Day #2-Counting

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December 2, 2020

I would like to thank Evan Chen for letting me use his LaTeX package.

Contents

§1 Counting

Example 1.1 (Warm Up-Dealing With Restrictions)
The McRoberts Debate Club has 6 English Delegates, 5 French Delegates and 3 Chinese Delegates. In how many ways can these 14 delegates sit in a row of 14 chairs, if each country's delegates must all sit next to each other?

Let us glue each country's delegates together. There are 3! ways to arrange the 3 "big" delegates. Then, for each of the delegates of the countries, there are 6!, 5! and 3! ways to arrange them. Thus, our final answer is $3! \cdot 6! \cdot 5! \cdot 3! = 3110400$

Example 1.2 (Correcting for Overcounting)
How many possible distinct arrangements are there of the letters in the word **BIG**?

There are 3! ways to arrange.

Example 1.3 (Correcting for Overcounting)
How many possible distinct arrangements are there of the letters in the word **BALL**?

If we pretend all the letters are different, there are 4! ways to arrange everything. However, there are 2 of the same L's. So, for every distinct arrangement, we counted it twice in the 4!. Thus, we have to divide by 2. Thus, our answer is $\frac{4!}{2} = 12$

A good example would be noticing that BAL_1L_2 and BAL_2L_1 are the same arrangement.

Example 1.4 (A Harder Problem)
How many possible distinct arrangements are there of the letters in the word **MAMA**?

If we pretend all the letters are different, there are 4! ways to arrange everything. However, there are 2 of the same L's and 2 of the same M's. So, for every distinct arrangement, we counted it twice in the 4!. Thus, we have to divide by $2 \cdot 2$. Thus, our answer is $\frac{4!}{2} = 12$

§2 Committees and Combinations

Example 2.1

In how many ways can a 2-person committee be chosen from a group of 4 people? The order in which we chose the 2 people does not matter.

There are $4 \cdot 3$ choices to choose the 1st and 2nd person. However, the order in which we choose the people does not matter. There are 2! ways to arrange the 2 people, so our final answer is $\frac{4 \cdot 3}{2!} = 6$

Example 2.2

In the McRoberts Student Council, how many ways can a a group of 3 representatives be chosen from a group of 8 students?

There are $8 \cdot 7 \cdot 6$ choices to choose the 1st, 2nd and 3rd person. However, the order in which we choose the students does not matter. There are 3! ways to arrange the 3 people, so our final answer is $\frac{8 \cdot 7 \cdot 6}{3!} = 56$