# Scaling Hidden Markov Models

March 12, 2021

# Hidden Markov Models in NLP

- Simplest latent variable models for time series data
- Are thought to be very poor language models
- We show they are not!

# Scaling HMMs with Emission Sparsity

# Lessons from Large Neural Language Models

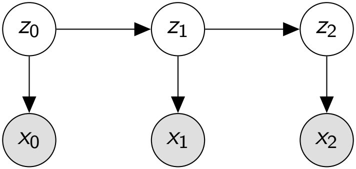
Large models perform better but are . . .

1. Slow to train
2. Prone to overfitting

We must overcome these issues when scaling HMMs

# HMMs

For times $t$, model states $z_t \in [Z]$, and tokens $x_t \in [X]$,




We wish to optimize

$$p(x) = \sum_z p(x, z),$$

computed via the forward algorithm

# 3 Techniques for Training Large HMMs

- Block-sparse emission constraints

  ⇧ Speed

- Compact neural parameterization

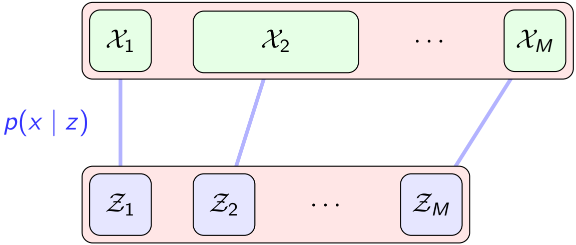
  ⇧ Generalization

- State dropout

  ⇧ Speed ⇧ Generalization

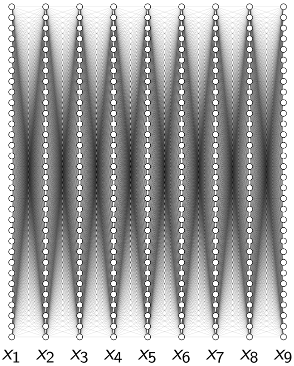
# Technique 1: Block-Sparse Emission Constraints

- Reduce cost of marginalization by enforcing structure
- Partition words and states jointly
- Words can only be emit by states in same group

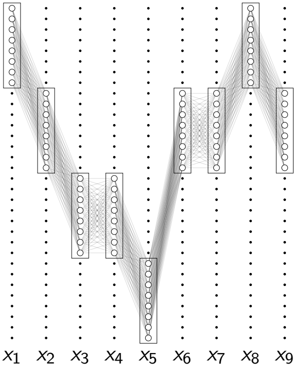


# Block-Sparse Emissions: Effect on Inference

Given each word $x_t$, only the states in the correct group can occur



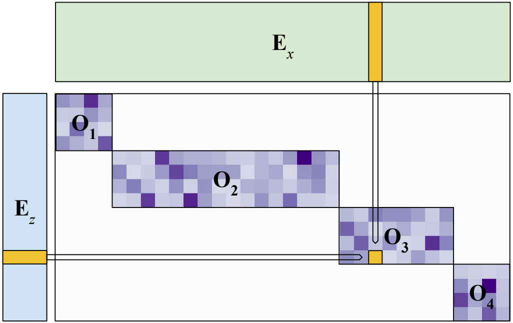

(a) No constraints




(b) Block-sparse emission

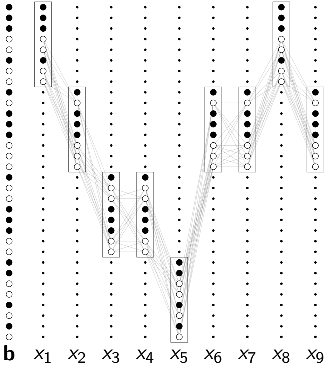
# Technique 2: Neural Parameterization

- A neural parameterization allows for parameter sharing
- Generate conditional distributions from state $\mathbf{E}_z$ and token representations $\mathbf{E}_x$



# Technique 3: State Dropout

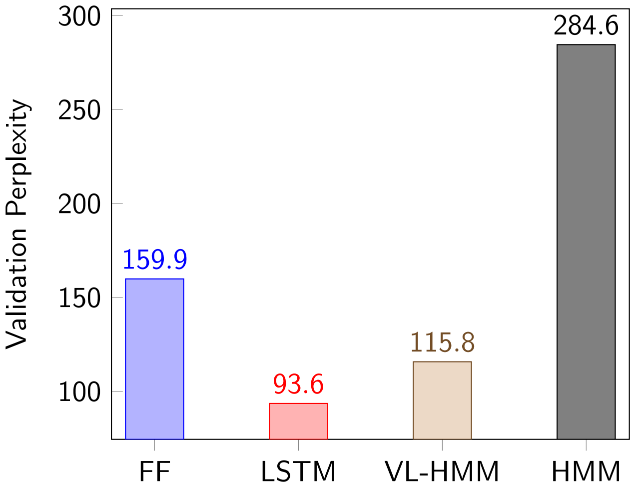
- State dropout encourages broad state usage
- At each batch, sample dropout mask $\mathbf{b} \in \{0,1\}^{Z}$



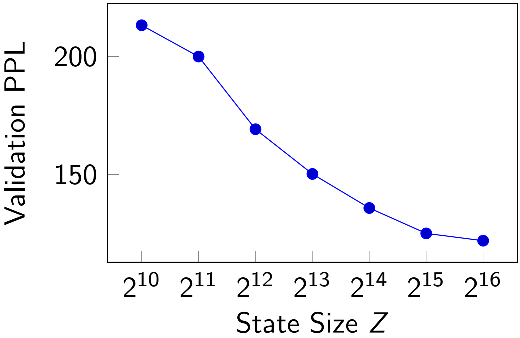
# Experiments

- Language modeling on Penn Treebank
- Baselines
  - Feedforward 5-gram model
  - 2-layer LSTM
  - A 900 state HMM (Buys et al 2018)
- Model
  - $2^{15}$ (32k) state very large HMM (VL-HMM)
  - $M = 128$ groups (256 states per type), obtained via Brown Clustering
  - Dropout rate of 0.5 during training

