1. We consider the heart attack data. We shall assume normality and control in the data for the purposes of the assignment. First, we compute $\hat{C}_p$ as,

$$\begin{aligned}
\hat{\sigma} &= \frac{\bar{s}}{c_4} \\
&= \frac{1}{0.9693} \\
&= 1.03167 \\
\hat{C}_p &= \frac{USL - LSL}{6\hat{\sigma}} \\
&= \frac{36-30}{1.03167} \\
&= 0.9693 \\
&\to 0.97
\end{aligned}$$

Next, we consider $\hat{C}_{pk}$. To compute, we consider the following calculations,

$$\begin{aligned}
\zeta(USL) &= |36-32| \\
&= 4 \\
\zeta(LSL) &= |30-32| \\
&= 2
\end{aligned}$$

Because we see that the second of these calculations is the smallest, we shall use it in the calculation of our $\hat{C}_{pk}$ value as,

$$\begin{aligned}
\hat{C}_{pk} &= \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}} \\
&= \frac{32-30}{3(1.03167)} \\
&= \frac{2}{3.09501} \\
&= 0.6462 \\
&\to 0.65
\end{aligned}$$

Finally, we compute the DPMO as,

$$\begin{aligned}
P(X < LSL) &= \Phi(\frac{30-32}{\hat{\sigma}}) \\
&= 0.026275032 \\
P(X > USL) &= 1 - \Phi(\frac{36-32}{\hat{\sigma}} \\
&= 0.0000528327 \\
P(Defect) &= 0.026275032 + 0.0000528327 \\
&= 0.026327865 \\
DPMO &= P(Defect) \times 10^6 \\
&= 26327.86453 \\
&\to 26327.9
\end{aligned}$$

2. Next, we consider the Chocolate data. Again, we assume that the process is normally distributed and in control. Then, as

before, we compute,

$$
\begin{aligned}
\hat{\sigma} &= \frac{\bar{R}}{d_2} \\
&= \frac{2}{2.059} \\
&= 0.971345313 \\
\hat{C}_p &= \frac{USL - LSL}{6\hat{\sigma}} \\
&= \frac{29-23}{0.971345313} \\
&= 1.0295 \\
&\to 1.03
\end{aligned}
$$

Next, we consider $\hat{C}_{pk}$. To compute, we consider the following calculations,

$$
\begin{aligned}
\zeta(USL) &= |29-25| \\
&= 4 \\
\zeta(LSL) &= |23-25| \\
&= 2
\end{aligned}
$$

Because we see that the second of these calculations is the smallest, we shall use it in the calculation of our $\hat{C}_{pk}$ value as,

$$
\begin{aligned}
\hat{C}_{pk} &= \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}} \\
&= \frac{25-23}{3(0.971345313)} \\
&= \frac{2}{2.914035939} \\
&= 0.68633 \\
&\to 0.69
\end{aligned}
$$

Finally, we compute the DPMO as,

$$
\begin{aligned}
P(X < LSL) &= \Phi(\frac{23-25}{\hat{\sigma}}) \\
&= 0.019747119 \\
P(X > USL) &= 1 - \Phi(\frac{29-25}{\hat{\sigma}} \\
&= 0.0000191087 \\
P(Defect) &= 0.019747119 + 0.0000191087 \\
&= 0.019766228 \\
DPMO &= P(Defect) \times 10^6 \\
&= 19766.22798 \\
&\to 19766.2
\end{aligned}
$$

3. Next, we consider the strut data. Again, we assume that the process is normally distributed and in control. Then, as

before, we compute,

$$\begin{aligned}
\hat{\sigma} &= \frac{\bar{R}}{d_2} \\
&= \frac{1.66}{2.059} \\
&= 0.80621661 \\
\hat{C}_p &= \frac{USL - LSL}{6\hat{\sigma}} \\
&= \frac{90 - 70}{6 * 0.80621661} \\
&= 4.134538151 \\
&\to 4.14
\end{aligned}$$

