Political Methodology III: Model Based Inference

Justin Grimmer

Associate Professor
Department of Political Science
Stanford University

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Supervised Learning: Ensemble Learning

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    - Learn a Conditional Expectation Function

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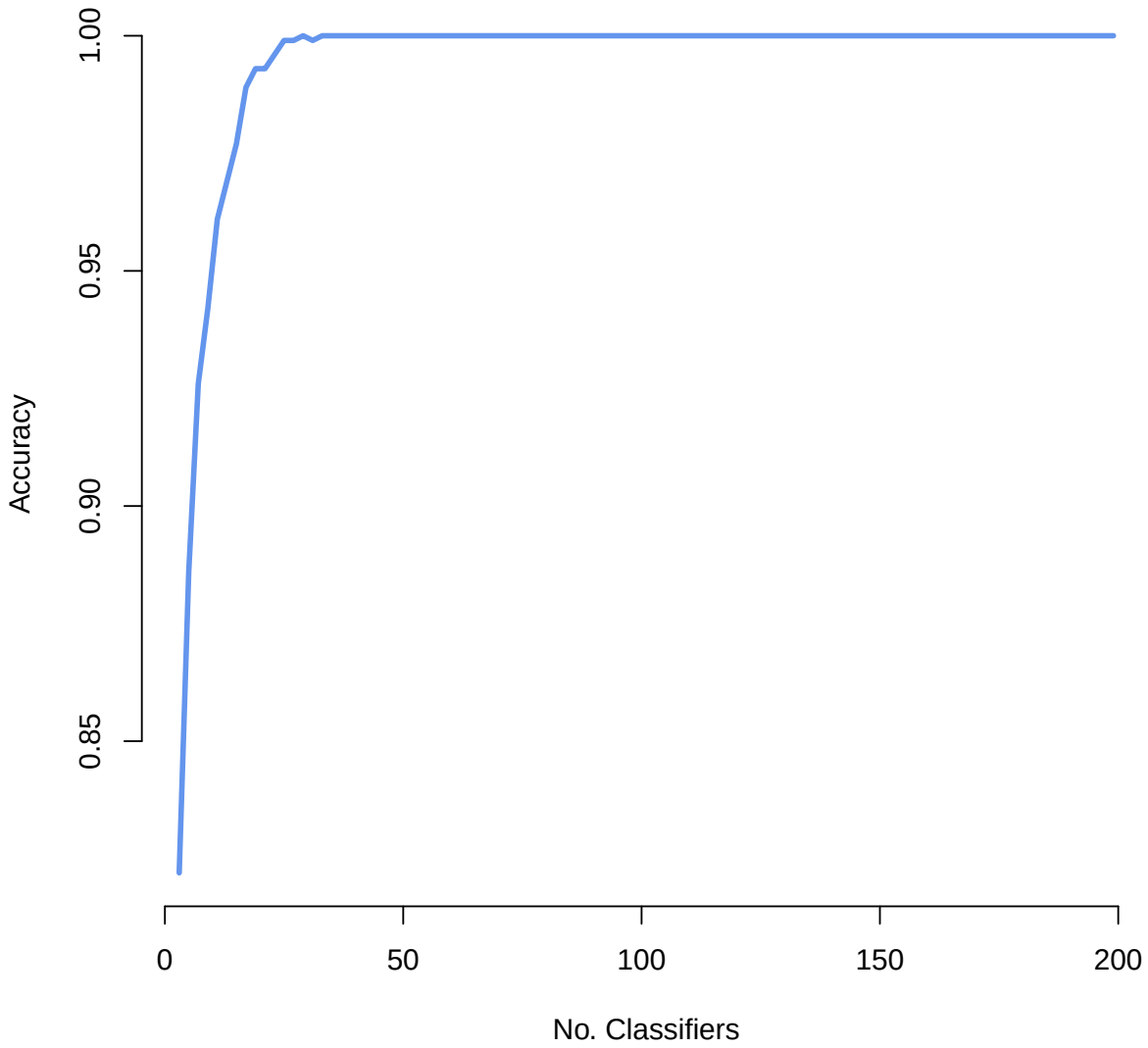

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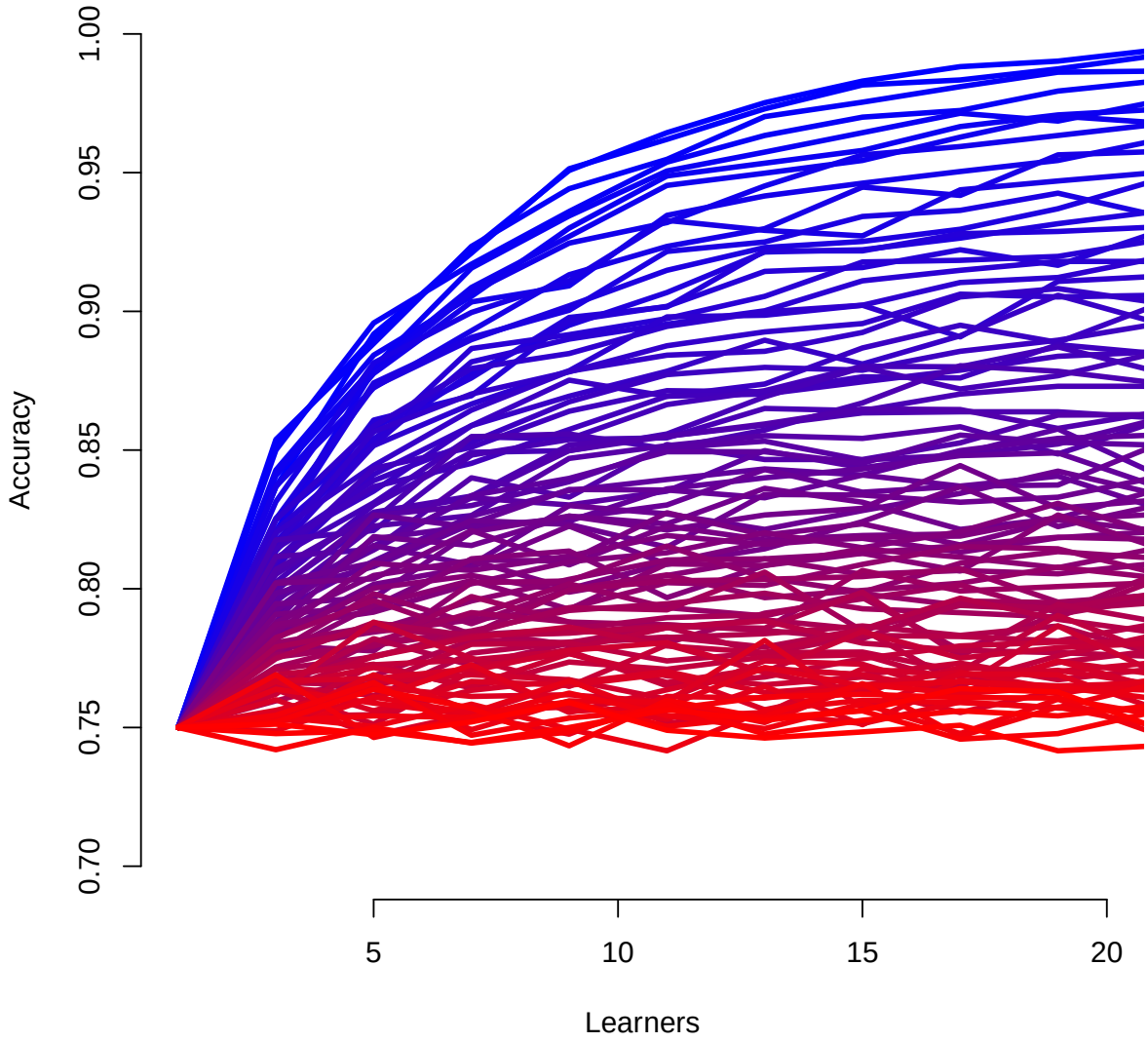
Diverse and Accurate matter.



