

# Mathematical Modelling

## QuestQ

### Initial model & Proposed alternative model

In both initial and alternative models we tracked the following population parameters:

1. Arrival times of customers entering the system. The arrival time of the  $i$ th customer is denoted by  $A_i$ .
2. Departure times of customers leaving queuing system after collecting order. The departure time of the  $i$ th customer is denoted by  $D_i$ .
3. Service times for customers in the system. The service time for the  $i$ th customer is given by  $S_i$ .
4. Total number of customers that have entered/left the system is denoted by  $n$ . (Note: We do not account for customers leaving the system, without making an order hence the total number of arrivals = total number of departures)

From the above, we derived the following:

1. Inter-arrival times between customers entering the queuing system. The  $j$ th inter-arrival time is given by:

$$A_i - A_{i-1}$$

2. Inter-departure time between customers leaving the system. The  $j$ th inter-departure time is given by:

$$D_i - D_{i-1}$$

3. Total waiting time since entering the system, till customers depart from their system after collecting an order. The total waiting time of the  $i$ th customer is given by:

$$D_i - A_i$$

4. Total waiting time prior to making an order for the  $i$ th customer:

$$D_i - A_i - S_i$$

5. The total waiting time for all customers:

$$\sum_i^n D_i - A_i$$

6. Average waiting, during the 45 minute peak period:

$$\frac{\sum_i^n D_i - A_i}{n}$$

7. Rate of change in queue length (Expected number of people in the queue):

$$\frac{dD}{dt} - \frac{dA}{dt}$$