Scaling Hidden Markov Models

April 19, 2021

Latent Variables Models in NLP

- NLP benchmarks are dominated by fully observed models
- But most tasks are fully supervised
- Evaluation of unsupervised is also difficult
- Make latent variable models competitive on an existing task

Language Modeling

- Given the words seen so far, predict the next word
- Language requires modeling long-range phenomena

PICTURE

Research Question

- How far can we scale simple latent variables models?

- Under the assumption that tasks will only be developed if models are reasonably performant

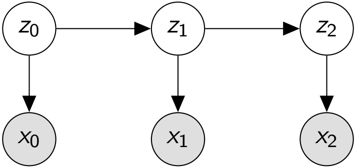
Hidden Markov Models in NLP

- Simplest latent variable models for time series data
- Are thought to be very poor language models
- We show they are better than previously thought once scaled

Background: HMMs

HMM Notation

For times t, model states $z_t \in [Z]$, and tokens $x_t \in [X]$,



We wish to optimize

$$p(x) = \sum_{z_1} \cdots \sum_{z_T} p(x, z),$$

computed via the forward algorithm

Inference

Given observed $x = (x_1, \ldots, x_T)$

- Start $\boldsymbol{\pi} \in [0,1]^Z$, with

$$[\boldsymbol{\pi}]_{z_1} = p(x_1 \mid z_1)p(z_1)$$

- Transition operators $\Lambda \in [0,1]^{Z \times Z}$, with

$$[\Lambda_t]_{z_{t-1}, z_t} = p(x_t \mid z_t)p(z_t \mid z_{t-1})$$

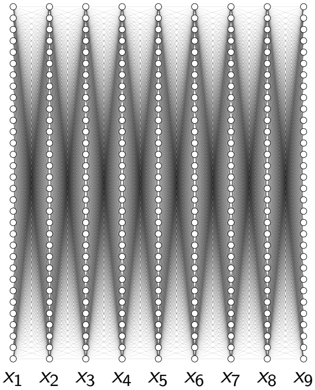
- Forward algorithm computes evidence

$$p(x) = \underbrace{\boldsymbol{\pi}^\top \Lambda_2 \Lambda_3}_{\alpha_3} \cdots \Lambda_T \mathbf{1}$$

- Each $[\alpha_t]_{z_t} = p(z_t, x_{\leq t})$

Inference

- Nodes correspond to states
- Edges to entries in Λ_t
- Sequentially compute posterior state probabilities



Scaling HMMs

Lessons from Large Neural Language Models

Large models perform better but are . . .

1. Slow to train
2. Prone to overfitting

We must overcome these issues when scaling HMMs

4 Techniques for Training Large HMMs

- Compact neural parameterization

  ⇧ Generalization

- State dropout

  ⇧ Speed ⇧ Generalization

- Block-sparse emission constraints

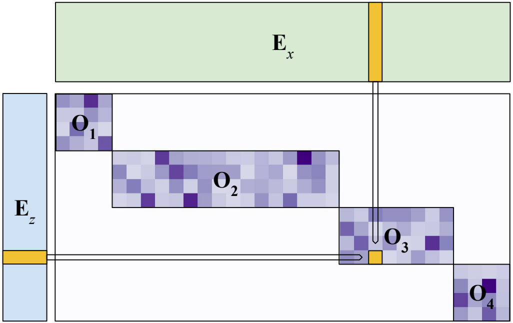
  ⇧ Speed

- Kernel-based generalized softmax

  ⇧ Speed

Technique 1: Neural Parameterization

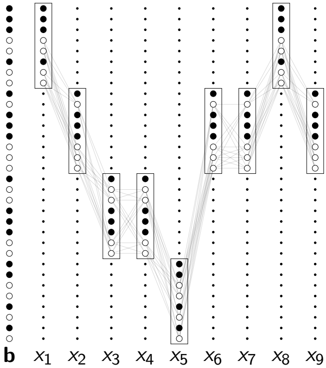
- A neural parameterization allows for parameter sharing
- Generate conditional distributions from state $\mathbf{E}_z$ and token representations $\mathbf{E}_x$



REDO. picture should show $p(x \mid z) = \exp(LR^\top)$, generating L and R from neural networks