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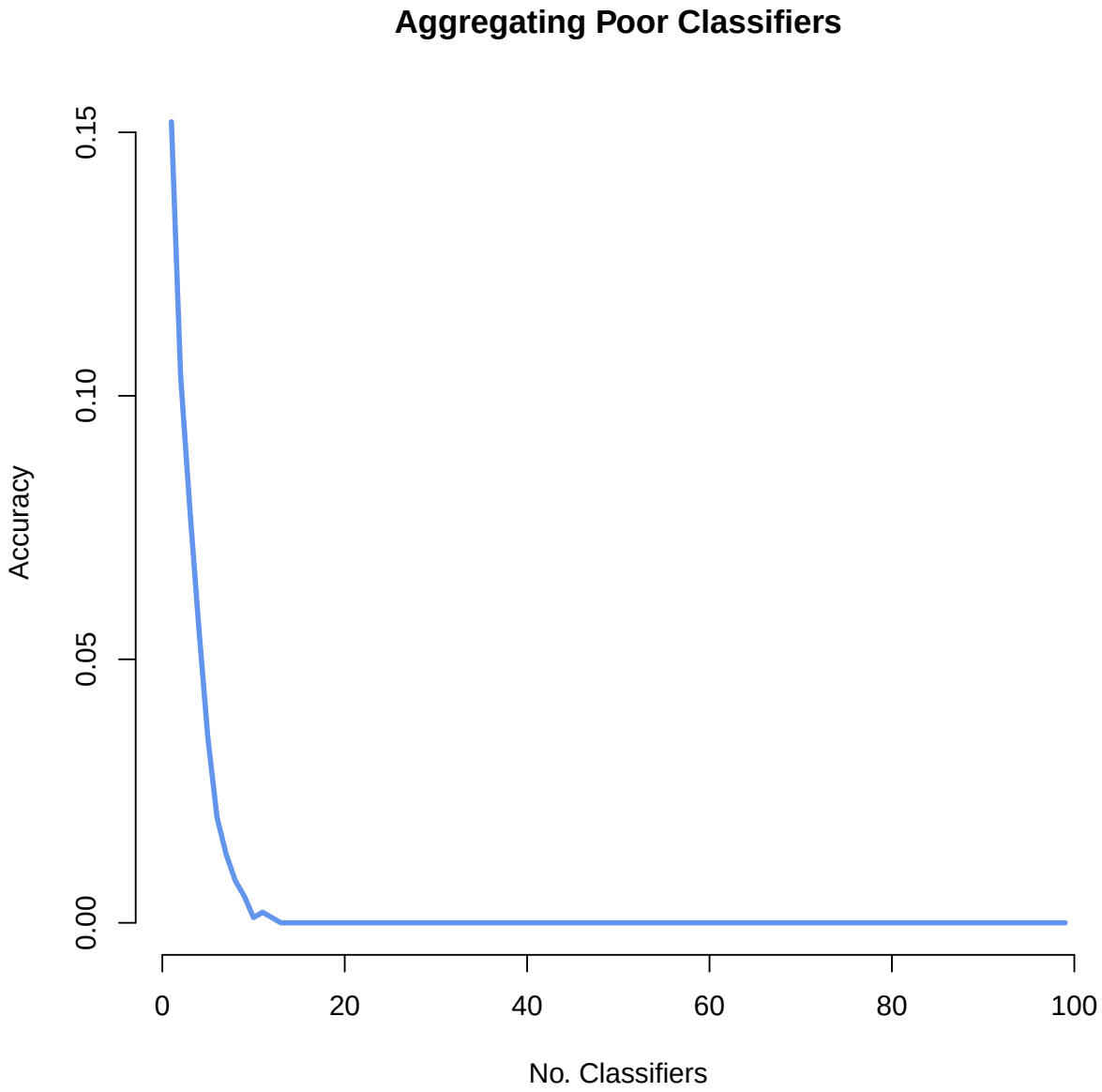

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$$\lim_{M \to \infty} P(\bar{B} > 0.5) = 1$$

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&= \underbrace{\rho\sigma^2}_{\text{Resolve with independence}} + \underbrace{\frac{1-\rho}{M}\sigma^2}_{\text{Resolve with } \uparrow\text{classifiers}}
\end{aligned}$$

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- Strong Correlation between classifiers (recall optimal division from previous slide)

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Curse of dimensionality(!!!)

Approximate with regions$\rightsquigarrow$ search for splits of data to approximate stratification

Classification and Regression Trees (CART): Objective function

Labels $\boldsymbol{Y}_i$ and documents $\boldsymbol{x}_i$

$$\begin{aligned} E[Y|\boldsymbol{x}_i] &= \widehat{f}(\boldsymbol{x}_i) \\ &= \sum_{p=1}^{P} c_p I(\boldsymbol{x}_i \in R_p) \end{aligned}$$

where:

- R_p describes a region $\rightsquigarrow$ node
- c_p describes values of Y_i for document in R_p

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Suppose we are in some node (perhaps at the start).

