Sunday Special #16

Send me Your Answer!

Let $f(x) = (x - a_1)\dots(x - a_n) + 1$, where $a_1, \dots, a_n$ are distinct integers. Show that (i) if n is odd, then $f(x)$ is irreducible over $\mathbb{Z}$ i.e. $f(x)$ cannot be factorized in the form $f(x) = p(x)q(x)$ where $p(x)$ and $q(x)$ are polynomials with integer coefficients and their degrees are less than the degree of $f(x)$ (Here the degree of $f(x)$ is n.) and (ii) if n is even, then either $f(x)$ is irreducible over $\mathbb{Z}$ or is the square of a polynomial with integer coefficients.

ONLY ELEMENTARY SOLUTIONS ALLOWED

$$\sum_{i=0}^{Creative} Math_i = Solving$$