

I. Compute the mass moment of inertia matrices of the rocket,  $\mathbf{I}^r$ , and reaction wheel,  $\mathbf{I}^w$ , about their mutual center of mass in reference frame  $\mathbf{F}_c$ . Note this requires that you determine the location of the center of mass of rocket/reaction wheel system.

a)  $\vec{r}_{oc}$ : center of mass relative to bottom center of rocket

$$\cdot L=4, R=0.2S, h=1, t=0.04, r=0.2 \text{ [m]}$$

$$\cdot M_b=600, m=10, M_n=50 \text{ [kg]}$$

$\vec{r}_{ob}$ : center of mass for rocket body relative to bottom center of rocket

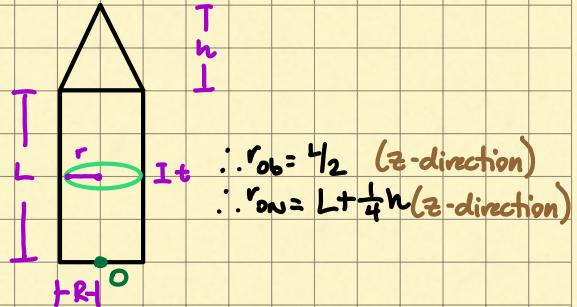
$\vec{r}_{on}$ : center of mass for nose relative to bottom center of rocket

$$(M_b + M_n) \vec{r}_{oc} = M_b \vec{r}_{ob} + M_n \vec{r}_{on}$$

$$\therefore \vec{r}_{oc_2} = \frac{M_b \vec{r}_{ob} + M_n \vec{r}_{on}}{M_b + M_n}$$

$$= \frac{(600 \text{ kg})(2 \text{ m}) + (50 \text{ kg})(4 + \frac{1}{4} \text{ m})}{(600 \text{ kg}) + (50 \text{ kg})}$$

$$\vec{r}_{oc_2} = 2.173$$



$$\therefore \vec{r}_{ob} = \frac{L}{2} \text{ (z-direction)}$$

$$\therefore \vec{r}_{on} = L + \frac{r}{4} \text{ (z-direction)}$$

$\vec{r}_b$ : vector from center of mass to center of mass of body

$\vec{r}_n$ : vector from center of mass to center of mass of cone

$$\therefore \vec{r}_b = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2.173 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -0.173 \end{pmatrix} \text{ m}$$

$$\vec{r}_n = \begin{pmatrix} 0 \\ 0 \\ 4.25 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2.173 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2.077 \end{pmatrix} \text{ m}$$

$$\mathbf{I}_r = \mathbf{J}_b + \mathbf{J}_N$$

$$\text{*from notes } \mathbf{J}_b = \mathbf{I}_b - M_b \vec{r}_b \times \vec{r}_b \times$$

$$\mathbf{I}_b = \begin{bmatrix} \frac{mR^2}{4} + \frac{mh^2}{12} & 0 & 0 \\ 0 & \frac{mR^2}{4} + \frac{mh^2}{12} & 0 \\ 0 & 0 & \frac{mR^2}{2} \end{bmatrix}$$

$$\begin{bmatrix} 600 \cdot 0.25^2/4 + 50 \cdot 4^2/12 & 0 & 0 \\ 0 & 600 \cdot 0.25/4 + 50 \cdot 4^2/12 & 0 \\ 0 & 0 & 600 \cdot 0.25^2/2 \end{bmatrix} - 600 \cdot \begin{bmatrix} 0 & 0.173 & 0 \\ -0.173 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.173 & 0 \\ -0.173 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 809.375 & 0 & 0 \\ 0 & 809.375 & 0 \\ 0 & 0 & 18.75 \end{bmatrix} - \begin{bmatrix} -17.957 & 0 & 0 \\ 0 & -17.957 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{J}_b = \begin{bmatrix} 827.33 & 0 & 0 \\ 0 & 827.33 & 0 \\ 0 & 0 & 18.75 \end{bmatrix}$$

$$J_n = I_n - m_n \vec{r}_n \times \vec{r}_n \times$$

$$I_n = \begin{bmatrix} \frac{3m_n R^2}{20} + \frac{3m_n n^2}{80} & \phi & \phi \\ \phi & \frac{3m_n R^2}{20} + \frac{3m_n n^2}{80} & \phi \\ \phi & \phi & \frac{3m_n R^2}{10} \end{bmatrix} \text{ #from HW 5}$$