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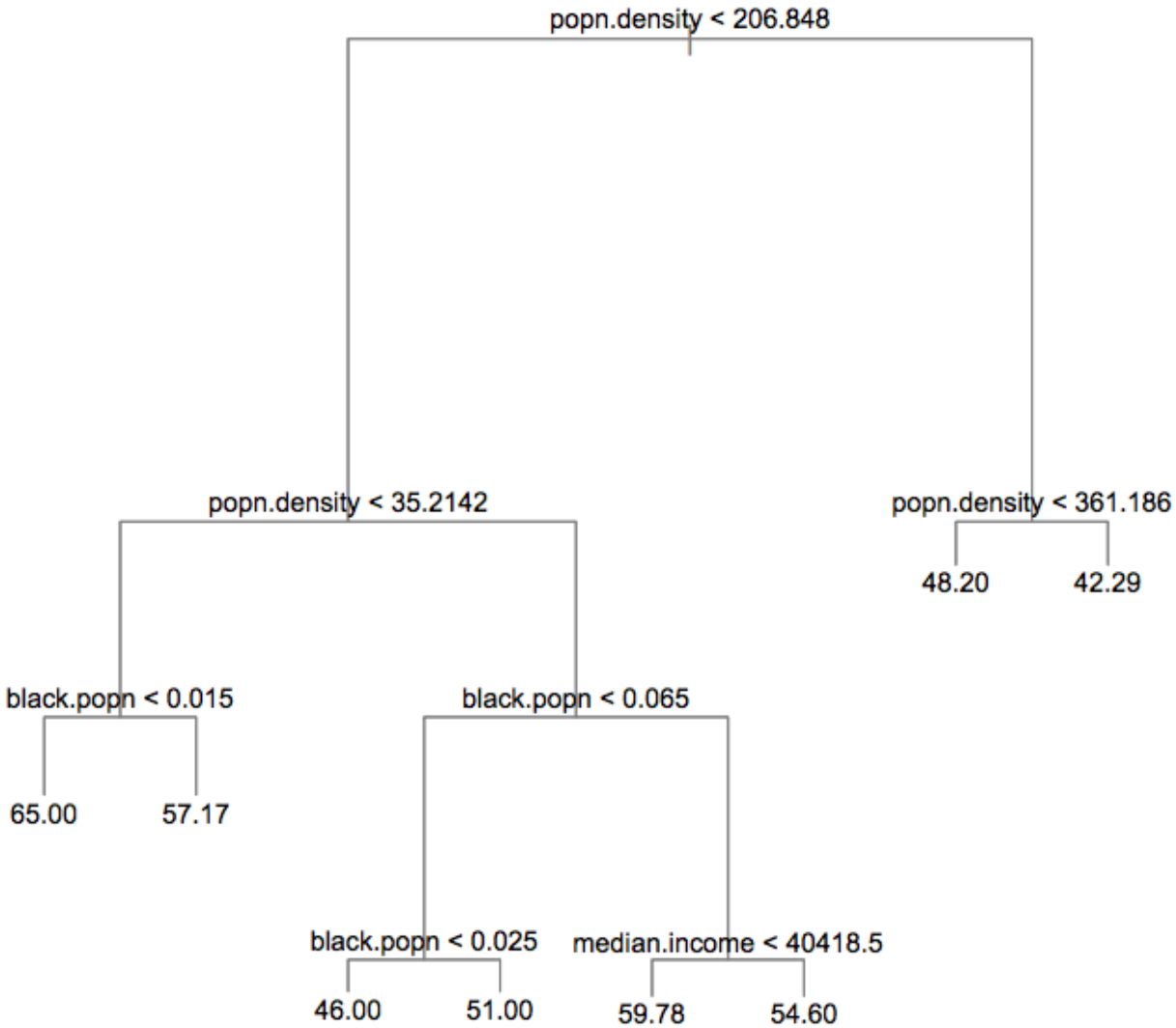
$$(j^*, s^*) = \text{arg min}_{j,s} \left[\underbrace{\text{min}_{c_1} \sum_{i=1}^{N} I(x_{ij} < s)(Y_i - c_1)^2}_{\text{“cost” group 1}} + \underbrace{\text{min}_{c_2} \sum_{i=1}^{N} I(x_{ij} > s)(Y_i - c_2)^2}_{\text{“cost” group 2}} \right]$$

Classification and Regression Trees (CART): Algorithm

- Start in Node
- Partition according to Greedy algorithm
- Continue until some stopping rule: number of observations per node

CART Picture (Spirling 2008)



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- Average$\rightsquigarrow$ reduces variance, but will be correlated
- Random forest$\rightsquigarrow$ introduce additional sampling to induce independence$\rightsquigarrow$ Only split on subset of variables

Random Forest Algorithm (ESL, 588)

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- Wager and Athey (2015): Random Forest for estimating heterogeneous effects

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Weighted ensemble: weights determined by (unique) out of sample predictive performance

Committee Methods:
Fit many methods, average with equal weights

- Voting (classification)
- Averaging (predictions)

Problem: many poor methods may overwhelm high quality fit (remember earlier figures)
Solution: learn weights via cross validation

Weighted Ensemble to Classify Documents

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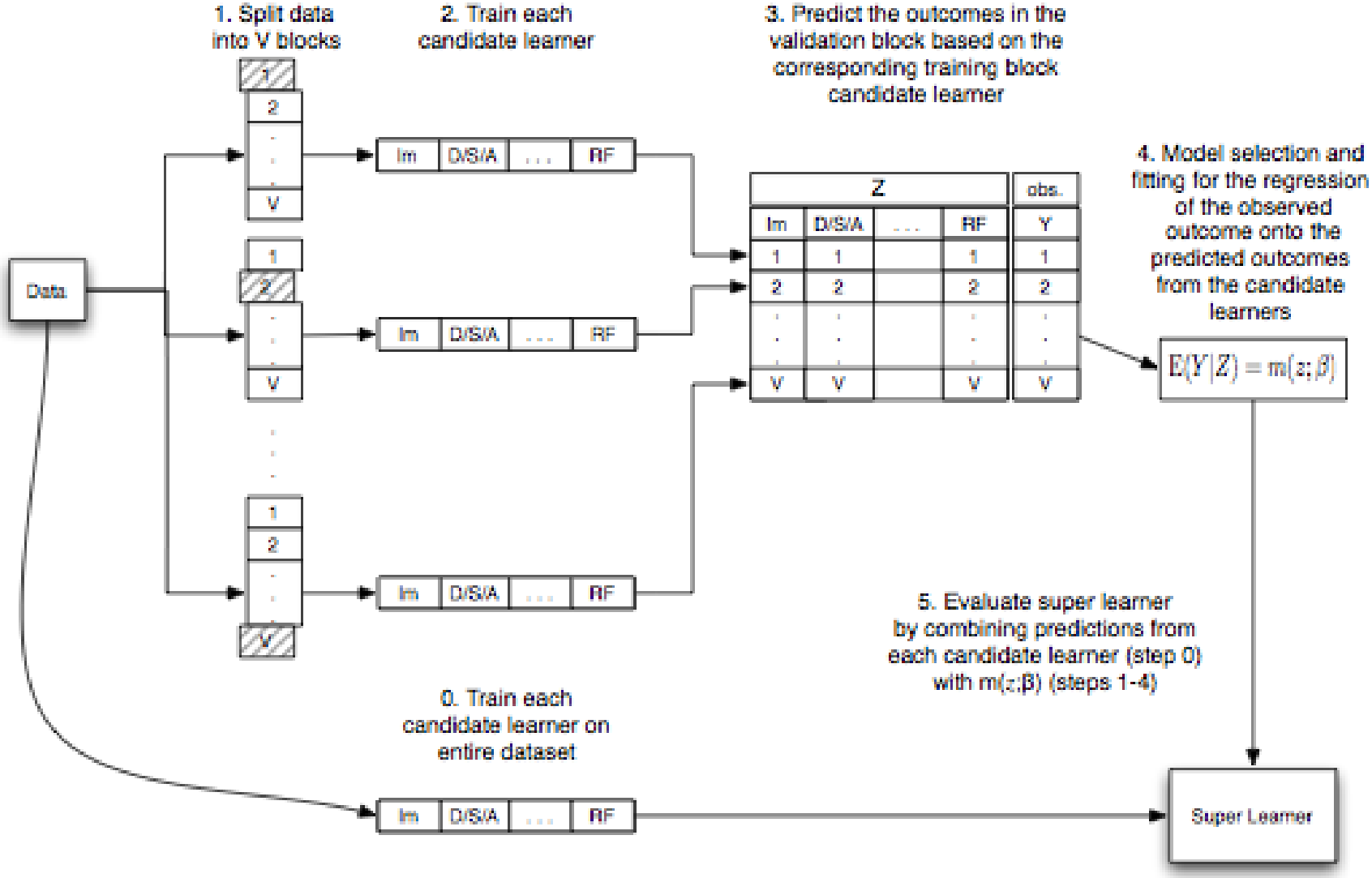

