Question 1.

a. Given that the process is modeled by a Poisson distribution with $\lambda = 0.02$,

$$P(X = 1) = \frac{(0.02)^1 e^{-0.02}}{1!} \approx 0.019604$$

b. We now compute the probability of one or more defects as,

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{(0.02)^0 e^{-0.02}}{0!} = 1 - e^{-0.02} \approx 0.019801$$

c. Given that our new mean is $\lambda = 0.01$, we again compute

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{(0.01)^0 e^{-0.01}}{0!} = 1 - e^{-0.012} \approx 0.00995017$$

This new probability is effectively half of our original probability.

Question 2.

Consider the random variable with pdf:

$$p(X = x) = \begin{cases} \frac{1+3k}{3} & x = 1 \\ \frac{1+2k}{3} & x = 2 \\ \frac{.5+5k}{3} & x = 3 \end{cases}$$

We now note the following,

$$\begin{aligned} \sum_x P(X = x) = 1 & & \Rightarrow \\ 1 &= \frac{1+3k}{3} + \frac{1+2k}{3} + \frac{.5+5k}{3} & \Rightarrow \\ 3 &= 10k + \frac{5}{2} & \Rightarrow \\ k &= \frac{1}{20} \end{aligned}$$

Question 3.

We note that this process can be modeled by a binomial distribution with $p = 0.01$ and $n = 25$. This process is stopped when $x \geq 1$. Thus,

$$P(x \geq 1) = 1 - P(X = 0) = 1 - \binom{25}{0}(0.01)^0(1 - 0.01)^{25-0} = 1 - 0.77782 \approx 0.2222$$

Hence, we see that this decision model will call for a stoppage around 22.2% of the time. For a company to have a fault or stoppage over one fifth of the time would be quite expensive and time consuming.

Question 4.

The new distribution is now a binomial with $p = 0.04$. Given that the process is stopped for one or more defects, we compute the probability of a stop as,

$$P(x \geq 1) = 1 - P(X = 0) = 1 - \binom{25}{0}(0.04)^0(1 - 0.04)^{25-0} \approx 0.639603$$

The average number of runs then is,

$$N = \frac{1}{0.639603} = 1.56347$$

Rounding, we see that this defect should be found in around 2 runs of the process with the new defect rate.

Question 5.

Given that the strength is normally distributed, we may compute the probability that a part does not meet the minimum limit of 35 pounds of tensile strength with,

$$P(x < 35) = \int_{-\infty}^{35} \frac{e^{-(x-40)^2/50}}{5\sqrt{2\pi}} \approx 0.158655$$

Given that we have sampled 50000 parts, we expect,

$$E(x) = 50000 * 0.158655 = 7932.76 \to 7933$$

Parts to be defective.
Considering parts that have strength over 48 pounds, we compute,

$$P(x > 48) = 1 - \int_{-\infty}^{48} \frac{e^{-(x-40)^2/50}}{5\sqrt{2\pi}} = 1 - 0.945201 \approx 0.0547993$$

Which results in an expected number of parts,

$$E(x) = 50000 * 0.0547993 = 2739.96 \to 2740$$

Question 6.

Given the normal distribution of the voltage, we may compute the probability of the voltage being within the stipulations by computing,

$$P(4.95 \leq x \leq 5.05) = \int_{4.95}^{5.05} \frac{e^{-(x-5)^2/(2*.02^2)}}{.02\sqrt{2\pi}} \approx 0.987581$$

To find the variance required to reduce the failure rate, we consider that the probability of a failure from being lower than the minimum value is, $\frac{1}{2}\frac{1}{1000}$ Due to symmetry. So, we know that our z-score equation should be,

$$\begin{aligned} P(Z < \frac{4.95-5}{\sigma}) &= 0.0005 \\ \frac{4.95-5}{\sigma}) &= -3.29 \\ \sigma &= \frac{-.05}{-3.29} \\ &= 0.0152 \end{aligned}$$

Hence, our variance would be,

$$\sigma^2 = 2.3104 \times 10^{-4}$$

Question 7.

To begin, we consider process 1. A normally distributed process with mean 7500 and deviation of 1000h as specified. We then compute the probability of a non usable part as,

$$P(reject) = 1 - \int_{5000}^{10000} \frac{e^{-(x-7500)^2/(2*1000^2)}}{1000\sqrt{2\pi}} \approx 0.01242$$

So, we consider the cost of using process 1.

$$C_1 = Construction\ Cost + Repair\ Cost = P * n + 5 * .01242 * n = (P + .0621)n$$

Similarly, for process 2, we compute the same values,

$$P(reject) = 1 - \int_{5000}^{10000} \frac{e^{-(x-7500)^2/(2*500^2)}}{500\sqrt{2\pi}} \approx 0$$

The Cost is then,

$$C_2 = 2P * n + 5 * 0 * n = 2Pn$$

So, we may now compare the production costs to find the transition point in terms of production cost,

$$\begin{aligned} 2Pn &= (P + 0.0621)n \\ 2P &= P + 0.0621 \\ P &= 0.0621 \end{aligned}$$

So, the manufacturer decision is then,

$$\begin{cases} 0 < P < 0.0621 & \text{Manufacturer 2} \\ P > 0.0621 & \text{Manufacturer 1} \end{cases}$$