Committee Forming Problems

Meeting #3

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Outline

Review

Definition

$\binom{n}{r}$ is the number of ways to choose an r-person committee from a total of n people, **where order does not matter**.

Finding the formula

Problem

Consider a math circle that has n students. Find the number of ways to choose r different students from the n students, where order matters.

Problem

Find the number of ways that any given r people can be assigned to be volunteers.

The formula

Theorem

$$\binom{n}{r} = \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot (n-4) \cdot \ldots \cdot (n-r+1)}{r!}$$

Finding a Better Formula

That was pretty messy. Can we come up with a better formula?
Notice that $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot (n-4) \cdot \ldots \cdot (n-r+1) = \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot (n-4) \cdot \ldots \cdot (n-r+1) \cdot (n-r) \cdot (n-r-1) \cdot \ldots \cdot 2 \cdot 1}{(n-r) \cdot (n-r-1) \cdot \ldots \cdot 2 \cdot 1} = \frac{n!}{(n-r)!}$ So,

Theorem

$$\binom{n}{r} = \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot (n-4) \cdot \ldots \cdot (n-r+1)}{r!} = \frac{\frac{n!}{(n-r)!}}{r!} = \frac{}{r}$$

Problems

Problem (Computational)

Compute

$$\binom{10}{3}$$

Problem (Another computation)

Compute

$$\binom{10}{7}$$

Grid Walking

Problem

How many possible distinct paths are there on this 2×2 grid that start from the bottom left corner to the to right and consists of only 4 moves?

More Grid Paths

Problem

How many possible distinct paths are there on this 5×12 *grid that start from the bottom left corner to the to right and consists of only 17 moves?*

Casework or Complementary Counting

Problem

In how many ways can Ms. Wolbers separate her 10 students into literature circles of 4 and 6 students if Tom and Jerry have to be in different groups?

Combinatorial Geometry

Problem

There are lines drawn in a plane. What is the largest possible number of points in the plane at which at least two of the nine lines intersect?

Challenging Problem for Committee forming

Problem

The United States' Denate committee has 6 Republicans and 10 Democrats. In how ways can we form a subcommittee of 2 Republicans and 3 Democrats?

Problem (AMC 12A/5 2017)

At a gathering of 30 *people, there are* 20 *people who all know each other and* 10 *people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur?*

Problem

We call a descending number if each digit is strictly smaller than the digit that comes before it. How many 3-digit descending numbers are there?

Resources

Art of Problem Solving-artofproblemsolving.com

1. Problems
2. Alcumus Game
3. Problem Solving Books
4. Classes