Figure 1: Flow Diagram for Super Learner

Why Super Learn?

van der Laan et al (2007) prove:

- Asymptotically: super learners will perform as well the best candidates for data
- Oracle: performs like the best possible method among candidate methods
  - Asymptotically outperforms constituent methods
  - Performs as well as optimal combinations of those methods

Practical questions:

- Final regression:
  - Logistic
  - Linear
  - Could super learn again!
- How Many Folds?
  - van der Laan et al's proofs rely on growing folds with N (but slowly)
  - Use 10-fold cross validation for simulations

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Estimate: $Y_i \in \{\text{Credit}, \text{Not Credit}\}$

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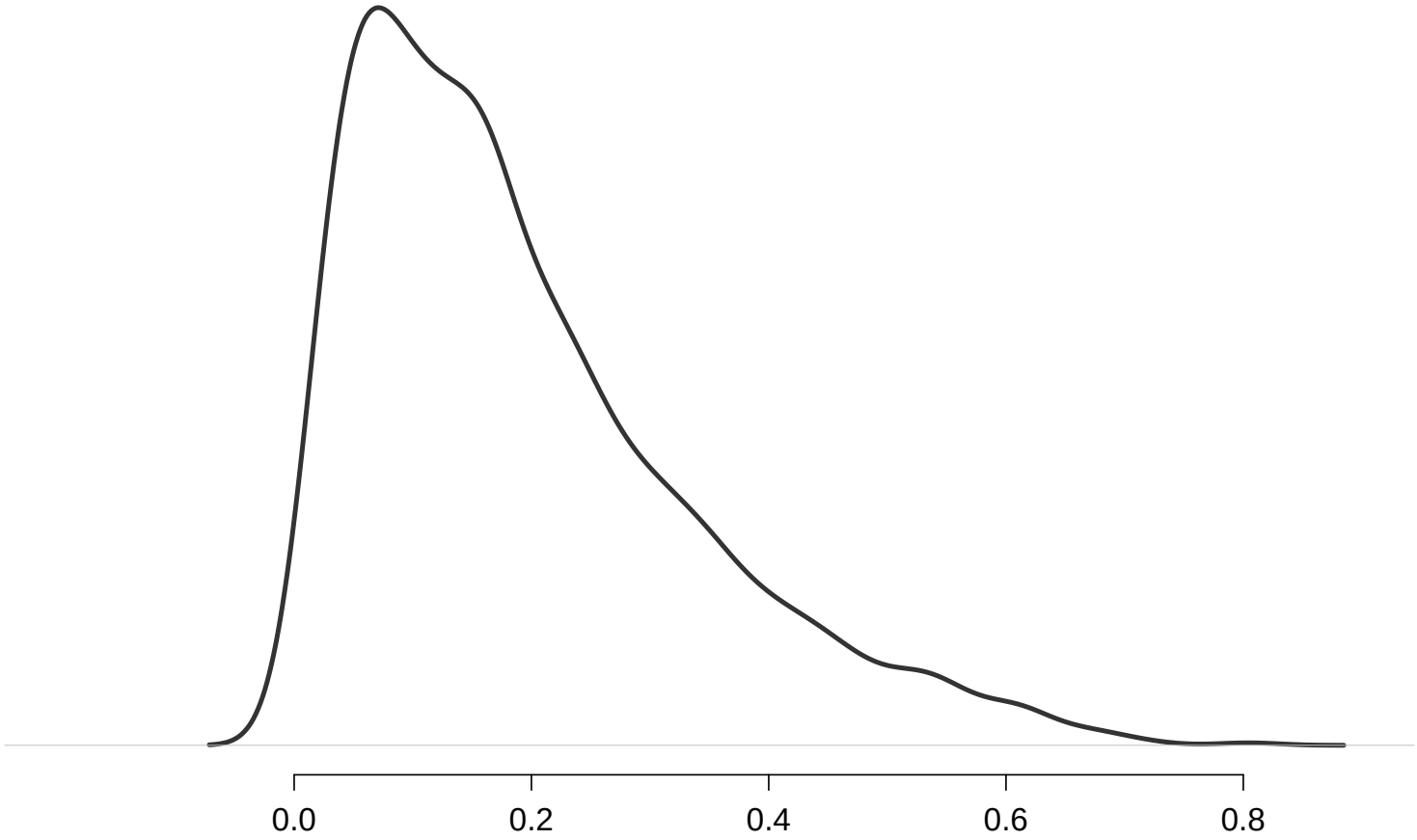
- LASSO 0
- Elastic-Net 23%
- Random Forest 61%
- A Support Vector Machine16%
- Kernel Regularized Least Squares (KRLS, Hainmueller and Hazlett 2014) 0

Strategic Credit Claiming to Build a Personal Vote



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John McGroff: "voted for every spending bill " that went through the office"

