1. A rule of thumb (derived by Einstein) is that the diffusion coefficient for a globular molecule satisfies $D \approx M^{-1/3}$ where M is the molecular weight. Determine how well this relationship holds for the substances listed in Table 1 by plotting D and M on a log-log plot.

Substance	Molecular Weight	$D(\text{cm}^2/\text{s})$
hydrogen	1	4.5×10^{-5}
oxygen	32	2.1×10^{-5}
carbon dioxide	48	1.92×10^{-5}
glucose	192	6.60×10^{-6}
insulin	5,734	2.10×10^{-6}
Cytochrome c	13,370	1.14×10^{-6}
Myoglobin	16,900	5.1×10^{-7}
Serum albumin	66,500	6.03×10^{-7}
hemoglobin	64,500	6.9×10^{-7}
Catalase	247,500	4.1×10^{-7}
Urease	482,700	3.46×10^{-7}
Fibrinogen	330,000	1.97×10^{-7}
Myosin	524,800	1.05×10^{-7}
Tobacco mosaic virus	40,590,000	5.3×10^{-8}

2. The cross-sectional area of the small intestine varies periodically in space and time due to peristaltic motion of the gut muscles. Suppose that at position x (where x = length along the small intestine) the area can be described by

$$A(x,t) = \frac{a}{2}[2 + \cos(x - vt)],$$

where v is a constant.

   (a) Write an equation of balance for $c(x,t)$, the concentration of digested material at location x.

   (b) Suppose there is a constant flux of material throughout the intestine from the stomach (that is, $\mathbf{J}(x,t) = 1$) and that material is absorbed from the gut into the bloodstream at a rate proportional to its concentration for every unit area of intestinal wall. Give the appropriate balance equation.

   (c) Show that even if $\mathbf{J}(x,t) = 0$ and $\sigma(x,t)$=0, the concentration $c(x,t)$ appears to change.

3. Using $\int_{-\infty}^{\infty} \exp(-x^2)dx = \sqrt{\pi}$, verify that $\int_{-\infty}^{\infty} p(x,t)dx = 1$ for all time, where $p(x,t)$ is given by

$$p(x,t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right)$$

4. Numerically simulate a random walk with $\alpha = 0.25$. Estimate the variance for this process as a function of time and compare this estimate with the theoretically derived variance for this process.

5. Suppose a particle moves to the right with probability α and to the left with probability β and stays put with probability $1 - \alpha - \beta$. Following the arguments from class, formulate this as a discrete random walk process and determine

   (a) The limiting partial differential equation.

   (b) The mean and the variance for the process using the limiting partial differential equation.

   (c) Numerically simulate this random walk with $\alpha = 0.1, \beta = 0.2$. Estimate the mean and variance for this process as a function of time and compare these with the mean and the variance derived directly from the partial differential equation.

   (d) Write the limiting differential equation in conservation form, and identify the flux terms.

6. Suppose that particles in discrete boxes of size Δx leave box j to box $j \pm 1$ at the rate $\lambda_j/(\Delta x)^2$, where $\lambda_j = \lambda(j\Delta x)$ for some smooth function $\lambda(x)$. Derive the limiting diffusion equation, written in conservation form. Identify the different flux terms.

7. Suppose that particles in discrete boxes of size Δx leave box j to box $j \pm 1$ at the rate $\lambda_{j\pm1}/(\Delta x)^2$, where $\lambda_j = \lambda(j\Delta x)$ for some smooth function $\lambda(x)$. Derive the limiting diffusion equation, written in conservation form. Identify the different flux terms.