1. Webwork Complete.

2. Consider the system of equations,

$$0.003x_1 + 59.14x_2 = 59.17$$
$$5.291x_1 - 6.130x_2 = 46.78$$

First, we test that $[10, 1]^T$ is a solution,

$$\begin{bmatrix} 0.003 & 59.14 \\ 5.291 & -6.130 \end{bmatrix} \begin{bmatrix} 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.03 + 59.14 \\ 52.91 - 6.130 \end{bmatrix} = \begin{bmatrix} 59.17 \\ 46.78 \end{bmatrix}$$

As desired. We now apply gaussian elimination to the system with 4-digit arithmetic:

$$\left[\begin{array}{cc|c} 0.003 & 59.14 & 59.17 \\ 5.291 & -6.130 & 46.78 \end{array}\right] \to \left[\begin{array}{cc|c} 0.003 & 59.14 & 59.17 \\ 0.001 & -104300 & -104400 \end{array}\right] \qquad R_2 - 1764R_1$$
$$\to \left[\begin{array}{cc|c} 0.003 & 59.14 & 59.17 \\ 0.000 & -104300 & -104400 \end{array}\right] \qquad R_2 - \frac{1}{3}R_1$$
$$\to \left[\begin{array}{cc|c} 0.003 & 59.14 & 59.17 \\ 0.000 & 1 & 1.001 \end{array}\right] \qquad \frac{-1}{104300}R_2$$
$$\to \left[\begin{array}{cc|c} 1 & 19710 & 19720 \\ 0.000 & 1 & 1.001 \end{array}\right] \qquad \frac{1}{0.003}R_1$$
$$\to \left[\begin{array}{cc|c} 1.000 & 0.000 & -10 \\ 0.000 & 1 & 1.001 \end{array}\right] \qquad R_1 - 19710R_2$$

We see that the result is then $[-10, 1.001]^T$. The four digit arithmetic without pivots leads to dramatically incorrect results.

3. Now, we apply the partial pivot method,

$$\left[\begin{array}{cc|c} 0.003 & 59.14 & 59.17 \\ 5.291 & -6.130 & 46.78 \end{array}\right] \to \left[\begin{array}{cc|c} 5.291 & -6.130 & 46.78 \\ 0.003 & 59.14 & 59.17 \end{array}\right] \qquad \text{Move largest value to R1}$$
$$\to \left[\begin{array}{cc|c} 5.291 & -6.130 & 46.78 \\ 0.000 & 59.14 & 59.14 \end{array}\right] \qquad R_2 - 0.006R_1$$
$$\to \left[\begin{array}{cc|c} 5.291 & -6.130 & 46.78 \\ 0.000 & 1.000 & 1.00 \end{array}\right] \qquad \frac{1}{59.14}R_2$$
$$\to \left[\begin{array}{cc|c} 5.291 & 0.000 & 52.91 \\ 0.000 & 1.000 & 1.000 \end{array}\right] \qquad R_1 + 6.130R_2$$
$$\to \left[\begin{array}{cc|c} 1.000 & 0.000 & 10 \\ 0.000 & 1.000 & 1.000 \end{array}\right] \qquad \frac{1}{5.291}R_1$$

We see that the partial pivoting method retains more information for solving. This method is more numerically stable than simple elimination.

4. See Jupyter Notebook

5. See Jupyter Notebook

6. See Jupyter Notebook