Finding the Index of Refraction of Air

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Overview:

Purpose:

- Experimentally calculate the index of refraction of air using mathematical theory and two given variables

To do this, I will...

- Review the Index of Refraction & Wave Interference
- Introduce the Michelson Interferometer
- Outline the Experiment
- Present Results

What is an Index of Refraction?

- The index of refraction is the ratio of the speed of light in a vacuum to its speed in a different medium:

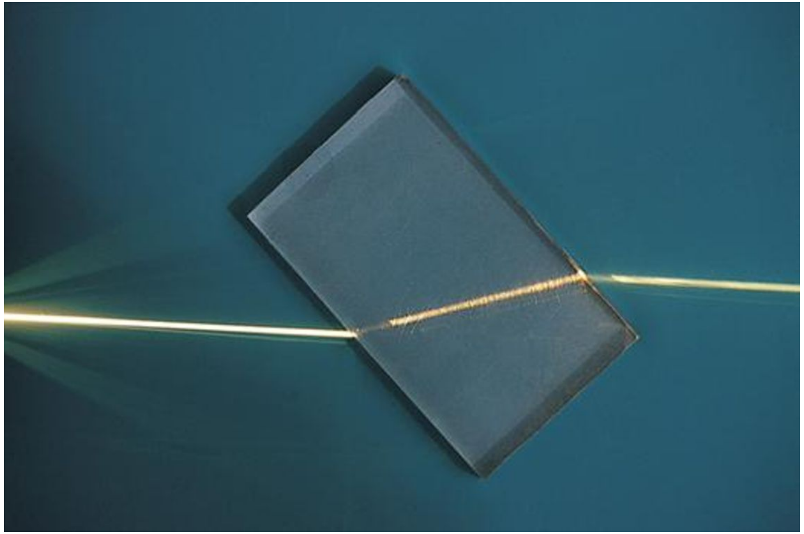
$$n = \frac{c}{v}$$

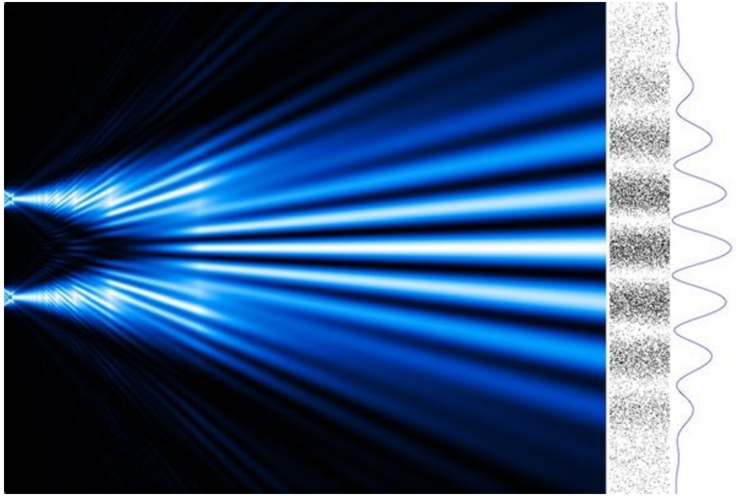
c = speed of light in vacuum
v = speed of light in medium

- Because the frequency of the wave does not change equation can be written as:

$$\lambda = \frac{\lambda_0}{n}$$

λ = wavelength in medium
λo = wavelength in vacuum





Double Slit Experiment

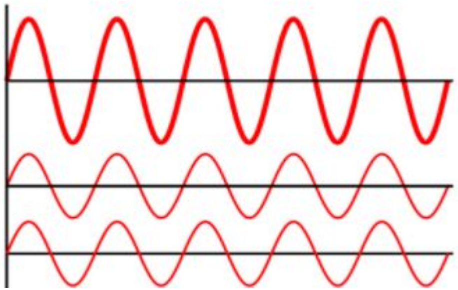
What is Wave Interference?

- Wave interference is superposition of light waves with similar frequencies

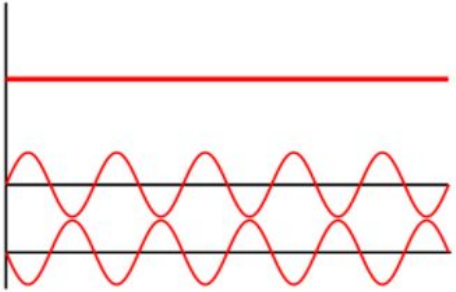
- The interference can increase or decrease the amplitude equivalent light, this is called constructive and destructive interference

- An example of wave interference is Thomas Young's double slit experiment shown left.

