Scaling Hidden Markov Models

April 26, 2021

Latent Variables Models in NLP

- NLP benchmarks are dominated by fully observed models
  - Transformers
  - Previously, Recurrent Neural Networks
- We instead explore latent variable models

Latent Variable Models

- Maintain uncertainty over latent representations
- Focus on models that admit tractable inference (say to avoid error due to approximations during learning)
- Research question 1: Does uncertainty in the model improve performance?
- Research question 2: To what extent is the performance of tractable latent variable models limited by scale and choices in parameterization?

Language Modeling

To answer these questions, we focus on the task of language modeling

- Given the words seen so far, predict the next word
- Language requires modeling long-range phenomena
- Focus on the smallest dataset, Penn Treebank, and the simplest latent variable model

PICTURE

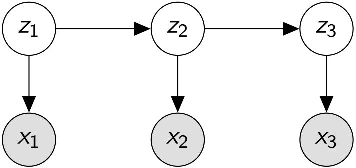
Hidden Markov Models in NLP

- Simplest latent variable models for time series data
- Are thought to be very poor language models
- We show they are better than previously thought once scaled
  - But still not as performant as fully observed models

Background: Hidden Markov Models

Hidden Markov Models (HMMs)

For times t, model states $z_t \in [Z]$, and tokens $x_t \in [X]$,



This yields the joint distribution

$$p(x, z) = \prod_t p(x_t \mid z_t) p(z_t \mid z_{t-1}),$$

with distributional parameters

start vector	$\pi \in [0,1]^Z$, with	$[\pi]_{z_0} = p(z_0)$,
transition matrix	$A \in [0,1]^{Z \times Z}$, with	$[A]_{z_{t-1}, z_t} = p(z_t \mid z_{t-1})$,
emission matrix	$O \in [0,1]^{Z \times X}$, with	$[O]_{z_t, x_t} = p(x_t \mid z_t)$

Inference

Given observed $x = (x_1, \dots, x_T)$ We wish to maximize

$$p(x) = \sum_{z_1} \cdots \sum_{z_T} p(x, z) = \alpha_1^\top \Lambda_2 \Lambda_3 \cdots \Lambda_T \mathbf{1},$$

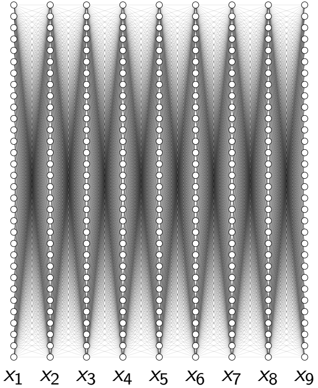
where we have the

$$\begin{aligned} &\text{start,} && \alpha_1 = \pi[O]_{\cdot, x_1}, \\ &\text{and transition operators,} && \Lambda_t = A \operatorname{diag}([O]_{\cdot, x_t}) \end{aligned}$$

The result of each matvec has the alphas of the forward algorithm, i.e. $\alpha_3 = \alpha_1 \Lambda_2 \Lambda_3$ gives $p(z_3, x_{1:3})$

Inference

Given the start and transition operators, we can visualize inference



- Each node corresponds to a state
- Each edge to an entry in the transition operator matrix

Scaling HMMs

Lessons from Large Neural Language Models

Large models perform better but are ...

1. Slow to train
2. Prone to overfitting

We must overcome these issues when scaling HMMs

4 Techniques for Training Large HMMs

- Compact neural parameterization

  ⇧ Generalization

- State dropout

  ⇧ Speed ⇧ Generalization

- Block-sparse emission constraints

  ⇧ Speed

- Kernel-based generalized softmax for transitions

  ⇧ Speed

Technique 1: Neural Parameterization

For both the transition and emission matrices, we use a nonlinear low-dimensional decomposition

$$\boxed{W} \propto \exp\left(\boxed{U} \times \boxed{V^\top}\right)$$

- Fewer parameters than directly parameterizing W
- Can further parameterize U or $V = \mathrm{MLP}(E_u)$
- This particular parameterization does not affect inference

Technique 2: State Dropout

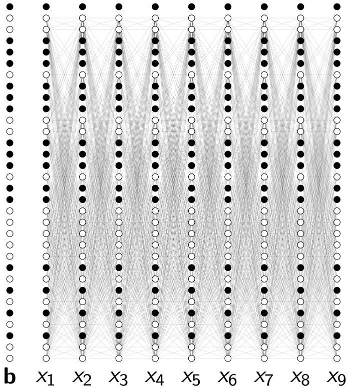
- State dropout encourages broad state usage
- At each batch, sample dropout mask $\mathbf{b} \in \{0,1\}^Z$
- Compute distributional parameters by indexing into embeddings U, V

$$\text{Transition } A \leftarrow \left(\mathbf{b} \circ \boxed{U_A}\right) \times \left(\mathbf{b} \circ \boxed{V_A}\right)^\top$$

$$\text{Emission } O \leftarrow \left(\mathbf{b} \circ \boxed{U_O}\right) \times \boxed{V_O}^\top$$