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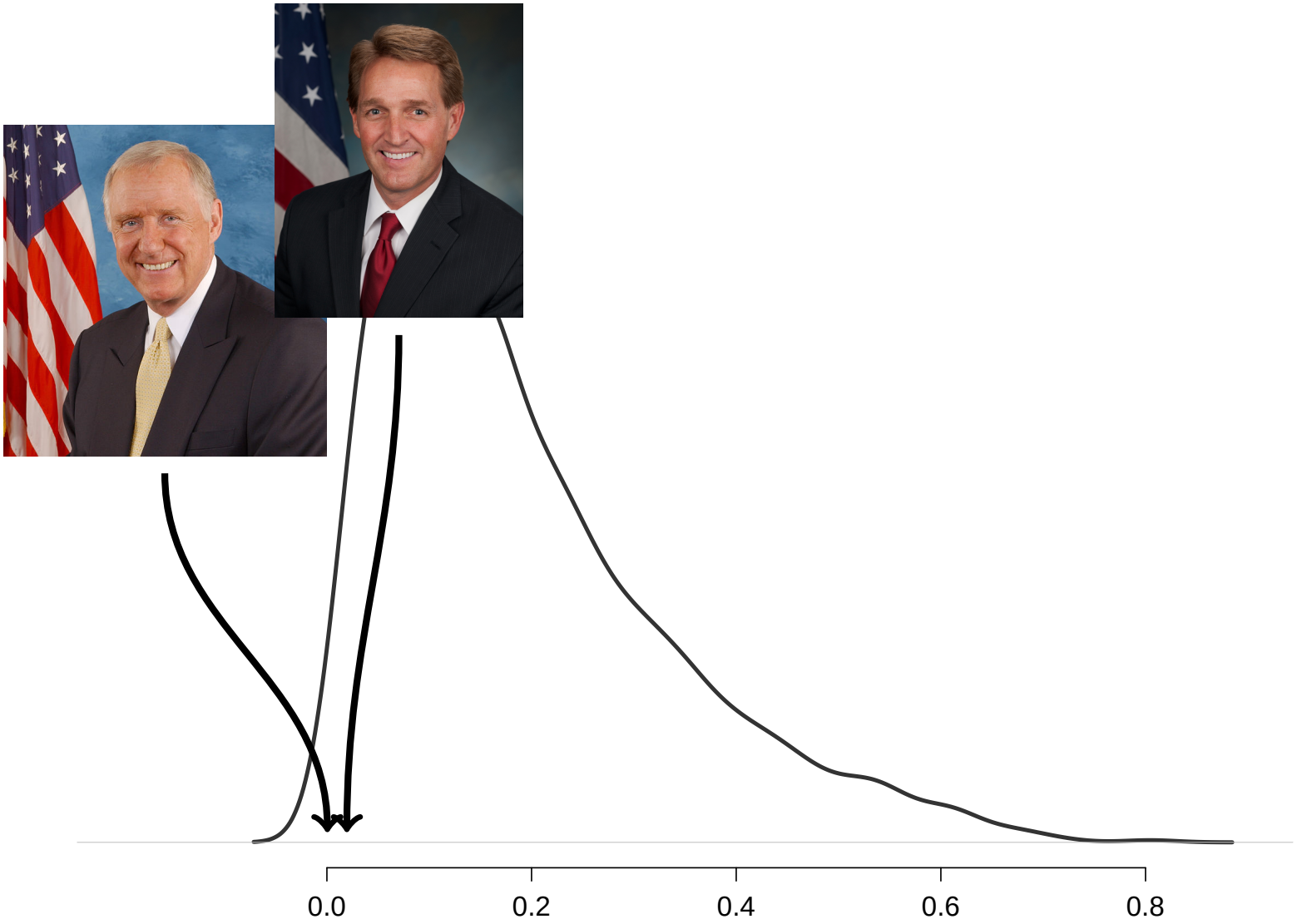
John McGroff: "Not the actions of a fiscally "
conservative congressman who
cares about personal responsibility"

Strategic Credit Claiming to Build a Personal Vote



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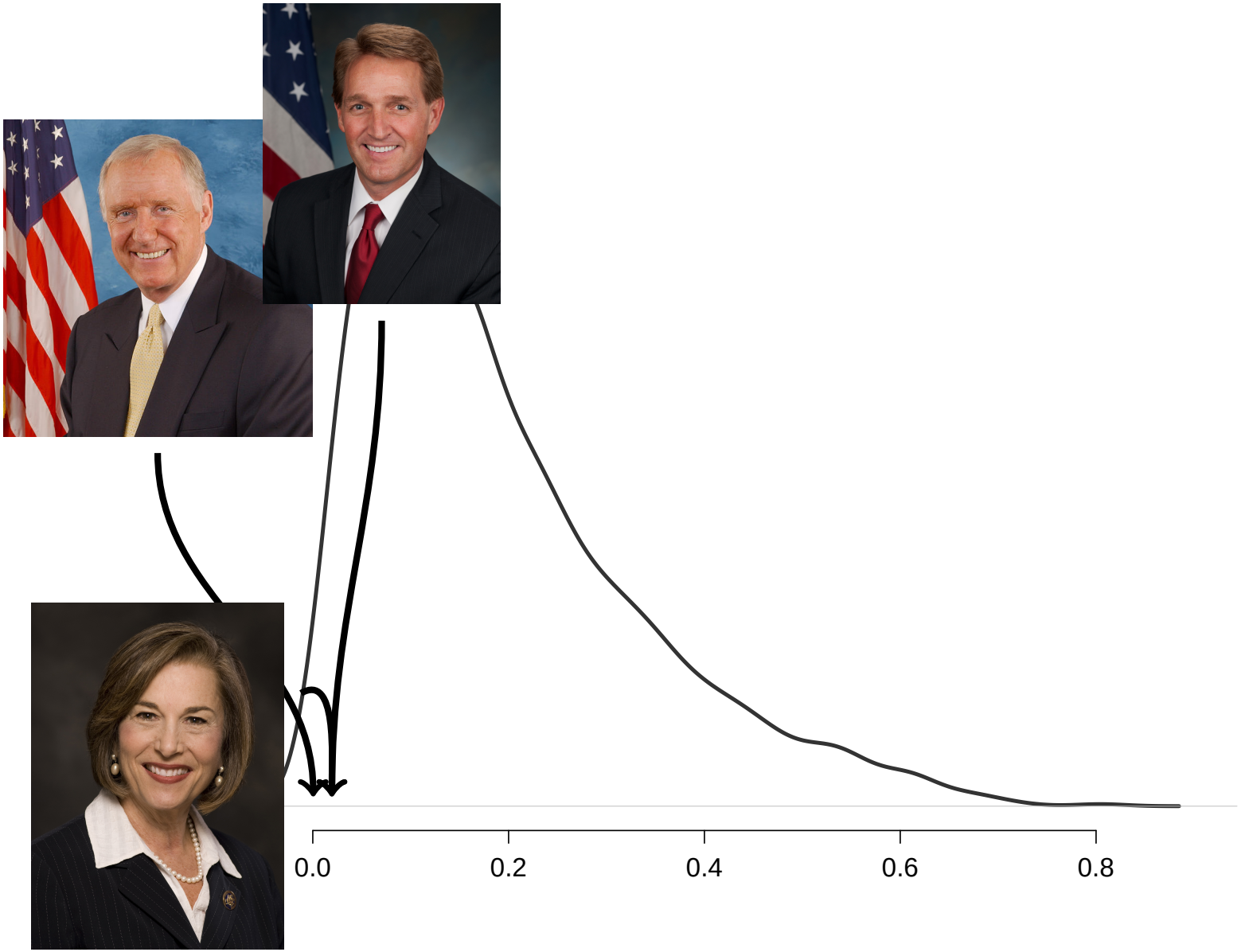