$$J_n = \begin{bmatrix} \frac{3 \cdot 50 \cdot 0.25^2}{20} + \frac{3 \cdot 50 \cdot 1^2}{80} & \phi & \phi \\ \phi & \frac{3 \cdot 50 \cdot 0.25^2}{20} + \frac{3 \cdot 50 \cdot 1^2}{80} & \phi \\ \phi & \phi & \frac{3 \cdot 50 \cdot 0.25^2}{10} \end{bmatrix} - 50 \begin{bmatrix} \phi & -2.077 & \phi \\ 2.077 & \phi & \phi \\ \phi & \phi & \phi \end{bmatrix} \cdot \begin{bmatrix} \phi & -2.077 & \phi \\ 2.077 & \phi & \phi \\ \phi & \phi & \phi \end{bmatrix}$$

$$= \begin{bmatrix} 2.344 & \phi & \phi \\ \phi & 2.344 & \phi \\ \phi & \phi & 0.9375 \end{bmatrix} - \begin{bmatrix} -215.70 & \phi & \phi \\ \phi & -215.70 & \phi \\ \phi & \phi & \phi \end{bmatrix}$$

$$J_n = \begin{bmatrix} 218.04 & \phi & \phi \\ \phi & 218.04 & \phi \\ \phi & \phi & 0.9375 \end{bmatrix}$$

$$I_r = J_b + J_n$$

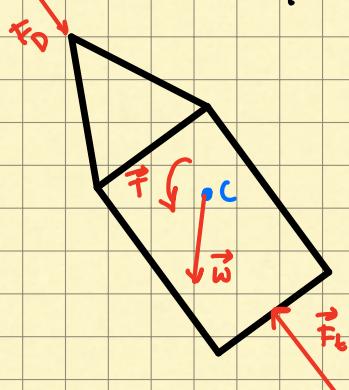
$$\begin{bmatrix} 827.33 & \phi & \phi \\ \phi & 827.33 & \phi \\ \phi & \phi & 18.75 \end{bmatrix} + \begin{bmatrix} 218.04 & \phi & \phi \\ \phi & 218.04 & \phi \\ \phi & \phi & 0.9375 \end{bmatrix} = \begin{bmatrix} 1045.37 & \phi & \phi \\ \phi & 1045.37 & \phi \\ \phi & \phi & 19.68 \end{bmatrix} \text{ kg} \cdot \text{m}^2 = I_r$$

$$J_w = I_w = \text{for reaction wheel}$$

$$J_w = I_w = \begin{bmatrix} \frac{mv^2}{4} + \frac{mr^2}{12} & \phi & \phi \\ \phi & \frac{mv^2}{4} + \frac{mr^2}{12} & \phi \\ \phi & \phi & mr^2/2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{10 \cdot 0.2^2}{4} + \frac{10 \cdot 0.04^2}{12} & \phi & \phi \\ \phi & \frac{10 \cdot 0.2^2}{4} + \frac{10 \cdot 0.04^2}{12} & \phi \\ \phi & \phi & \frac{10 \cdot 0.2^2}{2} \end{bmatrix} = \begin{bmatrix} 0.1013 & \phi & \phi \\ \phi & 0.1013 & \phi \\ \phi & \phi & 0.2 \end{bmatrix} \text{ kg} \cdot \text{m}^2 = I_w$$

b. Draw a FBD of the system.



c. Express absolute angular velocity of angular velocity of reaction wheel,  $\vec{\omega}_w$  in terms of  $\vec{\omega}$  and  $\vec{\omega}_{rel}$

$\vec{\omega}_w = \vec{\omega}_{rel} + \vec{\omega}$

$\cdot \vec{\omega}_w$ : angular velocity of reaction wheel

$\cdot \vec{\omega}_{rel}$ : angular velocity of reaction wheel relative to rocket

$\cdot \vec{\omega}$ : angular velocity of reaction wheel relative to inertial frame.

d. Write conservation of angular momentum equation in reference frame  $\vec{F}_c$  in terms of  $\vec{T}'$ ,  $\vec{I}^w$ ,  $\omega$ ,  $\omega_{rel}$ , and  $T_d$ , where  $\omega_{rel}$  denotes the components of physical vector  $\vec{\omega}_{rel}$  in reference frame  $\vec{F}_c$ .

$$\sum \vec{T} = \vec{T}_c + \vec{T}_d = \vec{h}_c = \vec{h}_c r + \vec{h}_c \omega$$

