

Introduction of Generative Adversarial Network (GAN)

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Generative Adversarial Network (GAN)

- How to pronounce “GAN”?



Google 小姐

Yann LeCun's comment

What are some recent and potentially upcoming breakthroughs in unsupervised learning?



Yann LeCun, Director of AI Research at Facebook and Professor at NYU

Written Jul 29 · Upvoted by Joaquin Quiñonero Candela, Director Applied Machine Learning at Facebook and Huang Xiao



Adversarial training is the coolest thing since sliced bread.

有史以來

↳ 有些東西問也時會用這個語

I've listed a bunch of relevant papers in a previous answer.

Expect more impressive results with this technique in the coming years.

What's missing at the moment is a good understanding of it so we can make it work reliably. It's very finicky. Sort of like ConvNet were in the 1990s, when I had the reputation of being the only person who could make them work (which wasn't true).

Yann LeCun's comment

What are some recent and potentially upcoming breakthroughs in deep learning?



Yann LeCun, Director of AI Research at Facebook and Professor at NYU

Written Jul 29 · Upvoted by Joaquin Quiñonero Candela, Director Applied Machine Learning at Facebook and Nikhil Garg, I lead a team of Quora engineers working on ML/NLP problems



.....

The most important one, in my opinion, is adversarial training (also called GAN for Generative Adversarial Networks). This is an idea that was originally proposed by Ian Goodfellow when he was a student with Yoshua Bengio at the University of Montreal (he since moved to Google Brain and recently to OpenAI).

GAN和他的种种变形是近年来他觉得ML最有前途的idea

This, and the variations that are now being proposed is the most interesting idea in the last 10 years in ML, in my opinion.

<https://www.quora.com/What-are-some-recent-and-potentially-upcoming-breakthroughs-in-deep-learning>

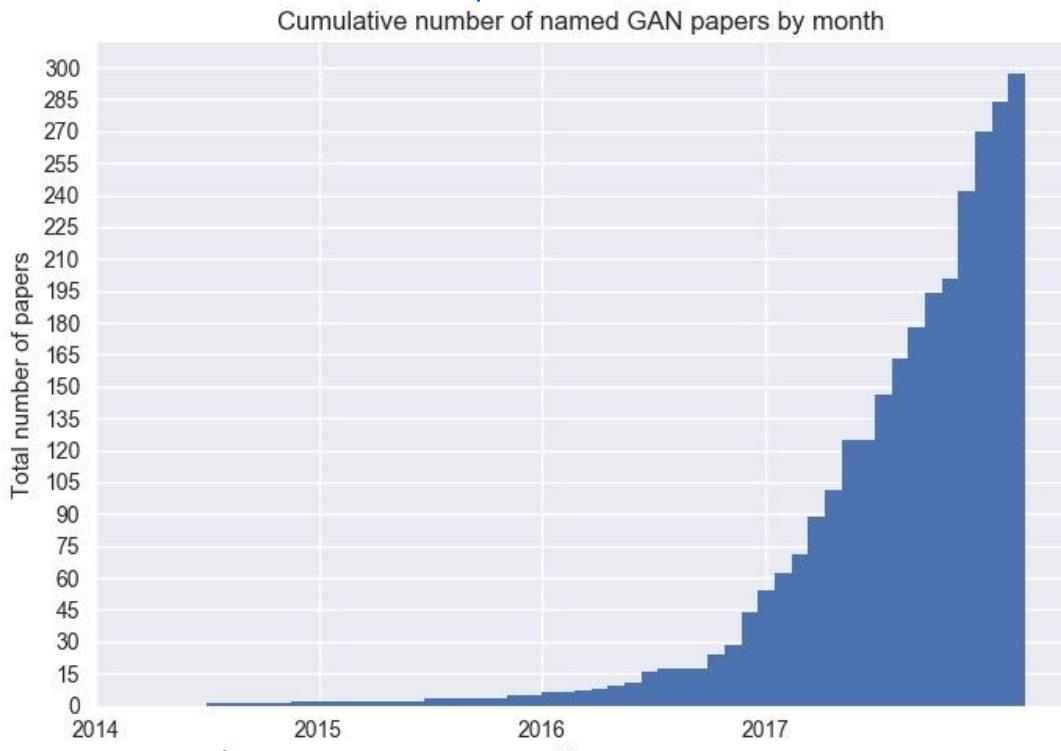
All Kinds of GAN ...

<https://github.com/hindupuravinash/the-gan-zoo>



LSGAN < Least square GAN
Lose square GAN

GAN
ACGAN
BGAN
CGAN
DCGAN
EBGAN
fGAN
GoGAN
⋮
⋮



⇒ 統計至今已經有300種的GAN了

Mihaela Rosca, Balaji Lakshminarayanan, David Warde-Farley, Shakir Mohamed, "Variational Approaches for Auto-Encoding Generative Adversarial Networks", arXiv, 2017

AGAN or AEGAN ⇒ but都被用掉了 ⇒ α -GAN

²We use the Greek α prefix for α -GAN, as AEGAN and most other Latin prefixes seem to have been taken
<https://deephunt.in/the-gan-zoo-79597dc8c347>.

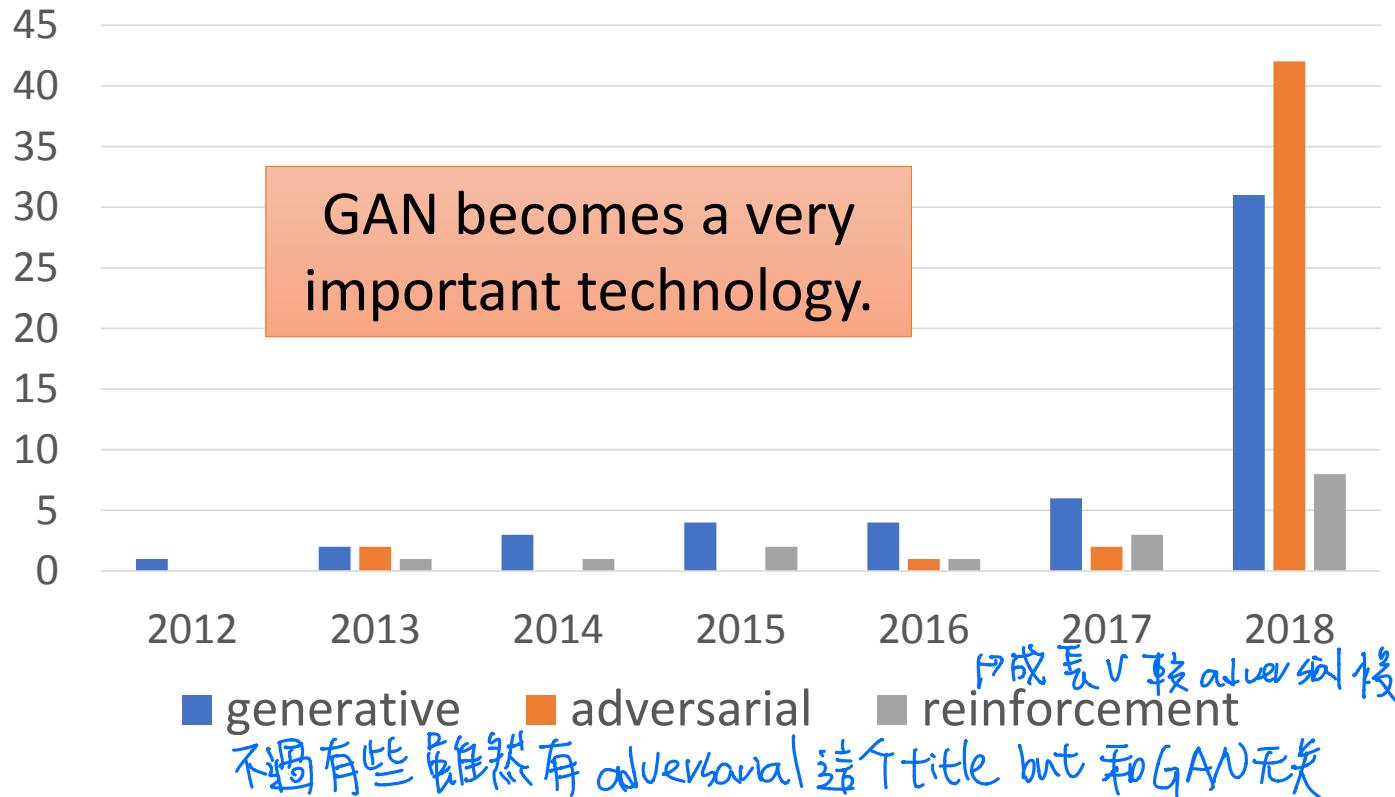
→ is Signal Processing conference

ICASSP

Keyword search on session index page,
so session names are included.

⇒ 近年 ICASSP 和 GAN 有跟的 paper

Number of papers whose titles include the keyword



Outline

* 給一個大的概念讓我們知道這個技術運作的情形

Basic Idea of GAN

GAN as structured learning

Can Generator learn by itself?

Can Discriminator generate?

A little bit theory

* GAN 想做的事 = 讓机器生成東西 影像、句子、文章

Generation

need → 訓練一個 generator

We will control what to generate latter. → Conditional Generation (較有用)

可以輸入一些條件 影文字讓機器產生正確的圖

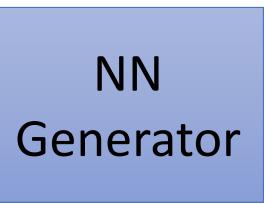
可以控制的文字/影像

Image Generation

有一个 Gaussian distribution 产生 random sample 一个 vector, then 把 vector 去到 generator

$$\begin{bmatrix} 0.3 \\ -0.1 \\ \vdots \\ -0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ -0.1 \\ \vdots \\ 0.7 \end{bmatrix} \begin{bmatrix} -0.3 \\ 0.1 \\ \vdots \\ 0.9 \end{bmatrix}$$

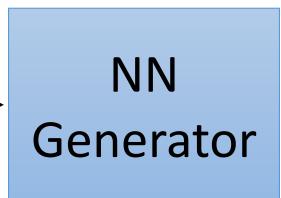
In a specific range
(不是 Vector) →



產生不同 image

Sentence Generation

$$\begin{bmatrix} 0.3 \\ -0.1 \\ \vdots \\ -0.7 \end{bmatrix} \begin{bmatrix} 0.1 \\ -0.1 \\ \vdots \\ 0.2 \end{bmatrix} \begin{bmatrix} -0.3 \\ 0.1 \\ \vdots \\ 0.5 \end{bmatrix}$$



How are you?
Good morning.
Good afternoon.

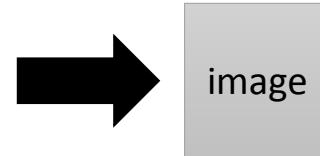
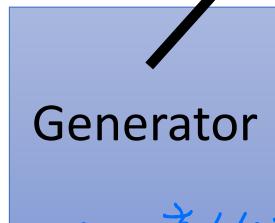
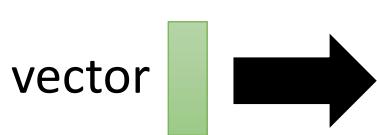
輸入隨機的向量 → output 想要的 object

其實沒有很大的用處

Basic Idea of GAN

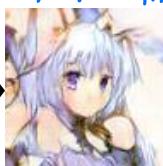
It is a neural network (NN), or a function.

→ 輸入一个東西輸出一个東西



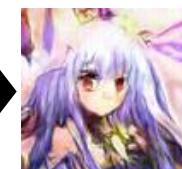
high dimensional vector

Ex 在影像生成，其 output 是高維的向量 (vector) → 会对应到每个 pixel 的 color

$$\begin{bmatrix} 0.1 \\ -3 \\ \vdots \\ 2.4 \\ 0.9 \end{bmatrix}$$


Each dimension of input vector represents some characteristics.

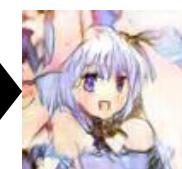
→ = 次元人物頭像生成

$$\begin{bmatrix} 3 \\ -3 \\ \vdots \\ 2.4 \\ 0.9 \end{bmatrix}$$


Longer hair

$$\begin{bmatrix} 0.1 \\ 2.1 \\ \vdots \\ 5.4 \\ 0.9 \end{bmatrix}$$


blue hair

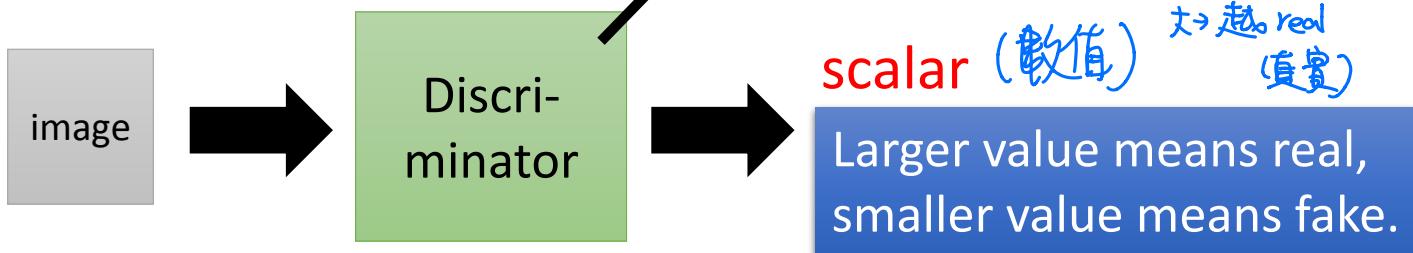
$$\begin{bmatrix} 0.1 \\ -3 \\ \vdots \\ 2.4 \\ 3.5 \end{bmatrix}$$


Open mouth

(每个维度会对应到图片的某种特征)

Basic Idea of GAN

It is a neural network (NN), or a function.



