Automata and Countability, Kozen - Solutions

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Chapter 1

Finite Automata and Regular Sets

Chapter 2

Pushdown Automata and Context-Free Languages

69. In the grammar a 'b' is generated with the non-terminal B and B is always replaced by bA. Hence for every b in any word of the grammar there will be a a immediately after b. Hence. Now a is generated by the non-terminal A. And the production rule $A \to aA \mid a$ ensures that there can be any number of a's consecutively.

    (a) Hence $aabaab$ is not in $L(G)$ because there is no a after the last b

    (b) $aaaaba$ is in $L(G)$ and

    $$S \to AB \to aAB \to aaAB \to aaaAB \to aaaaB \to aaaabA \to aaaaba$$

    (c) $aabbaa$ is not in $L(G)$ because there is no a just after the first b.

    (d) $abaaba$ is in $L(G)$ and

    $$S \to ABS \to aBS \to abAS \to abaS \to abaAB \to abaaB \to abaabA \to abaaba$$

70. We can change the grammar by removing ϵ-productions

$$\begin{aligned} S &\to aAB \mid aBA \mid bAA \\ A &\to aS \mid bAAA \mid a \\ B &\to aABB \mid aBAB \mid aBBA \mid bS \mid b \end{aligned}$$

Now at any stage if previously $\#a + \#A = 2(\#b + \#B)$ then is we use any production rule replacing S the number of $\#a + \#A$ is increased by 2 and number of $\#b + \#B$ is increased by 1 so still the relation $\#a + \#A = 2(\#b + \#B)$ is maintained. If any production rule replacing A is used then either no a, A, b or B is added or number of $\#a + \#A$ is increased by 2 and number of $\#b + \#B$ is increased by 1. Hence the relation $\#a + \#A = 2(\#b + \#B)$ is maintained. And if B is replaced then either number of $\#a + \#A$ is increased by 2 and number of $\#b + \#B$ is increased by 1 or no a, A, b or B is added. Hence the relation $\#a + \#A = 2(\#b + \#B)$ is maintained. is satisfied in every level of the parse tree. Therefore $L(G)$ contains the set L.

Now consider the function $f(w) = \#_a(w) - 2\#_b(w)$. Now we know for all word $w \in L$, $f(w) = 0$. Now for any word if we plot the graph of f as it gradually reads the whole word we may consider upward diagonal movement by $\frac{1}{2}$ unit if it reads a and downward diagonal movement by one unit if it reads b. WLOG suppose the first letter is a. Then if the last letter is a then the function f must have reached the x-axis at some point after the first letter and the before the last letter. Hence $w = w_1 w_2$ where both w_1, w_2 has $\#a = 2\#b$. By induction $w \in L(G)$.

If the last letter is b. Then if the second letter is b then we have touched the x-axis. So $w = ab$

71.

$$\begin{aligned} S &\to aSbb \mid T \mid abb \\ T &\to bTaa \mid S \mid baa \end{aligned}$$