Constructive Interference

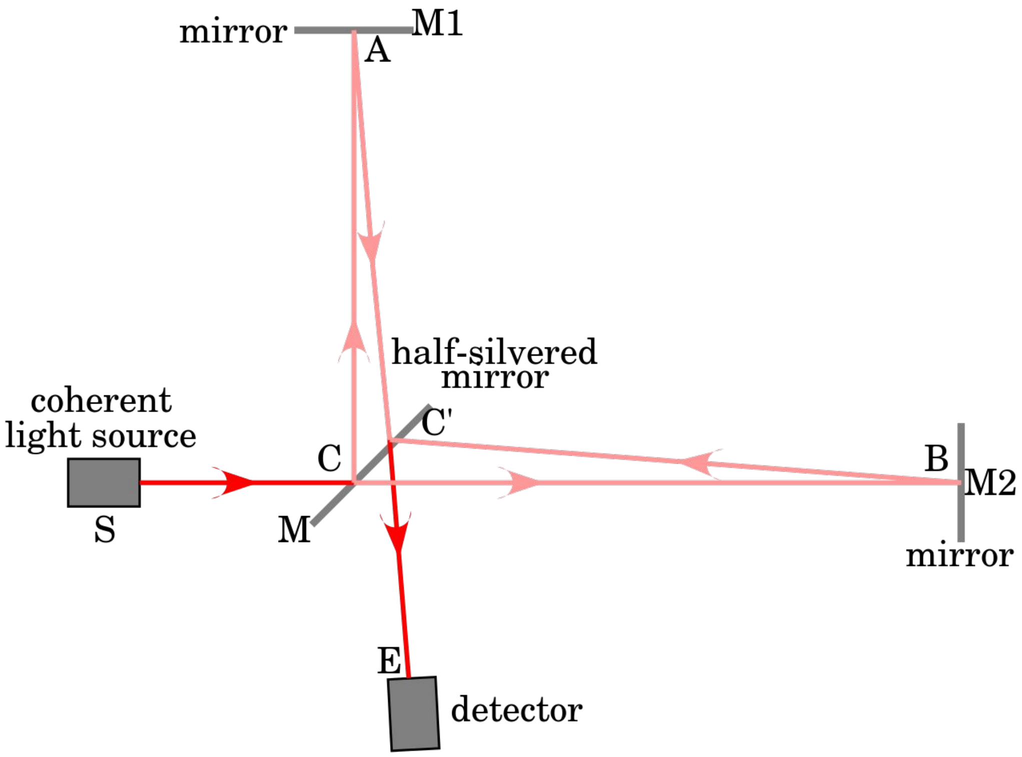


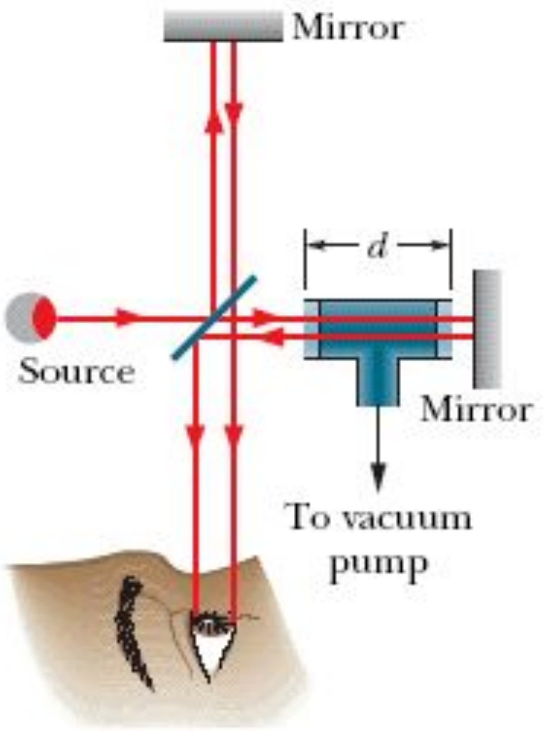
Destructive Interference



Michelson Interferometer

- Used in the Michelson-Morley Experiment 1887
- Beams produce an interference pattern at the detector
- If the path length of either beam changes, the interference pattern will change





