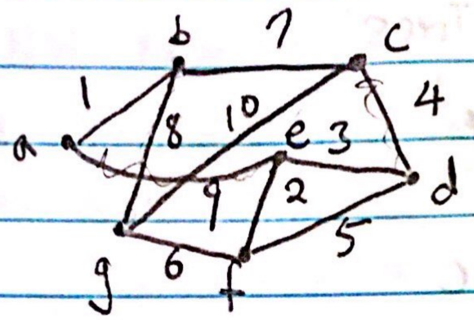
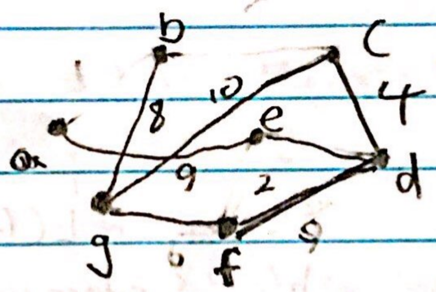
Justin Kim.

1 a)



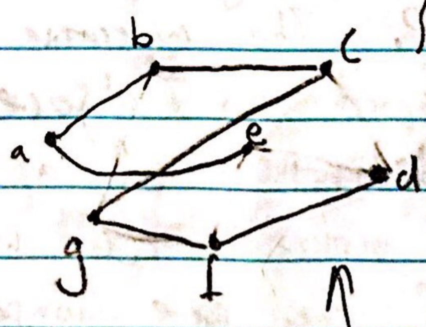
1)

cut set = { (a,e), (b,g), (g,c), (c,d) }



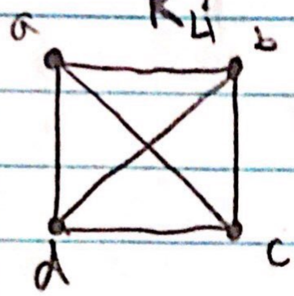
2)

edges that produce cycles: { (e,d), (e,f), (f,d) }
{ (b,c), (b,g), (c,g) }
{ (a,b), (b,c), (c,d), (d,e), (e,a) }
{ (g,c), (c,d), (d,f), (f,g) }

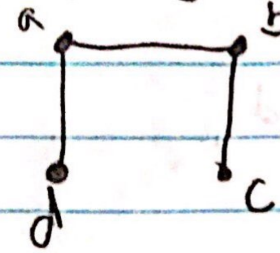


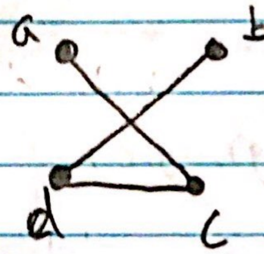
no cycles

b) K_4

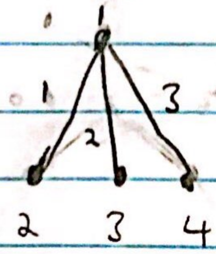


Spanning Trees

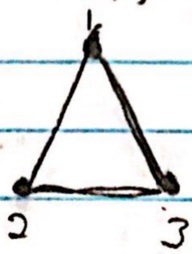




2 a) $G = K_{1,3}$



$L(G)$



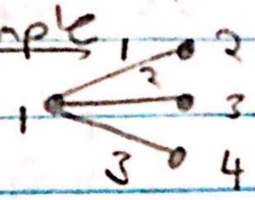
3 vertices in $L(G)$ b/c 3 edges in $G = K_{1,3}$

b). Let's have Eulerian path P of graph G. By definition, every edge of G occurs exactly once in P. The consecutive edges of P will correspond to adjacent vertices in L(G) by definition of a line graph. Moreover, adjacent vertices in L(G) would share an edge if they were consecutive in G. Thus, every vertex appears only once in tour in L(G), which makes L(G) is Hamiltonian.

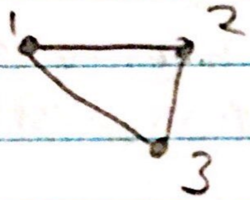
c) Not necessarily.

In the example,

$G = K_{1,3} \rightarrow$



$L(G) \rightarrow$

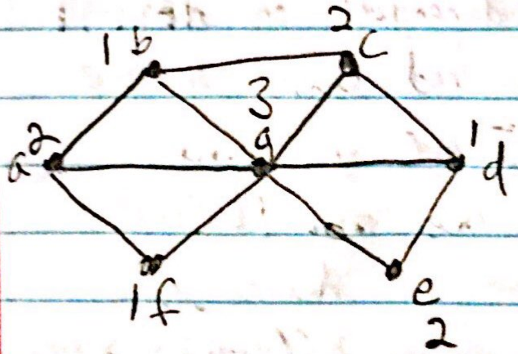


$L(G)$ is Eulerian and Hamiltonian

However, G is neither.