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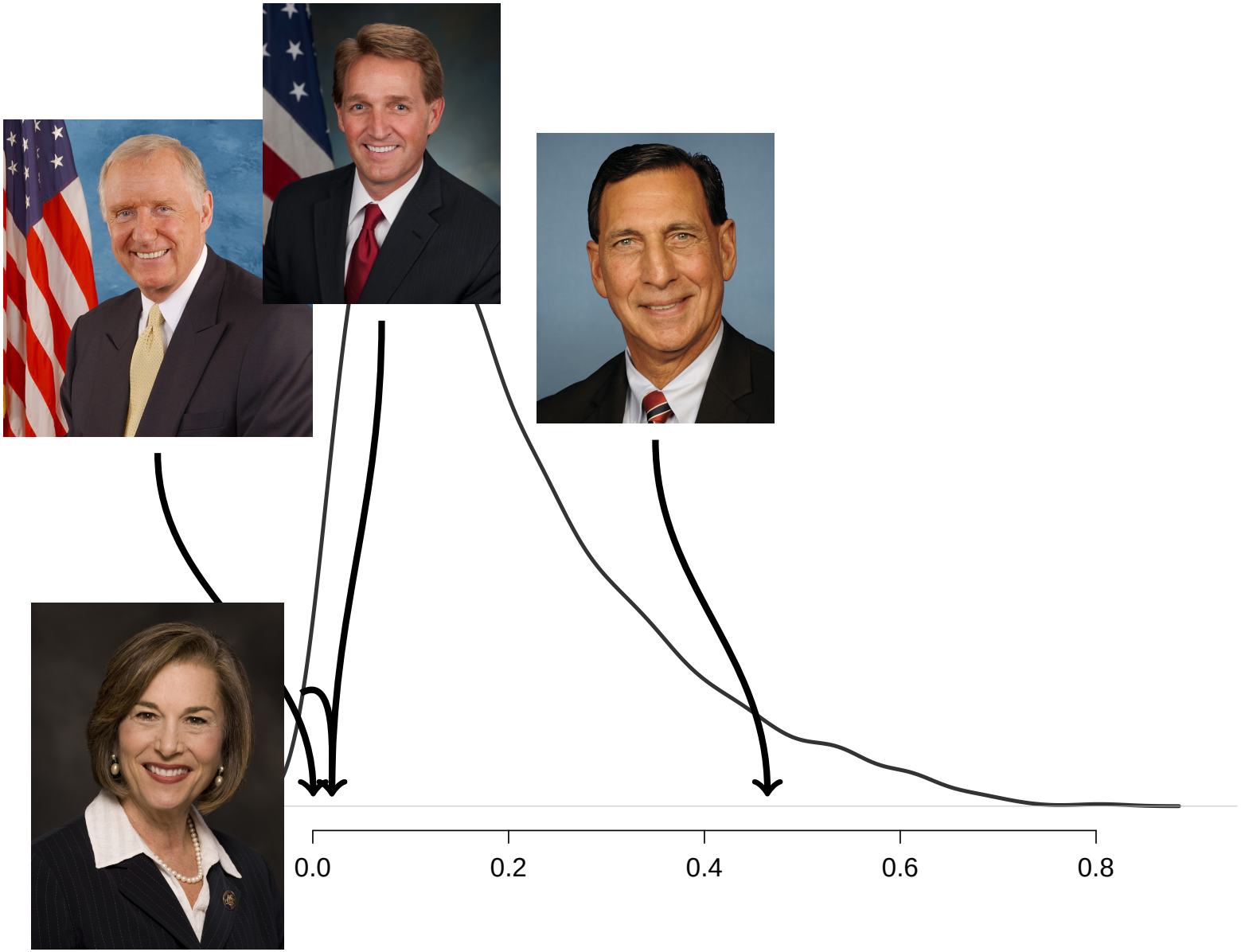

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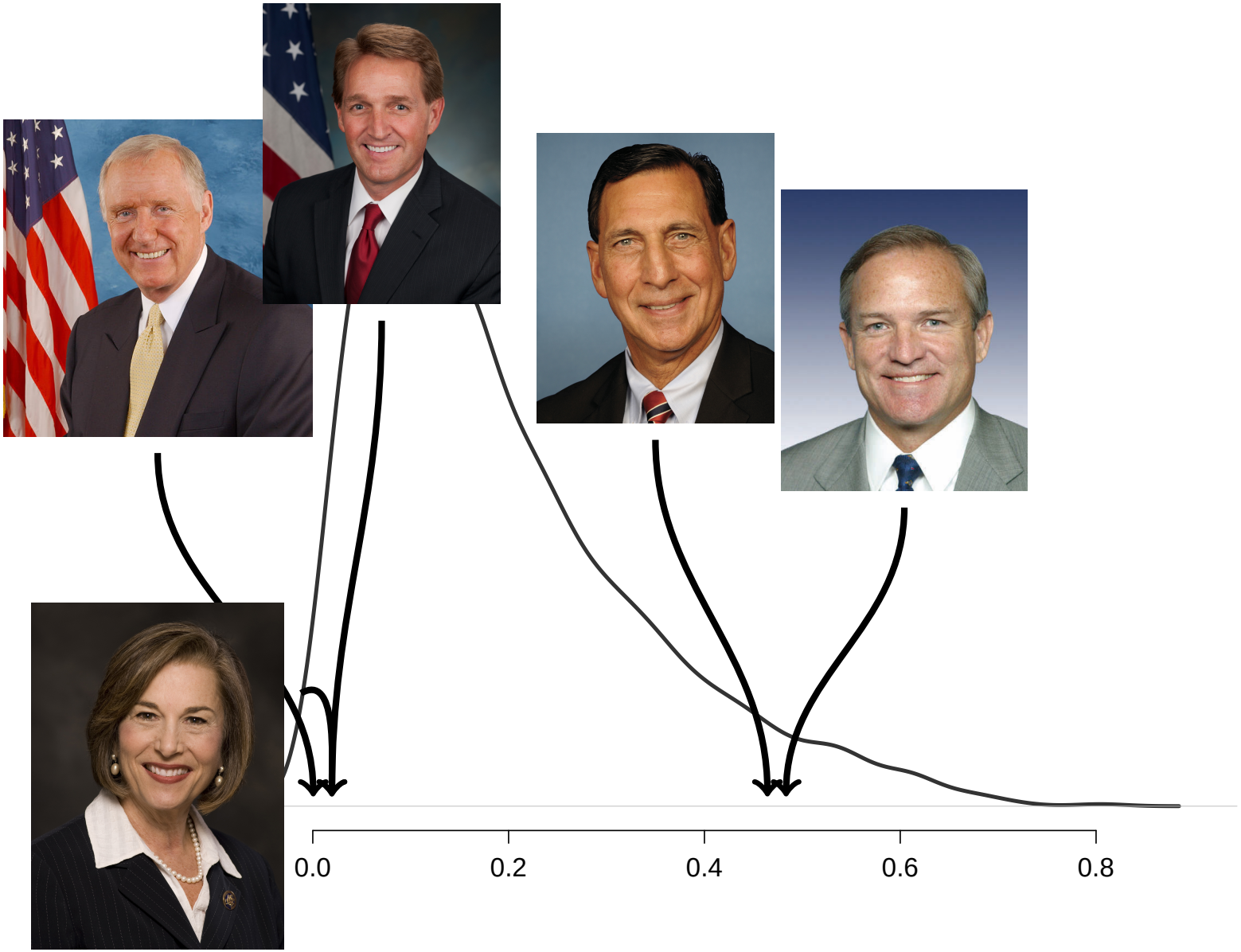

