CMI ALGEBRA 1 (2021) ASSIGNMENT 2 - T. R. Ramadas

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1. For the linear map $\hat{A} : \mathbb{R}^3 \to \mathbb{R}$ such that
$$\hat{A}((x, y, z)) = x + y + z$$
it has a right inverse $\hat{B} : \mathbb{R} \to \mathbb{R}^3$ such that $\hat{A} \circ \hat{B} = I_{\mathbb{R}}$. If we take
$$\hat{B}(x) = \left(\frac{x}{3}, \frac{x}{3}, \frac{x}{3}\right)$$
then
$$\hat{A} \circ \hat{B}(x) = \hat{A}(\hat{B}(x)) = \hat{A}\left(\left(\frac{x}{3}, \frac{x}{3}, \frac{x}{3}\right)\right) = \frac{x}{3} + \frac{x}{3} + \frac{x}{3} = x = I_{\mathbb{R}}(x)$$
Hence $\hat{B}$ here is a right inverse of $\hat{A}$.

The linear map $\hat{A} : \mathbb{R}^3 \to \mathbb{R}$ has no left inverse.

2. For the linear map $\hat{A} : \mathbb{R}^2 \to \mathbb{R}^3$ such that
$$\hat{A}((x, y)) = (x - y, x + y, 0)$$
has no right inverse.

The linear map $\hat{A} : \mathbb{R}^2 \to \mathbb{R}^3$, it has a left inverse $\hat{C} : \mathbb{R}^3 \to \mathbb{R}^2$ such that $\hat{C} \circ \hat{A} = I_{\mathbb{R}^2}$. If we take
$$\hat{C}((x, y, z)) = \left(\frac{x + y}{2}, \frac{y - x}{2}\right)$$
then
$$\hat{C}\circ\hat{A}((x, y)) = \hat{C}(\hat{A}((x, y))) = \hat{C}((x-y, x+y, 0)) = \left(\frac{(x - y) + (x + y)}{2}, \frac{(x + y) - (x - y)}{2}\right) = (x, y) = I_{\mathbb{R}^2}((x, y))$$
Hence $\hat{C}$ here is a left inverse of $\hat{A}$.

3. For the linear map $\hat{A} : \mathbb{R}^3 \to \mathbb{R}^3$ such that
$$\hat{A}((x, y, z)) = (x - y, y - z, z - x)$$
has no left and right inverse.

4. For the linear map $\hat{A} : \mathbb{R}^3 \to \mathbb{R}^3$ such that
$$\hat{A}((x, y, z)) = (x, 2y, 3z)$$
it has a right inverse $\hat{B} : \mathbb{R}^3 \to \mathbb{R}^3$ such that $\hat{A} \circ \hat{B} = I_{\mathbb{R}^3}$. If we take
$$\hat{B}((x, y, z)) = \left(x, \frac{y}{2}, \frac{z}{3}\right)$$
then
$$\hat{A} \circ \hat{B}((x, y, z)) = \hat{A}(\hat{B}((x, y, z))) = \hat{A}\left(\left(x, \frac{y}{2}, \frac{z}{3}\right)\right) = \left(x, 2 \cdot \frac{y}{2}, 3 \cdot \frac{z}{3}\right) = (x, y, z) = I_{\mathbb{R}^3}((x, y, z))$$
Hence $\hat{B}$ here is a right inverse of $\hat{A}$.

The linear map $\hat{A} : \mathbb{R}^3 \to \mathbb{R}^3$ has a left inverse $\hat{C} : \mathbb{R}^3 \to \mathbb{R}^3$ such that $\hat{C} \circ \hat{A} = I_{\mathbb{R}^3}$. If we take
$$\hat{C}((x, y, z)) = \left(x, \frac{y}{2}, \frac{z}{3}\right)$$
then
$$\hat{C} \circ \hat{A}((x, y, z)) = \hat{C}(\hat{A}((x, y, z))) = \hat{C}((x, 2y, 3z)) = \left(x, \frac{2y}{2}, \frac{3z}{3}\right) = (x, y, z) = I_{\mathbb{R}^3}((x, y, z))$$
Hence $\hat{C}$ here is a left inverse of $\hat{A}$.