Homework 3

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It helps to visualize this as a function of y with respect to x.

$$ax + by + c = 0$$
$$y = -\frac{a}{b} - \frac{c}{b}$$

First find the perpendicular line.

$$\frac{bx}{a} + [v - \frac{bu}{a}] = y$$

We then want to find where these lines intersect. That is we set these lines equal to each other and solve for x and in turn solve for y.

We get $x = b^2u - ac - abv$ and $y = -abu + \frac{a^2c}{b} + a^2v - \frac{c}{b}$.

We continue to find the distance between these two points by using the Euclidean distance. So,

$$\sqrt{[v-y]^2 + [u-x]^2}$$

$$= \sqrt{[v - (-abu + \frac{a^2c}{b} + a^2v - \frac{c}{b})]^2 + [u - (b^2u - ac - abv)]^2}$$

From here we can come to the following result using algebra and the $a^2+b^2 = 1$ property that is given:

$$\sqrt{b^4v^2 + a^4u^2 + 2abuv + 2b^3vc + 2a^3uc + 2ab^2cu + 2a^2bcv + a^2b^2u^2 + a^2b^2v^2 + c^2}$$

Using the $a^2 + b^2 = 1$ property again we are able to find

$$2b^3vc + 2a^2bcv = 2bcv$$
$$2a^3uc + 2ab^2cv = 2auc$$

$$b^4v^2 + a^2b^2v^2 = b^2v^2$$

$$a^4u^2 + a^2b^2u^2 = a^2u^2$$

and so we have

$$\sqrt{b^4v^2 + a^4u^2 + 2abuv + 2b^3vc + 2a^3uc + 2ab^2cu + 2a^2bcv + a^2b^2u^2 + a^2b^2v^2 + c^2}$$

$$= \sqrt{(bv)^2 + (au)^2 + 2abuv + 2bvc + 2auc + c^2}$$

$$= |au + bv + c|$$

**can also be done using line integral

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2.1

Circles are represented as $(x - a)^2 + (y - b)^2 = r^2$. This means that our Hough space will be 3 dimensions (a, b, r) assuming we don't have fixed radii. The result of the 3 dimensional Hough space is a cone.

2.2

The Hough circles I implemented was naive Hough transform starts with the Canny edge detector and using the edges for the voting process of different centers and radii. This of course took a while to run so just to check if it worked I only evaluated the top half of the image. There is no condition to concentric circles or minimum radius, the only restriction is the largest radii can be 60. After the voting process we just check the 20 most voted (a, b, r).

2.3

The code does seem to work to a certain degree. The results of my implementation and trying to find decent parameters for cv2.HoughCircles respectively can be found on the next page.

It appears that both have trouble distinguishing which edges belong to which circles. My implementation goes to the first 20 most voted circles and as we can clearly see, it has difficulty finding the appropriate circles. I believe that we could resolve this problem by implementing a concentric circles circumstance but it still be difficult to determine the most fit circle. We can also see that it uses the top edge of the shoe as a circle. So we need to define some threshold to determine true circles. As for cv2.Houghcircles it is really dependent on the given parameters. Parameters that I found to work somewhat well with this image were param1 = 150, param2 = 30, min radius = 0, max radius = 80. cv2 definitely can have difficulty determining true circles like my implementation without good parameters. Since cv2 cannot have the minimum distance between

