1. If $0 \leq n\alpha - \left\lfloor \frac{n\alpha}{2\pi} \right\rfloor \times 2\pi < \frac{\pi}{2}$ sign changes 0 times.
2. If $\frac{\pi}{2} \leq n\alpha - \left\lfloor \frac{n\alpha}{2\pi} \right\rfloor \times 2\pi < \frac{3\pi}{2}$ sign changes 1 times
3. If $\frac{3\pi}{2} \leq n\alpha - \left\lfloor \frac{n\alpha}{2\pi} \right\rfloor \times 2\pi < 2\pi$ sign changes 2 times

Hence

$$\max\{V_n(\alpha)\} = 2\left\lfloor \frac{n\alpha}{2\pi} \right\rfloor + 2$$

and

$$\min\{V_n(\alpha)\} = 2\left\lfloor \frac{n\alpha}{2\pi} \right\rfloor$$