Hw8 Discrete Mathematics

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April 2019

Problem 1

(a) 925554 anagrams
Let X_T = total number of anagrams that can be made from SLEEPLESSNESS
$|X_T| = \frac{13!}{5!4!2!} = 1081080$
Let sets $X_{SLEEP}, X_{LESS}, X_{NESS}$ respectively represent the set of anagrams containing SLEEP, LESS, and NESS.
$|X_{SLEEP}| = \frac{9!}{4!2!} = 7560$
$|X_{LESS}| = \frac{10!}{3!3!} = 100800$
$|X_{NESS}| = \frac{10!}{3!3!2!} = 50400$
Let $X_{SLEEP} \cap X_{LESS}$ be the the intersection of X_{SLEEP}, X_{LESS}
$|X_{SLEEP} \cap X_{LESS}| = \frac{6!}{2!} = 360$
Let $X_{SLEEP} \cap X_{NESS}$ be the the intersection of X_{SLEEP}, X_{NESS}
$|X_{SLEEP} \cap X_{NESS}| = \frac{6!}{2!} = 360$
Let $X_{LESS} \cap X_{NESS}$ be the the intersection of X_{LESS}, X_{NESS}
$|X_{LESS} \cap X_{NESS}| = \frac{7!}{2!} = 2520$
Let $X_{SLEEP} \cap X_{LESS} \cap X_{NESS}$ be the intersection of sets $X_{SLEEP}, X_{LESS}, X_{NESS}$
$|X_{SLEEP} \cap X_{LESS} \cap X_{NESS}| = \frac{3!}{1} = 6$
$X_{SLEEP} \cup X_{LESS} \cup X_{NESS}$ is the union of $X_{SLEEP}, X_{LESS}, X_{NESS}$
$|X_{SLEEP} \cup X_{LESS} \cup X_{NESS}| = 7560 + 100800 + 50400 - 360 - 360 - 2520 + 6 = 155526$ (because inclusion-exclusion)
$|X_T| - |X_{SLEEP} \cup X_{LESS} \cup X_{NESS}| = 1081080 - 155526 = 925554$

(b) 917784 anagrams
Let X_T = total number of anagrams that can be made from SLEEPLESSNESS
$|X_T| = \frac{13!}{5!4!2!} = 1081080$
Let sets $X_{LEER}, X_{LEETWICE}$ respectively represent the set of anagrams containing LEE with repetition and LEE appearing twice, respectively.
$|X_{LEER}| = \frac{11!}{5!2!} = 166320$
$|X_{LEETWICE}| = \frac{9!}{5!} = 3024$
$|X_{LEER}| - |X_{LEETWICE}| = 166320 - 3024 = 163296$
$|X_T| - (|X_{LEER}| - |X_{LEETWICE}|) = 1081080 - 163296 = 917784$

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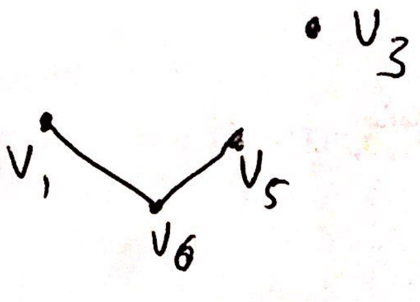
2 a) max. degree = 4
min. degree = 1

b)



c) $\omega(G) = 3$

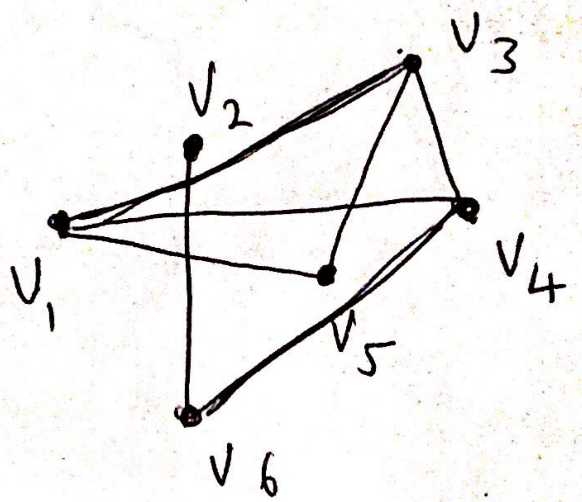
$\{v_2, v_4, v_5\}$

d) $\alpha(G) = 3$

$\{v_1, v_4, v_3\}$
$\{v_5, v_3, v_1\}$

$\{v_3, v_4, v_6\}$

e)



