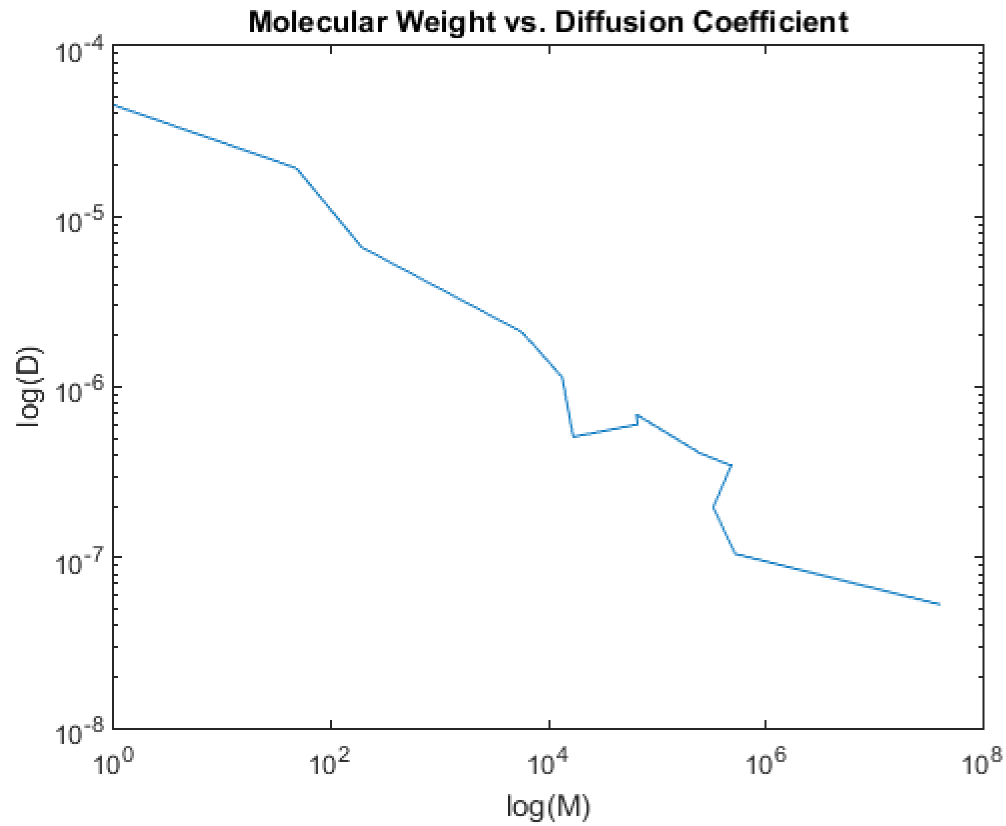
1.

We consider the table of molecular weights and diffusion coefficients. Our rule of thumb for computing D from M is $D \approx M^{-1/3}$. Plotting the table on a log-log plot yields the result below.



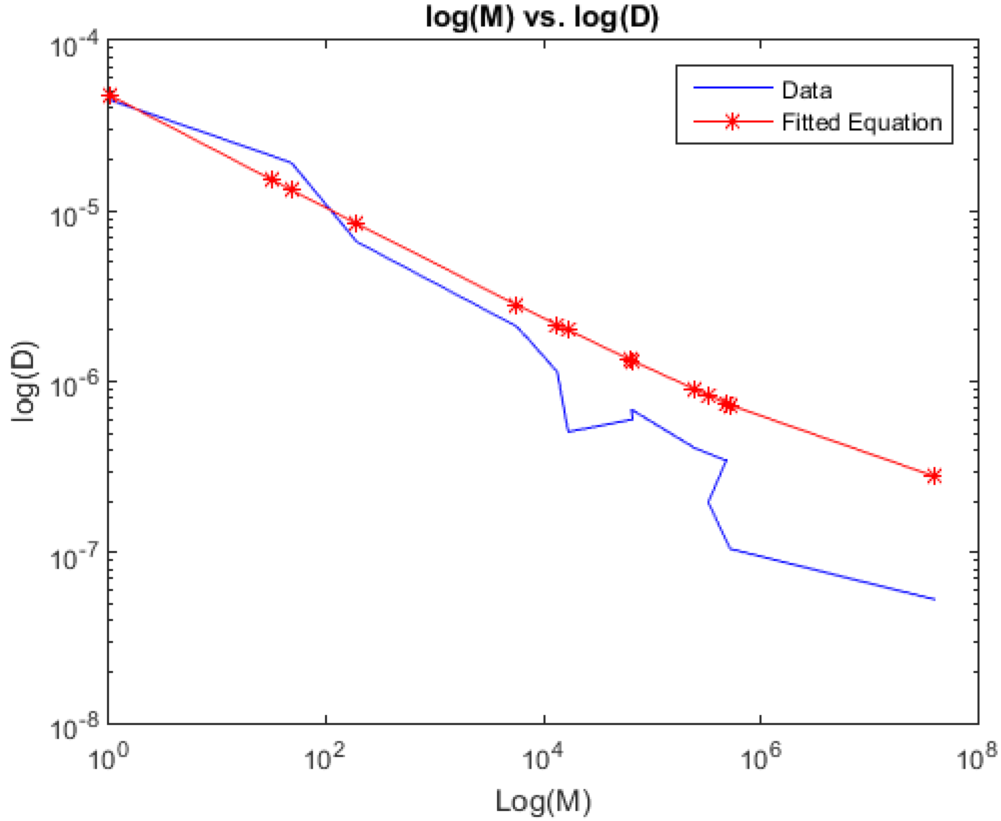
We note that the plot is linear with values decreasing as M increases, as expected. Given that we can assume the data is proportional by the rule of thumb, we consider a line of best fit to be of the form,

