Probability and Set Notation

Meeting #4

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Outline

An Introductory Example

Many problems ask you to find the probability of a certain event happening.

Example (Flipping Coins)

Ethan that a fair coin with two sides: heads and tails. What is the probability that when he flips the coin, he gets tails?

Example (Rolling a dice)

Ethan also has a fair dice with six sides. What is the probability that when he rolls an even number? How about an odd number?

Definition of Probability

Definition (Probability)

When all outcomes are equally likely,
The *probability of success* is defined as:

$$\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

In the example above, flipping a heads or tails are **equally likely** because the probability of flipping heads and tails are the same.

Another easy problem

Problem

We have a 8-sided die with 6 blue faces and 2 red faces. What is the probability of rolling a blue face? How about a red face?

What happens when you add the two probabilities together?

A Simple Bound on Probability

Theorem (Probability Bound)

For any event A, the probability of event A happening is between $[0, 1]$. That is,

$$0 \leq \mathbb{P}(A) \leq 1$$

where $\mathbb{P}(A)$ denotes the probability of event A occuring.

Example (Raining)

The probability of it raining on a Saturday can be $\frac{1}{5}$ or 1 but it **cannot** be 3 or -1.

A Typical Problem

Problem (Sitting in a circle)

There are 7 doctors sitting in a circle for a meeting. We randomly chose 3 doctors. What is the probability that they were all sitting together?

Problem (Sitting in a row)

There are 7 doctors sitting in a row of chairs for a meeting. We randomly chose 3 doctors. What is the probability that they were all sitting together?

Another Fundamental Property

Problem (Rain in Vancouver)

If the probability of it raining tomorrow is $\frac{2}{3}$*, what is the probability that it doesn't rain?*

Complementary Probabilities

Theorem (Complementary Probabilities add up to 1)

Let A be an event. Then we have:

$$\mathbb{P}(A \textit{ does not occur}) = 1 - \mathbb{P}(A \textit{ occurs})$$

Definition (Complement)

Let B be an event happening. Then we call B', the event not happening, the complement of B. So, we have: $\mathbb{P}(B') = 1 - \mathbb{P}(B)$

Sitting on Chairs

Problem

John and Amanda need to sit down in a row of 5 chairs. John refuses to sit next to Amanda. What is the probability of this occurring?

Problem (Complementary)

John and Amanda need to sit down in a row of 5 chairs. John refuses to sit next to Amanda. What is the probability that John ends up sitting next to ?

Mutually Exclusive Events

Definition (Mutually Exclusive Events)

Events that cannot occur at the same time are called *mutually exclusive*.

Example

For example, flipping a head and flipping a tails on a coin are *mutually exclusive events* because you can't flip a tail and heads at the same time!

Mutually Exclusive Events and Probabilities

Theorem

If A and B are mutually exclusive events, then

$$\mathbb{P}(A \text{ or } B) = \mathbb{P}(A) + \mathbb{P}(B)$$

Example

Let A be the event that it rains, B be the event that it doesn't rain and C the event that there is a rainbow.
$\mathbb{P}(A) = \frac{1}{2}, \mathbb{P}(B) = \frac{1}{10}, \mathbb{P}(C) = \frac{2}{5}$
What is the probability that it either rains *or* is a rainbow?

Independent Events and Probabilities

Theorem

If A and B are independent events, then

$$\mathbb{P}(A \text{ and } B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$$

Example

Let A be the event that we flip a heads on a coin on the first flip, B be the event that we flip heads again on the second flip. What is the probability of A and B happening?
That is, what is $\mathbb{P}(A \cap B)$?

AMC 12A 2017 #10

Problem

Chloé chooses a real number uniformly at random from the interval $[0, 2017]$*. Independently, Laurent chooses a real number uniformly at random from the interval* $[0, 4034]$*. What is the probability that Laurent's number is greater than Chloe's number?*

AMC 12A 2016 #13-Patterns with Probability

Problem

Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

Resources

Art of Problem Solving-artofproblemsolving.com

1. Problems
2. Alcumus Game
3. Problem Solving Books
4. Classes