

1. (a) Find the flatness ratio  $\gamma$  for the following shapes
  - i. An ellipsoid of dimensions  $a \times b \times c$ .
  - ii. A sphere of radius  $R$ .
  - iii. A long cylinder of radius  $r$  and height  $h$  (neglect the top and bottom caps).
  - iv. A cone whose radius is  $r$  when its height is  $h$ .
- (b) Make a summary of the various ways in which organisms overcome diffusional limitations, and illustrate these with examples drawn from the biological literature.
2. In two dimensions consider the radially symmetric region with sink of radius  $a$  and a source of radius  $L$  (as drawn in class – a circle of radius  $a$  inside a circle of radius  $L$ ). Assume that  $c(a) = 0$  and  $c(L) = C_0$ .

- (a) Solve the steady-state 2D diffusion equation

$$0 = \frac{D}{r} \frac{\partial}{\partial r} \left( \frac{\partial c}{\partial r} \right)$$

- (b) Define

$$N = \iint_{\text{disk}} c dA = \int_0^{2\pi} \int_a^L c(r) r dr d\theta.$$

Compute this integral and interpret its meaning.

- (c) Define

$$\begin{aligned} F &= \text{flux} \times \text{circumference of circle} \\ &= \left( D \frac{\partial c}{\partial r} \right) (2\pi L) \end{aligned}$$

- (d) Find  $\tau = N/F$ , and compare this with the value given in class.

3. Consider 1D diffusion between a absorber at  $x = a$  and an absorber at  $x = c$ . Suppose  $a < b < c$  and that a particle is released at  $x = b$ .
  - (a) If  $D$  is the diffusion coefficient, what is the probability the particle will be absorbed at  $x = a$  before it is absorbed at  $x = c$ ? What happens to this probability as  $c \rightarrow \infty$ ?
  - (b) How does the result in (a) differ from that for diffusion from  $r = b$  to a spherical absorber of radius  $a$  in an infinite space?
  - (c) If the absorber at  $x = a$  is replaced by a reflecting boundary, what is the average number of trips the particle makes from  $x = b$  to  $x = a$  before reaching  $x = c$ ?