Therefore

$$2\left\lfloor \frac{n\alpha}{2\pi} \right\rfloor + 2 \geq V_n(\alpha) \geq 2\left\lfloor \frac{n\alpha}{2\pi} \right\rfloor$$

Now,

$$\lim_{n\to\infty} \frac{2\left\lfloor \frac{n\alpha}{2\pi} \right\rfloor}{n} = \frac{\alpha}{\pi}$$

and

$$\lim_{n\to\infty} \frac{2\left\lfloor \frac{n\alpha}{2\pi} \right\rfloor + 2}{n} = \frac{\alpha}{\pi}$$

Hence by Sandwich Theorem We get

$$\lim_{n\to\infty} \frac{V_n(\alpha)}{n} = \frac{\alpha}{\pi}$$