State Dropout: Inference

The cost of inference is reduced by state dropout



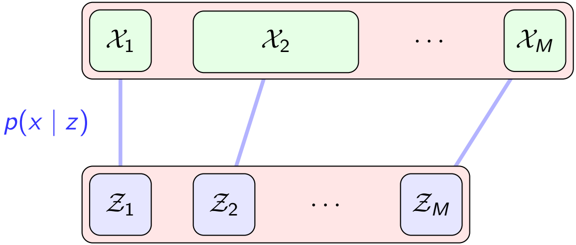
- Shaded nodes depict dropped states
- Ignore dropped states during inference

Technique 3: Block-Sparse Emission Constraints

This slide sucks, redo

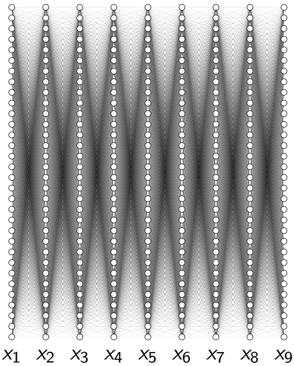
- Reduce cost of marginalization by enforcing structure
- Partition words and states jointly
- Words can only be emit by states in the aligned group

Block-Sparse Emission Constraints: Inference

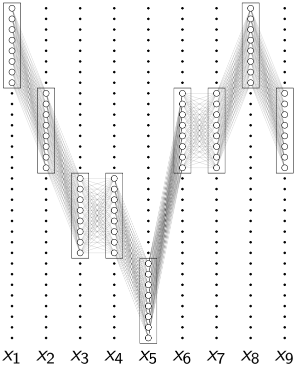


Block-Sparse Emissions: Inference

Given each word x_t, only the states in the correct group can occur



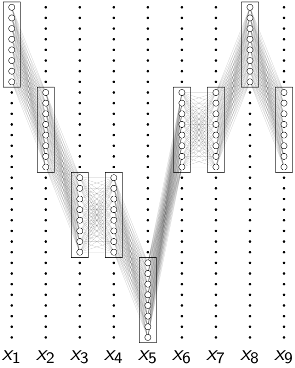
(a) No constraints



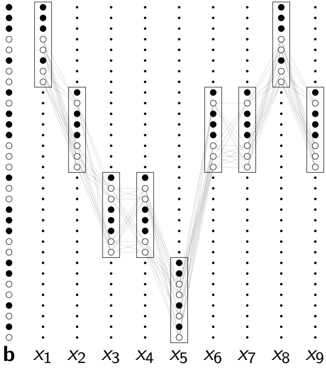
(b) Block-sparse emission

Block-Sparse Emissions: Inference

Given each word x_t, only the states in the correct group can occur



(a) Block-sparse emission



(b) With state dropout

Technique 4: Generalized Softmax

Focusing on the transition distribution,

- Softmax

$$p(z_t \mid z_{t-1}) = \frac{\exp(\mathbf{u}_{z_{t-1}}^\top \mathbf{v}_{z_t})}{\sum_z \exp(\mathbf{u}_{z_{t-1}}^\top \mathbf{v}_z)}$$

- Generalized Softmax

$$p(z_t \mid z_{t-1}) = \frac{K(\mathbf{u}, \mathbf{v})}{\sum_z K(\mathbf{u}, \mathbf{v}_z)} = \frac{\phi(\mathbf{u})^\top \phi(\mathbf{v})}{\sum_z \phi(\mathbf{u})^\top \phi(\mathbf{v}_z)},$$

  for positive kernel $K : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}_+$ and feature map $\phi : \mathbb{R}^d \to \mathbb{R}^f$

Generalized Softmax: Inference

- The key $O(Z^2)$ step in the forward algorithm:

$$p(z_t \mid x_{<t}) = \sum_{z_{t-1}} p(z_t \mid z_{t-1}) p(z_{t-1} \mid x_{<t})$$

- In matrix form,

$$\boldsymbol{\gamma}_t = \underbrace{\boldsymbol{\alpha}_{t-1}}_{\mathbb{R}^Z} \underbrace{\Lambda}_{\mathbb{R}^{Z \times Z}},$$

where we have the probability of the

current state,	$[\boldsymbol{\gamma}_t]_{z_t} = p(z_t \mid x_{<t}),$
last state,	$[\boldsymbol{\alpha}_{t-1}]_{z_{t-1}} = p(z_{t-1} \mid x_{<t}),$
transition probability,	$[\Lambda]_{z_{t-1}, z_t} = p(z_t \mid z_{t-1})$

Generalized Softmax: Inference

- Use generalized softmax in transition distribution

$$[\Lambda]_{z_{t-1},z_t} = p(z_t \mid z_{t-1}) \propto \phi(\mathbf{u}_{z_{t-1}})^\top \phi(\mathbf{v}_{z_t})$$