The Experiment:

- The number of wavelengths in the vacuum chamber:

$$N = \frac{2d}{\lambda} = \frac{2nd}{\lambda_0}$$

- By definition, as the pressure decreases, n gets closer to 1. This causes N to decrease

- For every whole number that N decreases, the interference pattern will go through one phase transition

The Experiment (cont.):

From:

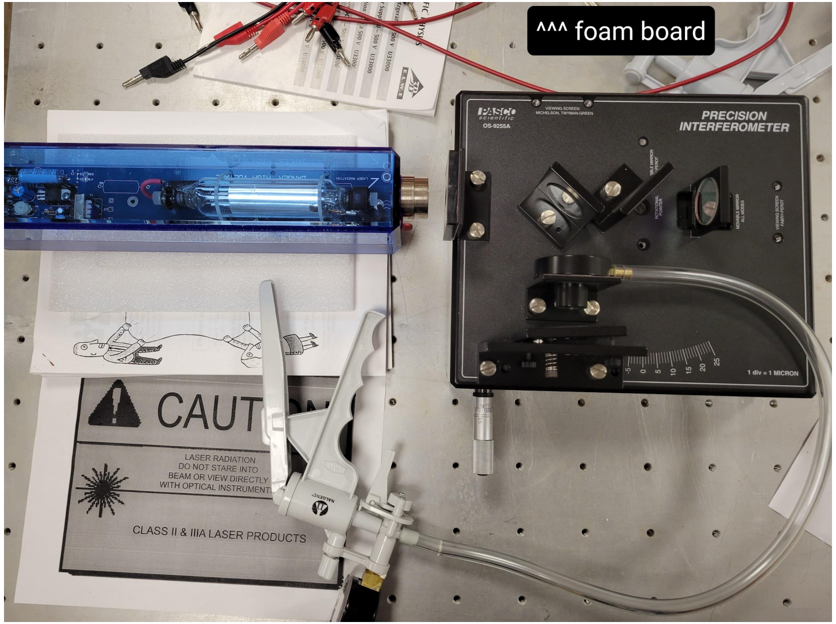
$$N = \frac{2nd}{\lambda_0}$$

The ratio of the change in “n” to change in P can be found to be:

$$\frac{\Delta n}{\Delta P} = \frac{\Delta N \lambda_0}{2d\Delta P}$$

Knowing that @P = 0 (abs) the index of refraction “n” is 1, the index of refraction of air at pressure P is described by:

$$n = \frac{\Delta N \lambda_0}{2d\Delta P} P + 1$$

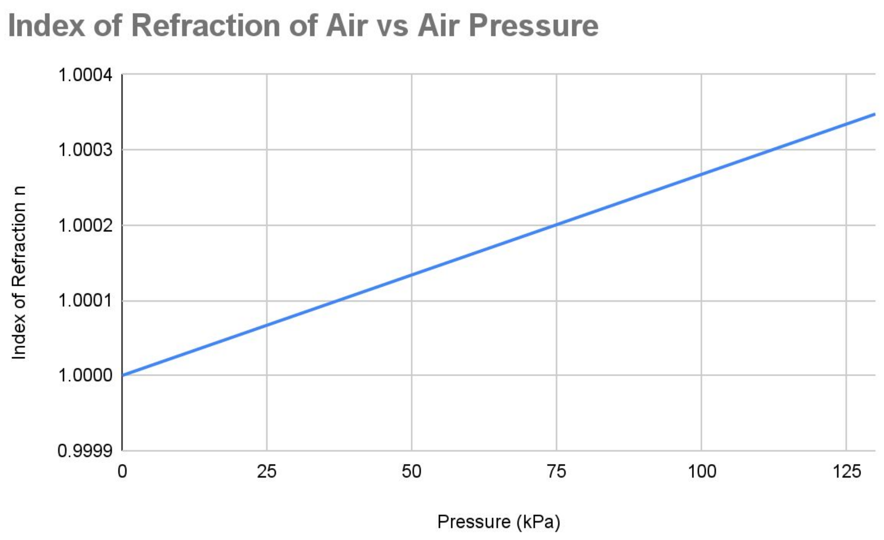


Results:

With ΔP = 30 +/- 2 kPa (error based on equipment)

A ΔN of 8 +/- 0.5 was recorded.