What is the relation between generator & discriminator.

Basic Idea of GAN

就好像 獵食者 & 獵物 之間的關係

对抗
合作



Generator

Brown

有葉脈的路
veins
有別的標準來判斷
不吃

Butterflies are
not brown

Butterflies do
not have veins

.....



Discriminator

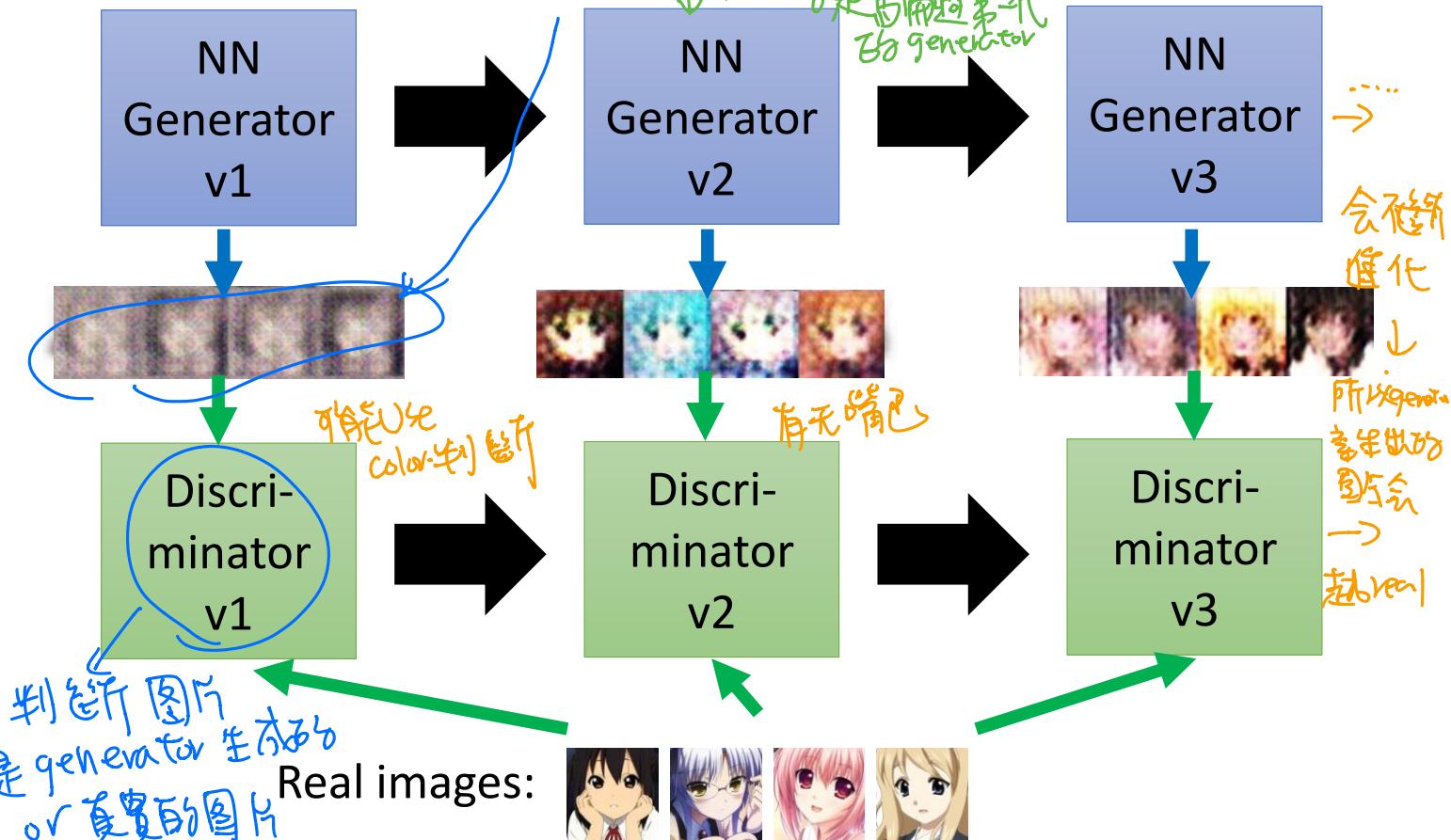
麻雀會吃枯葉蝶：在天擇的壓力下，變成棕色不會被吃
→ 也會演化

First 準備 database 內有很多真實人物的頭像
 - 開始參數是 random \Rightarrow 也不知道何產生

This is where the term “adversarial” comes from.
 反抗 (兩個 network 反抗)
 You can explain the process in different ways.....

Basic Idea of GAN

二次元頭像 \Rightarrow 只能產生雜訊



Basic Idea of GAN (和平的比喻)

Generator
(student)

Discriminator
(teacher)



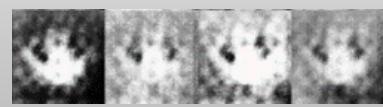
Generator
v1



Discriminator
v1

沒有兩個圈

Generator
v2



Discriminator
v2

沒有彩色

Generator
v3

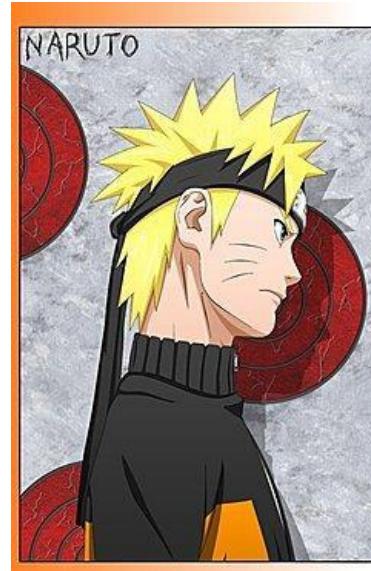


為什麼不自己學？

為什麼不自己做？

Generator v.s. Discriminator

- 寫作敵人，唸做朋友



Algorithm (GAN) → 操作原理

1. 參數 random initialized generator & discriminator
 2. iterative 的 train generator & discriminator.
- generator 參數固定，只調 discriminator 參數



- Initialize generator and discriminator

- In each training iteration:

兩組圖片 [generator 生產物 database sample 資料]

Step 1: Fix generator G, and update discriminator D



sample

generated objects

randomly sampled



G

then 調整 discriminator 參數
→ how to 調用？ database sample
↓
AND if image is 真實的
Update 雖然高分

D
if is generator 產生
就給低分

Fix
:: input is random
:: output 不會特別好

可以看作是 regression problem
[classification]

目的：訓練 network，使其 output ($t = \text{database}$)

(上) \star output 要離 t 越近越好
D \rightarrow (下)

Discriminator learns to assign high scores to real objects
and low scores to generated objects.

Algorithm

- Initialize generator and discriminator

G

D

- In each training iteration:

∴ discriminator 訓練好了 :: how fix

Step 2: Fix discriminator D, and update generator G

Generator learns to “fool” the discriminator

hope generator 產生出來的圖片 discriminator 可以給比較高的分數 \Rightarrow generator 產生的是較真實的



實際上在 write code 時會把 generator & discriminator 合起來當作是一個巨大的 network
train network 時，固定最後幾個 hidden layer，只調前幾個 hidden layer large network \Rightarrow 目標 output 值越越
好
* 不是 Gradient descent \rightarrow 目標越來越好了

Algorithm

Initialize θ_d for D and θ_g for G

- In each training iteration:

- Sample m examples $\{x^1, x^2, \dots, x^m\}$ from database
- Sample m noise samples $\{z^1, z^2, \dots, z^m\}$ from a distribution // 可能沒有最真實影響 = 可以是 Gaussian / uniform
→ 產生m張圖片 $p_x \tilde{x}(\text{tilda})$
- Obtaining generated data $\{\tilde{x}^1, \tilde{x}^2, \dots, \tilde{x}^m\}$, $\tilde{x}^i = G(z^i)$

Learning D

只是一个範例
搞不好用其他的
也不错
要執行几次是
可以調整的

原初GAN paper寫的

$$\begin{aligned} \tilde{V} &= \frac{1}{m} \sum_{i=1}^m \log D(x^i) + \frac{1}{m} \sum_{i=1}^m \log (1 - D(\tilde{x}^i)) \\ \theta_d &\leftarrow \theta_d + \eta \nabla \tilde{V}(\theta_d) \end{aligned}$$

learning rate 取何等值 = 可用 Adam 方法
→ D 的值会落在 0-1 之间
if "—" = descent
if want loss 越小越好
假意的圖片 ↓
signal
logistic
會有問題

- Sample m noise samples $\{z^1, z^2, \dots, z^m\}$ from a distribution

- Update generator parameters θ_g to maximize

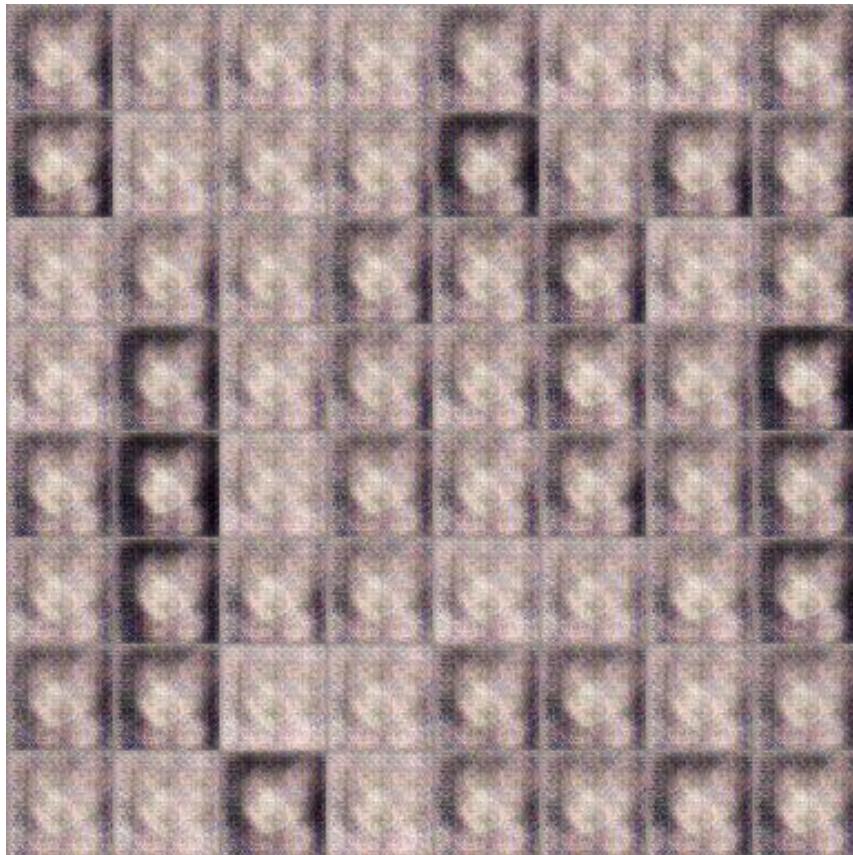
$$\begin{aligned} \tilde{V} &= \frac{1}{m} \sum_{i=1}^m \log (D(G(z^i))) \\ \theta_g &\leftarrow \theta_g - \eta \nabla \tilde{V}(\theta_g) \end{aligned}$$

產生-張圖
數值

Learning G

Anime Face Generation

100 updates



Source of training data: <https://zhuanlan.zhihu.com/p/24767059>

Anime Face Generation



1000 updates

Anime Face Generation

muñecos



2000 updates

Anime Face Generation

key



5000 updates

Anime Face Generation

手書き
おめでたす



10,000 updates

Anime Face Generation



20,000 updates

Anime Face Generation



50,000 updates



The faces
generated by
machine.

