Problem 1

1. A shipment from Vendor 1 consists of 10000 parts, wherein we assume that 1.5% are defective. Thus we conclude that on an average shipment from Vendor 1 the shipment will have 9850 working parts and 150 defective parts. We then consider the conditions for a sample to pass inspection. As stated in the prompt, the only success conditions are when

$$\#\ of\ defectives < 2$$

Thus, we consider how to compute the probability of this. To compute whether a certain number of defective parts is in a random sample of 100, we construct a hypergeometric distribution. By the independence of the distribution, we may write,

$$P(X<2)=\sum_{i=0}^{1} P(X=i)$$

The PMF of the hypergeometric distribution is,

$$P(X=x)=\frac{\binom{R}{x}\binom{N-R}{n-x}}{\binom{N}{n}}$$

For our problem,

$$P(X=x)=\frac{\binom{150}{x}\binom{9850}{100-x}}{\binom{10000}{100}}$$

Thus,

$$P(pass)=\sum_{i=0}^{1} P(X=i)=\frac{\binom{150}{0}\binom{9850}{100}}{\binom{10000}{100}}+\frac{\binom{150}{1}\binom{9850}{99}}{\binom{10000}{100}}$$

Using R, we find this to be,

$$P(pass)=0.218941+0.336798=0.555739$$

Thus, we see that the probability of accepting a shipment from Vendor 1 is approximately 55.6%.

2. On the other hand, a shipment from Vendor 2 consists of 5000 parts, wherein we assume that 0.5% are defective. Thus, an average shipment from Vendor 2 will have 4975 working parts, and 25 defective parts. We then consider the conditions for a shipment to fail inspection. As stated a fail condition occurs when,

$$\#\ of\ defectives \geq 2$$

Under this sampling plan, we will again use the hypergeometric distribution and its independence to write,

$$1=\sum_{i} P(X=i),\ i\in 0,1,2,\ldots,100$$

$$\Leftrightarrow$$

$$1-(P(X=0)+P(X=1))=\sum_{i=2}^{100} P(X=i)$$

I.e., $1-P(pass)=P(fail)$. Using the PMF for the hypergeometric distribution, we write,

$$P(reject)=1-\frac{\binom{25}{0}\binom{4975}{100}}{\binom{5000}{100}}-\frac{\binom{25}{1}\binom{4975}{99}}{\binom{5000}{100}}=1-0.602724-0.309026=0.08825$$

Thus, we see that the probability of a shipment being rejected from Vendor 2 is approxiamtely 8.83%

Problem 2

Given that the produced samples follow a normal distribution, we may easily compute the probabilities associated with the part being defective. This computation is an integral of the CDF for the normal distribution,

$$P(X\leq x)=\int_{-\infty}^{x}\frac{1}{\sigma\sqrt{2\pi}}e^{-(t-\mu)^2/(2\sigma^2)}dt$$

For this problem, the engineer has computed that,

$$\mu = 10.48,\ \sigma = 0.0142$$

Thus, we have two computations that we must compute. The first being the probability that the dimension is less than 10.45. This computation is computed using R and its built in CDF integrator to obtain,

$$P(dim \leq 10.45) = 0.017314$$

The second computation is slightly more complex, as we want the area to the right of the curve, thus,

$$P(dim > 10.55) = 1 - p(dim \leq 10.55)$$

Using R,

$$P(dim > 10.55) = 1 - 0.9999996 = 4.1204 \times 10^{-7}$$

The probability of failure is then,

$$P(dim \leq 10.45) + P(dim > 10.55) = 0.017314 + 4.1204 \times 10^{-7} = 0.01731466$$

Problem 3

The distribution of spots on these circuit boards follows a Poisson distribution. To begin the computation of probabilities, we must compute,

$$\hat{\lambda} = \frac{97}{1000}$$

From this, we may say that the number of flaws, $X \sim Poisson(\hat{\lambda})$. Thus, we may use the PMF of the poisson distribution to compute,

$$P(X = x) = \frac{e^{-\hat{\lambda}}\hat{\lambda}^x}{x!}$$

1. So,

   $$P(X = 0) = \frac{e^{-\hat{\lambda}}\hat{\lambda}^0}{0!} = e^{-\hat{\lambda}} = 0.907556$$

   and

   $$P(X = 1) = \frac{e^{-\hat{\lambda}}\hat{\lambda}^1}{1!} = \hat{\lambda}e^{-\hat{\lambda}} = 0.088033$$

   Then,

   $$P(pass) = P(X = 0) + P(X = 1) = 0.99559$$

2. If the customer orders 500 Boards, we would expect that,

   $$500 * P(pass) = 497.79 \approx 498$$

   Boards will pass inspection.