\*  $\vec{T}_c$  is external torque and is  $\emptyset$  in this case

$$\begin{aligned}\vec{T}_d &= \vec{h}_{rc} + \vec{h}_{rw} \\ \vec{h}_{rc} &= \vec{h}_{rc} + \vec{\omega} \times \vec{h}_{rc} \quad \vec{h}_w = \vec{h}_{rw} + \vec{\omega}_w \times \vec{h}_{rw}\end{aligned}$$

$$\vec{T}_d = \vec{h}_{rc} + \vec{\omega} \times \vec{h}_{rc} + \vec{h}_w + \vec{\omega}_w \times \vec{h}_w$$

-since  $\vec{h}_c = \vec{I}^w \vec{\omega}$ , the above takes the form  
 $I\dot{\omega} + \omega \times I\omega = T_c$

$$\vec{T}_d = I_r \dot{\omega} + \omega \times I_r \omega + I_w \dot{\omega}_w + \omega \times I_w \omega_w$$

\* recall  $\omega_w = \omega_{rel} + \omega$

$$\vec{T}_d = I_r \dot{\omega} + \omega \times I_r \omega + I_w (\omega_{rel} + \omega) + (\omega_{rel} + \omega) \times I_w (\omega_{rel} + \omega)$$

e) What is rotational kinetic energy of the rocket/reaction wheel system?

\* rotational energy:  $\frac{1}{2} \vec{\omega}^T \vec{I} \vec{\omega}$

$$T = \frac{1}{2} \vec{\omega}^T \vec{I} \vec{\omega}$$

$$T = T_r + T_w$$

\* in a principal axes frame

$$T_r = \frac{1}{2} (I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2)$$

$$T_b = \frac{1}{2} I_r \omega_r^2 \quad T_w = \frac{1}{2} I_w \omega_w^2$$

\*  $\omega_y + \omega_x$  are taken to be zero

\*  $I_r$ : inertia matrix for body

\*  $I_w$ : inertia matrix for reaction wheel

$$\begin{aligned}T_{sys} &= T_r + T_w \\ &= \frac{1}{2} I_r \omega_r^2 + \frac{1}{2} I_w (\omega_{rel} + \omega)^2\end{aligned}$$

2. Assume no external moment applied to the system, i.e.,  $\vec{T}_d$  to be zero for now. For a given change in the angular velocity of the reaction wheel relative to rocket, what is the resulting change in the angular velocity in the angular velocity of rocket? In other words, for  $\omega_{rel} \neq 0$ , use the result of Question 1(d) to find  $\dot{\omega}_r$ . (Assume all angular velocity components in the  $\vec{F}_c$  and  $\vec{F}_c'$  directions are zero.)

- in absence of external torque acting on spacecraft, the total angular momentum of spacecraft remains constant.

$$\vec{T}_d = I_r \dot{\omega}_r + \omega \times I_r \omega + I_w (\omega_{rel} + \omega) + (\omega_{rel} + \omega) \times I_w (\omega_{rel} + \omega)$$

\*  $\vec{T}_d = \emptyset$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_r \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ \omega_{rel} \end{pmatrix} + \begin{bmatrix} 0 & -\omega_r & 0 \\ \omega_r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_r \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ \omega_{rel} \end{pmatrix} +$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_w \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ \omega_{rel} + \omega \end{pmatrix} + \begin{bmatrix} 0 & -\omega_{rel} - \omega & 0 \\ \omega_{rel} - \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_w \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ \omega_{rel} + \omega \end{pmatrix} \rightarrow \emptyset$$

$$\dot{\theta} = I_r \dot{\omega}_z + I_w (\dot{\omega}_{rel} + \dot{\omega})$$

$$= I_r \dot{\omega}_z + I_w \dot{\omega}_{rel} + I_w \dot{\omega}_z$$

$$= \dot{\omega}_z (I_r + I_w) + I_w \dot{\omega}_{rel}$$

$$\frac{-I_w \dot{\omega}_{rel}}{I_r + I_w} = \dot{\omega}$$

3. Solve for angular speed & displacement of rocket if reaction wheel is accelerating at constant rate 0.05 rad/s. Assume initial angular displacement of rocket is zero

$$\vec{T}_d = I_r \dot{\omega}_r + \omega^2 I_r \omega_r + I_w (\dot{\omega}_{rel} + \dot{\omega}) + (\dot{\omega}_{rel} + \dot{\omega})^2 I_w (\dot{\omega}_{rel} + \dot{\omega})$$

$$\vec{T}_d - \omega^2 I_r \omega_r - (\dot{\omega}_{rel} + \dot{\omega})^2 I_w (\dot{\omega}_{rel} + \dot{\omega}) = I_r \dot{\omega}_r + I_w (\dot{\omega}_{rel} + \dot{\omega}) \\ \dot{\omega} (I_r + I_w) + I_w \cdot \dot{\omega}_{rel}$$

$$\dot{\omega} = (I_r + I_w)^{-1} \cdot (\vec{T}_d - \omega^2 I_r \omega_r - (\dot{\omega}_{rel} + \dot{\omega})^2 I_w (\dot{\omega}_{rel} + \dot{\omega}) - I_w \cdot \dot{\omega}_{rel})$$

