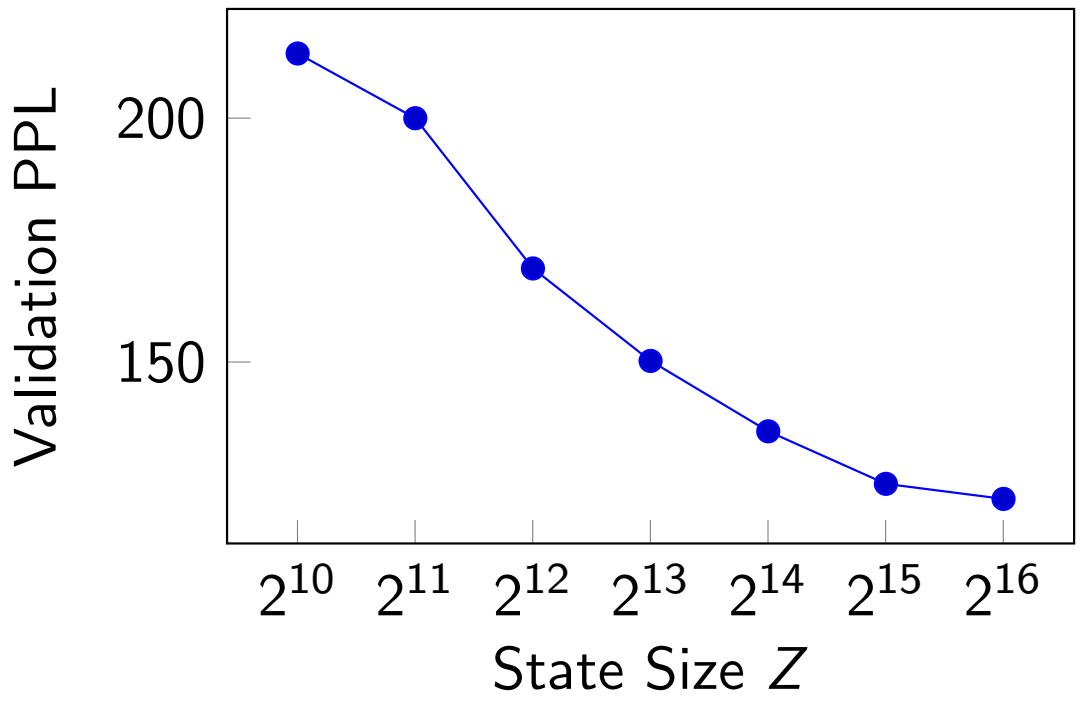
Experiments

- Language modeling on Penn Treebank
- Baselines
  - Feedforward 5-gram model
  - 2-layer LSTM
  - A 900 state HMM (Buys et al 2018)
- Model
  - 2^{15} (32k) state very large HMM (VL-HMM)
  - $M = 128$ groups (256 states per type), obtained via Brown Clustering
  - Dropout rate of 0.5 during training

Results on PTB Validation Data



State Size Ablation



Validation perplexity on PTB by state size ($\lambda = 0.5$ and $M = 128$)

Other Ablations

Model	Param	Train	Val
VL-HMM (2^{14})	7.2M	115	134
- neural param	423M	119	169
- state dropout	7.2M	88	157

Scaling HMMs with Kernel Methods

Embedded Structure Prediction

- The previous work relied on emission sparsity constraints
- Can we scale with weaker assumptions?
- Reduce the quadratic dependence of inference on number of states to linear with kernel trick

Generalized Softmax

Focusing on the transition distribution,

- Softmax

$$p(z_t \mid z_{t-1}) = \frac{\exp(\mathbf{u}_{z_{t-1}}^\top \mathbf{v}_{z_t})}{\sum_z \exp(\mathbf{u}_{z_{t-1}}^\top \mathbf{v}_z)}$$

- Generalized Softmax

$$p(z_t \mid z_{t-1}) = \frac{K(\mathbf{u}, \mathbf{v})}{\sum_z K(\mathbf{u}, \mathbf{v}_z)} = \frac{\phi(\mathbf{u})^\top \phi(\mathbf{v})}{\sum_z \phi(\mathbf{u})^\top \phi(\mathbf{v}_z)},$$

for positive kernel $K : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}_+$ and feature map $\phi : \mathbb{R}^d \to \mathbb{R}^f$

Inference

- The key $O(Z^2)$ step in the forward algorithm:

$$p(z_t \mid x_{<t}) = \sum_{z_{t-1}} p(z_t \mid z_{t-1}) p(z_t \mid x_{<t})$$

- In matrix form,

$$\gamma_t = \underbrace{\alpha_{t-1}}_{\mathbb{R}^Z} \underbrace{\Lambda}_{\mathbb{R}^{Z \times Z}},$$

where

$$\begin{aligned}
[\gamma_t]_{z_t} &= p(z_t \mid x_{<t}) \\
[\alpha_{t-1}]_{z_{t-1}} &= p(z_{t-1} \mid x_{<t}) \\
[\Lambda]_{z_{t-1}, z_t} &= p(z_t \mid z_{t-1})
\end{aligned}$$

