Hockey Hockey Hockey
and more Hockey

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Introduction

As with any professional sporting league, the number of teams and their spatial distribution relative to one another is very important, and for the National Hockey League (NHL), this is no exception. *In fact, it is possibly even more important given the seasonal and temperature dependent nature of the game itself.* When new teams are added to the league, they may have negative impacts on nearby teams, as well as struggle to be profitable if placed in an unfitting locale. Of all leagues, it seems that the NHL feels these impacts the most, as hockey requires just as much specialized equipment, if not more than other sports, but the NHL also lacks many of the national TV contracts that other professional sports use as a source of revenue[1]. As such, expansion of the NHL is a complex problem that encompasses a large number of varibles. ************* MIL vs ORD/MIL **

Methods

SIR Model

Given that the sucess of a team in the NHL is mostly dependent on fan base size and growth, we consider fan loyalty to spread as in a traditional SIR disease model as in Light *et al.*[1] The model for two geographically close teams follows, the isolated geographic case follows quite easily.

$$\begin{aligned}
\dot{S} &= -S[\beta_1(I_1 + C_1) + \beta_2(I_2 + C_2)] \\
\dot{I}_1 &= \beta_1 S(I_1 + C_1) - \gamma I_1 - \alpha_1 I_1 + \delta_{21}[(I_1 + C_1)I_2] - \delta_{12}[I_1(I_2 + C_2)] \\
\dot{I}_2 &= \beta_2 S(I_2 + C_2) - \gamma I_2 - \alpha_2 I_2 + \delta_{12}[(I_2 + C_2)I_1] - \delta_{21}[I_2(I_1 + C_1)] \\
\dot{C}_1 &= \alpha_1 I_1 \\
\dot{C}_2 &= \alpha_2 I_2 \\
\dot{R} &= \gamma(I_1 + I_2) \\
N &= S + I_1 + I_2 + C_1 + C_2 + R
\end{aligned} \tag{1}$$

As in [1], we consider a model of disease spread that includes a "Chronically infected" fan category. As the SIR model is well studied, we know that we may compute the basic reprodictive ratio $\mathcal{R}_0$ as,

$$\mathcal{R}_0 = \frac{\beta N}{\gamma}$$

with β the mean rate of transmission of fan allegiance for a given team, N is the population drawn from, and $1/\gamma$ is the mean transmission period, or 9 months (1 Season). Knowing that $\mathcal{R}_0 > 1$ means that the infected population is growing, and that $\mathcal{R}_0 < 1$ is a dying infected population, we consider our computation of β subject to this constraint.

Beta

β is a product of many factors, the main being team success, physical locale (weather, population, income), and the social environment[1].

Team Success

The success of a team can be quantified as a function of its successes in previous post seasons following the following equation,

$$S_t = \frac{1.875PSA + 3.75PSW + 30SC}{N} \tag{2}$$

Where, PSA is the number of series appearances by a team, PSW is the number of wins, and SC is the number of stanley cups won over N seasons. Taking the analysis of 2001-2015 from Light *et al.*[1], and extending it to include the seasons that have passed in the interim, we arrive at the updated Tables 1, 2 below.

Locale

The location of a team is very important for hockey. A number of teams in the NHL consistently struggle in terms of revenue, seemingly due to what Light *et al.* and Jones call low locational quality[1][4]. In Jones and Ferguson [4], the variable,

$$H_t = \frac{1}{4}\frac{a_t^2}{b_t^2} \tag{3}$$

is defined to be the locational quality, dependent on the equations,

$$\log(a_t) = \alpha_0 + \alpha_1(CAN) + \alpha_2 \log(POP) + \alpha_3 \log(INC) \tag{4}$$

$$\log(b_t) = \beta_0 + \beta_1(CAN) + \beta_2 \log(POP) + \beta_3 \log(INC) \tag{5}$$