# Results on PTB Validation Data



# State Size Ablation




Validation perplexity on PTB by state size ($\lambda = 0.5$ and $M = 128$)

# Other Ablations

| Model | Param | Train | Val |
|---|---|---|---|
| VL-HMM ($2^{14}$) | 7.2M | 115 | 134 |
| - neural param | 423M | 119 | 169 |
| - state dropout | 7.2M | 88 | 157 |

# Scaling HMMs with Kernel Methods

# Embedded Structure Prediction

- The previous model relied on emission sparsity constraints
- Can we scale with weaker assumptions?
- Reduce the quadratic dependence of inference on number of states to linear with kernel trick

# Generalized Softmax

Focusing on the transition distribution,

- Softmax

$$p(z_t \mid z_{t-1}) = \frac{\exp(\mathbf{u}_{z_{t-1}}^\top \mathbf{v}_{z_t})}{\sum_z \exp(\mathbf{u}_{z_{t-1}}^\top \mathbf{v}_z)}$$

- Generalized Softmax

$$p(z_t \mid z_{t-1}) = \frac{K(\mathbf{u}, \mathbf{v})}{\sum_z K(\mathbf{u}, \mathbf{v}_z)} = \frac{\phi(\mathbf{u})^\top \phi(\mathbf{v})}{\sum_z \phi(\mathbf{u})^\top \phi(\mathbf{v}_z)},$$

for positive kernel $K : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}_+$ and feature map $\phi : \mathbb{R}^d \to \mathbb{R}^f$

# Inference

- The key $O(Z^2)$ step in the forward algorithm:

$$p(z_t \mid x_{<t}) = \sum_{z_{t-1}} p(z_t \mid z_{t-1}) p(z_{t-1} \mid x_{<t})$$

- In matrix form,

$$\boldsymbol{\gamma}_t = \underbrace{\boldsymbol{\alpha}_{t-1}}_{\mathbb{R}^Z} \underbrace{\Lambda}_{\mathbb{R}^{Z \times Z}},$$

where we have the probability of the

$$\begin{aligned}
&\text{current state} && [\boldsymbol{\gamma}_t]_{z_t} = p(z_t \mid x_{<t}), \\
&\text{last state} && [\boldsymbol{\alpha}_{t-1}]_{z_{t-1}} = p(z_{t-1} \mid x_{<t}), \\
&\text{transition probability} && [\Lambda]_{z_{t-1}, z_t} = p(z_t \mid z_{t-1})
\end{aligned}$$

# Embedded Inference

- Use generalized softmax in transition distribution

$$[\Lambda]_{z_{t-1},z_t} = p(z_t \mid z_{t-1}) \propto \phi(\mathbf{u}_{z_{t-1}})^\top \phi(\mathbf{v}_{z_t})$$

- Allows us to apply associative property of matrix multiplication

$$\begin{aligned}\boldsymbol{\gamma}_t &= \boldsymbol{\alpha}_{t-1}\Lambda \\ &= \boldsymbol{\alpha}_{t-1}(\mathrm{diag}(d)\phi(U)\phi(V)^\top) \\ &= \underbrace{(\boldsymbol{\alpha}_{t-1} \circ d)}_{\mathbb{R}^Z} \underbrace{\phi(U)}_{\mathbb{R}^{Z \times f}} \underbrace{\phi(V)^\top}_{\mathbb{R}^{f \times Z}},\end{aligned}$$

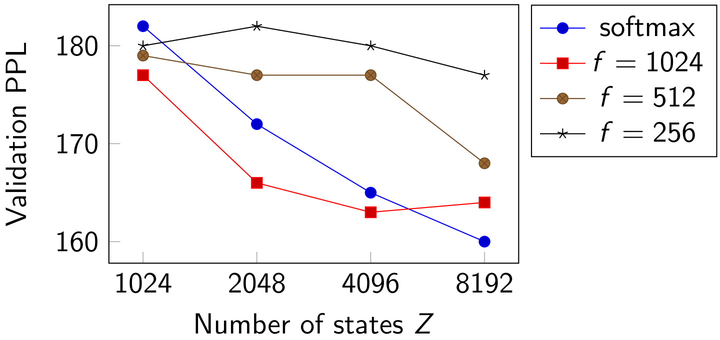
  with stacked embeddings $\phi(U), \phi(V) = [\phi(\mathbf{v}_1), \ldots, \phi(\mathbf{v}_Z)]$ and normalizing constants $d$,

- Takes $O(Zf)$ time from left to right!

# Experiments

- Language modeling on PTB
- Work directly with feature map $\phi(\mathbf{x}) = \exp\left(W\mathbf{x} - \frac{\|x\|^2}{2}\right)$, with learned $W \in \mathbb{R}^{d \times f}$
- No dropout or sparsity constraints

# Results on PTB Validation




- Holding number of features fixed, perplexity mostly improves or remains the same with an increasing number of states
- Achieve similar performance as softmax with around 4:1 state to feature ratio (also holds for 8k and 16k states)

# Conclusion

- Hopeful that HMMs can be competitive language models
- Introduced 4 techniques for tackling speed and overfitting
- Future work will extend to other discrete latent variable models

# EOS

# Citations