Problem 3

(a) Let G be a graph with $|V(G)|$ vertices and $|E(G)|$ edges. There exists edge $e(G)$ with endpoints $v_i \in V(G)$ and $v_j \in V(G)$. By virtue of the fact that e connects v_i and v_j e adds 1 to the degree of v_i and 1 to the degree of v_j . Therefore, e contributes 2 to the total degree of G. Thus for all edges contribute 2 to the total degree. Therefore, the total degree of G is $2|E(G)|$, which by definition must be even.

(b) Proof by Contradiction: Let G be a finite simple graph with $n = |V(G)|$ vertices, none of which have the same degree. $n \geq 2$. For any vertex $v \in V(G), 0 \leq d(v) \leq n-1$. If $v_1, v_2 \in V(G)$ such that $d(v_1) = 0, d(v_2) = n-1$, then v_1 cannot be adjacent to any vertex of G, whereas v_2 must be adjacent to every vertex of G (except for itself), including v_1. This presents us with a contradiction. Therefore, the graph G must have two vertices of the same degree.

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a)

False

Counterexample:

Let $V(G) = V(H) = \{2, 4\}$

$E(G) = \emptyset$

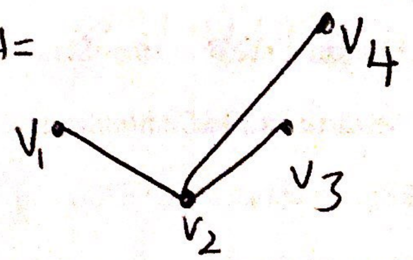
$E(H) = \{\{2, 4\}\}$

$\alpha(G) = 2 > \alpha(H) = 1$

b) False

Counterexample.

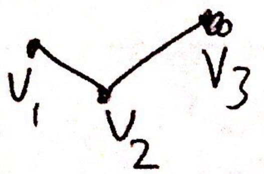
$H =$



$\alpha(H) = 3$

$\{v_1, v_3, v_4\}$

$G =$



$\alpha(G) = 2$

$\{v_1, v_3\}$

$\therefore\ \alpha(H) = 3 \geq \alpha(G) = 2$

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a) $2^{\frac{n(n-1)}{2}}$

The maximum number of edges with n vertices is $\frac{n(n-1)}{2}$. Thus, to find the # of graphs, we do $2^{\frac{n(n-1)}{2}}$

b) $\frac{n(n-1)}{2}$

For every n vertices we have, $n-1$ edges per vertex. However, to avoid double-counting we divide by 2.

c) $2^{\frac{n(n-1)}{2}}$

Same as part A.

d) 2^n.

This is the same as how many subsets can we have of set N in which $|N| = n$.

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c) Justin Kim,

TRUE

Proof: - Let $m = \omega(G)$

- Let set M be the corresponding clique such that $|M| = m$
- $M \subseteq V(G) \subseteq V(H) \rightarrow M \subseteq V(H)$
- All vertices in M are adjacent in H
- $E(G) \subseteq E(H) \rightarrow$ all vertices in M are adjacent in H b/c they are already adjacent in G.
- Therefore M is a clique within H and $\omega(H) \geq m$
- $\therefore \omega(G) \leq \omega(H)$

d) FALSE

Counterexample:

Let $V(G) = V(H) = \{2, 4\}$

Let $E(G) = \emptyset$

Let $E(H) = \{\{2, 4\}\}$

$\omega(G) = 1 < \omega(H) = 2$

~~$\omega(G) > \omega(H)$~~

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