\* what our omega is gonna be in MATLAB

$$\dot{\omega}_{rel} = 0.05 \text{ rad/s}$$

$$\int \frac{-I_w \dot{\omega}_{rel}}{I_r + I_w} dt = \int \dot{\omega} (t) \quad * z \text{ components}$$

$$\dot{\omega}_z(t) = \frac{-0.2(0.05)}{19.8875} t, \text{angular speed}$$

$$\int_0^t \omega(t) dt = \int_0^t \frac{-0.2}{19.8875} (0.05)t dt$$

$$s_z(t) = \frac{-0.2}{19.8875} (0.05)t^2, \text{angular displacement}$$

4. Done in MATLAB.

Zhitai Guo & Justin Ferrales

Dr. Deffo

12 November 2022

Aero 320

### ICGE 2 Discussion Questions

1. What is your interpretation of the equation developed in question 2 of the analysis section?

When there are no external torques acting upon the system the angular acceleration of the rocket is reliant on the reaction wheel's angular acceleration. In order for the total angular acceleration of the rocket to increase the reaction wheel's angular acceleration must increase as well, and vice versa for deceleration.

2. Discuss the results of Question 3 in the analysis section?

When the reaction wheel is accelerating at a constant rate, there is also a linear relationship between the entire rocket's angular velocity over time. As seen in the graph, the rocket starts at 0 angular speed and reaches -0.05 rad/s, which was the applied acceleration placed on the reaction wheel, portraying the conservation of angular momentum. We are given a constant value for angular acceleration, so we should expect a linear value for angular velocity, and a quadratic plot for angular displacement.

3. Discuss the impact on the angular velocity of the rocket and the reaction wheel in question 4 of the analysis question.

- a) In part A, we have zero angular velocity of the rocket and zero angular velocity of the reaction wheel relative to the rocket. In doing so we have a linear change in angular velocity reflecting the external disturbance moment over a period of 1000 seconds.
- b) In part B, we have zero angular velocity of the reaction wheel relative to the rocket and an initial angular velocity of the rocket. This causes small oscillations in the x and y components of the angular velocities but the angular velocity in the z direction remains proportional to the external disturbance moment.
- c) In part C, the reaction wheel rotates 100 rad/s in the z direction and the initial angular velocity of the rocket is zero. The z component of the angular velocity remains at zero throughout the time span. There are quite a bit of oscillations in the x and y direction and are considerably larger than the previous problem where there was zero angular velocity of the wheel.
- d) In part D, the reaction wheel rotates 100 rad/s in the z direction and the initial angular velocity of the rocket is 0.1 rad/s in the z direction. The angular velocity in the z-direction remains constant, with some oscillations in the x and y directions. The oscillations in part D and part B are both similar in frequency and amplitude.

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# Justin Ferrales Zhitai Guo ICGE 2

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```
clear all;
close all;
clc;
% Global Variables, Inertia matrices for rocket and the wheel
Ir = [1045.37 0 0; 0 1045.37 0; 0 0 19.68];
Iw = [.1013 0 0; 0 .1013 0; 0 0 0.2];
```

## Problem 3

```
time = [0 100];
Ir = [1045.37 0 0; 0 1045.37 0; 0 0 19.68];
Iw = [0.1013 0 0; 0 0.1013 0; 0 0 0.2];

torque = [0; 0; 0]; % assuming no torque for problem 3

omegaRelDot = [0; 0; 0.05]; % 0.05 rad/s^2 given in prob 3

init_state = zeros(9,1); % saying all initial values in state is going to be
zero, will change later on in prob 4

options = odeset('RelTol', 1e-8, 'AbsTol', 1e-8);

[tout1, stateNew1] = ode45(@ICGE2func, time, init_state, options, omegaRelDot,
Ir, Iw, torque);

func = @(t) (-0.2*0.05*t.^2) / (Ir(3,3) + Iw(3,3));
timel = linspace(0,10,100);
func_vals = func(timel);

figure
subplot(2,1,1)
plot(tout1,stateNew1(:,1:3))
grid on
title('Problem 3: Angular Velocity vs Time');
xlabel('Time (s)');
ylabel('Angular Velocity (rad/s)');
legend('omega_x','omega_y','omega_z','Location','best');
```

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```

subplot(2,1,2)
plot(time1,func_vals)
grid on
title('Problem 3: Angular Displacement vs Time');
xlabel('Time (s)');
ylabel('rad');

tspan = [0 1000];
torque = [0.1; 0; 0]; % given torque for problem 4
omegaRelDot2 = [0; 0; 0];
% no changes needed to the init state w_rel and w are 0 vectors.
[tout2, stateNew2] = ode45(@ICGE2func, tspan, init_state, options,
omegaRelDot2, Ir, Iw, torque);