Where CAN is a binary variable, POP is the population of the metro area in millions, and INC is GDP per capita of the area. For our analysis, we shall take the convention of Light *et al.*, and replace the CAN variable with a variable called $WINT$, which will be a binary value describing if a location has a colder mean monthly temperature than 6 °C. Using the data from Table 3 of [1], we perform a least squares regression on the locational data in an attempt to compute the coefficents $\alpha_i,\ \beta_i$ from 4 and 5. The coefficients we obtain are,

$$\log(a_t) = 14.1 + 0.2205(WINT) + .091 \log(POP) - 0.3766 \log(INC) \tag{6}$$

$$\log(b_t) = 9.171 + .03(WINT) + .028 \log(POP) - .1879 \log(INC) \tag{7}$$

Each of the coefficents of these regressions are significant at the $\alpha = 10^{-6}$ level, so we know to keep them for our analysis.

Social Environment

Team	Revenue[a]	Playoff Appearances[b]	Playoff Wins[c]	Stanley Cup Wins[d]
Anaheim Ducks	134	26	15	1
Arizona Coyotes	96	6	2	0
Boston Bruins	191	26	14	1
Buffalo Sabres	128	10	5	0
Calgary Flames	132	12	4	0
Carolina Hurricanes	109	15	11	1
Chicago Blackhawks	201	24	16	3
Colorado Avalanche	119	19	10	1
Columbus Blue Jackets	111	6	1	0
Dallas Stars	144	15	6	0
Detroit Red Wings	171	30	17	2
Edmonton Oilers	145	8	4	0
Florida Panthers	99	3	0	0
L.A. Kings	193	18	11	2
Minnesota Wild	142	13	4	0
Montreal Canadians	239	19	8	0
Nashville Predators	132	19	7	0
New Jersey Devils	166	22	12	1
New York Islanders	107	10	2	0
New York Rangers	253	22	11	0
Ottawa Senators	124	21	10	0
Philadelphia Flyers	186	23	10	0
Pittsburgh Penguins	185	33	22	3
San Jose Sharks	148	23	10	0
St. Louis Blues	148	32	18	1
Tampa Bay Lightning	146	23	14	1
Toronto Maple Leafs	232	12	4	0
Vancouver Canucks	168	18	7	0
Washington Capitals	194	22	10	1
Winnipeg Jets	135	5	2	0
Las Vegas Knights	180	5	3	0

Revenue and Success Rates of NHL Teams 2001-2019

Table 1: [a]in Millions of USD as of 2018 [2]; [b]Playoff series appearances 2001-2019 [3]; [c]Playoff series wins 2001-2019 [3]; [d]Stanley Cup wins 2001-2019 [3]

Computed and Normalized Success Scores by Team		
Team	S_t^a	s_t^b
Anaheim Ducks	7.50	1.63
Arizona Coyotes	1.04	0.23
Boston Bruins	7.29	1.59
Buffalo Sabres	2.08	0.45
Calgary Flames	2.08	0.45
Carolina Hurricanes	5.52	1.20
Chicago Blackhawks	10.83	2.36
Colorado Avalanche	5.73	1.25
Columbus Blue Jackets	0.83	0.18
Dallas Stars	2.81	0.61
Detroit Red Wings	10.00	2.18
Edmonton Oilers	1.67	0.36
Florida Panthers	0.31	0.07
L.A. Kings	7.50	1.63
Minnesota Wild	2.19	0.48
Nashville Predators	3.44	0.75
New Jersey Devils	6.46	1.40
New York Islanders	1.46	0.32
New York Rangers	4.58	1.00
Ottawa Senators	4.27	0.93
Philadelphia Flyers	4.48	0.97
Pittsburgh Penguins	13.02	2.83
San Jose Sharks	4.48	0.97
St. Louis Blues	8.75	1.90
Tampa Bay Lightning	6.98	1.52
Toronto Maple Leafs	2.08	0.45
Vancouver Canucks	3.33	0.73
Washington Capitals	6.04	1.31
Winnipeg Jets	0.94	0.20
Las Vegas Knights	1.15	0.25

Table 2: ^{a}Table 1 and Eq. 2; ^{b}Normalized by mean of S_t

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