圖片生成：
吳宗翰、謝濬丞、
陳延昊、錢柏均

3次元
人物
生成

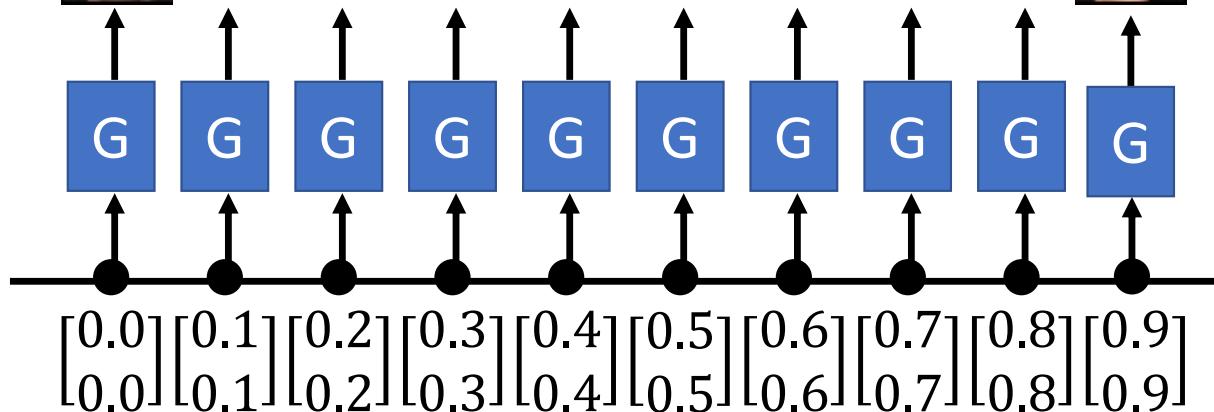


input的vector代別輸出圖片的特性



圈裏

→ 不需要物理模型、規則
就可以做出來



机器可以生成沒有距離的圖

感謝陳柏文同學提供實驗結果

Outline

Basic Idea of GAN

GAN as structured learning (原理)

Can Generator learn by itself?

Can Discriminator generate?

A little bit theory

Object Structured Learning

Structured Learning

Machine learning is to find a function f

$$f : X \rightarrow Y$$

Regression: output a scalar

Classification: output a “class” (one-hot vector)

1	0	0
---	---	---

Class 1

0	1	0
---	---	---

Class 2

0	0	1
---	---	---

Class 3

Structured Learning/Prediction: output a sequence, a matrix, a graph, a tree

Output is composed of components with dependency

Output Sequence

$$f : X \rightarrow Y$$

Machine Translation

X ：“機器學習及其深層與
結構化”
(sentence of language 1)

Y ：“Machine learning and
having it deep and structured”
(sentence of language 2)

Speech Recognition

X ：
(speech)

Y ：感謝大家來上課
(transcription)

Chat-bot

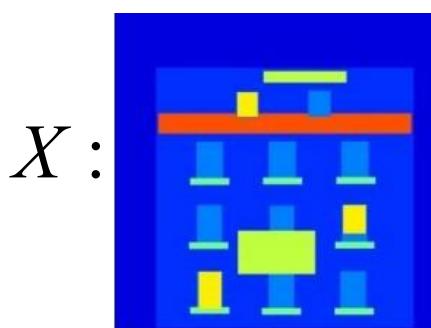
X ：“How are you?”
(what a user says)

Y ：“I'm fine.”
(response of machine)

Output Matrix

$$f : X \rightarrow Y$$

Image to Image



$Y :$



Colorization:



Ref: <https://arxiv.org/pdf/1611.07004v1.pdf>

Text to Image

$X :$ “this white and yellow flower
have thin white petals and a
round yellow stamen”

$Y :$



ref: <https://arxiv.org/pdf/1605.05396.pdf>

Why Structured Learning Challenging?

可以被視為極端的

→ 根本沒有範例 or 很少很少範例

- 假設我們把機器每一种可能的 output 視為一个 class
structured learning 是一個極端的 one shot learning or zero-shot

- One-shot/Zero-shot Learning:

- In classification, each class has some examples.
- In structured learning,
 - If you consider each possible output as a “class”
⇒ 表示在 training data 只會出現一次 testing data 中的 class 可能在 training 數據中
 - Since the output space is huge, most “classes” do not have any training data.
 - Machine has to create new stuff during testing.
• 如何學到一般化，如何學著輸出從沒有看過的東西變成很重要
 - Need more intelligence

↓
機器要學會創造 才能用 structure learning
(training
沒 seen)

Why Structured Learning Challenging?

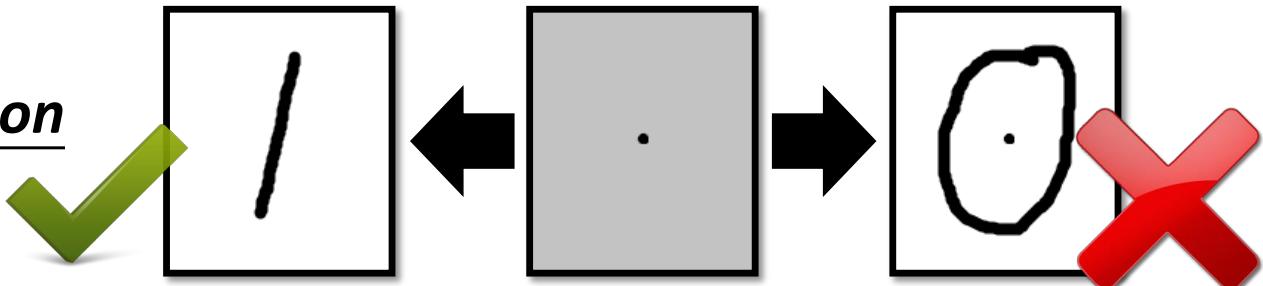
- Machine has to learn to do **planning**
 - Machine generates objects component-by-component, but it should have a big picture in its mind.
 - Because the output components have dependency, they should be considered globally.

重要的不是產生了什麼 component
而是 component 之間的關係
⇒ 只看一部份不會知道 output 整體
↓
機器要產生很複雜的物件

是由簡單的
Component 組成

need 看全局

Image Generation



Sentence Generation

這個婆娘不是人

九天玄女下凡塵



GAN 是 Structured Learning 的 sol.

Structured Learning Approach

Generator

Learn to generate
the object at the
component level

- 一个个 component 分開去產生這個物件
(可能會失去大局观)

2个方法

1.



Generative
Adversarial
Network (GAN)

Discriminator

Evaluating the
whole object, and
find the best one

從產生完一個完整的物件以后
再去評定整體來看產生的物件好不好 (很難做 generation)

2.



but

Outline

Basic Idea of GAN

GAN as structured learning

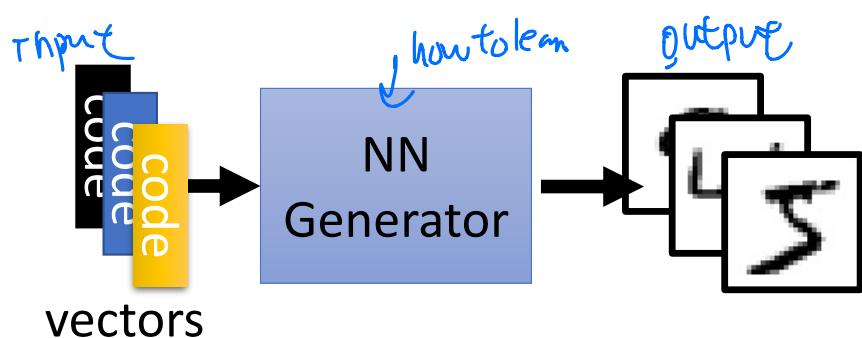
Can Generator learn by itself?

Can Discriminator generate?

A little bit theory

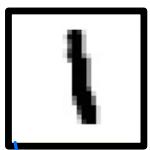
T傳統先知 input & output
就會知道結果

Generator



code:
[0.1]
[-0.5]

Image:



[0.1]
[0.9]



[0.2]
[-0.1]



[0.3]
[0.2]



和 train Supervised learning
用 gradient descent train

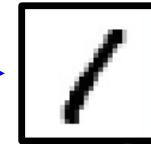
Same

[0.1]
[0.9]

NN
Generator

As close as possible

image



實際上就是輸出元的向量

As close as possible

y_1
 y_2
⋮

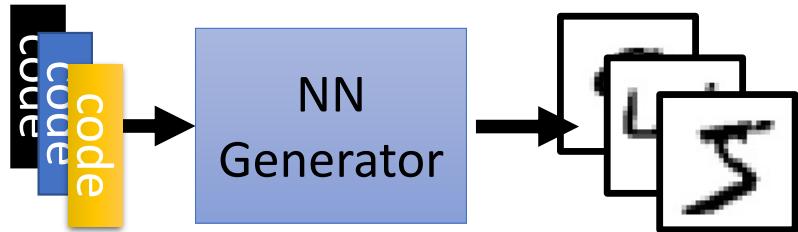
[1]
[0]
⋮

c.f.



NN
Classifier

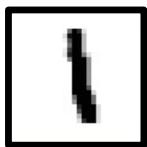
Generator



可将多维希望 input 的 Vectors
和 out put 特征数有关

code:
(where does they
come from?)

Image:



$$\begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$$

$$\begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}$$



.. learn encoder

Encoder in auto-encoder
provides the code 😊

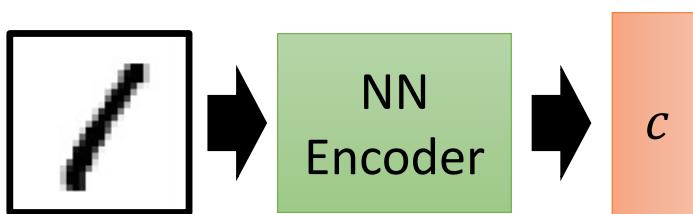
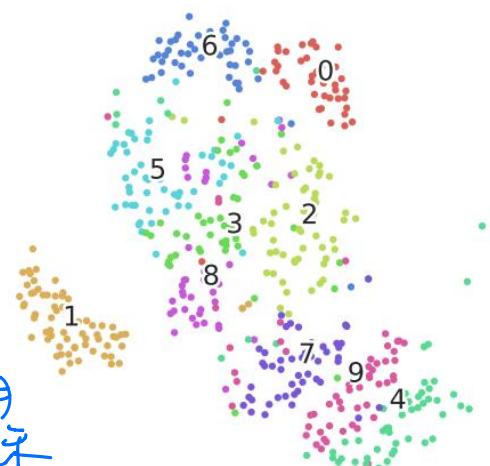
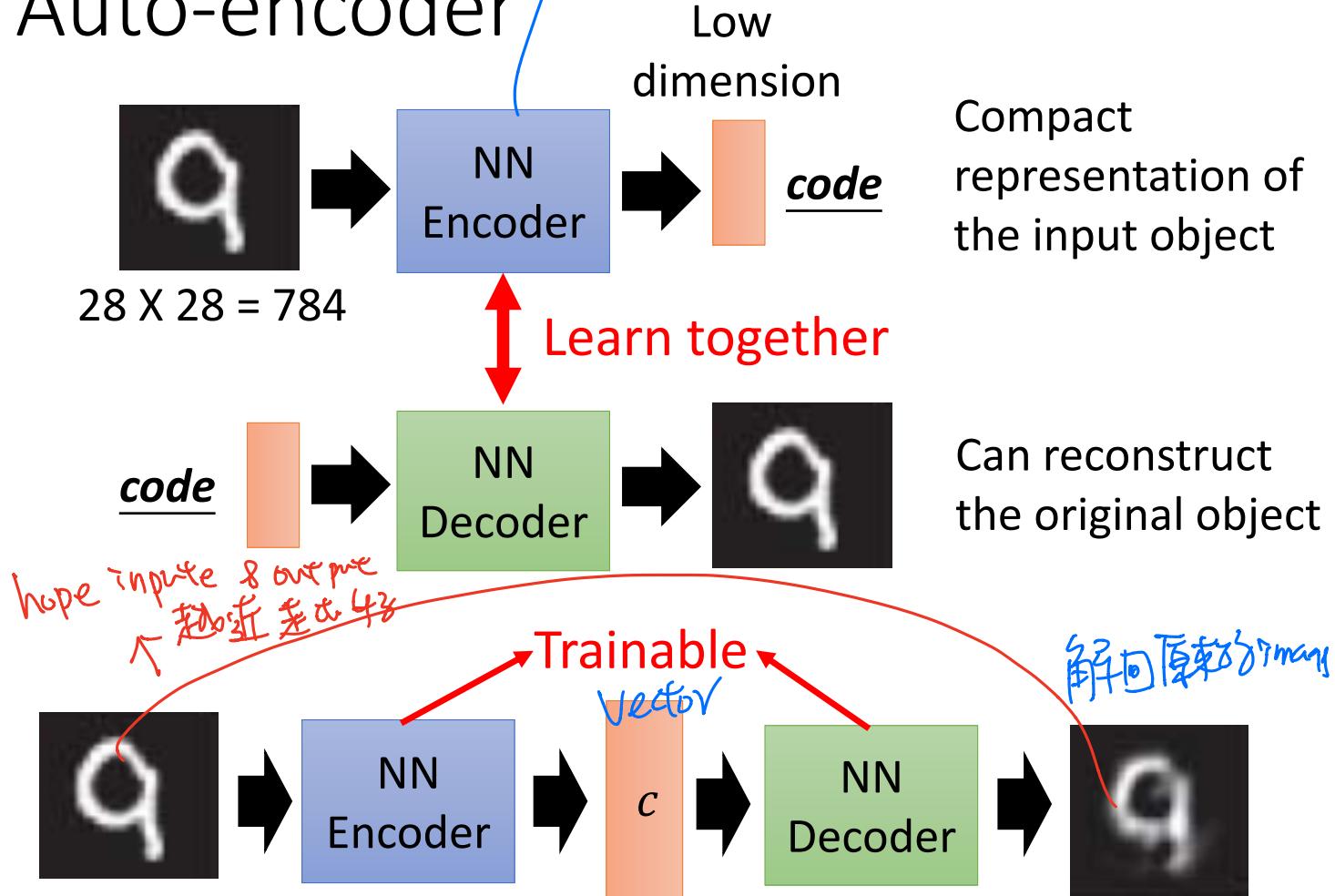