```

```

figure
subplot(2,1,1)
plot(tout2,stateNew2(:,1:3))
grid on
title('Problem 4a: Angular Velocity vs Time');
xlabel('Time (s)');
ylabel('Angular Velocity (rad/s)');
legend('omega_x','omega_y','omega_z','Location','best');

subplot(2,1,2)
plot(tout2,stateNew2(:,7:9));
grid on
title('Problem 4a: Euler Angles vs Time');
xlabel('Time (s)');
ylabel('Angles (rad)');
legend('phi','theta','psi','Location','best');

```

## Part 4b

```

tspan = [0 1000];
torque = [0.1; 0; 0];
omegaRelDot2 = [0; 0; 0];
init_state1 = init_state;
init_state1(3) = 0.1; % changing z component of omega by making a copy

[tout3, stateNew3] = ode45(@ICGE2func, tspan, init_state1, options,
omegaRelDot2, Ir, Iw, torque);

figure
subplot(2,1,1)
plot(tout3,stateNew3(:,1:3))
grid on
title('Problem 4b: Angular Velocity vs Time');
xlabel('Time (s)');
ylabel('Angular Velocity (rad/s)');
legend('omega_x','omega_y','omega_z','Location','best');

```

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---

```

subplot(2,1,2)
plot(tout3,stateNew3(:,7:9));
grid on
title('Problem 4b: Euler Angles vs Time');
xlabel('Time (s)');
ylabel('Angles (rad)');
legend('phi','theta','psi','Location','best')

```

## Part 4c

```

tspan = [0 1000];
torque = [0.1; 0; 0];
omegaRelDot2 = [0; 0; 0];
init_state2 = init_state;
init_state2(6) = 100; % changing z component of omega_rel by making a copy

[tout4, stateNew4] = ode45(@ICGE2func, tspan, init_state2, options,
                           omegaRelDot2, Ir, Iw, torque);

figure
subplot(2,1,1)
plot(tout4,stateNew4(:,1:3))
grid on
title('Problem 4c: Angular Velocity vs Time');
xlabel('Time (s)');
ylabel('Angular Velocity (rad/s)');
legend('omega_x','omega_y','omega_z','Location','best');

subplot(2,1,2)
plot(tout4,stateNew4(:,7:9));
grid on
title('Problem 4c: Euler Angles vs Time');
xlabel('Time (s)');
ylabel('Angles (rad)');
legend('phi','theta','psi','Location','best')

```

## Part 4d

```

tspan = [0 1000];
torque = [0.1; 0; 0];
omegaRelDot2 = [0; 0; 0];
init_state3 = init_state;
init_state3(3) = 0.1; % changing z component of omega by making a copy
init_state3(6) = 100; % changing z component of omega_rel by making a copy

[tout6, stateNew6] = ode45(@ICGE2func, tspan, init_state3, options,
                           omegaRelDot2, Ir, Iw, torque);

figure
subplot(2,1,1)
plot(tout6,stateNew6(:,1:3))
grid on

```

---

---

```

title('Problem 4d: Angular Velocity vs Time');
xlabel('Time (s)');
ylabel('Angular Velocity (rad/s)');
legend('omega_x','omega_y','omega_z','Location','best');

subplot(2,1,2)
plot(tout6,stateNew6(:,7:9));
grid on
title('Problem 4d: Euler Angles vs Time');
xlabel('Time (s)');
ylabel('Angles (rad)');
legend('phi','theta','psi','Location','best')