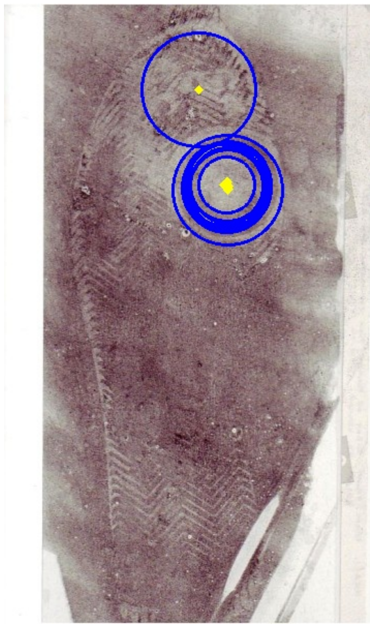

Figure 1: My implementation

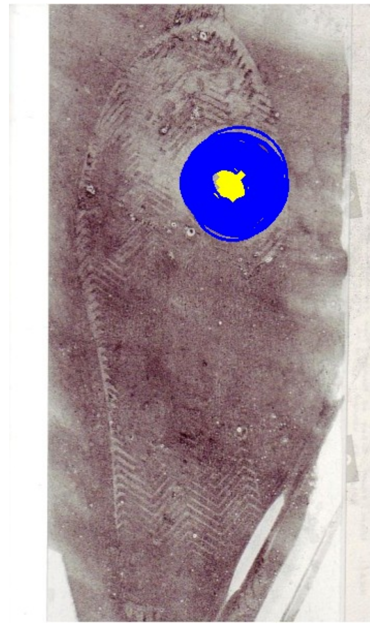

Figure 2: Hough Circles 1, distance very small

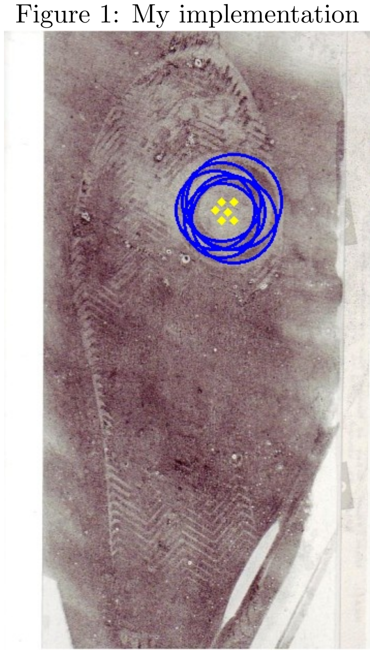

Figure 3: Hough Circles 2, distance = 10

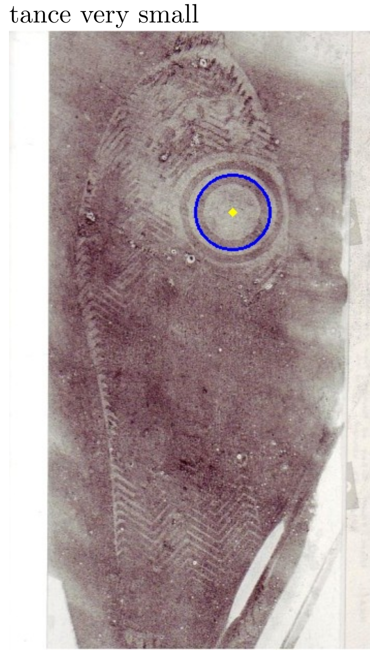

Figure 4: Hough Circles 3 distance = 20

circles be 0, we can approximate by setting it to a very small number. As we can see from our figures, cv2 also has difficulty finding true centers and radii. As we increase the parameter that defines distance between circles, we can see it has doesn't look as clustered and it'll more accurately find circles.

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First we will simplify E with

$$E = ||Y - WX||^2 + \lambda||W||^2$$

$$E = Tr[(Y - WX)^T(Y - WX)] + \lambda Tr[W^T W]$$

Before we derive with respect to W we will need the following property:

1.
$$\frac{dTr(BXC)}{dX} = B^T C^T [1]$$

2.
$$\frac{dTr(AXb + c)(AXb + c)^T}{dX} = 2A^T(AXb + c)b^T [1]$$

3.
$$\frac{dTr(x^2)}{dX} = 2X[1]$$

Continuing we get,

$$\frac{dE}{dW} = -2YX^T + 2WXX^T + 2\lambda W = 0 = -YX^T + XX^T W + \lambda W$$

With algebra,

$$-YX^T + XX^T W + \lambda W = -YX^T + [XX^T + \lambda I]W$$

$$YX^T = [XX^T + \lambda I]W$$

$$[XX^T + \lambda I]^{-1} YX^T = W = YX^T [XX^T + \lambda I]^{-1}$$

4 References

[1] Petersen, Kaare Brandt, and Michael Syskind Pedersen. "PDF." 5 Jan. 2005.