图 19

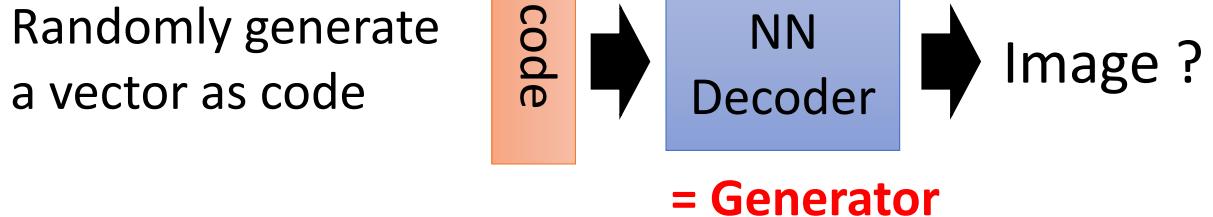
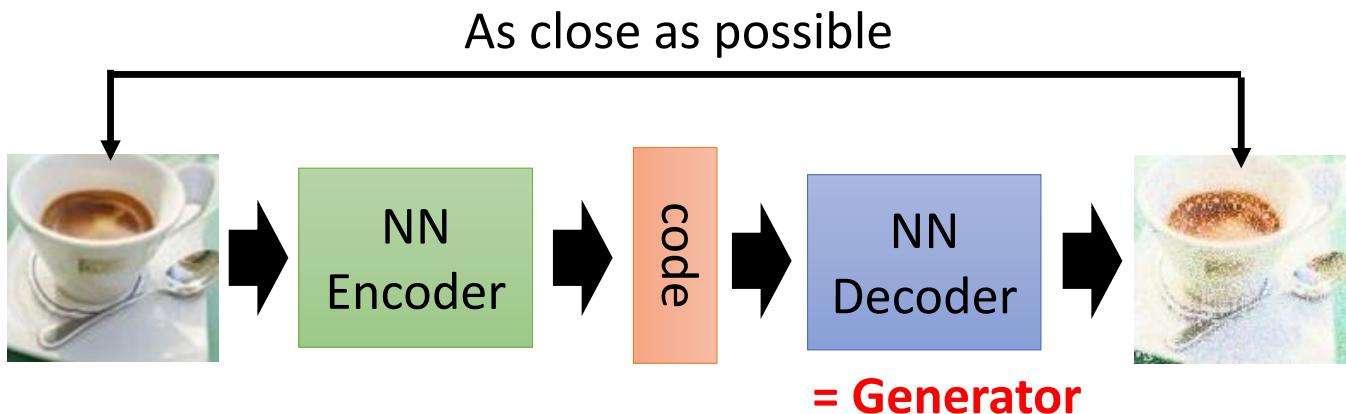
把特征放用
一个向量表示



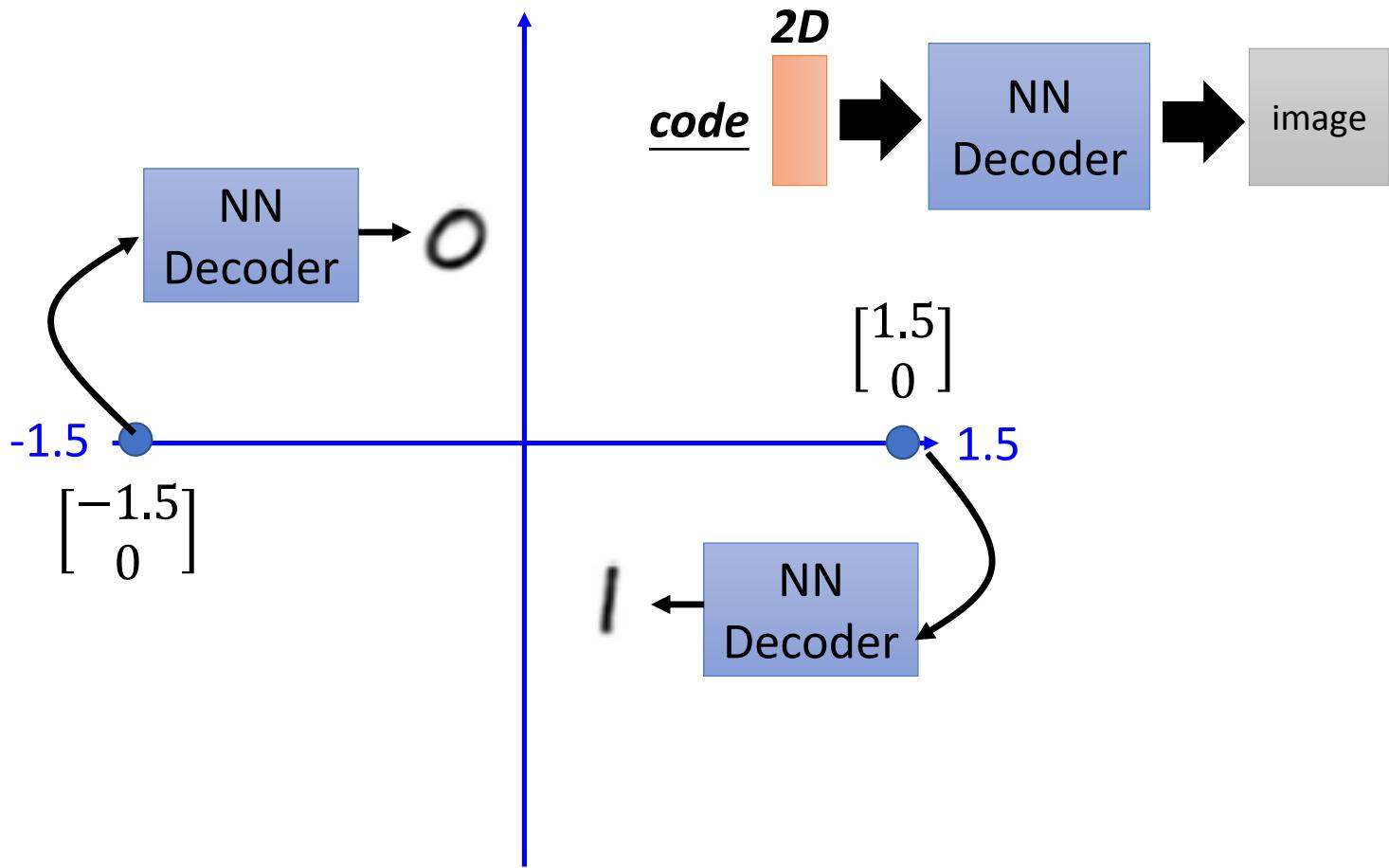
Auto-encoder



Auto-encoder



Auto-encoder



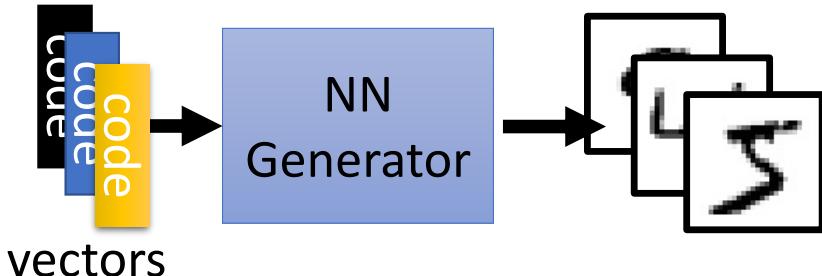
Auto-encoder

和蔼有关

和
蔼
有
关

5	5	5	5	3	3	3	3	3	3
6	6	6	6	3	3	3	3	3	3
0	6	5	5	3	3	3	3	2	2
0	0	5	5	3	3	3	3	2	2
0	0	0	8	3	3	3	8	8	8
0	0	0	0	8	8	8	8	8	8
0	0	0	0	2	6	6	8	8	8
0	0	0	0	0	5	7	7	1	1
0	0	0	0	0	2	4	7	7	1
0	0	0	0	0	4	4	7	7	1

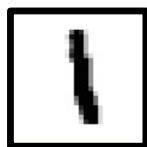
Auto-encoder



code:
[0.1]
[-0.5]

(where does them
come from?)

Image:



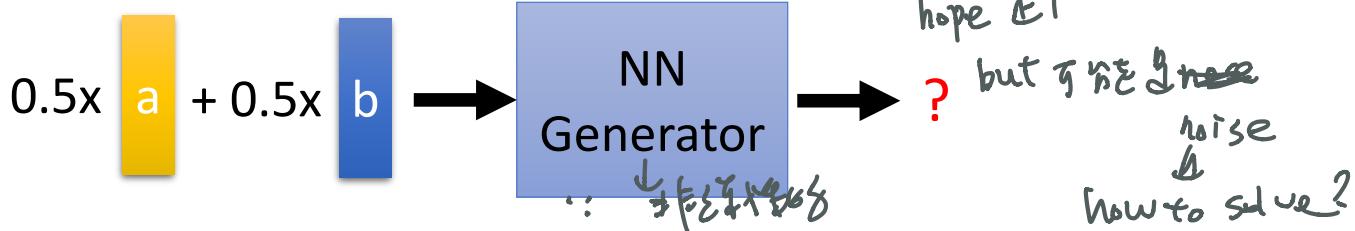
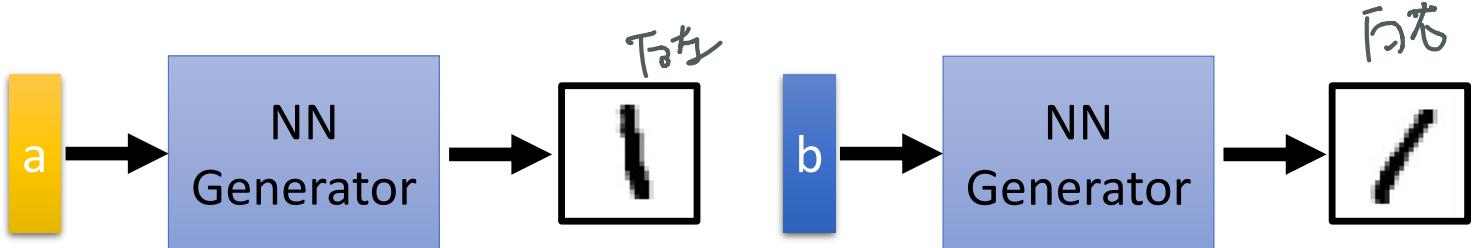
[0.1]
[0.9]



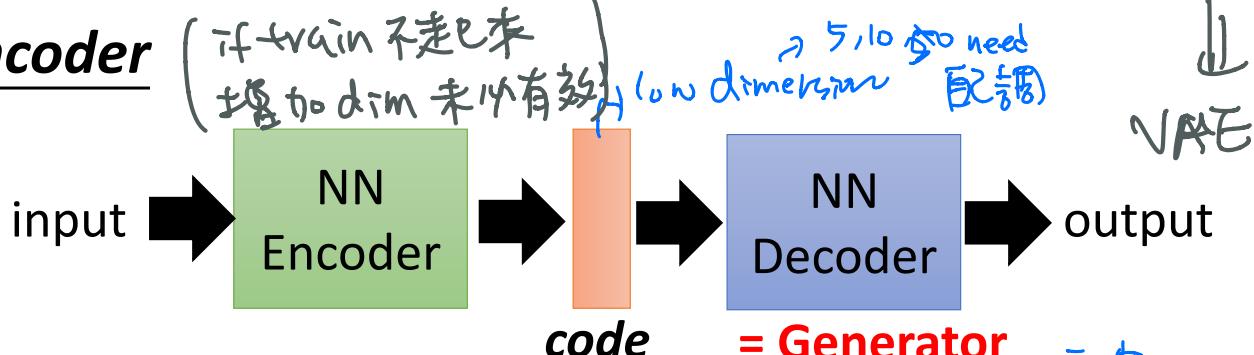
[0.2]
[-0.1]



[0.3]
[0.2]