Thus, $L(G)$ being Eulerian/Hamiltonian $\not\rightarrow$ we can say anything about G.

3 a)



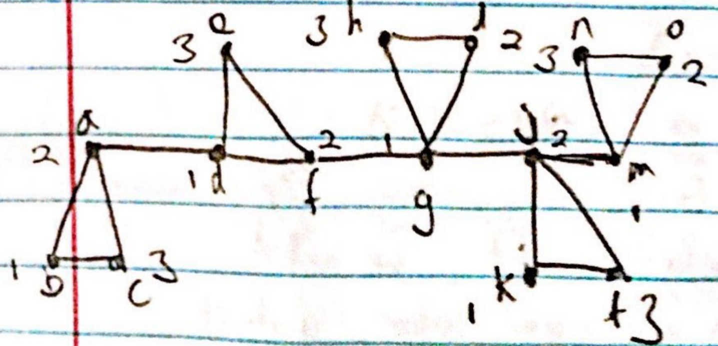
1 = blue

2 = green

3 = red

$\chi(G) = 3$

b)



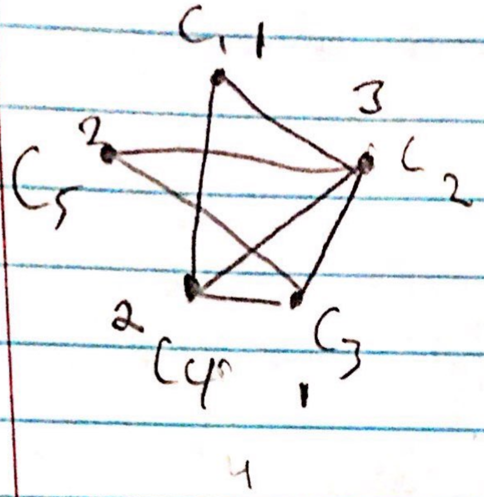
1 = blue

2 = red

3 = green

$\chi(G) = 3$

c)



1 = red
2 = green
3 = yellow

$\chi(G) = 3$

3 meeting times must be scheduled

4 a) $\chi(G-v) \leq \chi(G) \leq \chi(G-v)+1$

$\chi(G-v) \leq \chi(G) \wedge \chi(G) \leq \chi(G-v)+1$

Prove $\chi(G-v) \leq \chi(G)$
By definition, $\chi(G)$ is dependent on $|E(G)|$ because colors are determined based on the adjacency of vertices. In $G-v$, where a vertex is removed, the ~~...~~ $|E(G)|$ will either stay the same or decrease. Thus it makes sense that $\chi(G-v) \leq \chi(G)$

Prove $\chi(G) \leq \chi(G-v)+1$
Let $\chi(G) = k$ and $\chi(G-v) \leq k-2$
We can color graph G by coloring G-V using at most k-2 colors. If we add V, we can assign a new color, which gives us $\chi(G) = k-2+1 = k-1 \neq k$. Thus

This therefore gives us either $\chi(G-v) \geq \chi(G) - 1$

or $\chi(G-v) = \chi(G)$

Therefore.
$\chi(G-v) + 1 \geq \chi(G)$

$\chi(G) \leq \chi(G-v) + 1 \wedge \chi(G-v) \leq \chi(G) \Rightarrow$

$\chi(G-v) \leq \chi(G) \leq \chi(G-v) + 1$

b) Prove $\chi(G) \geq \omega(G)$ order of

By definition $\omega(G)$ is largest subgraph where all vertices are adjacent. No 2 colors can be adjacent, every vertex in a clique needs to have a unique color.

$\therefore \chi(G) \geq \omega(G)$

Prove $\chi(G) \geq n/\alpha(G)$

$\alpha(G)$

$V_i = \{v : c(v) = i\}$ such that $i = 1, 2, \dots k$

V_i is an independence set.

$\alpha(G) \geq |V_i|$

$\chi(G)\,\alpha(G) \geq \underbrace{|V_1| + |V_2| + \dots |V_k|}_{n}$

$\chi(G)\,\alpha(G) \geq n$

$\chi(G) \geq \dfrac{n}{\alpha(G)}$