Winter 2020 Problem Set

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I would like to thank Evan Chen for letting me use his LaTeX package.

Contents

§1 Useful Facts

Theorem 1.1 (Multiplication and Independent Events)
We use **multiplication** to count a series of independent events.

Theorem 1.2 (The factorial)
The n^{th} factorial, $n!$,is the number of ways to arrange n things when **order matters**.

Theorem 1.3 (n choose r principle)
$\binom{n}{r}$ is the number of ways to choose an r-person committee from a total of n people, where order does not matter.

The formula is given by: $\binom{n}{r} = \frac{n\cdot(n-1)\cdot(n-2)\cdot(n-3)\cdot(n-4)\cdot\ldots\cdot(n-r+1)}{r!} = \frac{\frac{n!}{(n-r)!}}{r!} = \frac{n!}{r!(n-r)!}$

§2 Useful Ideas

Lemma 2.1
If there are restrictions in problems, deal with them *first*

Lemma 2.2

Don't use the $\binom{n}{r}$ principle randomly, especially if you don't why you are using it. In the future, problems won't be that easy to solve(just plugging into the formula) and will usually need some *modifications* to the principle. However, knowing how and when to use the principle is **very** important.

§3 Counting Warm-ups

Problem 3.1. How many ways are there to arrange 7 different Harry Potter books on a bookshelf if order matters?

Problem 3.2. Suppose I have 5 different books, 2 of which are math books. How many ways are there to stack the 5 books on the shelf if I always want a math book at the end of the stack?

Problem 3.3. What is an real-life example of 2 independent events?

Problem 3.4. How many ways are there for Justin to choose 2 socks to wear from his drawer that has a total of 10 socks? The order in which he chooses the socks *does not matter*.

§4 Counting Problems

Problem 4.1 (AMC 12A #1 2016)**.** What is the value of $\frac{11!-10!}{9!}$

Problem 4.2. The United States' Denate committee has 6 Republicans and 10 Democrats. In how ways can we form a *subcommittee* of 2 Republicans and 3 Democrats?

Problem 4.3. In how many ways can Ms. Wolbers separate her 10 students into literature circles of 4 and 6 students if Tom and Jerry have to be in different groups?

Problem 4.4 (AMC 12A #10 2016-Not a real counting problem)**.** Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

Problem 4.5. We call a *descending number* if each digit is strictly smaller than the digit that comes before it. How many 3-digit descending numbers are there?

Problem 4.6. How many possible distinct paths are there on this 6×13 grid that start from the bottom left corner to the to right and consists of only 19 moves?

Problem 4.7. There are lines drawn in a plane. What is the largest possible number of points in the plane at which at least two of the nine lines intersect?

Problem 4.8 (AMC 12A #11 2016-Requires an idea called:"principle of inclusion and exclusion".You should search it up if you want to solve this problem)**.** Each of the 100 students in a certain summer camp can either sing, dance, or act. Some students have more than one talent, but no student has all three talents. There are 42 students who cannot sing, 65 students who cannot dance, and 29 students who cannot act. How many students have two of these talents?