- Allows us to apply associative property of matrix multiplication

$$\begin{aligned}
\boldsymbol{\gamma}_t &= \boldsymbol{\alpha}_{t-1}\Lambda \\
&= \boldsymbol{\alpha}_{t-1}(\mathrm{diag}(d)\phi(U)\phi(V)^\top) \\
&= \underbrace{(\boldsymbol{\alpha}_{t-1} \circ d)}_{\mathbb{R}^Z}\underbrace{\phi(U)}_{\mathbb{R}^{Z\times f}}\underbrace{\phi(V)^\top}_{\mathbb{R}^{f\times Z}},
\end{aligned}$$

  with stacked embeddings $\phi(U), \phi(V) = [\phi(\mathbf{v}_1), \ldots, \phi(\mathbf{v}_Z)]$ and normalizing constants d

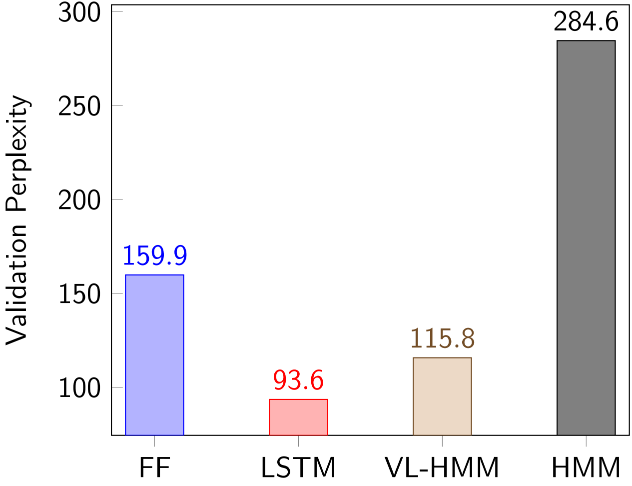
- Takes $O(Zf)$ time from left to right!

Experiments

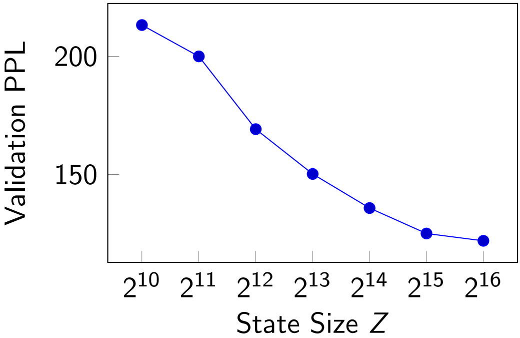
Experiments

- Language modeling on Penn Treebank
- Baselines
  - Feedforward 5-gram model
  - 2-layer LSTM
  - A 900 state HMM (Buys et al 2018)
- Model
  - 2^{15} (32k) state very large HMM (VL-HMM)
  - $M = 128$ groups (256 states per type), obtained via Brown Clustering
  - Dropout rate of 0.5 during training

Results on PTB Validation Data



State Size Ablation



Validation perplexity on PTB by state size ($\lambda = 0.5$ and $M = 128$)

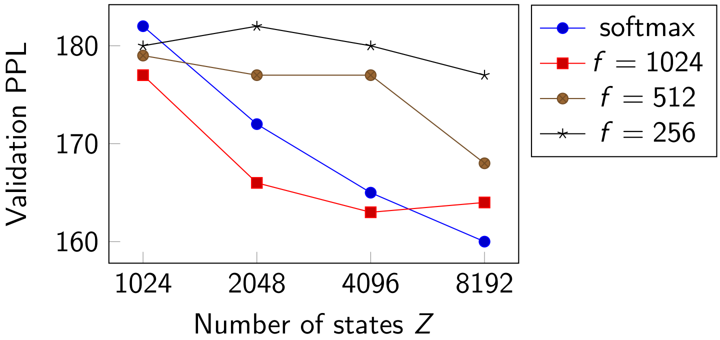
Other Ablations

Model	Param	Train	Val
VL-HMM (2^{14})	7.2M	115	134
- neural param	423M	119	169
- state dropout	7.2M	88	157

Experiments

- Language modeling on PTB
- Work directly with feature map $\phi(\mathbf{x}) = \exp\left(W\mathbf{x} - \frac{\|x\|^2}{2}\right)$, with learned $W \in \mathbb{R}^{d \times f}$
- No dropout or sparsity constraints

Results on PTB Validation



- Holding number of features fixed, perplexity mostly improves or remains the same with an increasing number of states
- Achieve similar performance as softmax with around 4:1 state to feature ratio (also holds for 8k and 16k states)

Conclusion

- Hopeful that HMMs can be competitive language models
- Introduced 4 techniques for tackling speed and overfitting
- Future work will extend to other discrete latent variable models

EOS

Citations