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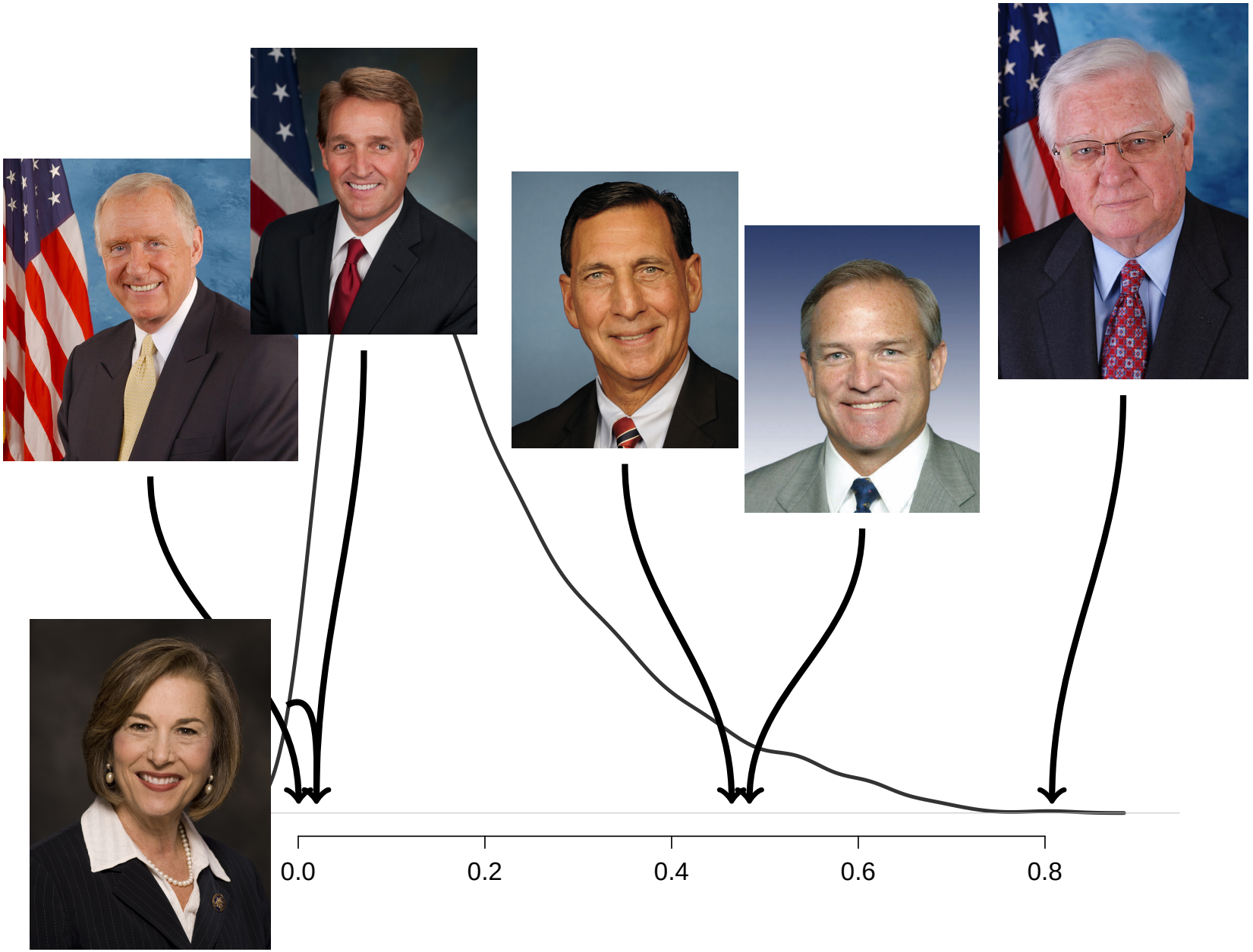