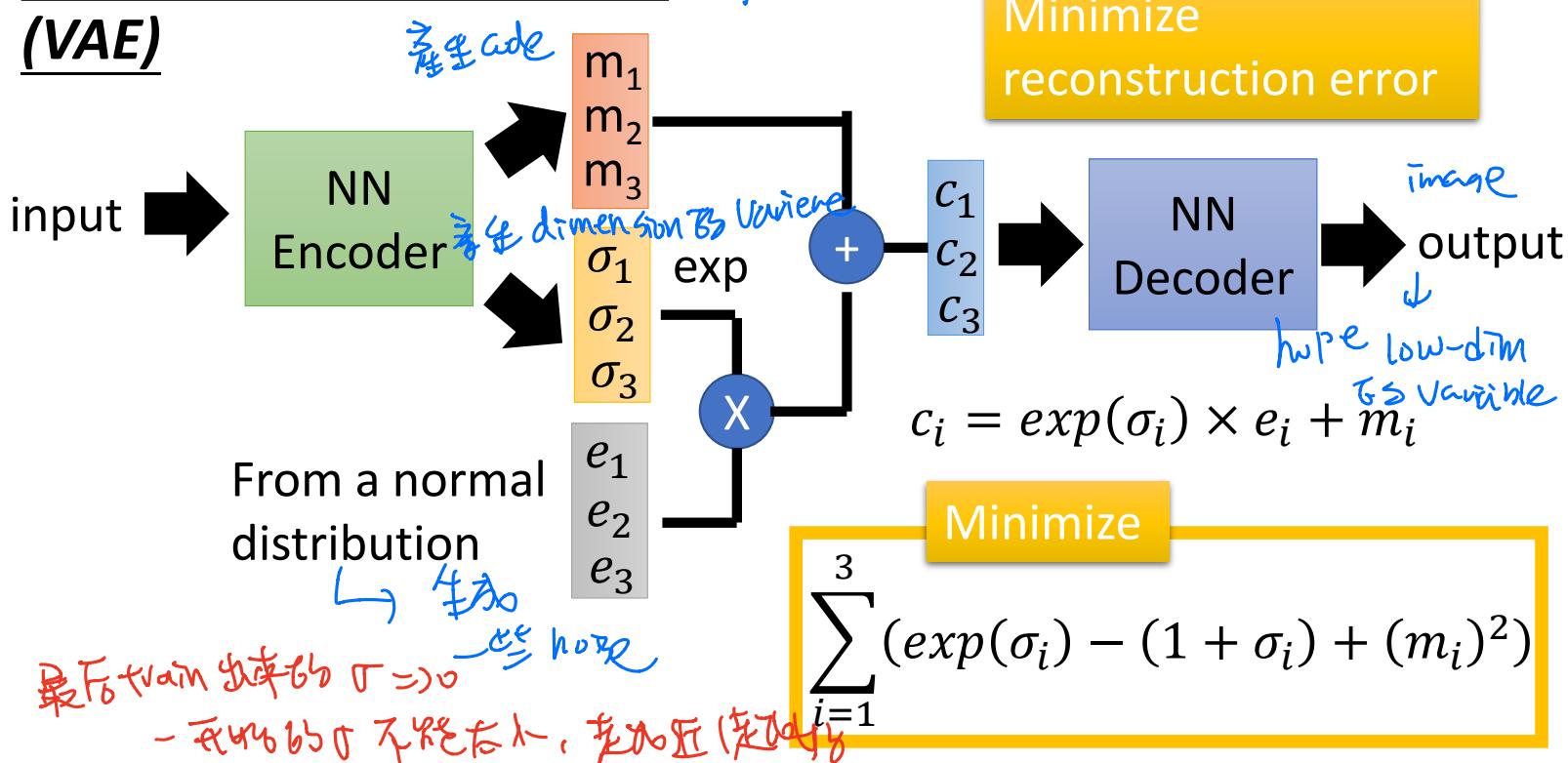
Auto-encoder



VAE

Variational Auto-encoder

(VAE)



Auto-encoding ↗ ↘ ?
What do we miss?

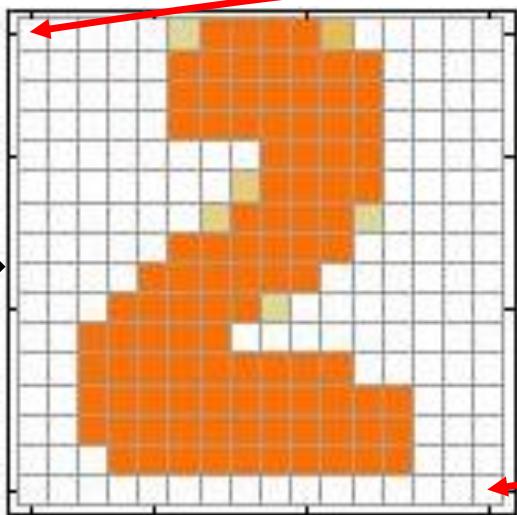
把這兩張圖片變成 vector

問 用一个 vector 表示 euclidean
distance \Rightarrow 這個就是要 minimizes 的
hope 走動到那邊

Generated Image

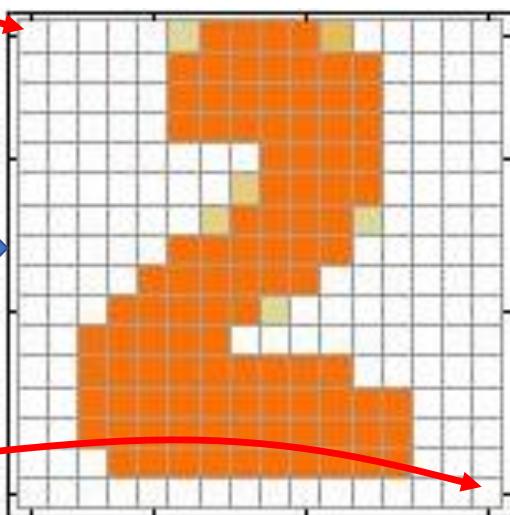
Target

G



as close as possible

盡量



→ 可能沒有辦法完全 copy image \Rightarrow 會有誤差 \Rightarrow 在哪些部份 \rightarrow 重要

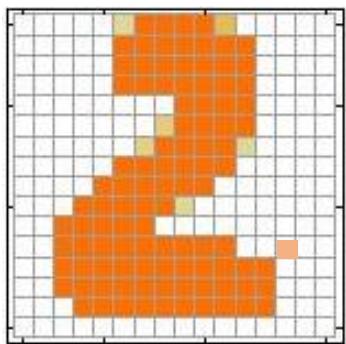
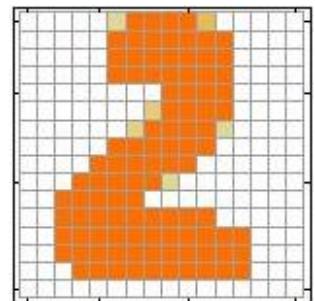
It will be fine if the generator can truly copy the target image.

What if the generator makes some mistakes

Some mistakes are serious, while some are fine.

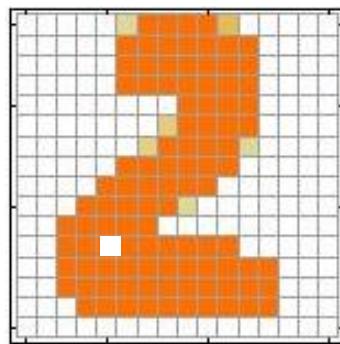
What do we miss?

Target



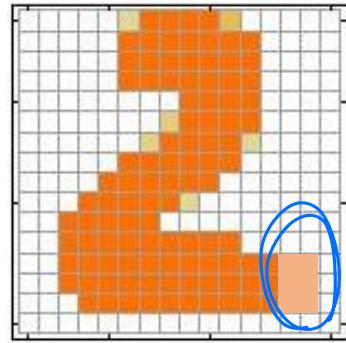
1 pixel error

我覺得不行



1 pixel error

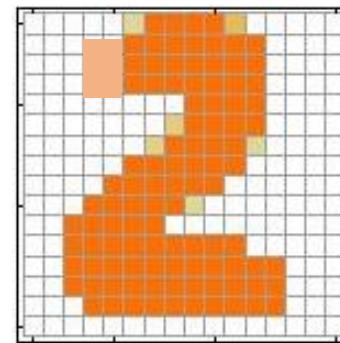
我覺得不行



6 pixel errors

我覺得其實
可以

→ 懂了怎麼用



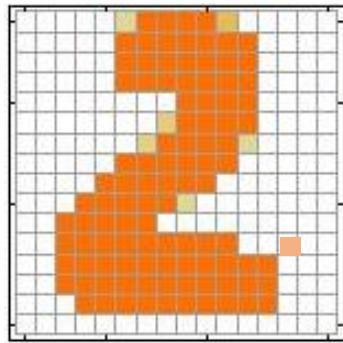
6 pixel errors

我覺得其實
可以

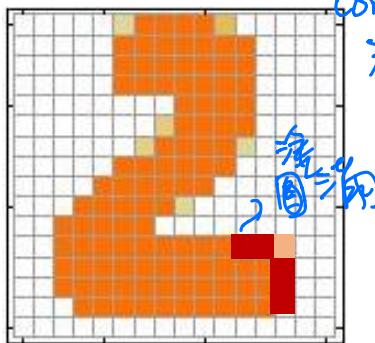
GAN train = 產生圖片
Auto-encoder train = need 複雜的 network
會導致一個 network 的架構 很複雜

What do we miss?

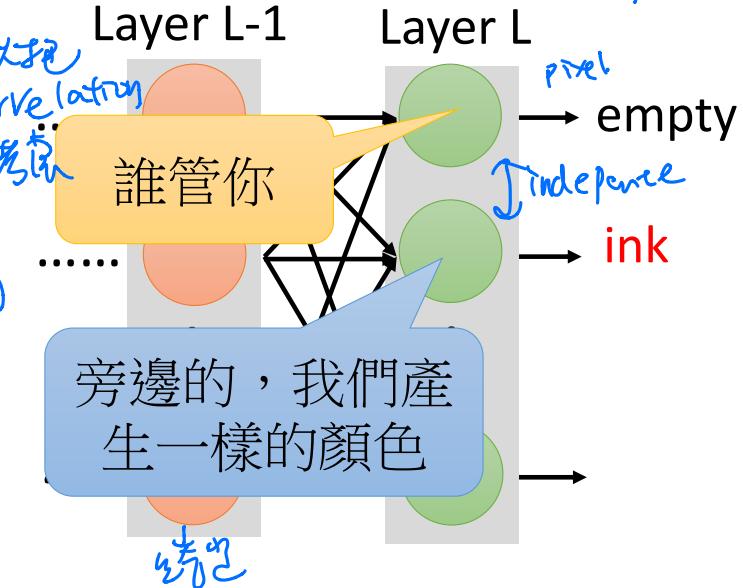
- 一個組織沒有多層 pixel & pixel 之間的關係 but 在 hidden layer 可以把 correlation 為甚麼 Layer L-1 Layer L



我覺得不行



我覺得其實可以



The relation between the components are critical.

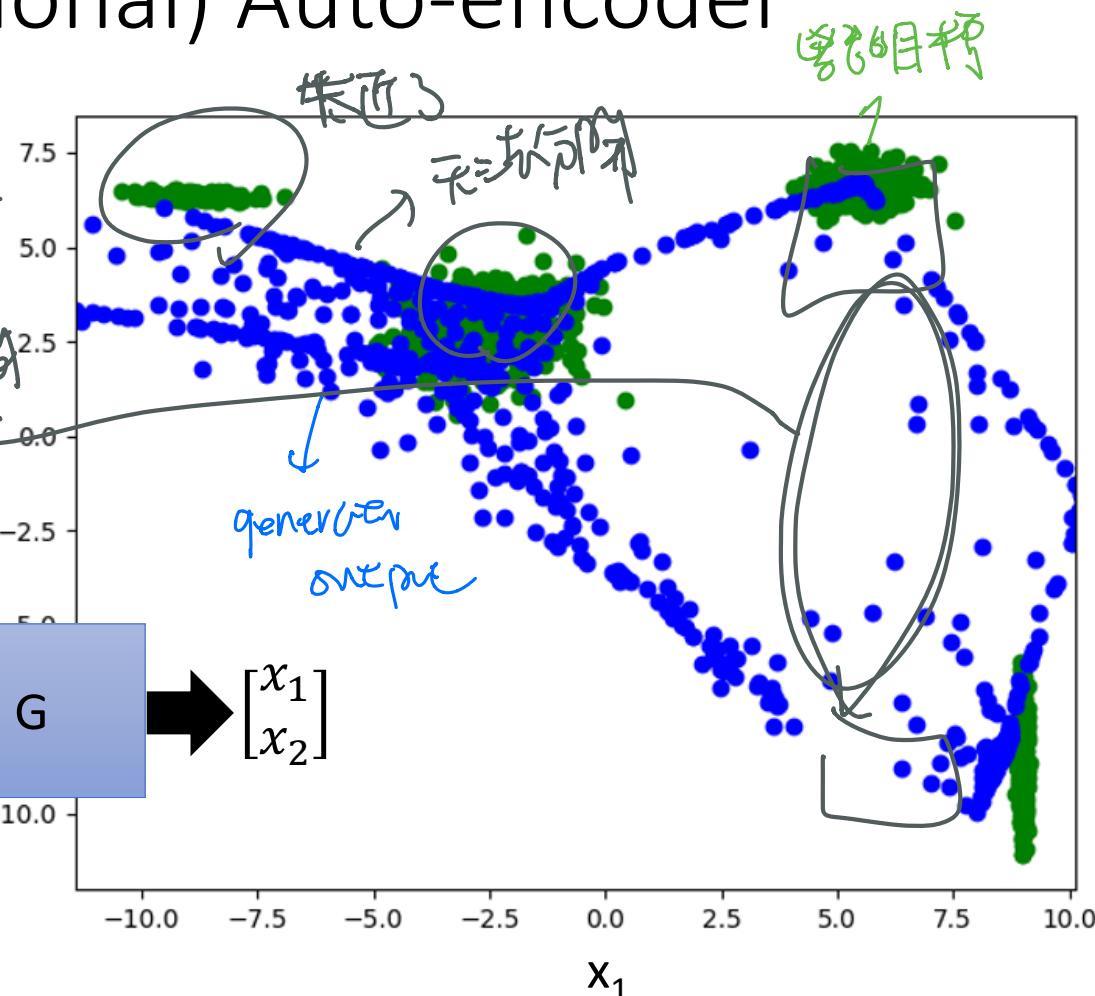
Although highly correlated, they cannot influence each other.

