PROBLEM 5.37(D)
PROJECTIVE PLANE CURVES

let P be a point of order 2.
let $P \in U_2 \implies P = (a, 1, c)$
if $c = 0 \implies a = 0$ so, $c \neq 0$
let L be the tangent of (a, c) at C_*
as P is order $2 \implies L$ passes through $(0, 0)$
let $X = mZ$ be the equation of L
now $M \neq \infty$ (then $Z = 0$ be the equation of L which is the tangent at $(0, 1, 0)$
so, $a = mc$
now, $c = a(a - c)(a - \lambda c) \implies c = mc(mc - c)(mc - \lambda c)$
$\implies c^2 m(m - 1)(m - \lambda) = 1$.....(1)
so, $c^2 = 1/(m(m - 1)(m - \lambda)$[as $(m(m - 1)(m - \lambda) \neq 0$] (by equation 1)
so, c has two distinct values which implies L intersect C_* in three distinct points which is not possible.
so, $P \notin U_2$
so, $P = (a, 0, c) = (m, 0, 1) \implies m = 0, 1, \lambda$
tangent at $(m, 0, 1)$ is $X = mZ \forall m \in \{0, 1, \lambda\}$
and the tangent passes through $(0, 1, 0)$
so there are 3 points of order 2.
(e)
so, any non-singular irreducible curve is projectively equivalent to the equation given in (c)
so, the group has 4 point .
so, it is isomorphic to $\mathbb{Z}/(2) \times \mathbb{Z}/(2)$