HW # 7

1) Proof by induction

Base $n(T)=1$ $h(T)=0$

$h(T) \geq \lfloor \lg n(T) \rfloor$

$0 \geq 0$ ✓

Induction Step $n(T) \geq 2$ $h(T) \geq \lfloor \lg n(T)$

It is given that $h(T) = 1 + \max(h(L), h(R))$

Assume that $n(R) \geq n(L)$

Thus $\max(\lfloor \lg(n(L)) \rfloor, \lfloor \lg(n(R)) \rfloor) = \lfloor \lg(n(R)) \rfloor$

Thus $h(T) = 1 + \lfloor \lg(n(R)) \rfloor$

$= \lfloor \lg(2n(L)+1) \rfloor$

$\geq \lfloor \lg n(T) \rfloor$

$\boxed{\text{Therefore } h(T) \geq \lfloor \lg(n(T)) \rfloor}$ ✓

2) Min Priority w/ min heap

<u>HeapMinimum(A)</u>

1) return A[1]

<u>HeapDecreaseKey(A, i, k)</u>

1) if $k < A[i]$
2) $A[i] = k$
3) while $i > 1$ and $A[parent(i)] > A[i]$
4) $A[i] \leftrightarrow A[parent(i)]$
$i = parent(i)$

<u>HeapInsert(A, k)</u>

1) heapSize[A]++
2) A[heapSize[A]] = $+\infty$
3) HeapDecreaseKey(A, heapSize[A], k)

3) Proof by induction

Base

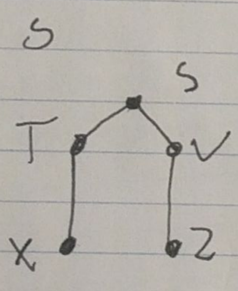
4) <u>BellmanFord(G, s)</u>

```
Initialize(G, s)
    for i=1 to V-1
        for each edge (u,v) ε E
            Relax(u, v)
for each (u, v) ε E
    if d[v] > d[u] + w(u, v)
        return false
return true
```

5)

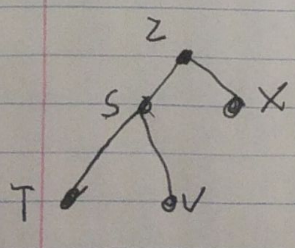
Vertex s

Queue : S t v x z



	d	p
s	0	N
t	3	S
v	5	S
x	9	T
z	11	V

Vertex Z



	d	p
s	3	Z
t	6	S
v	8	S
x	7	Z
z	0	N

G) Use dijkstra

Dijkstra (G, s)

1) Initalize (G, s)
2) $S = \emptyset$
3) $Q = V$
4) while $Q \neq 0$
5) $x = ExtractMin(Q)$
6) $S = S \cup \{x\}$
7) for all $y \in adj[x]$
8) Relax (x, y)