Need deep structure to catch the relation between components.

把 correlation 考慮進去 need deep network

(Variational) Auto-encoder

不容易知道
在值很大的
但小的時候
但值介於中間
→ 不夠



Outline

Basic Idea of GAN

GAN as structured learning

Can Generator learn by itself?

Can Discriminator generate?

A little bit theory

Discriminator

Evaluation function, Potential Function, Energy Function ...

- Discriminator is a function D (network, can deep)

$$D : X \rightarrow \mathbb{R}$$

- Input x: an object x (e.g. an image)
- Output D(x): scalar which represents how “good” an object x is



Can we use the discriminator to generate objects?

Yes.

Discriminator

可能是 \downarrow convolution
network

discriminator 相對 generator 有什麼優勢

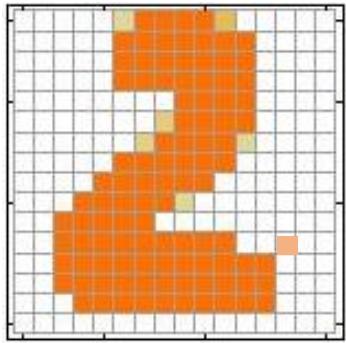
\downarrow 老實 Compute 之間的關係 一個 component 獨立生的

很 easy

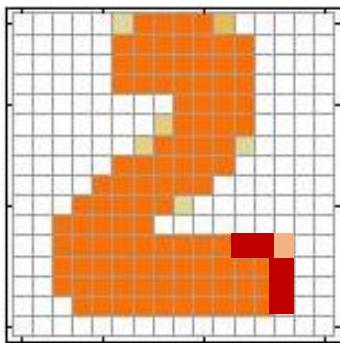
\downarrow 老實 Compute 之間的關係

- It is easier to catch the relation between the components by top-down evaluation.

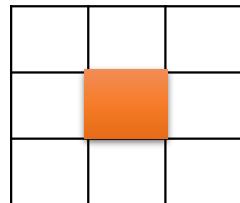
裡面有 \downarrow filter \rightarrow 看去 see 有夠 獨立的 filter. 可以分



我覺得不行



我覺得其實 OK



\Rightarrow 依

This CNN filter is
good enough.

Discriminator

input一个 x see 他是猫 or 狗

⇒ 密鑑所有可能的 x 一个个丢到 discriminator 中

- Suppose we already have a good discriminator

$D(x) \dots$ see 那个 x discriminator 会给他很高的分數
→ 最高分的那个 x 就是 Ans

Inference

- Generate object \tilde{x} that

$$\tilde{x} = \arg \max_{x \in X} D(x)$$

Enumerate all possible x !!!

It is feasible ???

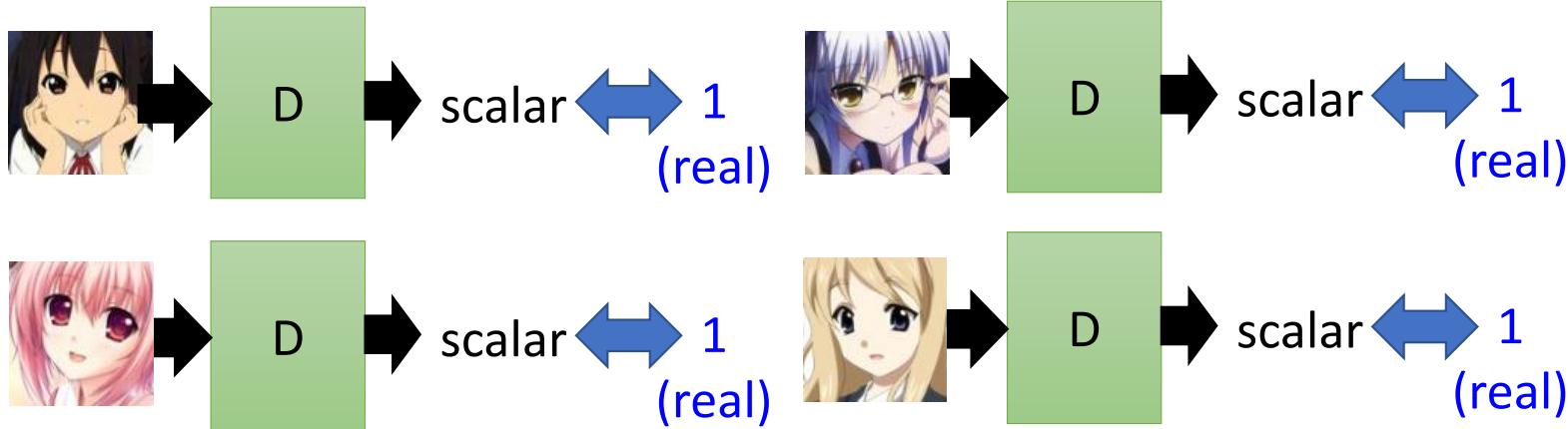
How to learn the discriminator?

問自己
能不能
東西

Discriminator - Training

- I have some real images

only have 1 (real)
沒有負例的

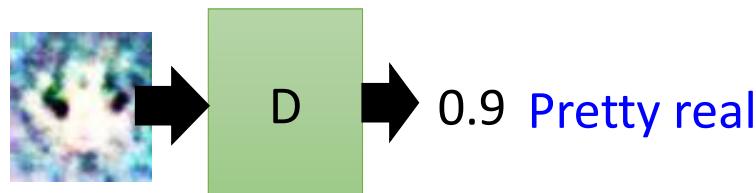
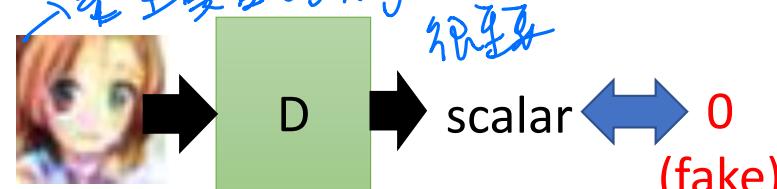
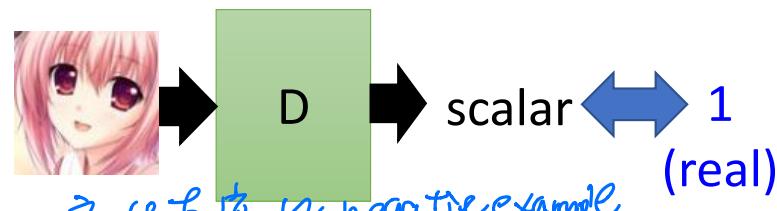
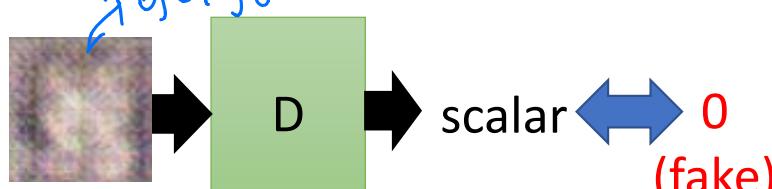
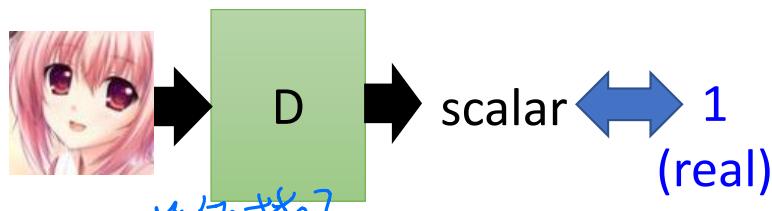


Discriminator only learns to output “1” (real).

Discriminator training needs some negative examples.

Discriminator - Training

- Negative examples are critical.



How to generate realistic
negative examples?

Use iterative 来解决

Discriminator - Training

- General Algorithm

- Given a set of **positive examples**, randomly generate a set of **negative examples**. *random sample.
加点噪音*



- In each iteration

- Learn a discriminator D that can discriminate positive and negative examples.

反复迭代



v.s.



- Generate negative examples by discriminator D



$$\tilde{x} = \arg \max_{x \in X} D(x)$$

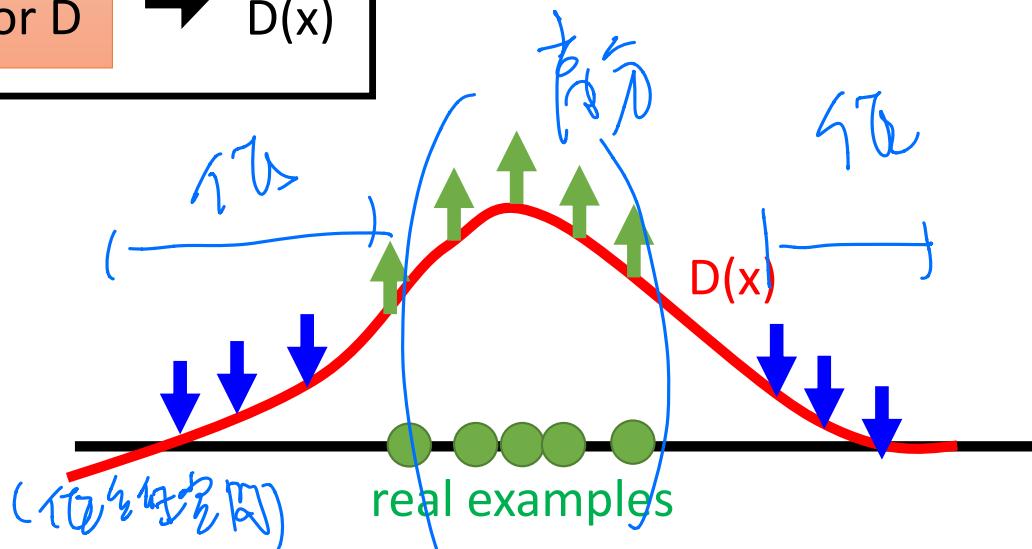
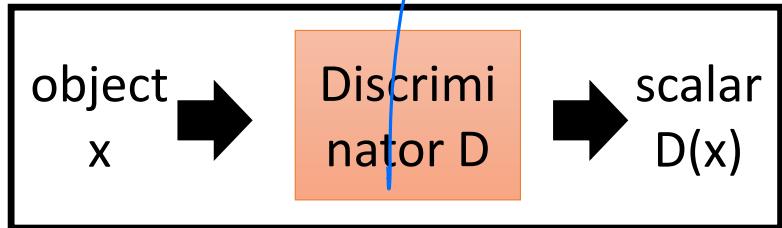
随机
采样

→ 会自动这个样子用法

D

→ 会自动这个样子用法

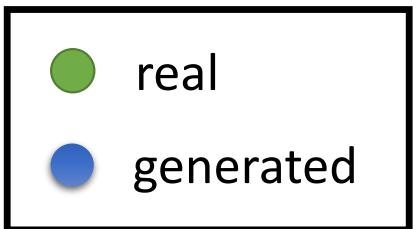
Discriminator - Training



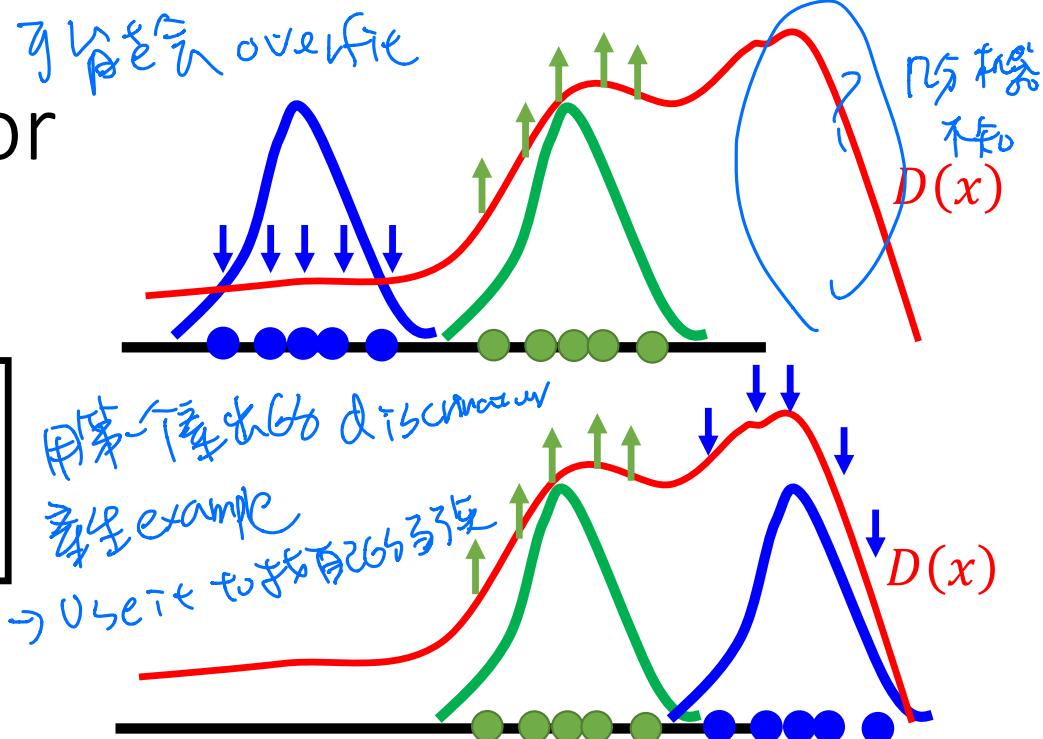
In practice, you cannot decrease all the x

