Soham Chatterjee
Email: sohamc@cmi.ac.in
Course: Analysis 2

Assignment - 2
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Problem 1 Rudin Chapt. 9 Problem 6

If $f(0,0)=0$ and

$$f(x,y)=\frac{xy}{x^2+y^2} \qquad \text{if } (x,y)\neq(0,0)$$

prove that $(D_1 f)(x,y)$ and $(D_2 f)(x,y)$ exist at every point of R^2, although f is not continuous at $(0,0)$.

Solution:When $(x,y)\neq 0$ then

$$(D_1 f)(x,y)=\frac{y(y^2-x^2)}{(x^2+y^2)^2} \text{ and } (D_2 f)(X,y)=\frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

Now at $(0,0)$

$$\begin{aligned}(D_1 f)(0,0) &= \lim_{h\to 0}\frac{f(h,0)-f(0,0)}{|h|} \\ &= \lim_{h\to 0}\frac{\frac{h\times 0}{h^2+0}-0}{|h|} \\ &= \lim_{h\to 0}\frac{0}{|h|} \\ &= 0\end{aligned}$$

$$\begin{aligned}(D_2 f)(0,0) &= \lim_{k\to 0}\frac{f(0,k)-f(0,0)}{|k|} \\ &= \lim_{k\to 0}\frac{\frac{0\times k}{0+k^2}-0}{|k|} \\ &= \lim_{k\to 0}\frac{0}{|k|} \\ &= 0\end{aligned}$$

Hence $(D_1 f)(x,y)$ and $(D_2 f)(x,y)$ exists at every point of $\mathbb{R}^2$.

Now if we approach $(0,0)$ along the line then it approaches to 0. But if we approach $(0,0)$ along the line $y=x$ then

$$\lim_{h\to 0} f(h,h)=\lim_{h\to 0}\frac{h^2}{2h^2}=\frac{1}{2}$$

Hence f is not continuous at (0,0)

$\square$

Problem 2 Rudin Chapt. 9 Problem 7

Suppose that f is a real-valued function defined in an open set $E\subset R^n$, and that the partial derivatives $D_1 f,\ldots,D_n f$ are bounded in E. Prove that f is continuous in E.

Hint: Proceed as in the proof of Theorem 9.21.

Solution:We have to show that $\forall\ \epsilon>0$

$\square$

Problem 3 Rudin Chapt. 9 Problem 8

Suppose that f is a differentiable real function in an open set $E\subset R^n$, and that f has a local maximum at a point $x\in E$. Prove that $f'(x)=0$.

Solution:

$\square$

Problem 4 Rudin Chapt. 9 Problem 10

If f is a real function defined in a convex open set $E \subset R^n$, such that $(D_1 f)(x) = 0$ for every $x \in E$, prove that $f(x)$ depends only on $x_2, \ldots, x_n$.

Show that the convexity of E can be replaced by a weaker condition, but that some condition is required. For example, if $n = 2$ and E is shaped like a horseshoe, the statement may be false.

Solution:

□

Problem 5 Rudin Chapt. 9 Problem 13

Suppose f is a differentiable mapping of R^1 into R^3 such that $|f(t)| = 1$ for every t. Prove that $f'(t) \cdot f(t) = 0$.

Interpret this result geometrically.

Solution:

□