

Basic Counting

Meeting #1

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Outline

- 1 Counting
 - Counting Multiple Events
 - Independent Events
- 2 Factorial Problems
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Counting Multiple Events

Many problems ask you to count the number of outcomes of a series of events.

Example

James has 3 shirts and 2 pants. How many outfits consisting of one shirt and one pants can he wear?

Independent Events

Definition

Independent Events:

A series of *independent event* are events that events that **do not depend on each other**

In the example above, choosing a shirt and choosing pants are **independent events** because each decision does not depend on the others.

Multiplication for Independent Events

Theorem

We use multiplication to count a series of independent events.

Thus, for the first example with James and his shirts and pants. There are $3 \cdot 2 = 6$ outfits.

Problem

Donald Trump wants to form an international committee with one representative from each of the following countries: Canada, Mexico, France and South Africa. There are 4 possible representatives from both Canada and Mexico, 3 possible representatives from France and 6 possible representatives from South Africa. How many different committees can Donald form?

Arranging Books

Here is another problem which asks you to count the number of outcomes of a series of events.

Example

In how many ways can Johnny arrange four different books on a shelf?

Arranging lots of Books

Let us generalize the problem above into a concept.

Example

In how many ways can Johnny arrange n different books on a shelf?

The Factorial

Definition

We define the n^{th} factorial as: $n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \dots \cdot 3 \cdot 2 \cdot 1$
This is particularly useful as a counting technique.

Example

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

The Factorial

The n^{th} factorial, $n!$, is the number of ways to arrange n things when **order matters**

Stacking Books with Restrictions

Suppose I have 6 different books, 2 of which are math books. How many ways are there to stack the 6 books on the shelf if I always want a math book at the end of the stack?

Stacking Books with Restrictions

Suppose I have 3 different math books and 5 different history books. How many ways are there to arrange the books on the shelf if I always want a math book at the end of the stack?

Girls and Boys Sitting on Chairs

In Ms. Yajima's chemistry class, there are 4 boys and 3 girls. In how many ways can they be seated in a row of 7 chairs such that all 3 girls **MUST** sit together?

Resources

Art of Problem Solving-artofproblemsolving.com

- ① Problems
- ② Alcumus Game
- ③ Problem Solving Books
- ④ Classes