other than real examples. object 跟著分布的位置不同 disom. 會跟著改變

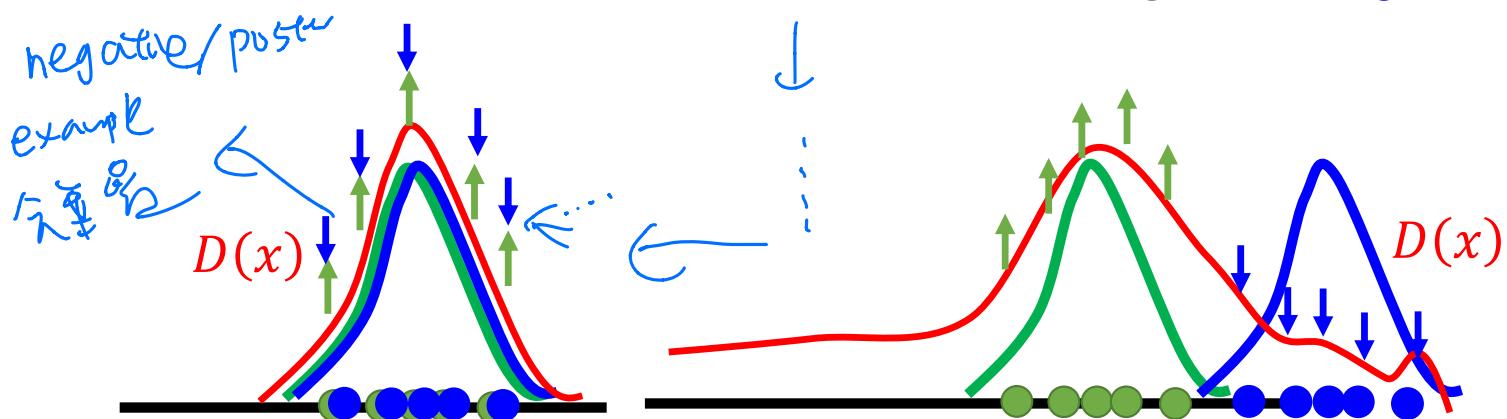
Discriminator - Training

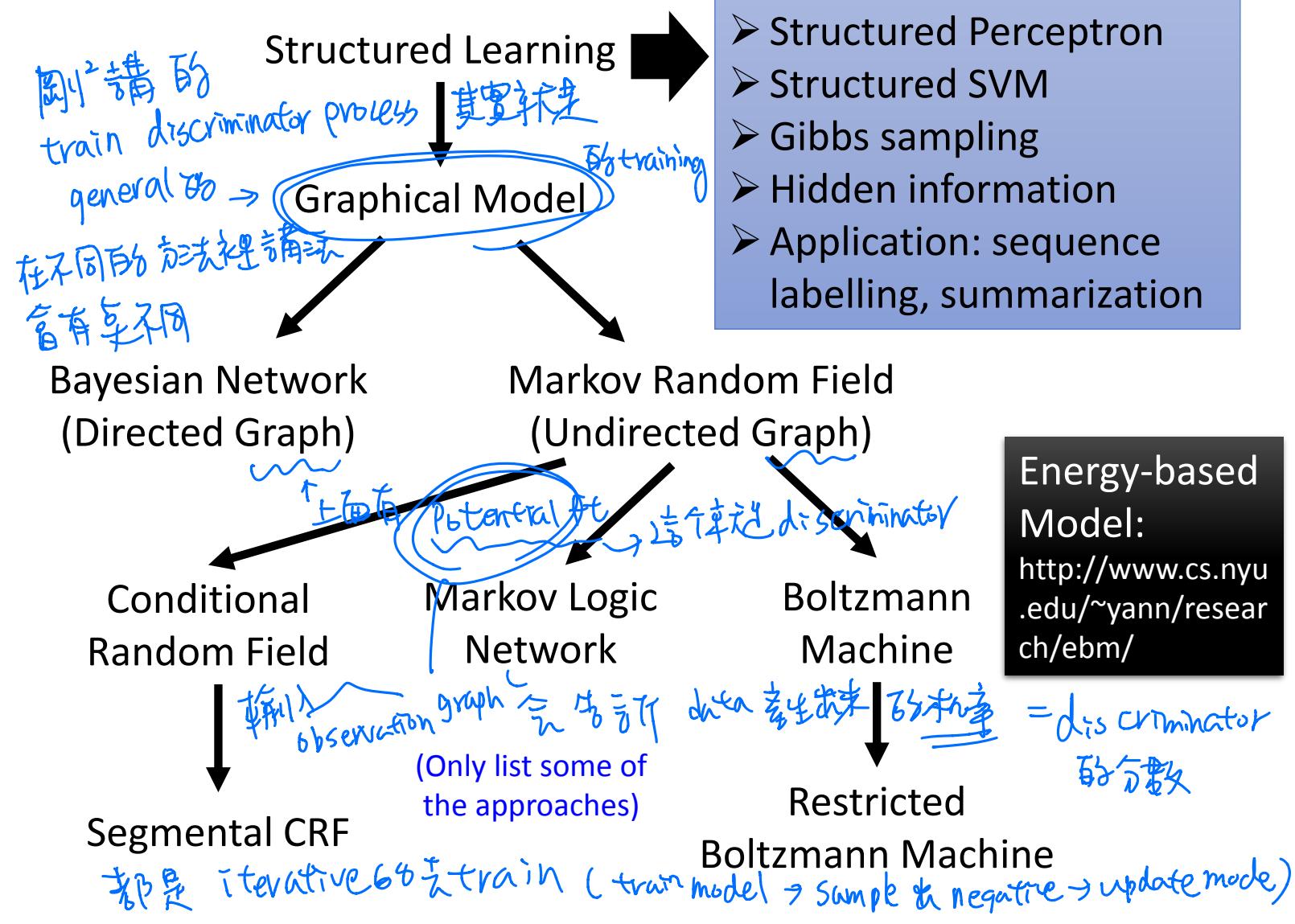


If data 亂序 且會溢出 overflow



In the end





Generator v.s. Discriminator

- **Generator**

生成快

- Pros:

- Easy to generate even with deep model

- Cons:

→ 只看到表面

- Imitate the appearance
- Hard to learn the correlation between components

- **Discriminator**

考全局

- Pros:

- Considering the big picture

- Cons:

→ 生成困难
of Arg max problem

- Generation is not always feasible
model need

- Especially when your model is deep

- How to do negative sampling?

→ 取得 Argmax BS problem (往々 discrimin 会得するものが image)

Generator + Discriminator

- General Algorithm

- Given a set of **positive examples**, randomly generate a set of **negative examples**.



- In each iteration
 - Learn a discriminator D that can discriminate positive and negative examples.

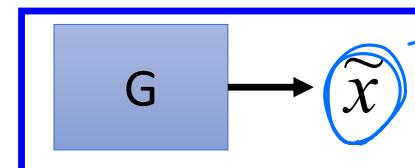


v.s.



D

- Generate negative examples by discriminator D



$$\tilde{x} = \arg \max_{x \in X} D(x)$$

Benefit of GAN

- From Discriminator's point of view
 - Using generator to generate negative samples

$$\boxed{G \rightarrow \tilde{x}} = \boxed{\tilde{x} = \arg \max_{x \in X} D(x)}$$

efficient ↳ loss 將是來自於由 discriminator

- From Generator's point of view
 - Still generate the object component-by-component
 - But it is learned from the discriminator with global view.

GAN

Top discriminator \hat{D}
generator \hat{G}_y

感謝 段逸林 同學提供結果

比 VAE 的 generator 好

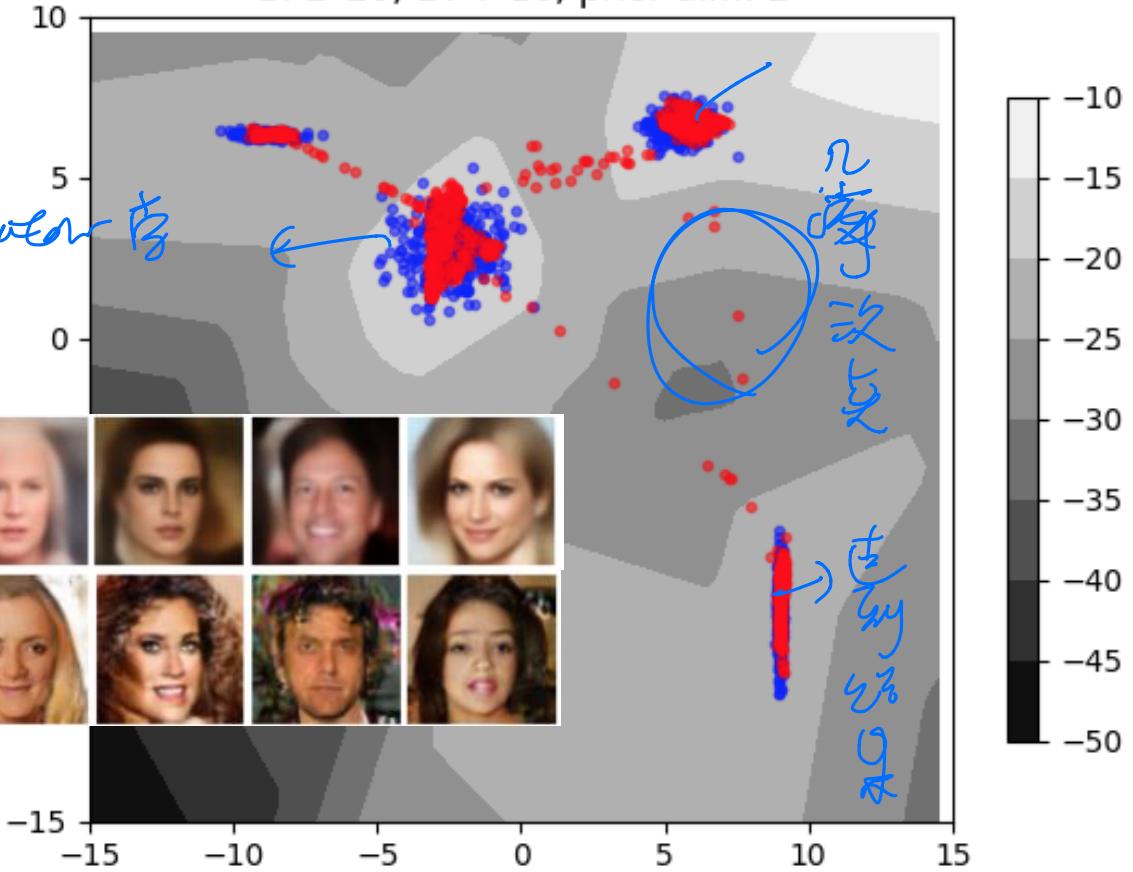
wgan-gp-sub1000-gauss4
Samples and Decision Boundary
G: 2*20; D: 4*10; prior dim: 2

