The Analytics Edge

Test your knowledge of LP and IP

The purpose of this set of exercises is to help build your familiarity with Julia and optimization

1. Billions of shares of stock, or fractions of ownership in a business, are traded on the stock market every day. Over half of all adults in the United States own stocks and 1.2 billion people worldwide invest in the stock market. Many people invest in stocks to increase their wealth and to increase their earnings beyond their salary. If the business that you own stock in does well, then your stock value will increase and you will make money. An individual who owns stock can sell their shares, or a fraction of their shares, to get cash that can be used for a down payment on a home, to buy a new car, or for any other purchase. However, when you sell stock, you have to pay both a transaction fee and tax on the money you gain. If you own many different stocks, you have to decide what stocks and how much to sell to make sure you have enough cash for your purchase. In this problem, we'll use linear optimization to decide which shares of stock and how many you need to sell in order to have enough cash to make your purchase, and to maintain a strong portfolio of stocks. Suppose that, last year, you purchased 150 shares of eight different stocks (for a total of 1200 shares). The Investment.csv lists the stocks that you purchased, the price you purchased them for last year, the current price, and the price estimate for next year. If you sell any shares, you have to pay a transaction cost of 1% of the amount transacted. In addition, you must pay a capital-gains tax at the rate of 30% on any capital gains at the time of the sale. For example, suppose that you sell 100 shares of a stock today at \$50 per share, which you originally purchased for \$30 per share. You would receive \$5,000. However, you would have to pay capital-gains taxes of $0.30 \times (\$5000 - \$3000) = \$600$ and you would have to pay \$50 in transaction cost (1% of \$5000). Therefore, by selling 100 shares of this stock, you would have a net cashflow of $\$5000 - \$600 - \$50 = \$4350$. Note that none of the stocks decreased in value since the time of purchase, so we don't have to deal with capital losses.

   (a) You would like to sell enough shares of stock today to generate \$10,000 to use as part of a down payment on a new home. You need to decide how many shares of which stocks to sell in order to generate \$10,000, after taxes and transaction costs, while maximizing the estimated value of your stock portfolio next year. Let's formulate this as a linear optimization problem.

      i. Describe the decision variables. How many decision variables should your model have?

      ii. We'll assume for this problem that you can't sell more shares of stock than you own, and you can't buy additional shares. What is the maximum value your decision variables can be? What is the minimum value your decision variables can be?

iii. Your objective is to maximize the estimated value of your stock portfolio next year. To do this, you should sum the estimated value of each stock next year. Suppose you sell x shares of your stock in Microsoft. What is the estimated value of your Microsoft stock next year?

iv. You need to make sure you get $10,000 in cash from selling your stocks, after taxes and transaction costs. How much would you get in cash, after taxes and transaction costs, if you sell 50 shares of your Intel stock?

(b) Formulate and solve this optimization problem in Julia as a linear program. Your objective is to maximize the estimated value of your stock portfolio next year. Make sure to include a constraint for the amount of cash you generate, and upper and lower bounds for the values of your decision variables.

i. What is the optimal objective value? Give the values of the decision variables.

ii. As an investor, we often see a diverse portfolio with all the eight different stocks since it diversifies your investment. In this case, are you happy with this solution? Why or why not?

(c) Support you would like to keep at least 75 shares of each of your eight stocks. Adjust the formulation so that you sell no more than 75 shares of each stock and solve the problem again. What is the optimal objective value? Give the values of the decision variables.

2. La Quinta Motor Inns is a mid-sized hotel chain headquartered in San Antonio, Texas. They are looking to expand to more locations, and know that selecting good sites is crucial to a hotel chain's success. Of the four major marketing considerations (price, product, promotion, and location), location has been shown to be one of the most important for multisite firms. Hotel chain owners who can pick good sites quickly have a distinct competitive advantage, since they are competing against other chains for the same sites. La Quinta used data on 57 existing inn locations to build a linear regression model to predict "Profitability", computed as the operating margin, or earnings before interest and taxes divided by total revenue. They tried many independent variables, such as "Number of hotel rooms in the vicinity" and "Age of the Inn". All independent variables were normalized to have mean zero and standard deviation 1. The final regression model is given by:
Profitability = 39.05 - (5.41 × State population per inn) + (5.86 × Price of the inn) - (3.09 × Square root of the median income of the area) + (1.75 × College students in the area)
The R-squared of the model is 0.51. In this problem, we'll use this regression model together with integer optimization to select the most profitable sites for La Quinta.

(a) Let us being by understanding the regression equation.

i. According to the regression equation given above, which variables positively affect Profitability? Which variables negatively affect Profitability?

ii. Using this regression equation, La Quinta created a model to predict profitability, and routinely uses it to screen potential real estate acquisitions. Suppose that La Quinta is looking to expand their locations in California, and has collected data for

16 different potential sites. This data is given in the file SelectingHotels.csv. For each hotel, it lists the location of the hotel, the price, and the value for each of the independent variables used in the regression equation (normalized to have mean zero and standard deviation one). Using the regression equation and the four normalized independent variables, what is the predicted profitability of hotel 1?

iii. Now use the regression equation and the data to compute the predicted profitability for all hotels. Which hotel has the highest profitability? The lowest?

(b) La Quinta has a budget of $10,000,000 to spend on hotels. Suppose we just used a "greedy" approach where we selected the most profitable hotels until we ran out of budget. So we would start by buying the hotel we predict to be the most profitable, and then if we had enough budget left, we would buy the hotel we predict to be the second most profitable, etc.

i. Which hotels would we purchase under this approach?

ii. What would our total predicted profitability be? (This is the sum of the predicted profitability of all hotels we purchase.)

(c) Now, build an optimization model to select hotels. The decision variables are whether or not a hotel is selected (binary variables). The objective is to maximize the total predicted profitability. We have two constraints: the decision variables should be binary, and the total cost should not exceed the budget of $10,000,000. Formulate and solve this model in Julia.

i. Write out the optimization model. What is the objective value of the solution? Give the optimal values of the decision variables. How many hotels are selected in the solution?

ii. How does it compare to the greedy solution?

(d) How many hotels are selected in the optimal solution in South Lake Tahoe? La Quinta thinks that buying too many hotels in one city is probably not a good idea, and would prefer to diversify in other cities, even though it will decrease the sum of the predicted profitability. Add a constraint to limit the number of hotels purchased in any city to 2.

i. What are the constraints you need to add to the model? Do you expected the new optimal objective function to be larger, smaller or the same as before?

ii. Solve the new model. What is the objective value of the solution? Give the optimal values of the decision variables.