```

## Function Calls

```

function output = ICGE2func(time,state,omegaRelDot,Ir,Iw,torque)

% initial states of the rocket
omega = [state(1); state(2); state(3)];
omegaRel = [state(4); state(5); state(6)];
omegaW = omega + omegaRel;

% omegadot = (Ir + Iw)^-1 * (torque - omegaCross*Ir*omega -
% (omegaW)cross*Iw(omega + omegarel

% Define cross matrices
omegaCross = [0 -omega(3) omega(2); omega(3) 0 -omega(1); -omega(2) omega(1)
0];
omegaWCross = [0 -omegaW(3) omegaW(2); omegaW(3) 0 -omegaW(1); -omegaW(2)
omegaW(1) 0];

% Find omegadot
omegaDot = ((Ir+Iw)^-1) * (torque - omegaCross*Ir*omega -
(omegaWCross*Iw*omegaW) - (Iw*omegaRelDot));

% Define euler angles
phi = state(7);
theta = state(8);

% Find euler angle rates
eulerRates = (1/cos(theta))*[cos(theta) sin(phi)*sin(theta)
cos(phi)*sin(theta); 0 cos(phi)*cos(theta) -sin(phi)*cos(theta); 0 sin(phi)
cos(phi)]*omega;

% output vector used to plot things
output(1:3,:) = omegaDot;
output(4:6,:) = omegaRelDot;
output(7:9,:) = eulerRates;

end

```

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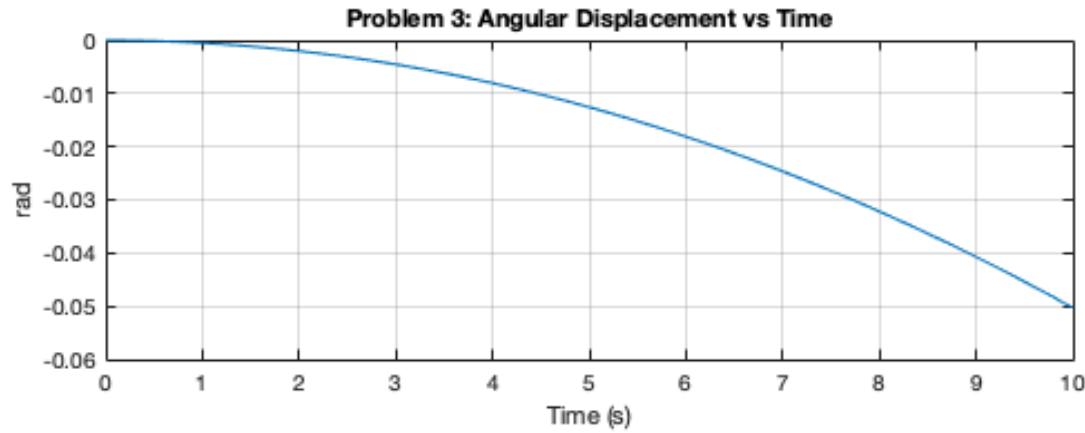
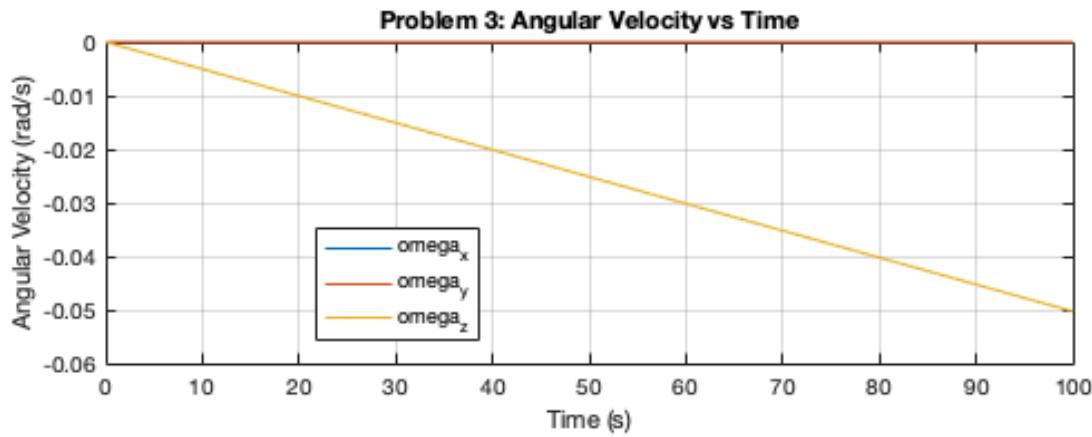
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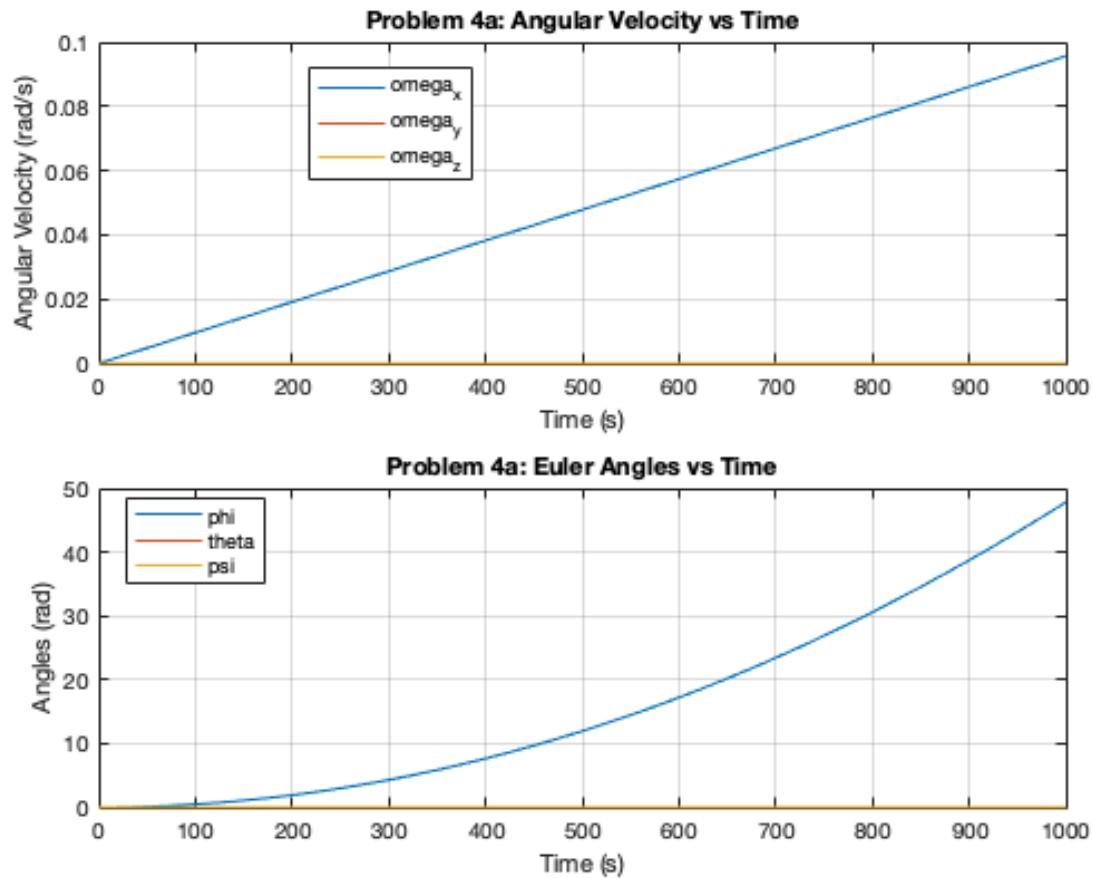
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## Problem 3