GAN

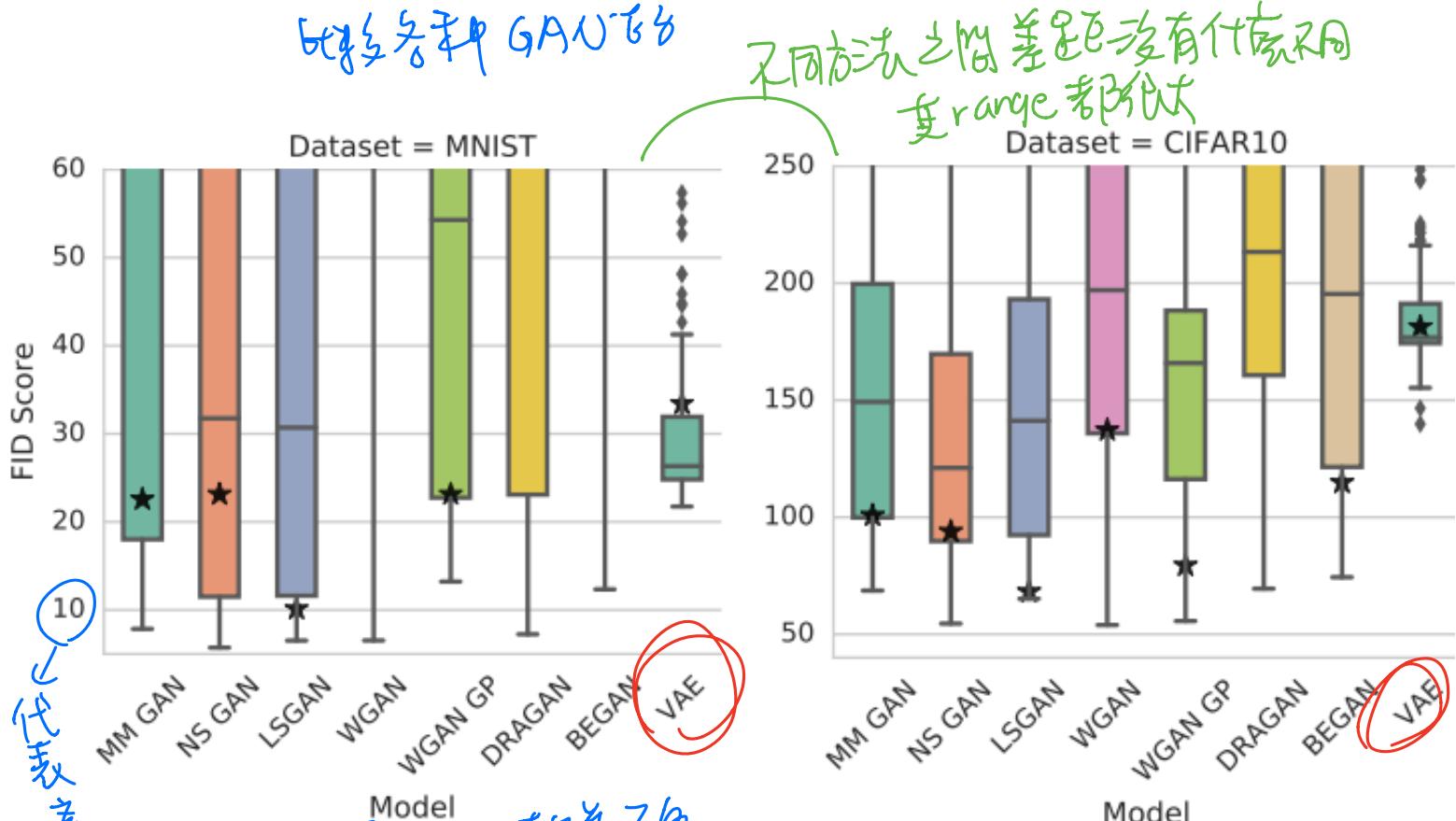
VAE

GAN

<https://arxiv.org/abs/1512.09300>



Iter: 99500; D loss: -0.04111; G loss: 20.36
KLD(r,g)=[0. 0.]; KLD(g,r)=[0.6510948 0.72137838]



FID [Martin Heusel, et al., NIPS, 2017]: Smaller is better

图片越real

- ① VAE 平差 (分數很集中)
- ② 最好結果 = GAN

Next Time

- Preview
 - <https://youtu.be/0CKeqXI5IY0>
 - <https://youtu.be/KSN4QYgAtao>

Outline

Basic Idea of GAN

GAN as structured learning

Can Generator learn by itself?

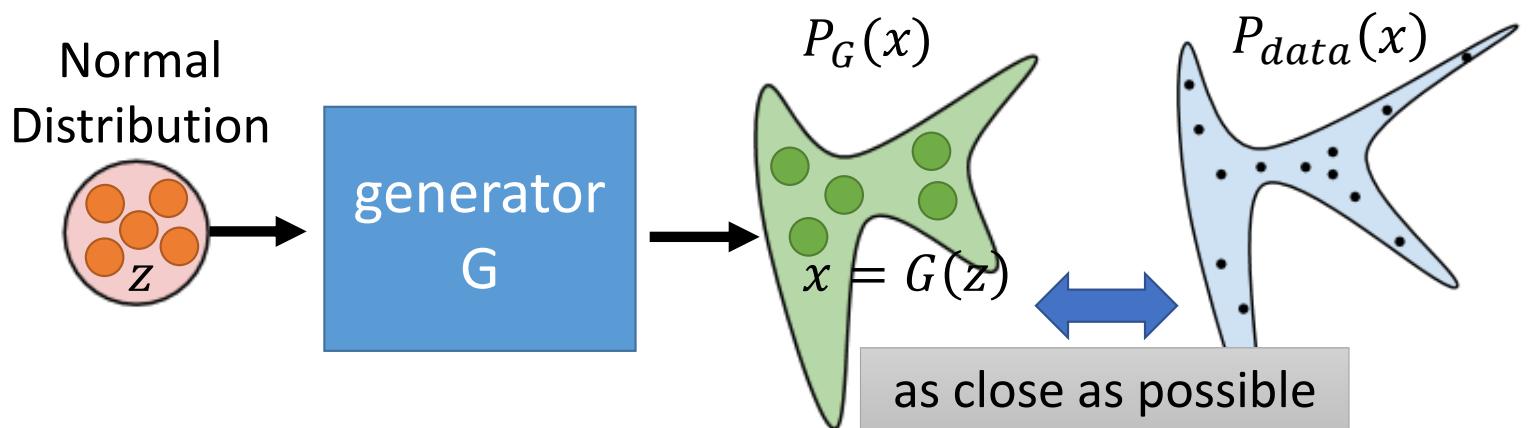
Can Discriminator generate?

A little bit theory

Generator

x : an image (a high-dimensional vector)

- A generator G is a network. The network defines a probability distribution P_G



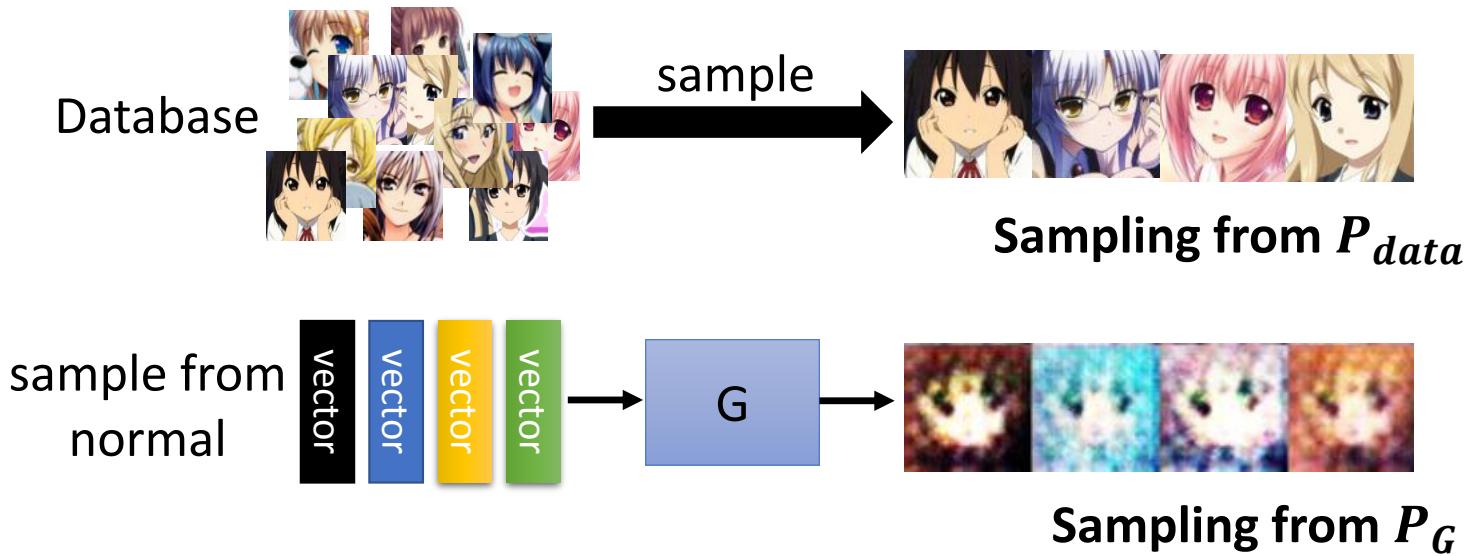
$$G^* = \arg \min_G \underline{\text{Div}}(P_G, P_{data})$$

Divergence between distributions P_G and P_{data}
How to compute the divergence?

Discriminator

$$G^* = \arg \min_G \text{Div}(P_G, P_{data})$$

Although we do not know the distributions of P_G and P_{data} , we can sample from them.



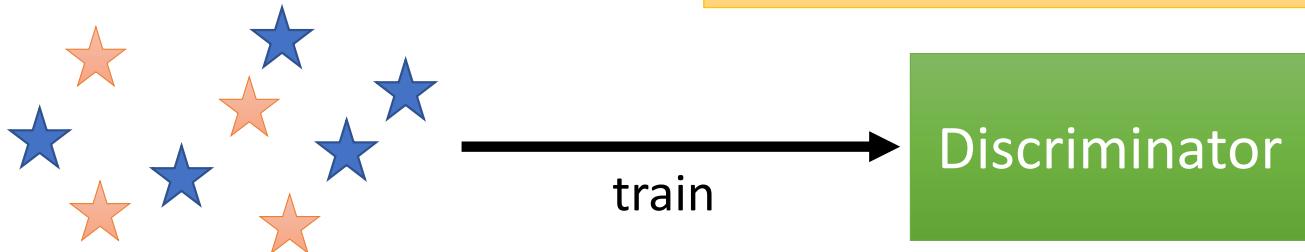
Discriminator

$$G^* = \arg \min_G \text{Div}(P_G, P_{data})$$

Blue star: data sampled from P_{data}

Orange star: data sampled from P_G

Using the example objective function is exactly the same as training a binary classifier.



Example Objective Function for D

$$V(G, D) = E_{x \sim P_{data}} [\log D(x)] + E_{x \sim P_G} [\log(1 - D(x))]$$

↑
(G is fixed)

Training: $D^* = \arg \max_D V(D, G)$

The maximum objective value is related to JS divergence.

Discriminator

$$G^* = \arg \min_G \text{Div}(P_G, P_{data})$$

blue star : data sampled from P_{data}

orange star : data sampled from P_G

Training:

$$D^* = \arg \max_D V(D, G)$$

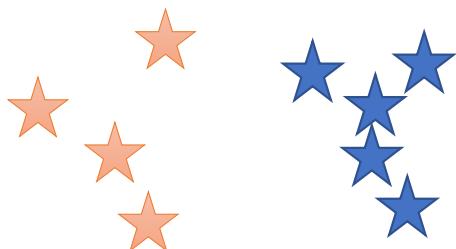


small divergence

train

Discriminator

hard to discriminate
(cannot make objective large)



large divergence

train

Discriminator

easy to discriminate

$$G^* = \arg \min_G \max_D V(G, D)$$

$$D^* = \arg \max_D V(D, G)$$

The maximum objective value is related to JS divergence.

- Initialize generator and discriminator
- In each training iteration:
 - Step 1:** Fix generator G, and update discriminator D
 - Step 2:** Fix discriminator D, and update generator G

Can we use other divergence?

Name	$D_f(P\ Q)$	Generator $f(u)$
Total variation	$\frac{1}{2} \int p(x) - q(x) dx$	$\frac{1}{2} u - 1 $
Kullback-Leibler	$\int p(x) \log \frac{p(x)}{q(x)} dx$	$u \log u$
Reverse Kullback-Leibler	$\int q(x) \log \frac{q(x)}{p(x)} dx$	$-\log u$
Pearson χ^2	$\int \frac{(q(x)-p(x))^2}{p(x)} dx$	$(u - 1)^2$
Neyman χ^2	$\int \frac{(p(x)-q(x))^2}{q(x)} dx$	$\frac{(1-u)^2}{u}$
Squared Hellinger	$\int \left(\sqrt{p(x)} - \sqrt{q(x)} \right)^2 dx$	$(\sqrt{u} - 1)^2$
Jeffrey	$\int (p(x) - q(x)) \log \left(\frac{p(x)}{q(x)} \right) dx$	$(u - 1) \log u$
Jensen-Shannon	$\frac{1}{2} \int p(x) \log \frac{2p(x)}{p(x)+q(x)} + q(x) \log \frac{2q(x)}{p(x)+q(x)} dx$	$-(u + 1) \log \frac{1+u}{2} + u \log u$
Jensen-Shannon-weighted	$\int p(x) \pi \log \frac{p(x)}{\pi p(x)+(1-\pi)q(x)} + (1 - \pi)q(x) \log \frac{q(x)}{\pi p(x)+(1-\pi)q(x)} dx$	$\pi u \log u - (1 - \pi + \pi u) \log(1 - \pi + \pi u)$
GAN	$\int p(x) \log \frac{2p(x)}{p(x)+q(x)} + q(x) \log \frac{2q(x)}{p(x)+q(x)} dx - \log(4)$	$u \log u - (u + 1) \log(u + 1)$

Name	Conjugate $f^*(t)$
Total variation	t
Kullback-Leibler (KL)	$\exp(t - 1)$
Reverse KL	$-1 - \log(-t)$
Pearson χ^2	$\frac{1}{4}t^2 + t$
Neyman χ^2	$2 - 2\sqrt{1-t}$
Squared Hellinger	$\frac{t}{1-t}$
Jeffrey	$W(e^{1-t}) + \frac{1}{W(e^{1-t})} + t - 2$
Jensen-Shannon	$-\log(2 - \exp(t))$
Jensen-Shannon-weighted	$(1 - \pi) \log \frac{1-\pi}{1-\pi e^{t/\pi}}$
GAN	$-\log(1 - \exp(t))$

Using the divergence
you like ☺