$$D = beta_1 M^{-\frac{1}{3}} + \beta_2$$

Using a linear fitting method, I determined the coefficients to be,

$$\beta_1 \approx 4.779 \times 10^{-5}$$
$$\beta_2 \approx 1.399 \times 10^{-7}$$

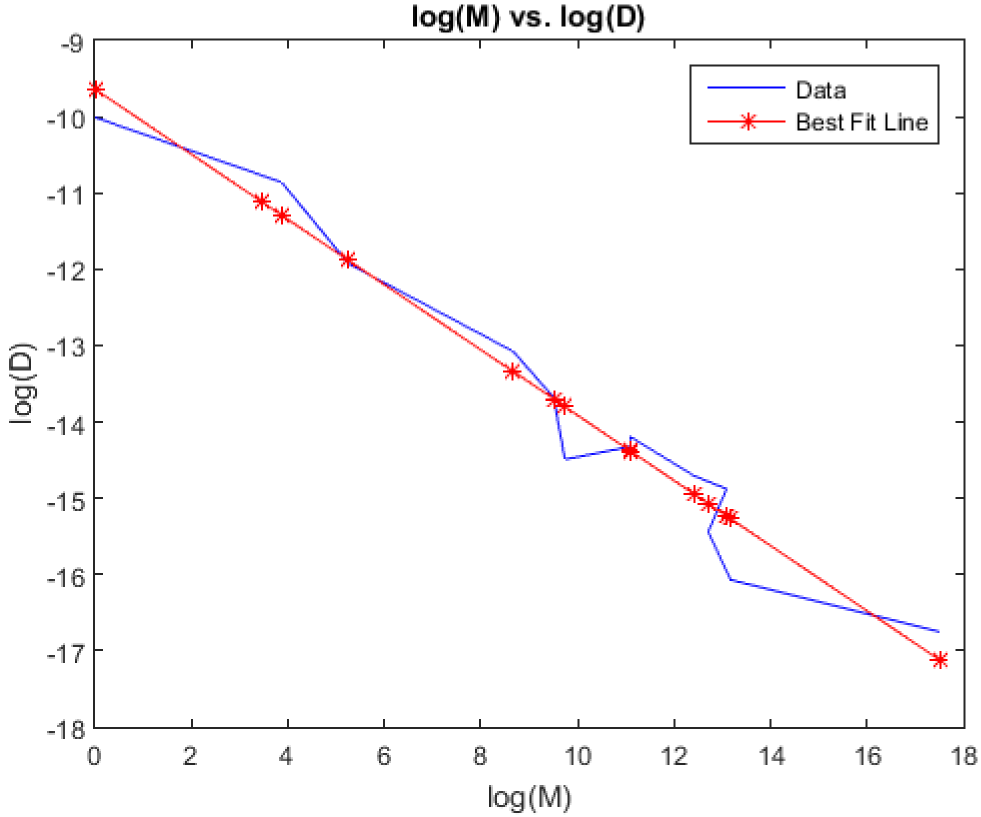
Plotting the resultant line over the data yields the following result; which, from a purely visual inspection seems to support the rule of thumb as being accurate.



In order to further validify the rule of thumb, we consider the slope of a line of best fit when we take the log of both sides. In this case, we construct a relation that looks like,

$$log(D) \approx \frac{-1}{3} log(M)$$

From this, we are able to see that a line of best fit should have a slope of $\frac{-1}{3}$. After running a model fit in R, I found that the best fit coefficient was $-.4274$. Plotting the resultant line over the data, we arrive at the following plot, further confirming our suspicions about the validity of the rule of thumb.



2.

