HW #5

1)

$T(n) = 7T(n/2) + n^2$

$= 49T\left(\frac{n}{2^2}\right) + 8n^2$

Compare n^2 to $n^{\log_4 49}$

Case 1 thus $T(n) = \Theta(n^{\log_4 49})$

$S(n) = aS(n/4) + n^2$

$= a^2 S\left(\frac{n}{4^2}\right) + an^2 + n^2$

Compare n^2 to $n^{\log_{16} a}$

Case 1 thus $S(n) = \Theta(n^{\log_{16} a})$

$a > 49$ such that $S(n) = o(T(n))$ ✓

2) Handshake lemma: The number of hands shaken at a party is twice the # of handshakes

Also can be written as:
The sum of degrees of all verticies in a graph G is twice the # of edges in g

For an odd degree graph all edges will sum up to be an odd #.

Any odd number x 2 is an even number

Thus the number of verticies of odd degree must be even

3) Prove by Induction

Base 0 m edges 1 n vertex

$m \geq n-1 \Rightarrow 0 \geq 1-1 \Rightarrow 0 \geq 0$ ✓

Induction (connected & disconnected case)

Connected Remove an edge from G

This new graph still has n vertices but now has m-1 edges

$m-1 \geq n-1 \Rightarrow m \geq n$ ✓

3 (cont) Disconnected. Remove an edge from G

This will result in two seperate graphs

The total number of vertices was m now it is the sum of the two ms plus the one we took out

$m = m_1 + m_2 + 1$

The number of vertices remains the same $n-1$

Thus

$m_1 + m_2 + 1 \geq n-1$

which satisfies $m \geq n-1$ ✓

Thus proven by induction for all cases.

4) Base $d=0$

I $i=j$ then $d=0$ this is the trivial walk and is valid ✓

Induction $d \geq 0$

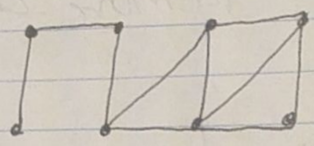
5) The distence from vertex i to vertex j is equal to $\min\{d \mid (A^d)_{ij} > 0\}$

If a vertex i is d away vertex j this means that they are not equal. Thus $(A^q)_{ij}$ must be greater than 0. Thus this is true.

Using the result from #4 we know that the ij^{th} entry is A^q number of walks away.

Thus both parts are true and $\min\{d \mid (A^q)_{ij} > 0\}$ ✓

6)



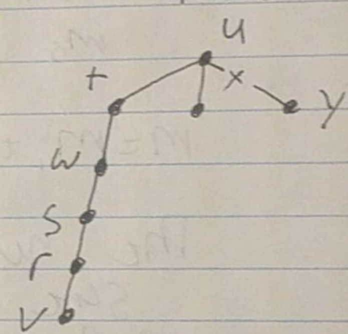
a) Source: u

Vert	d	P
r	4	s
s	3	w
t	1	u
u	0	no
v	5	r
w	2	t
x	1	u
y	1	u

Queue

u
t
x
y
w
s
r
v

Tree



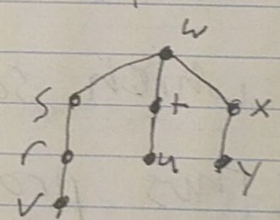
b) Source: w

Vert	d	P
r	2	s
s	1	w
t	1	w
u	2	t
v	3	r
w	0	no
x	1	w
y	2	x

Queue

w
s
t
x
r
u
y
v

Tree



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6 cont) c) Source: v

Vert	d	p
r	1	v
s	2	r
t	4	w
u	5	t
v	0	no
w	3	s
x	4	w
y	5	x

Queue
v
r
s
w
t
x
u
y

Tree