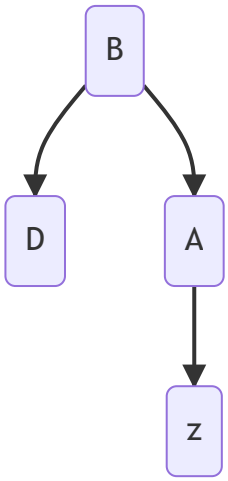
Chapter 13: Red-Black Tree Insertion

- **Red-Black Trees**
  - A *red-black tree* is a self-balancing binary search tree
  - A red-black tree will have the following properties
    - *Every* node is either red or black
    - The *root* is black
    - Each leaf, (NIL), is black
    - If a node is red, its children are black
- **Relationship of a node, z in a binary tree**



- Here, we call node A node z's parent, node B its grandparent, and node D its uncle
- **Strategy of Insertion in a Red-Black Tree**
  - Insert node z as a red node
  - Next, we re-color and rotate nodes in order to fix any violations of the rules of a red-black tree
  - There are four possible cases here
    1. If z is the root
       - All we need to do is insert the z and color it black instead of red

2. If z has a red uncle
    1. You must recolor the parent, grandparent, *and* uncle of node z
3. If z has a black uncle and forms a triangle with the parent (i.e. if the parent is a right child and z is a left child or if the parent is a left child and z is a right child)
    - We rotate z's parent with z
        - if z is the right child of A, now A will be the left child of z
4. If z has a black uncle and forms a line with the parent (i.e. if the parent is a right child and z is a right child or if the parent is a left child and z is a left child)
    - First, rotate z's grandparent
    - Then, recolor z's *original* parent and grandparent after rotation

- In a red-black tree, insertion and deletion operations each take $O(log(n))$ time because of tree rotation