HW # 2

1) $f(n)$ and $g(n)$ are asymptotically non-neg
Using def of Θ-notation prove that
$f(n) + g(n) = \Theta(\max(f(n), g(n))$

By definition of Θ-notation we know that

$$f(n) + g(n) \leq c * \max(f(n), g(n))$$

Individually we know that

$f(n) \leq \max(f(n), g(n))$ for $n > 0$
and
$g(n) \leq \max(f(n), g(n))$ for $n > 0$

thus we can infer that $c = 2$ for any n value greater than 0

Therefore we know that

$f(n) + g(n) = \Theta(\max(f(n), g(n))$ ✓

2) Why is the running time of algorithm A is at least $O(n^2)$ meaningless?

This statement doesn't give us any definitere information about the upper or lower bound as well as the average.

Thus we know nothing about the actual running time

3) a) $2^{n+1} = O(2^n)$

$2^{n+1} = 2 * 2^n$

we can drop the 2 thus

2^n therefore $2^{n+1} = O(2^n)$ is $\boxed{\text{true}}$

b) $2^{2n} = O(2^n)$

$2^n * 2^n$ is not the same as $O(2^n)$

thus $2^{2n} = O(2^n)$ is $\boxed{\text{false}}$

4)

$k \geq 1$

$\varepsilon > 0$

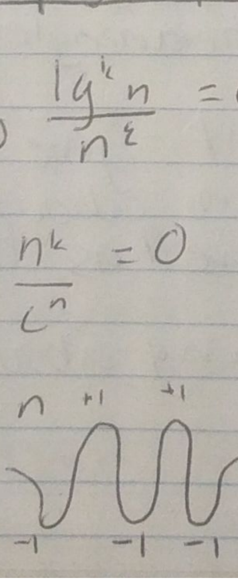
$c > 1$

	A	B	O	o	Ω	ω	Θ
a	$\lg^k n$	n^{ε}	Y	Y	N	N	N
b	n^k	c^n	Y	Y	N	N	N
c	$\sqrt{n}$	$n^{\sin n}$	N	N	N	N	N
d	2^n	$2^{n/2}$	N	N	Y	Y	N
e	$n^{\lg c}$	$c^{\lg n}$	Y	N	Y	N	Y
f	$\lg(n!)$	$\lg(n^n)$	Y	N	Y	N	Y

a) $\lim_{n \to \infty} \frac{\lg^k n}{n^{\varepsilon}} = 0$ thus $\lg^k n = o(n^{\varepsilon})$ and $O(n^{\varepsilon})$

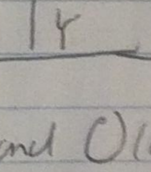
b) $\lim_{n \to \infty} \frac{n^k}{c^n} = 0$ thus $n^k = o(c^n)$ and $O(n^c)$

c) $n \sin n$



max and min val of $n^{\sin n}$ is outside range of $\sqrt{n}$

$\sqrt{n}$



thus None

#4 (cont) d) $\lim_{n\to\infty} \frac{2^n}{2^{n/2}} = \infty$ thus $2^n = \omega(2^{n/2})$ and $\Omega(n^{n/2})$

e) We know that $n^{\lg c} = c^{\lg n}$ thus all Θ Ω and O are "asymptotically tight"

f) Both $\lg(n!)$ and $\lg(n^n)$ can be rewritten as $n\lg(n)$. Thus both are equal and "asymptotically tight"

5) d) $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$

$f(n) \le c * g(n)$ $\qquad 2^{f(n)} \le c\, 2^{g(n)}$

$2^{cg(n)} \le c\, 2^{g(n)}$ say $c = 10$

$2^{10g(n)} \le 10 * 2^{g(n)}$

The left side will be much greater for most values.

Thus $\boxed{\text{disproved}}$

e) $f(n) = O((f(n))^2)$

For a sufficiently large n the upper bound running time for both sides will approach infinity.

Thus $\boxed{\text{proved}}$

5 (cont) f) $f(n) + o(f(n)) = \Theta(f(n))$

$f(n)$ covers the upper bound and everything below it.
$o(f(n))$ ~~only~~ allows the left side to be asymptotically tight

Thus $\boxed{\text{proved}}$ ✓

6) $f(n) = \Theta(n)$. Prove that $\sum_{i=1}^{n} f(i) = \Theta(n^2)$

$$\sum_{i=1}^{n} f(i) \le \sum_{i=1}^{n} n = n \cdot n = n^2 = O(n^2)$$

In addition $\sum_{i=1}^{n} f(i) \ge \sum_{i=\lceil n/2 \rceil}^{n} f(i)$

$$\ge \sum_{i=\lceil n/2 \rceil}^{n} f(\lceil n/2 \rceil)$$

$$= (n - \lceil n/2 \rceil + 1) \cdot f(\lceil n/2 \rceil)$$

$$= (\lfloor n/2 \rfloor - 1 + 1) \cdot f(\lceil n/2 \rceil)$$

$$= (n/2 - 1 + 1)(f(n/2))$$

$$= (1/2)^2 n^2$$

$$= \Omega\left(\frac{n^2}{4}\right)$$

Thus

$$\sum_{i=1}^{n} f(i) = \Theta(n^2)$$

7) $g(n)$ is asymptotically non-neg funct.
Prove $o(g(n)) \cap \Omega(g(n)) = \emptyset$

o

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

Ω

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} \Rightarrow 0 < \lim \leq \infty$$

Thus there is no intersection between $o(g(n))$ and $\Omega(g(n))$

$o(g(n)) \cap \Omega(g(n)) = 0$ ✓