Send me Your Answer!

Suppose $g(x)$ is a third degree polynomial with coefficient of x^3 as 1. Its all 3 roots are distinct (There was an error in the question post). Let $h(x) = x^2 + x + a$.(where a is some integer) If $g(h(x))$ has no real roots what is the lower bound of $g(a)$?

$$\sum_{i=0}^{Creative} Math_i = Solving$$

Solution $\rightarrow$

As $g(x)$ has 3 distinct roots let its 3 roots are p, q, r. Hence

$$g(x) = (x-p)(x-q)(x-r)$$

As the coefficient of x^3 is 1 hence as $x \to \infty$, $g(x) \to \infty$. Similarly $h(x) \to \infty$ as $x \to \infty$. As it is given that $g(h(x))$ has no real roots we can say $g(h(x)) > 0 \ \forall x \in \mathbb{R}$. Hence $h(x) - c$ has only imaginary roots where $c = \{p, q, r\}$. Hence the discriminant $1-4(a-c) < 0 \implies \min h(x) = a - \frac{1}{4} > c$ where $c = \{p, q, r\}$. Hence $g(a) = (a-p)(a-q)(a-r) > \frac{1}{64}$. Therefore $\frac{1}{64}$ is the infimum of $g(a)$.