Next, we consider $\hat{C}_{pk}$. To compute, we consider the following calculations,

$$\begin{aligned}
\zeta(USL) &= |90 - 73.58| \\
&= 16.42 \\
\zeta(LSL) &= |70 - 73.58| \\
&= 3.58
\end{aligned}$$

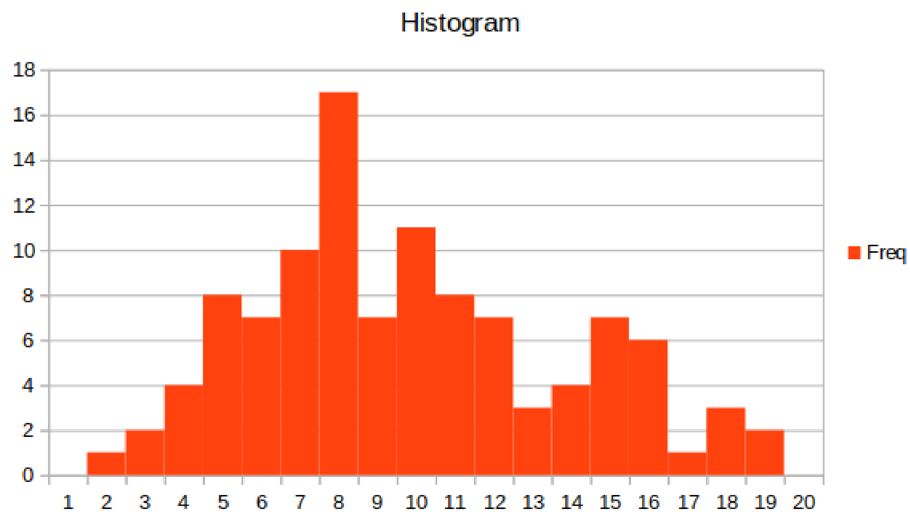
Because we see that the second of these calculations is the smallest, we shall use it in the calculation of our $\hat{C}_{pk}$ value as,

$$\begin{aligned}
\hat{C}_{pk} &= \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}} \\
&= \frac{73.58 - 70}{3(0.80621661)} \\
&= \frac{3.58}{2.41864983} \\
&= 1.480164659 \\
&\to 1.48
\end{aligned}$$