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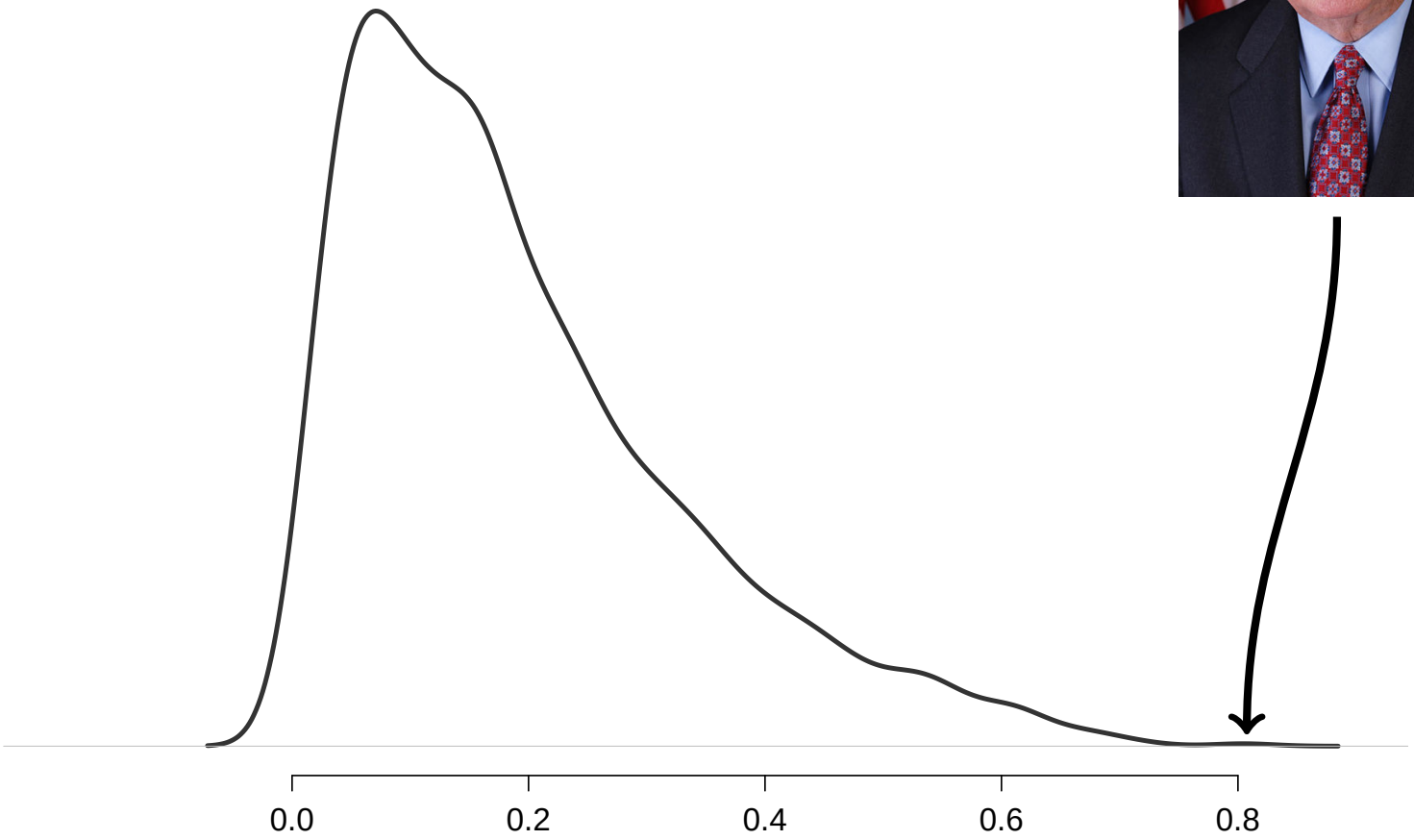
"We just can't afford luxuries like ideology"



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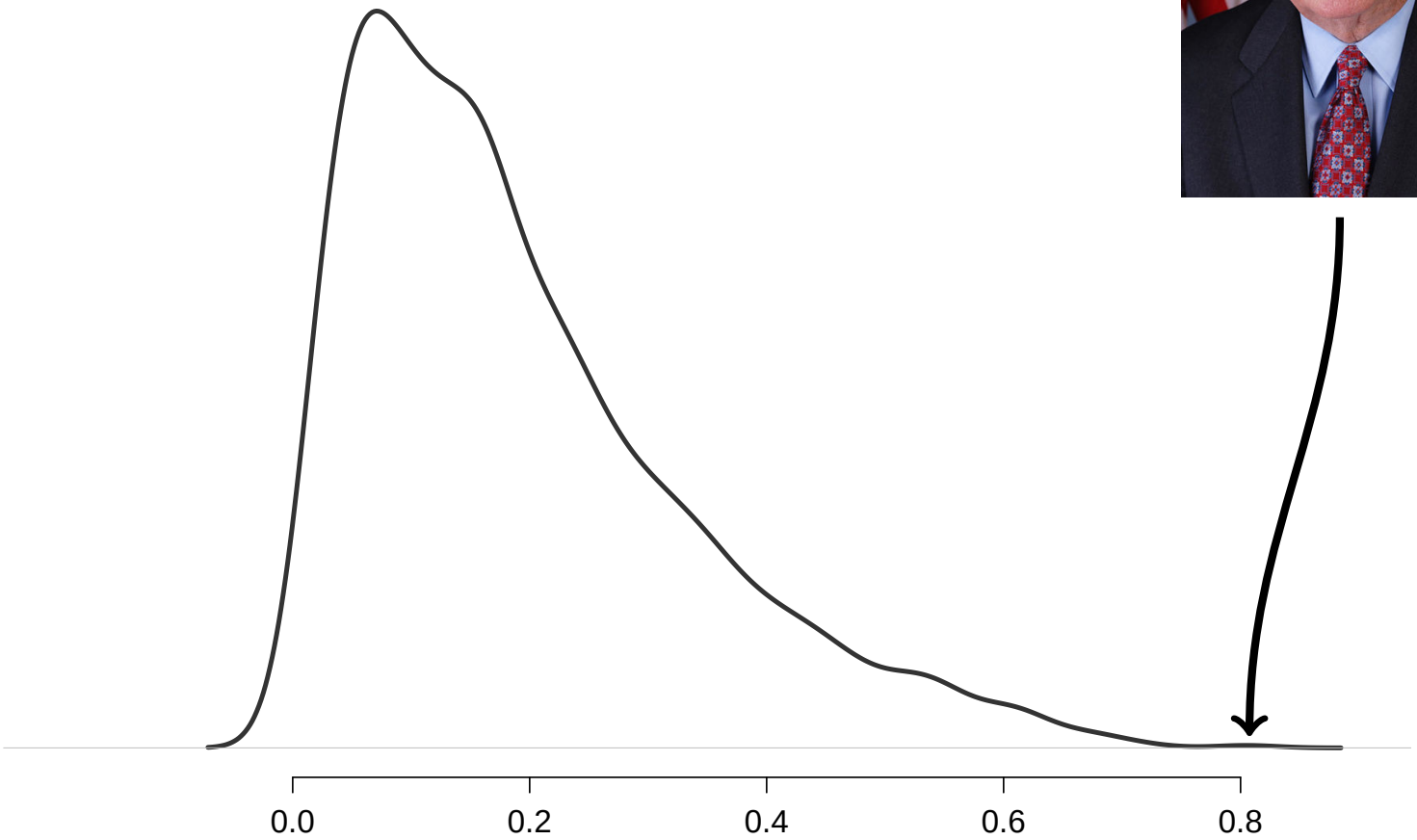
Lexington Herald-Leader: Prince of Pork



Other Reasons to Ensemble (Dietterich 2000)

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