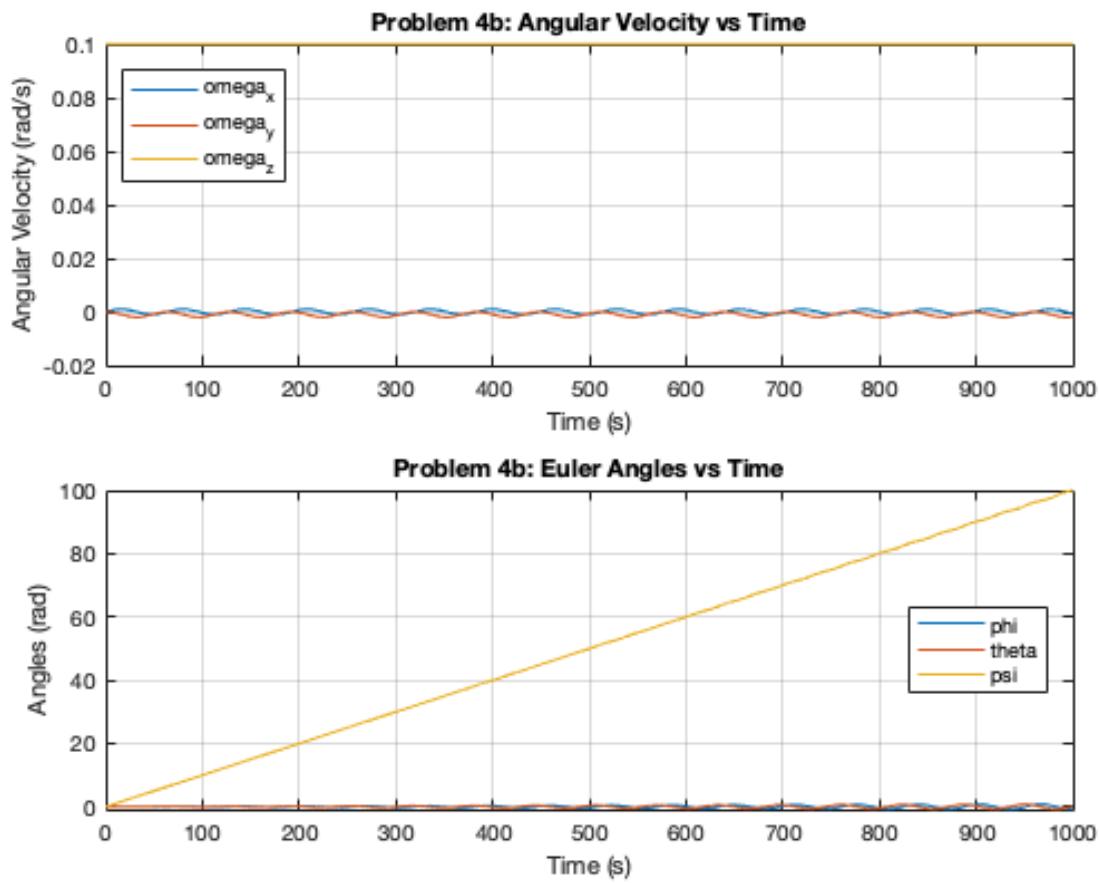
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## Part 4a)