Technique 2: State Dropout

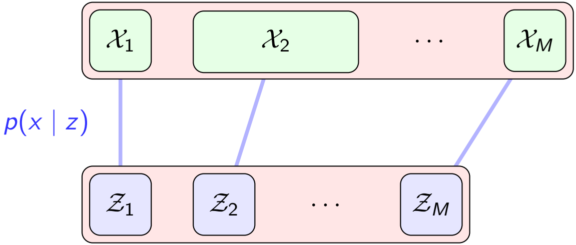
- State dropout encourages broad state usage
- At each batch, sample dropout mask $\mathbf{b} \in \{0, 1\}^Z$



Technique 3: Block-Sparse Emission Constraints

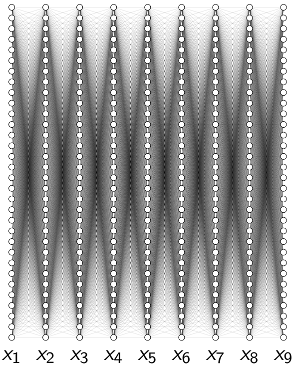
This slide sucks, redo

- Reduce cost of marginalization by enforcing structure
- Partition words and states jointly
- Words can only be emit by states in the aligned group

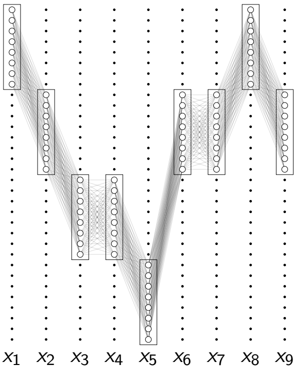


Block-Sparse Emissions: Effect on Inference

Given each word x_t, only the states in the correct group can occur



(a) No constraints



(b) Block-sparse emission

Technique 4: Generalized Softmax

Focusing on the transition distribution,

- Softmax

$$p(z_t \mid z_{t-1}) = \frac{\exp(\mathbf{u}_{z_{t-1}}^\top \mathbf{v}_{z_t})}{\sum_z \exp(\mathbf{u}_{z_{t-1}}^\top \mathbf{v}_z)}$$

- Generalized Softmax

$$p(z_t \mid z_{t-1}) = \frac{K(\mathbf{u}, \mathbf{v})}{\sum_z K(\mathbf{u}, \mathbf{v}_z)} = \frac{\phi(\mathbf{u})^\top \phi(\mathbf{v})}{\sum_z \phi(\mathbf{u})^\top \phi(\mathbf{v}_z)},$$

for positive kernel $K : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}_+$ and feature map $\phi : \mathbb{R}^d \to \mathbb{R}^f$

Generalized Softmax: Inference

- The key $O(Z^2)$ step in the forward algorithm:

$$p(z_t \mid x_{<t}) = \sum_{z_{t-1}} p(z_t \mid z_{t-1}) p(z_{t-1} \mid x_{<t})$$

- In matrix form,

$$\boldsymbol{\gamma}_t = \underbrace{\boldsymbol{\alpha}_{t-1}}_{\mathbb{R}^Z} \underbrace{\Lambda}_{\mathbb{R}^{Z \times Z}},$$

  where we have the probability of the

$$\begin{aligned}
&\text{current state,} \qquad & [\boldsymbol{\gamma}_t]_{z_t} &= p(z_t \mid x_{<t}), \\
&\text{last state,} \qquad & [\boldsymbol{\alpha}_{t-1}]_{z_{t-1}} &= p(z_{t-1} \mid x_{<t}), \\
&\text{transition probability,} \qquad & [\Lambda]_{z_{t-1}, z_t} &= p(z_t \mid z_{t-1})
\end{aligned}$$

Generalized Softmax: Inference

- Use generalized softmax in transition distribution

$$[\Lambda]_{z_{t-1},z_t} = p(z_t \mid z_{t-1}) \propto \phi(\mathbf{u}_{z_{t-1}})^\top \phi(\mathbf{v}_{z_t})$$

- Allows us to apply associative property of matrix multiplication

$$\begin{aligned}
\boldsymbol{\gamma}_t &= \boldsymbol{\alpha}_{t-1}\Lambda \\
&= \boldsymbol{\alpha}_{t-1}(\mathrm{diag}(d)\phi(U)\phi(V)^\top) \\
&= \underbrace{(\boldsymbol{\alpha}_{t-1} \circ d)}_{\mathbb{R}^Z}\underbrace{\phi(U)}_{\mathbb{R}^{Z\times f}}\underbrace{\phi(V)^\top}_{\mathbb{R}^{f\times Z}},
\end{aligned}$$

  with stacked embeddings $\phi(U), \phi(V) = [\phi(\mathbf{v}_1), \dots, \phi(\mathbf{v}_Z)]$ and normalizing constants d