We consider the area of a small intestine varying in time governed by,

$$A(x,t) = \frac{a}{2}\big[2+\cos(x-vt)\big]$$

Where v is a constant.
To write the equation of balance for the concentration $c(x,t)$, we first consider the known conservation equation for 1-D,

$$\frac{\partial}{\partial t}[c(x,t)A(x,t)] = -\frac{\partial}{\partial x}[J(x,t)A(x,t)] \pm \sigma(x,t)A(x,t)$$

Where J is the flux through the cross-sectional area A at position x and time t, and σ is the local rate of creation/destruction of particles. We now substitute into the equation, our known value fo A,

$$\frac{\partial}{\partial t}[c(x,t)(\frac{a}{2}\big[2+\cos(x-vt)\big])] = -\frac{\partial}{\partial x}[J(x,t)(\frac{a}{2}\big[2+\cos(x-vt)\big])] \pm \sigma(x,t)(\frac{a}{2}\big[2+\cos(x-vt)\big])$$

Using the product rule to expand both sides, we arrive at,

$$\frac{\partial c}{\partial t}(\frac{a}{2}\big[2+\cos(x-vt)\big]) + c(\frac{-av}{2}\big[-\sin(x-vt)\big])$$

on the left hand side, and

$$-\frac{\partial J}{\partial x}(\frac{a}{2}\big[2+\cos(x-vt)\big]) + J(\frac{a}{2}\big[-\sin(x-vt)\big]) \pm \sigma(\frac{a}{2}\big[2+\cos(x-vt)\big])$$

Simplifying, and solving for $\frac{\partial c}{\partial t}$, we get,

$$\frac{\partial c}{\partial t} = -\frac{\partial J}{\partial x} \pm \sigma + \frac{(J-vc)\sin(x-vc)}{2+\cos(x-vt)}$$

Thus, we have our balance for c.
We now consider a constant flux of material, and an absorption rate proportional to concentration. Thus, we have

$$J = 1$$
$$\sigma = -\alpha c$$

Substituting into the derived balance equation, we get,

$$\frac{\partial c}{\partial t} = -\alpha c + \frac{(1 - vc)\sin(x - vc)}{2 + \cos(x - vt)}$$

Thus we have updated the force balance.
In a similar way, we consider $J = \sigma = 0$. Substituting into the derived balance yields,

$$\frac{\partial c}{\partial t} = \frac{(vc)\sin(x - vc)}{2 + \cos(x - vt)}$$

This value is not necessarily zero, and thus, we see that the concentration will continue to change regardless.

3.

We consider the function,

$$p(x,t) = \frac{1}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}}$$

and the related integral,

$$\int_{-\infty}^{\infty} p(x,t)dx$$

We solve the integral, by considering the new variable and differential,

$$u = \frac{x}{\sqrt{4Dt}}$$
$$dx = \sqrt{4Dt}du$$

Substituting, we arrive at the following solution,

$$\begin{aligned}\int_{-\infty}^{\infty} p(x,t)dx &= \frac{1}{\sqrt{4\pi Dt}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{4Dt}} dx \\ &= \frac{1}{\sqrt{4\pi Dt}} \int_{-\infty}^{\infty} e^{-u^2} \sqrt{4Dt}du \\ &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-u^2} du \\ &= \frac{\sqrt{\pi}}{\sqrt{\pi}} \\ &= 1\end{aligned}$$

Thus, we see that the integral evaluates to one for all time.

6.

We consider the balance equation for an arbitrary box with concentration c_i,

$$\frac{\partial c_i}{\partial t} = \frac{1}{(\Delta x)^2}\left(\lambda_{i-1}c_{i-1} + \lambda_{i+1}c_{i+1} - 2\lambda_i c_i\right)$$

We now define the expression, $J_i = \lambda_i c_i$ and re-write the equation,

$$\frac{\partial c}{\partial t} = \frac{1}{(\Delta x)^2}\left(J_{i-1} - 2J_i + J_{i+1}\right)$$

Noting the second order difference equation, we re-write the equation, allowing the relation to hold for all boxes,

$$\frac{\partial c}{\partial t} = \frac{\partial^2 J}{\partial x^2}$$

Simplifying into conservation form, we arrive at,

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\partial J}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(\lambda c)\right)$$

Where the expressions in the parentheses are the flux terms.

7.

We consider the balance equation for an arbitrary box with concentration c_i,

$$\frac{\partial c_i}{\partial t} = \frac{1}{(\Delta x)^2}\left(\lambda_{(i-1)+1}c_{i-1} + \lambda_{(i+1)-1}c_{i+1} - lambda_{i+1}c_i - l\lambda_{i-1}c_i\right)$$

We then add and subtract the value of $\frac{2\lambda_i c_i}{(\Delta x)^2}$ to the above equation and re-arrange to arrive at,

$$\frac{\partial c_i}{\partial t} = \frac{1}{(\Delta x)^2}\left(\lambda_i\big(c_{i-1} - 2c_i + c_{i+1}\big) - c_i\big(\lambda_{i-1} - 2\lambda_i + \lambda_{i+1}\big)\right)$$

We may now see that there are two second order difference equations in this equation, and we now write them as such, and allow the relation to hold over all boxes.

$$\frac{\partial c}{\partial t} = \lambda\frac{\partial^2 c}{\partial x^2} - c\frac{d^2\lambda}{dx^2} + O(\Delta x^2)$$

We note that lambda is defined as only a function of x, and as such, we may simplify the partial into an ordinary derivative. Putting this into conservation form, we arrive at,

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(\lambda(x)\frac{\partial c}{\partial x} - c\frac{d\lambda}{dx}\right)$$

Here, the flux term is the inside of the parentheses.