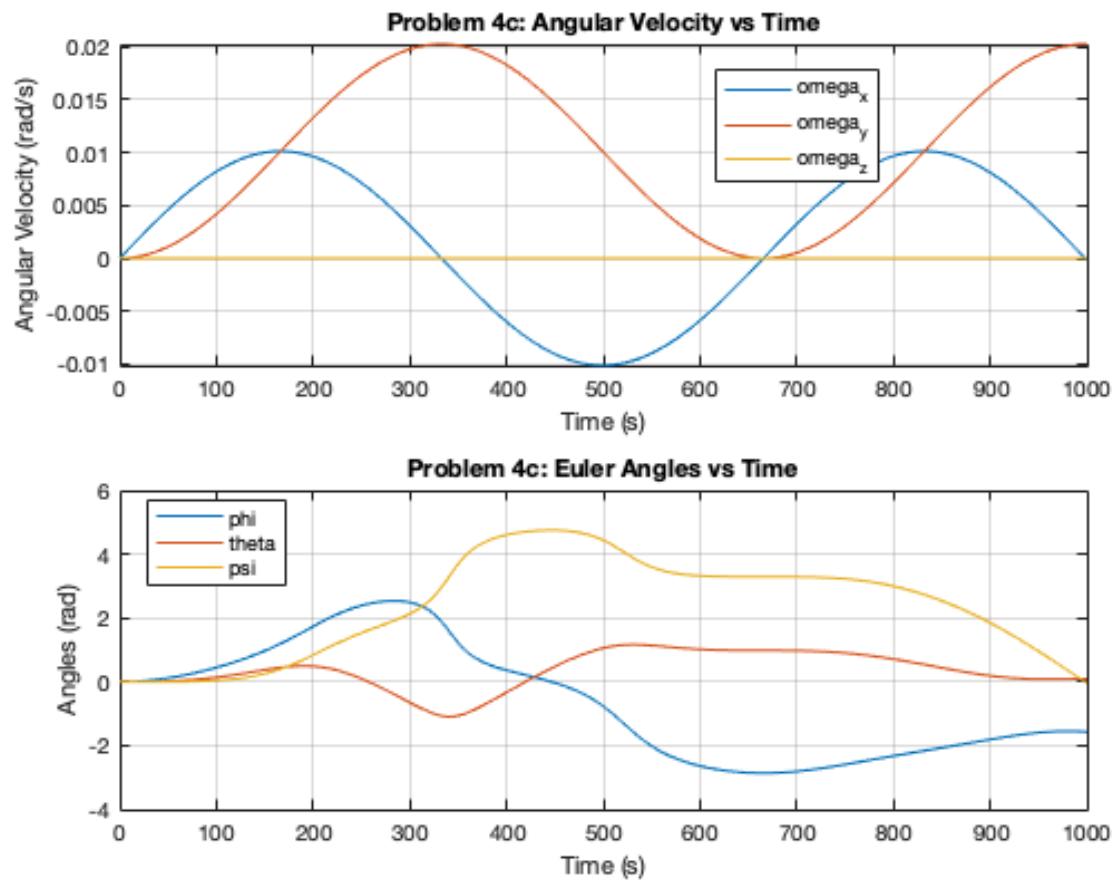
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## Part 4b



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## Part 4c



## Part 4d

### Function Calls

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