Embedded Inference

- Use generalized softmax in transition distribution

$$[\Lambda]_{z_{t-1},z_t} = p(z_t \mid z_{t-1}) \propto \phi(\mathbf{u}_{z_{t-1}})^\top \phi(\mathbf{v}_{z_t})$$

- In matrix form,

$$\gamma_t = \boldsymbol{\alpha}_{t-1}\Lambda = \underbrace{\boldsymbol{\alpha}_{t-1}}_{\mathbb{R}^Z} \underbrace{\mathrm{diag}(d)}_{\mathbb{R}^{Z\times Z}} \underbrace{\phi(U)}_{\mathbb{R}^{Z\times f}} \underbrace{\phi(V)^\top}_{\mathbb{R}^{f\times Z}},$$

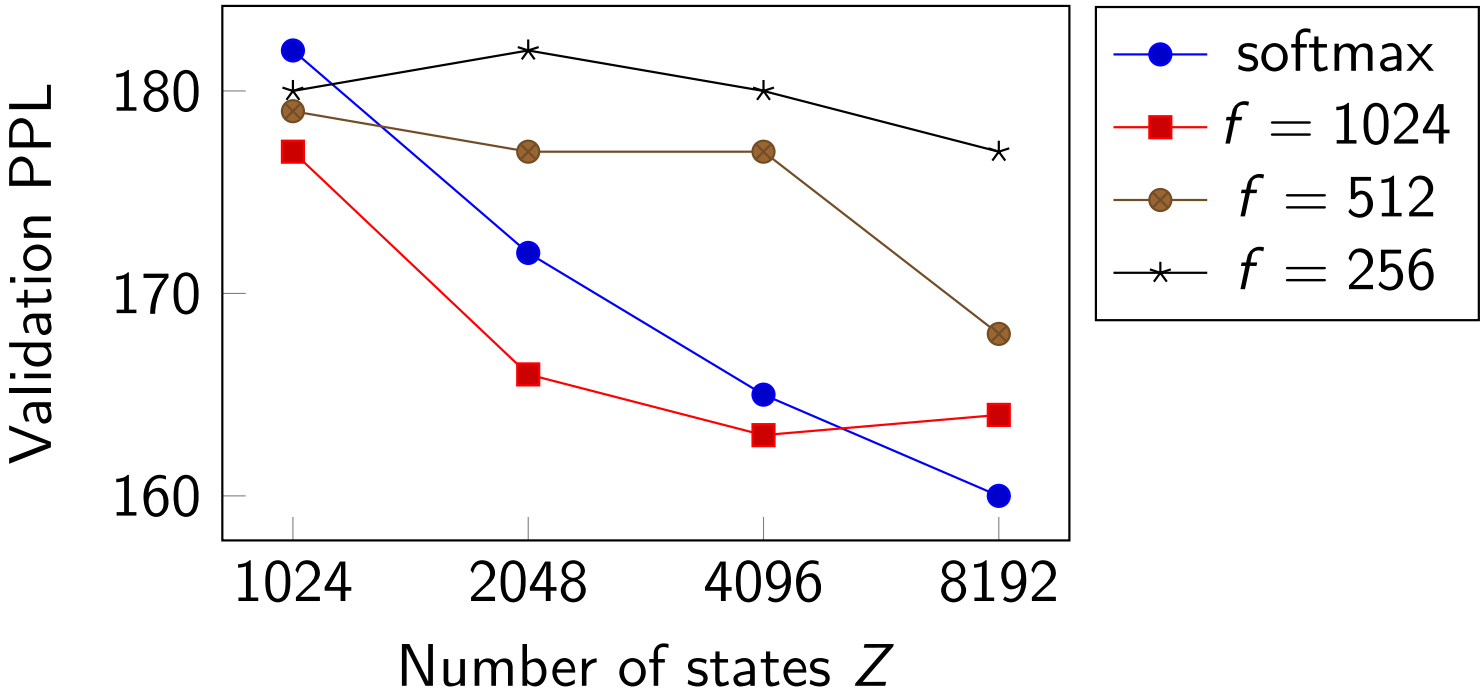
  with stacked embeddings $\phi(U), \phi(V)$ and normalizing constants d

- Takes $O(Zf)$ time from left to right!

Experiments

- Language modeling on PTB
- Work directly with feature map $\phi(\mathbf{x}) = \exp\left(W\mathbf{x} - \frac{\|x\|^2}{2}\right)$, with learned $W \in \mathbb{R}^{d \times f}$
- No dropout or sparsity constraints

Results on PTB Validation



- Holding number of features fixed, perplexity mostly improves or remains the same with an increasing number of states
- Achieve similar performance as softmax with around 4:1 state to feature ratio
- Results are a bit noisy, hoping dropout will clean things up
- Currently running that plus larger state sizes

Conclusion

- HMMs are competitive language models
- Introduced 4 techniques for tackling speed and overfitting
- A great time to revisit other discrete latent variable models

EOS

Citations