This produced the line shown in the figure



Solving for the Index of Refraction at standard atmospheric pressure (101.325 kPa):

n = 1.000271 +/- 0.000025

This calculation agrees with the accepted value of 1.000293

Sources:

https://en.wikipedia.org/wiki/Michelson_interferometer

https://en.wikipedia.org/wiki/Wave_interference

https://en.wikipedia.org/wiki/Refractive_index

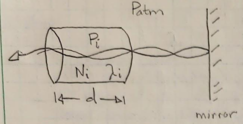
https://astronomy.com/news/2019/05/antimatter-acts-like-regular-matter-in-classic-double-slit-experiment

https://www.britannica.com/science/atmospheric-pressure

https://en.wikipedia.org/wiki/List_of_refractive_indices

Appendix: Theory and Calculations

Introduction to Index of Refraction of Air Calculation



let λ_o = wavelenght of the light in a vacuum

Initial State:

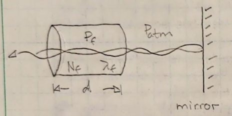
$P_i = P_{atm}$

light passes through the chamber twice

$$N_i = \frac{2d}{\lambda_i}$$

$$\lambda_i = \frac{\lambda_o}{n_i}$$

N_i = # of wavelengths in the chamber
d = length of the chamber
λ_i = wavelength inside the chamber
n_i = index of refraction within the chamber



$$N_f = \frac{2d}{\lambda_f}$$

$$\lambda_f = \frac{\lambda}{n_f}$$

Final State:
$P_f < P_{atm}$

From interference pattern,

N = the # of times the two beams go in and out of phase while changing the pressure

and,

$N = N_i - N_f$ → the change of the path length as a multiple of the wavelength

$$N = \frac{2d}{\lambda_i} - \frac{2d}{\lambda_f} = \frac{2n_i d}{\lambda_o} - \frac{2n_f d}{\lambda_o} = \frac{2d}{\lambda_o}(n_i - n_f)$$

$$\Rightarrow (n_i - n_f) = \frac{N\lambda_o}{2d} \underset{\text{relating to pressure}}{\Rightarrow} \left(\frac{n_i - n_f}{P_i - P_f}\right) = \frac{N\lambda_o}{2d(P_i - P_f)}$$

slope

with n = 1.000 @ P = 0 ✓

$$n = \frac{N\lambda_o}{2d(P_i - P_f)} P + 1.000$$

Results:

Measurment	$\Delta P \pm \Delta(\Delta P)$ kPa	$N \pm \Delta N$ #
1	30 ± 2	8 ± 0.5
2	30 ± 2	8 ± 0.5
3	30 ± 2	8 ± 0.5

from the scale of the pressure gauge?

d = 3.155 cm

λ_o = 632.8 nm

Error in the Slope

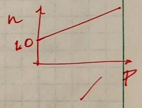
with $(P_i - P_f) = \Delta P$, error in measurements ΔP and N effect the slope calculation

letting $m = \frac{N\lambda_o}{2d\Delta P}$

$$\frac{\partial m}{\partial \Delta P} = -\frac{N\lambda_o}{2d(\Delta P)^2} \qquad \frac{\partial m}{\partial N} = \frac{\lambda_o}{2d\Delta P}$$

$$\Delta m = \left[\left(-\frac{N\lambda_o}{2d(\Delta P)^2}\Delta(\Delta P)\right)^2 + \left(\frac{\lambda_o}{2d(\Delta P)}\Delta N\right)^2\right]^{1/2}$$

include graph



With the data above

$m = 2.674 \times 10^{-6}\ (\text{kPa})^{-1}$ $\Delta m = 2.446 \times 10^{-7}\ (\text{kPa})^{-1}$

Using the index of refraction of air equation derived earlier...

$n = mP + 1.000$ with $m = \frac{N\lambda_o}{2d\Delta P}$

$\frac{\partial n}{\partial m} = P$ → $\Delta n = P\Delta m$

For P = 101.325 kPa, the accepted value of air pressure @ sea level

$n \pm \Delta n$ = 1.000271 ± 0.000025

Accepted value of the Index of refraction of air = 1.000293