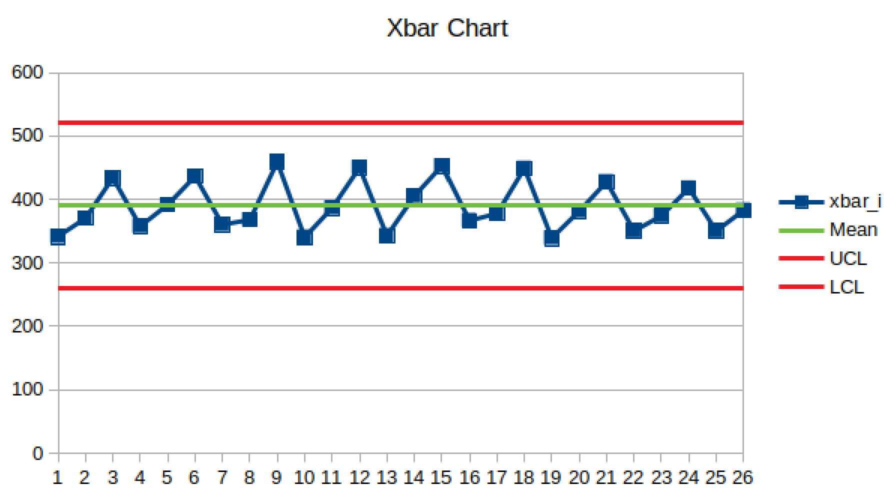
Finally, we compute the DPMO as,

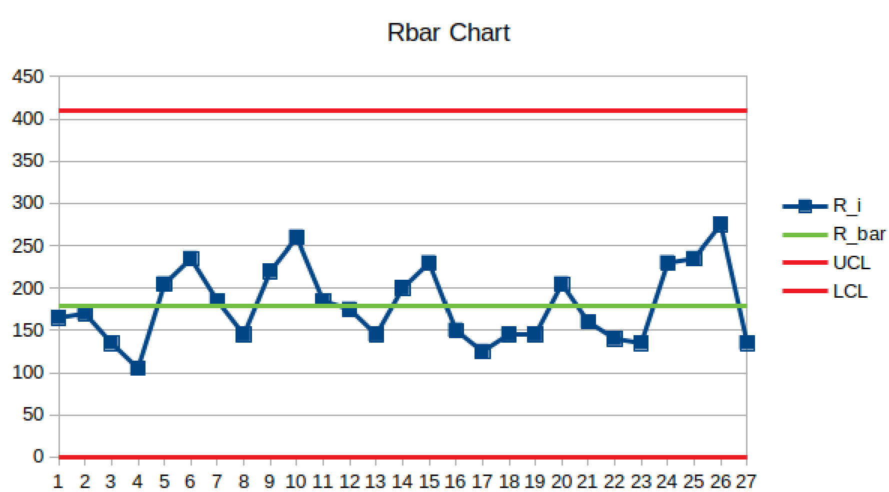
$$\begin{aligned}
P(X < LSL) &= \Phi(\frac{70 - 73.58}{\hat{\sigma}}) \\
&= 0.00000448763 \\
P(X > USL) &= 1 - \Phi(\frac{29 - 25}{\hat{\sigma}} \\
&\approx 0 \\
P(Defect) &= 0.00000448763 + 0 \\
&= 0.00000448763 \\
DPMO &= P(Defect) \times 10^6 \\
&= 4.48763 \\
&\to 4.5
\end{aligned}$$

4. Finally, we consider the air data provided. First, we compute a histogram of the data, and see that it is roughly normal,



We also compute the $\bar{X}$ and $\bar{R}$ charts,





We see that the process is well within control. Because of this, we continue our calculations. Now, we compute as before,

$$\begin{aligned}
\hat{\sigma} &= \frac{\bar{R}}{d_2} \\
&= \frac{179.2592593}{2.059} \\
&= 87.06132067 \\
\hat{C}_p &= \frac{USL - LSL}{6\hat{\sigma}} \\
&= \frac{500 - 300}{6 * 87.06132067} \\
&= 0.3831417625 \\
&\to 0.38
\end{aligned}$$

Next, we consider $\hat{C}_{pk}$. To compute, we consider the following calculations,

$$\begin{aligned}
\zeta(USL) &= |500 - 390.0925926| \\
&= 109.9074074 \\
\zeta(LSL) &= |390.0925926 - 300| \\
&= 90.09259259
\end{aligned}$$

Because we see that the second of these calculations is the smallest, we shall use it in the calculation of our $\hat{C}_{pk}$ value as,

$$\begin{aligned}
\hat{C}_{pk} &= \frac{\bar{\bar{x}} - LSL}{3\hat{\sigma}} \\
&= \frac{390.0925926 - 300}{3(87.06132067)} \\
&= \frac{90.0925926}{261.183962} \\
&= 0.3449392218 \\
&\to 0.35
\end{aligned}$$

Finally, we compute the DPMO as,

$$\begin{aligned}
P(X < LSL) &= \Phi(\frac{300 - 390.0925926}{\hat{\sigma}}) \\
&= 0.150377036 \\
P(X > USL) &= 1 - \Phi(\frac{500 - 390.0925926}{\hat{\sigma}} \\
&= 0.103399974 \\
P(Defect) &= 0.150377036 + 0.103399974 \\
&= 0.25377701 \\
DPMO &= P(Defect) \times 10^6 \\
&= 253777.0096 \\
&\to 253777.0
\end{aligned}$$