Let us define a sequence $\{b_k\}_{k\geq 1}$ such that $k\alpha \equiv b_k \pmod{2\pi}$ for all $k \in \{1, \cdots, n\}$. Now we can consider $\frac{V_n(\alpha)}{n}$ as the probability of sign change occurrence between $k\alpha$ and $(k+1)\alpha$. Hence it is same as the probability of occurrence of b_k in $\mathcal{I}$ which is equal to

$$\frac{\text{Length of } (\mathcal{I})}{\text{Length of } [0, 2\pi]} = \frac{\text{Length of } (\mathcal{I})}{2\pi}$$