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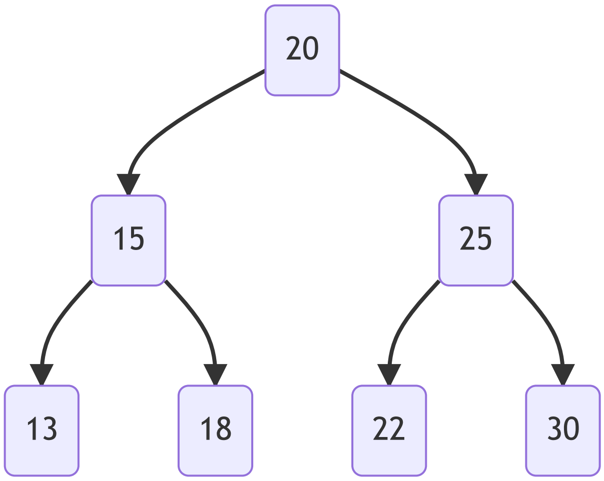
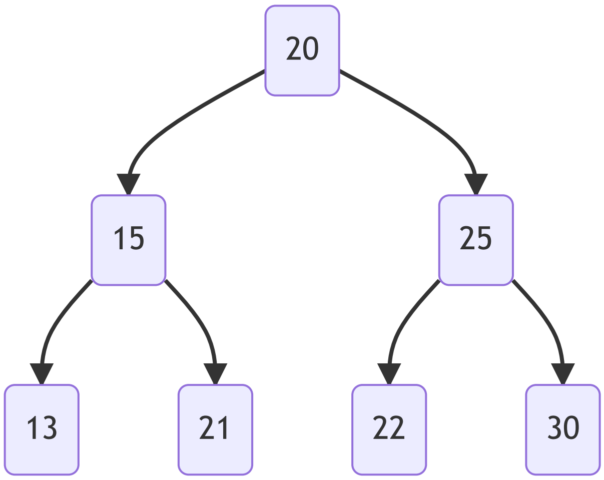
# CSCI 377 Computer Algorithms Video Notes

## Chapter 12: Binary Search Trees

- What is a *Binary Search Tree?*
  - A binary tree is a data structure in which each node maintains a key, or value, and 3 links:
    - One to the left child, *left*
    - One to the right child, *right*
    - One to the parent, *p*
  - If any link is empty, we say it is *NIL*
  - Root has $p = NIL$ since the root has no parent
  - Leaves have both left and right set as *NIL*
  - "NIL" here refers to the absence of a value
  - **Definition:** *Binary Search Tree*
    - A binary search tree is a tree in which for each node, the value of all the nodes in the left sub-tree are lesser or equal, and the value of all the nodes in the right subtree are greater or equal to the original node
    - For example, this *is* a binary search tree:




- And this *is not* a binary search tree




- **Tree Walks**
    - *In-order Tree Walk*
        - Visit left child, then parent, then right child
    - *Pre-order Tree Walk*
        - Visit parent, then left child, then right child
    - *Post-order Tree Walk*
        - Visit left child, then right child, then parent
- **More on Tree Walks**
    - For an in-order tree walk on a binary search tree with $n$ nodes, the operation takes $\Theta(n)$ time

- *In-order tree walk algorithm*

  ```
  inOrderTreeWalk(x)
  {
      if x != NIL
      {
          inOrderTreeWalk(x.left)
          print(x)
          inOrderTreeWalk(x.right)
      }
  }
  ```

- **Searching in a Binary Search Tree**
  - For example, to search for $n$
    1. Start at the root. If it is $n$, stop
    2. Compare $n$ with the root
       - If it is smaller look only at the left sub-tree
       - If it is larger look only at the right sub-tree
    3. Compare $n$ with with either the left or right child of the root, and effectively restart the processes treating the appropriate child as the new root in the search
- **Balance Trees**
  - We can call a tree *balanced* if for all nodes, the difference between the heights of the right and left sub-trees is not greater than $1$
  - In a balance tree, the running time for a search in a binary tree is $T(n) = \Theta(log(n))$
  - In a fully unbalanced tree, the running time will be $T(n) = O(n)$ since for each traversal down the tree, the search space is reduced by only one
- In a binary search tree, by definition, the smallest value is in the leftmost node and the greatest value is in the rightmost node
- **Operations on a Binary Search Tree**
  - *Insert(T, z)*
    - Walk through the tree starting at the root

        - Find the leaf position where z fits
    - *Delete(T, z)*
        - 3 Possible Cases
            1. If z has no children, delete z and modify its parent's link by setting it to NIL
            2. If z has one child, elevate the child to take z's position in the tree by replacing the parent's link to point at z's child
            3. If z has 2 children, we need to find the successor, y, to take z's place and attach z's right sub-tree to y's right sub-tree as well as z's left sub-tree to y's left sub-tree
                - This can be achieved *recursively*