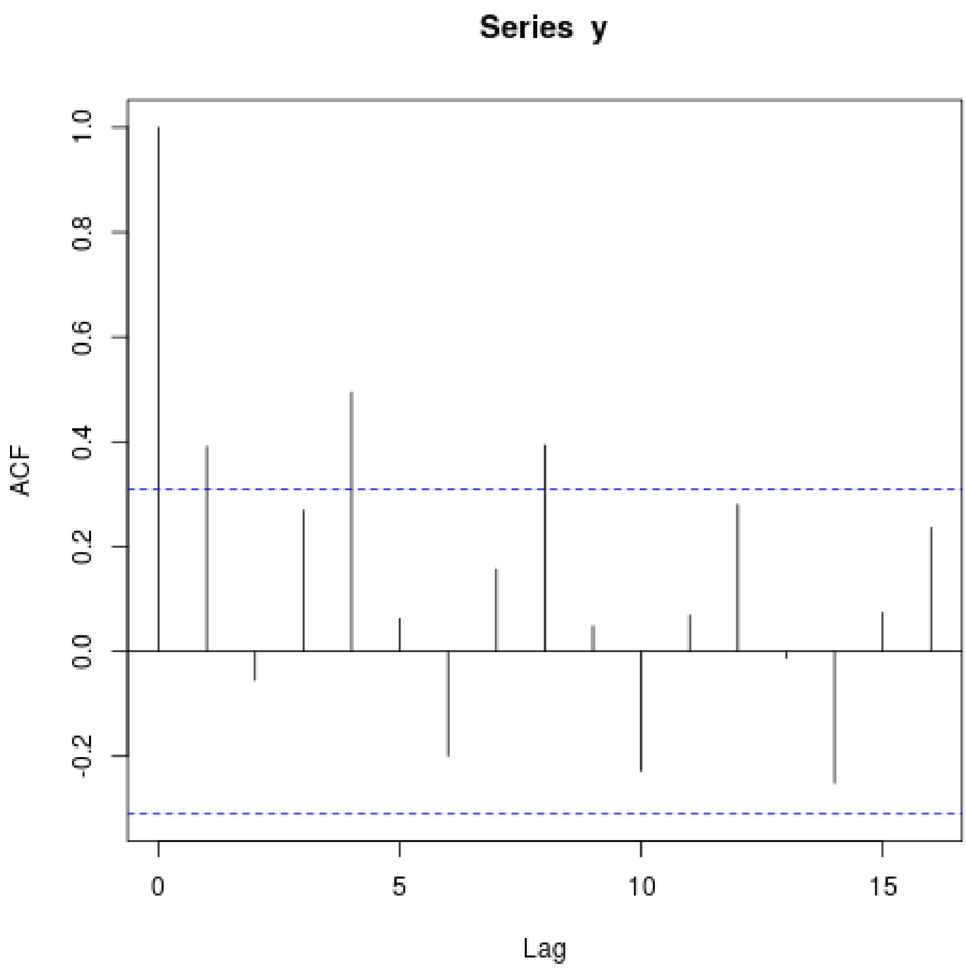
1. We consider the first-order autocorrelation of the Energy bills data. First, we consider the first order model of $y^* = \bar{y}$. Here, we now consider the equation for the test statistic,

$$d = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

Here, e_t is the the t^{th} residual of the model. When we compute this as above, we get the R output of

$$d = 0.3908$$

Using the built in ACF function in R, we obtain the following plot,



Next, we compute a linear model of energy cost as a function of time. This model will then be used to compute the residuals for the purposes of a first order test. We compute d as above to be equal to, $d = 1.369812$. So, we test the hypotheses,

$$H_0 : \text{The error is not autocorrelated}$$

$$H_A : \text{The error terms are positively autocorrelated}$$

With our computed d statistic, we arrive at the p value of $p = 0.01266$ which falls below our threshold of $\alpha = 0.05$. Thus, we reject our null hypothesis in favor of the alternative. So, we conclude that the data has positive first order autocorrelation.

2. Because our error terms are autocorrelated, we consider the following model,

$$y_t^* = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 Q_2 + \beta_4 Q_3 + \beta_5 Q_4 + \epsilon_t$$

Using $ARIMA$ in R, we obtain point estimates of b_i and ϕ_1.The results follow:

Estimator	Estimate
b_0	335.5962844
b_1	-7.8140844
b_2	0.3307836
b_3	-104.7870853
b_4	-195.5459704
b_5	-68.8726069
ϕ	0.5890558

3. We now compute the point prediction and prediction intervals of y_{41} and y_{42}. First, we compute,

$$s = \sqrt{\frac{\sum y_i - \bar{y}}{n-1}} = 111.666$$

Our computations are then as follows:

$$y_{41} = 335.5962844 - 7.8140844(41) + 0.3307836(41)^2 + 0.5890558 * 37.39 = 593.29$$

$$y_{42} = 335.5962844 - 7.8140844(42) + 0.3307836(42)^2 - 104.7870853(1) + 0.5890558 * 37.39 = 508.145$$

Our intervals are then,

$$I_{41} = [y_{41} \pm z_{.025} * s] = [593.29 \pm 1.96(111.666)] = [374.42464,\ 812.155]$$

$$I_{42} = [y_{42} \pm z_{.025} * s\sqrt{1+\phi^2}] = [254.130,\ 762.159]$$