

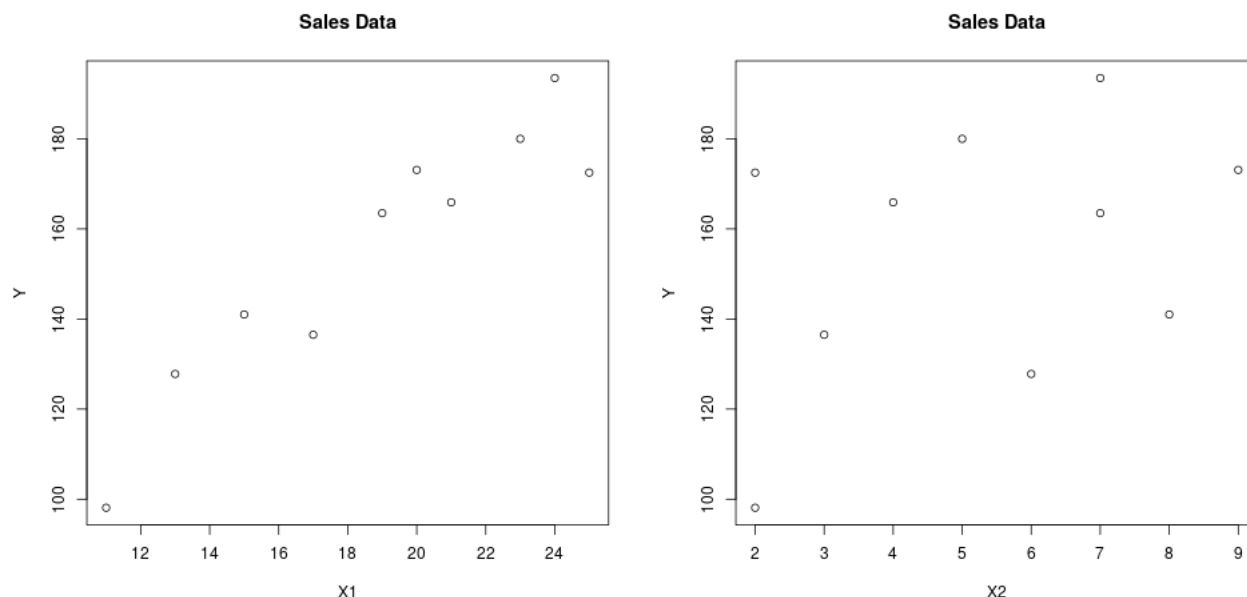
We consider the following model of the data,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$$

For the sales data, we have,

$$Y = 29.3468 + 5.6128X_1 + 3.8344X_2$$

1. We begin by plotting each of the independent variables against the Sales data in order to determine the nature of their relationship.



We see from these plots that  $X_1$  is clearly linearly related to  $Y$ , while the relationship between  $X_2$  and  $Y$  is more complex than a simple linear relationship. We consider the model,  $\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_2^2 + \epsilon$ . Computing, we find our model to be,

$$Y = 19.0737 + 5.5596X_1 + 9.2229X_2 - .5129X_2^2$$

2. We next consider a model that accounts for potential interactions between  $X_1$  and  $X_2$ ,  $\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_2^2 + \beta_4 X_1 X_2 + \epsilon$ . We compute this model to be,

$$Y = 27.43798 + 5.08130X_1 + 7.28995X_2 - .53110X_2^2 + .11473X_1X_2$$

3. In order to test the significance of this interaction model, we shall test,

$$H_0 : \beta_4 = 0$$

$$H_A : \beta_4 \neq 0$$

The  $p$ -value for our computed value of  $\beta_4$  is .014032. Thus, at  $\alpha = .05$ , we may accept that there is a non-zero level of interaction between the variables. At the  $\alpha = .01$  level however, we see that the interaction is not significant enough.

4. We now compare the  $R^2$  value for each of the computed models,

| Model | $R^2$  | Adjusted $R^2$ |
|-------|--------|----------------|
| 1     | 0.9901 | 0.9873         |
| 2     | 0.9975 | 0.9962         |
| 3     | 0.9993 | 0.9988         |

We see that as we have added a term to each of these models, the overall “correctness” of the successive models increases. Thus, we may conclude that Model 3 is the best overall at predicting the data.

5. Finally, we consider predicting the sales price(in thousands) using the inputs,  $X_1 = 16$  and  $X_2 = 8$ . The results for each model are below,

| Model | Point Prediction | 95% Interval     |
|-------|------------------|------------------|
| 1     | 149.83           | (141.17, 159.48) |
| 2     | 148.98           | (144.07, 153.90) |
| 3     | 147.75           | (144.70, 150.81) |

We note that each successive prediction interval is smaller than the last, implying that these models are successively more accurate than the last.