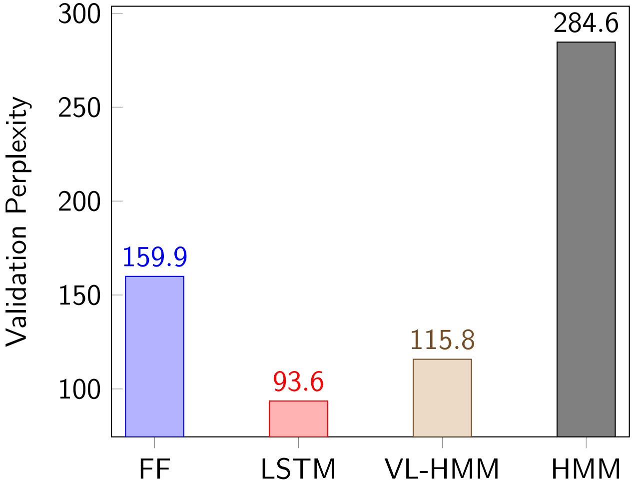
- Takes $O(Zf)$ time from left to right!

Experiments

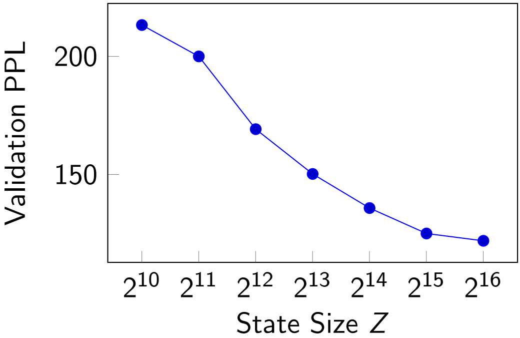
Experiments

- Language modeling on Penn Treebank
- Baselines
  - Feedforward 5-gram model
  - 2-layer LSTM
  - A 900 state HMM (Buys et al 2018)
- Model
  - 2^{15} (32k) state very large HMM (VL-HMM)
  - $M = 128$ groups (256 states per type), obtained via Brown Clustering
  - Dropout rate of 0.5 during training

Results on PTB Validation Data



State Size Ablation



Validation perplexity on PTB by state size ($\lambda = 0.5$ and $M = 128$)

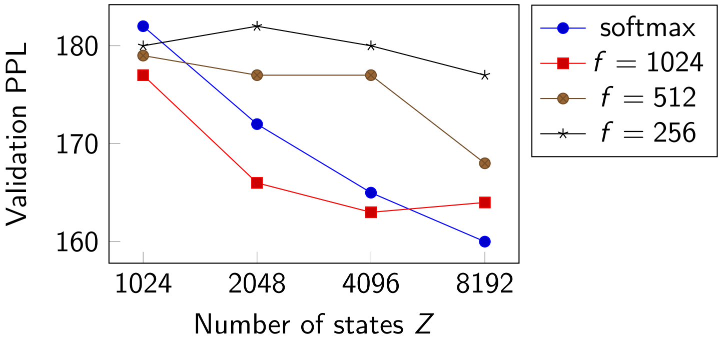
Other Ablations

Model	Param	Train	Val
VL-HMM (2^{14})	7.2M	115	134
- neural param	423M	119	169
- state dropout	7.2M	88	157

Experiments

- Language modeling on PTB
- Work directly with feature map $\phi(\mathbf{x}) = \exp\left(W\mathbf{x} - \frac{\|x\|^2}{2}\right)$, with learned $W \in \mathbb{R}^{d \times f}$
- No dropout or sparsity constraints

Results on PTB Validation



- Holding number of features fixed, perplexity mostly improves or remains the same with an increasing number of states
- Achieve similar performance as softmax with around 4:1 state to feature ratio (also holds for 8k and 16k states)

Conclusion

- Hopeful that HMMs can be competitive language models
- Introduced 4 techniques for tackling speed and overfitting
- Future work will extend to other discrete latent variable models

EOS

Citations