Classical Mechanics 1, Autumn 2021 CMI
Problem set 7
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1. (a) Given that $x(t) = \Re\left[Ae^{i\omega t}\right]$ where A is a complex number. Let $A = m + in$. Hence

$$\begin{aligned}
x(t) &= \Re\left[(m+in)e^{i\omega t}\right] \\
&= \Re\left[m\cos(\omega t) - n\sin(\omega t) + i(m\sin(\omega t) + n\cos(\omega t))\right] \\
&= m\cos(\omega t) - n\sin(\omega t) \\
&= \sqrt{m^2+n^2}\left(\frac{m}{\sqrt{m^2+n^2}}\cos(\omega t) - \frac{n}{\sqrt{m^2+n^2}}\sin(\omega t)\right) \\
&= \sqrt{m^2+n^2}\cos(\omega t + \alpha) \\
&= |A|\cos(\omega t + \alpha)
\end{aligned}$$

where $\cos\alpha = \frac{m}{\sqrt{m^2+n^2}}$ and $\sin\alpha = \frac{n}{\sqrt{m^2+n^2}}$. Hence amplitude of the simple harmonic motion is $|A|$.

(b) $\cos\alpha = \frac{m}{\sqrt{m^2+n^2}}$ and $\sin\alpha = \frac{n}{\sqrt{m^2+n^2}}$. Hence

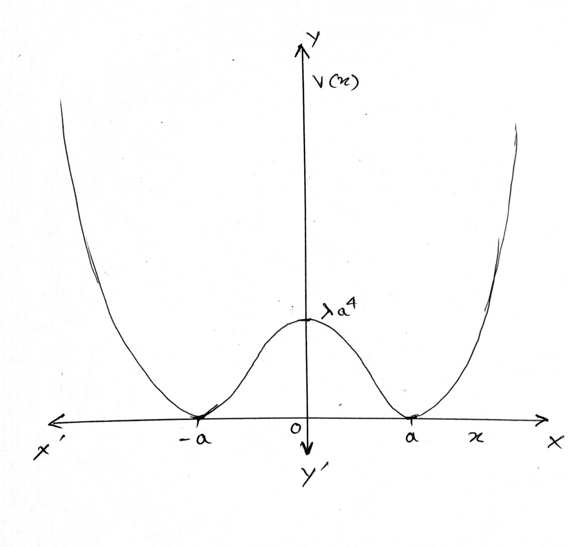
$$\begin{aligned}
\tan\alpha &= \frac{n}{m} \\
\implies \alpha &= \tan^{-1}\frac{n}{m} = \arg A
\end{aligned}$$

Hence the initial phase α of the motion is $\arg A$.

(c) $x(t) = |A|\cos(\omega t + \alpha) = m\cos(\omega t) - n\sin(\omega t)$. Hence $\dot{x}(t) = -|A|\omega\sin(\omega t + \alpha) = -m\omega\sin(\omega t) - n\omega\cos(\omega t)$. Therefore

$$\begin{aligned}
x(0) &= m\cos(\omega\times 0) - n\sin(\omega\times 0) = m\cos 0 - n\sin 0 = m = \Re(A) \\
&= |A|\cos(\omega\times 0 + \alpha) = |A|\cos\alpha \\
\dot{x}(0) &= -m\omega\sin(\omega\times 0) - n\omega\cos(\omega\times 0) = -m\omega\sin 0 - n\omega\cos 0 = -n\omega = -\omega\Im(A) \\
&= -|A|\omega\sin(\omega\times 0 + \alpha) = -|A|\omega\sin\alpha
\end{aligned}$$

2. (a) Given that $V(x) = \lambda(x^2 - a^2)^2 = \lambda(x-a)^2(x+a)^2$



(b) For static solution $\frac{d}{dx}V(x) = 0$. Therefore

$$\begin{aligned}
&\frac{d}{dx}\lambda(x^2 - a^2)^2 = 0 \\
\implies &4\lambda(x^2 - a^2)x = 0 \\
\implies &4\lambda x(x - a)(x + a) = 0
\end{aligned}$$

Hence the static solutions are $x = 0, a, -a$.

(c) At $x = a, -a$ $V(x) = 0$. And at $x = 0$ $V(x) = \lambda a^4 \neq 0$. Hence $a, -a$ are stable solutions and 0 is unstable solution.

3. x_1, x_2 are neighboring turning points. Hence the time interval between the two positions is $\frac{T}{2}$ where T is the time period of the oscillation. Hence when $x = x_1, x_2$, $E(x) = V(x)$ and when $x_1 < x < x_2$, $V(x) < E(x)$. Hence $E(x) = V(x) + \frac{1}{2}m\dot{x}^2$.

$$\begin{aligned}
&E(x) = V(x) + \frac{1}{2}m\dot{x}^2 \\
\implies &\frac{1}{2}m\dot{x}^2 = E(x) - V(x) \\
\implies &\frac{dx}{dt} = \sqrt{\frac{2(E(x) - V(x))}{m}} \\
\implies &\sqrt{\frac{m}{2}}\frac{dx}{\sqrt{E(x) - V(x)}} = dt \\
\implies &\sqrt{\frac{m}{2}}\int_{x_1}^{x_2}\frac{dx}{\sqrt{E(x) - V(x)}} = \int_0^{\frac{T}{2}} dt \\
\implies &\frac{T}{2} = \sqrt{\frac{m}{2}}\int_{x_1}^{x_2}\frac{dx}{\sqrt{E(x) - V(x)}}
\end{aligned}$$

4. (a) Force on the particle is $q\boldsymbol{v} \times \boldsymbol{B}(\boldsymbol{r})$. Hence the Newton's Equation of Motion is

$$m\ddot{\boldsymbol{r}}(t) = q\boldsymbol{v} \times \boldsymbol{B}(\boldsymbol{r}) \tag{1}$$

(b) Notice

$$\begin{aligned}
\frac{d}{d(-t)}\boldsymbol{r}(t) &= -\frac{d}{dt}\boldsymbol{r}(t) \\
\frac{d^2}{d(-t)^2}\boldsymbol{r}(t) &= \frac{d^2}{dt^2}\boldsymbol{r}(t)
\end{aligned}$$

Hence acceleration is time reversal invariant. $\boldsymbol{v}$ negates under time reversal and $\boldsymbol{B}$ does not depend on time. Let $t' = -t$ then

$$m\ddot{\boldsymbol{r}}(t') = -q\boldsymbol{v}(t') \times \boldsymbol{B}(\boldsymbol{r}) \tag{2}$$

Therefore the motion is not time invariant.

(c) With (**??**) and (**??**) we see that if we look at the path of the particle with charge q under time reversal then it is the path of the particle with charge $-q$ but their initial and final position interchanged.