1.

Calculations are performed in the attached Excel spreadsheet, but a summary of results follows,

Case	A	B	C
Number of Sample Groups	20	20	20
Number of Samples per group (n)	5	5	5
Center of X bar chart	10	9.74	9.74
Cetner of s chart	0.6	0.6	0.6
Target	10	10	10
USL	11	11	11
LSL	9	9	9
C_p	0.5222	0.5222	0.78333
C_{pk}	0.5222	0.386444	0.579667
$P(Defect)$	0.11719	0.147351	0.042551
DPMO	117192.6	147351.0	42550.6

2.

Calculations are performed in the attached Excel spreadsheet, but a summary of results follows,

Case	A	B	C	D
C_p	1	1.5	2	1
C_{pk}	1	1.5	2	1
Process Center	0	0	0	a
Sigma	1	1	1	4
Target	0	0	0	a
LSL	-3	-4.5	-6	a-12
USL	3	4.5	6	a+12
P(Defect)	0.0026998	6.79535E-06	1.97318E-09	0.0026998
DPMO	2699.8	6.8	$0.002 \to 0$	2699.8

3A.

We are given that $C_p = 1.6$ and $C_{pk} = 1.6$. We shall assume normality and statistical control for to solve. To begin, we note that $C_p = C_{pk}$ as such, we note that the process is centered. We shall also assume a standard normal model, $\sim N(0,1)$. Because the process is centered, we see that the target is equal to the process center. We then consider the following,

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} \Rightarrow USL - LSL = C_p 6\hat{\sigma} \Rightarrow USL - LSL = 1.6(6) \Rightarrow USL - LSL = 9.6$$

Because the process is centered, we may then compute,

$$\begin{aligned} LSL &= 0 - \frac{9.6}{2} \\ &= -4.8 \\ USL &= 0 + \frac{9.6}{2} \\ &= 4.8 \end{aligned}$$

Finally, we compute,

$$P(Defect) = \Phi(Z < -4.8) + (1 - \Phi(Z > 4.8)) = 1.586656E-06$$

Then,

$$DPMO = 1000000P(Defect) = 1.58870534 \to 1.6$$

3B.

e are given that $C_p = 1.5$ and $C_{pk} = 1$. We shall assume normality and statistical control for to solve. To begin, we note that $C_p \neq C_{pk}$ as such, we note that the process is not centered. We shall also assume a standard normal model, $\sim N(0,1)$. We now consider the following,

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} \Rightarrow USL - LSL = C_p 6\hat{\sigma} \Rightarrow USL - LSL = 1.5(6) \Rightarrow USL - LSL = 9$$

We now also consider,

$$C_{pk} = \frac{\min(USL - \bar{\bar{x}}, \bar{\bar{x}} - LSL)}{3\hat{\sigma}} \Rightarrow \min(USL - \bar{\bar{x}}, \bar{\bar{x}} - LSL) = 3$$

without loss of generality, due to the symmetry of the normal distribution, we shall assume that,

$$USL - \bar{\bar{x}} = 3 \Rightarrow USL = 3$$

Then,

$$3 - LSL = 9 \Rightarrow LSL = -6$$

Finally, we compute,

$$P(Defect) = \Phi(Z < -6) + (1 - \Phi(Z > 3)) = 1.349899E - 03$$

and,

$$DPMO = 1000000P(Defect) = 1349.9$$