Day #1-Counting

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§1 Counting

Example 1.1 (Counting Multiple Events)

James has 3 shirts and 2 pants. How many outfits consisting of one shirt and one pants can he wear?

Let us denote the 3 shirts by S_1, S_2 and S_3 and the 2 pants by P_1 and P_2. We can list out all the possible cases.
1.P$_1$, S_1
2.P_1, S_2
3.P_2, S_1
4.P_2, S_2
5.P_3, S_1
6.P_3, S_2
We can safely conclude that there are 6 possible ways.

Example 1.2 (Counting Multiple Events)

Donald Trump wants to form an international committee with one representative from each of the following countries: Canada, Mexico, France and South Africa. There are 4 possible representatives from both Canada and Mexico, 3 possible representatives from France and 6 possible representatives from South Africa. How many different committees can Donald form?

Choosing the representatives from each of the countries is an independent event. Thus, we use multiplication to find the answer. There are: $4 \cdot 4 \cdot 3 \cdot 6 = 288$

§2 Factorial Problems

Example 2.1 (Arranging Books)
In how many ways can Johnny arrange four different books on a shelf?

Let us place the books from left to right. There are 4 choices for the leftmost, 3 choices for the 2nd leftmost, 2 choices for the 3rd leftmost and only 1 choice for the rightmost position.
There are $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ possibilities

Example 2.2 (Arranging lots of Books)
In how many ways can Johnny arrange n different books on a shelf?

Let us label the n possible positions from 1 to n starting from left to right. There are n choices for the 1st position, $(n-1)$ choices for the 2nd position, $(n-2)$ choices for the 3rd position and etcetera. Continuing in a similar fashion, we can conclude that there are: $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$

§3 Counting with Restrictions

Example 3.1 (Stacking Books with Restrictions)
Suppose I have 6 different books, 2 of which are math books. How many ways are there to stack the 6 books on the shelf if I always want a math book at the end of the stack?

For problems with restrictions, the key idea is to **deal with the restrictions first**. We can arrange the 2 math books on the ends of the stack in $2! = 2 \cdot 1 = 2$ ways and there are $4! = 24$ ways to arrange the other 4 books

Example 3.2 (Stacking Books with Restrictions)
Suppose I have 3 different math books and 5 different history books. How many ways are there to arrange the books on the shelf if I always want a math book at the end of the stack? There are a total of $2 \cdot 4! = 48$

There are 3 choices for the leftmost book, and 2 choices for the rightmost book. Next, there are $5! = 120$ ways to arrange the 5 history books in between the two math books. There are a total of $3 \cdot 2 \cdot 5! = 720$

Example 3.3 (Girls and Boys Sitting on Chairs)
In Ms. Yajima's chemistry class, there are 4 boys and 3 girls. In how many ways can they be seated in a row of 7 chairs such that all 3 girls MUST sit together?

This is a pretty tricky question and needs some clever insight. Let us glue all the 3 girls together to form a "big girl". Let us treat the bundle of girls as a single person. There are 5! ways to arrange the big girl and the 4 boys. Next, we will unglue the bundle of girls. There are 3! ways to arrange the 3 girls for a total of $5! \cdot 3! = 120 \cdot 6 = 720$