Hw7 Discrete Mathematics

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Problem 1

(a) $\binom{20}{5}\binom{15}{7}\binom{8}{8} = 99768240$
There exist $\binom{20}{5}$ ways to choose 5 zeros. There exist $\binom{15}{7}$ ways to choose 7 ones. There exist $\binom{8}{8}$ ways to choose 8 twos. This is because we need to have 5+7+8=20. Thus, we need to multiply $\binom{20}{5}\binom{15}{7}\binom{8}{8}$ and we get 99768240 ways to do this.

(b) $\binom{13}{3} = 286$
This problem can be modeled in terms of a stars and bars problem. We have 10 elements, and by virtue of us having 4 different types of coins, so we have 10 elements separated by 4-1=3 bars. Therefore we have $\binom{10+4-1}{4-1} = 286$ ways to select 10 coins from a jar of 20 pennies,

(c) $\binom{9}{3} = 84$
This problem can be modeled in terms of a stars and bars problem. We have 10 elements, and by virtue of us having 4 different types of milk teas, we have 10 elements separated by 4-1=3 bars. Thus we have in total 10+4-1=13 different elements. The caveat here is that there must be one of each tea, so we subtract 4 from 13. This gives us $\binom{9}{3} = 84$ ways to do this.

(d) $\binom{8}{2}+\binom{7}{2}+\binom{6}{2} = 64$
We can divide this into 3 different cases. In the first case, we have 1 Taro tea. In this case, we have $\binom{8}{2}$ ways. In the 2nd case we have 2 Taro teas. In this case, we have $\binom{7}{2}$ ways. In the 3rd case we have 3 Taro teas. In this case, we have $\binom{6}{2}$ ways. This gives us $\binom{8}{2}+\binom{7}{2}+\binom{6}{2} = 64$ ways accomplish this.

Problem 2

(a) $1+2+3+4+5+6+7+8=36$
If $x_1=2$, then $x_2+x_3=11$. This means when $x_2=2$, $x_3=9$. When $x_2=3$, $x_3=8$, and so on. This gives us 8 different permutations of (x_2, x_3) when $x_1=2$. If $x_1=3$, then $x_2+x_3=10$. This means when $x_2=2$, $x_3=8$. When $x_2=3$, $x_3=7$, and so on. This gives us 7 different permutations of (x_2, x_3) when $x_1=3$. If $x_1=4$, then $x_2+x_3=9$. This means when $x_2=2$, $x_3=7$. When $x_2=3$, $x_3=6$, and so on. This gives us 6 different permutations of (x_2, x_3) when $x_1=4$. If we iterate this process for $x_1=5,6,7$, etc, the number of permutations we get for (x_2, x_3) decreases by increments of 1. This thus gives us $8+7+6+5+4+3+2+1=36$.

(b) $6+6+6+6+6=30$
Holding each value of x_1 constant, there exists 6 different solutions for x_3 because x_2 can take 6 different values. Because x_1 can take 5 different values, we get $(5)(6)=30$ different solutions.

(c) $\binom{13+4-1}{4-1}=560$
This can be modeled in terms of a stars and bars problem. If we introduce a 4th variable n in which $x_1+x_2+x_3=n$, then we can reform the inequality to $x_1+x_2+x_3=13-n$. If we take into account the leftover resulting from 13-n, then we get $\binom{13+4-1}{4-1}=560$ ways.

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a) $$\left(\!\!\binom{2n}{n}\!\!\right) = \binom{2n+n-1}{n} = \frac{(2n+n-1)!}{n!\,((2n+n-1)-(n))!}$$

$$= \frac{(2n+n-1)!}{(n)!\,(2n-1)!}$$

$$= \frac{(2n)}{(2n)} \cdot \frac{(2n+n-1)!}{n!\,(2n-1)!}$$

$$= \frac{2\,(2n+n-1)!}{(2n)!\,(n-1)!}$$

$$= 2 \cdot \frac{(n+2n-1)!}{(2n)!\,(n-1)!}$$

$$= 2 \cdot \frac{(n+2n-1)!}{(2n)!\,((n+2n-1)-(2n))!}$$

$$= 2\binom{n+2n-1}{2n}$$

$$= 2\left(\!\!\binom{n}{2n}\!\!\right)$$

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b) $\left(\!\binom{n}{k}\!\right) = \binom{n+k-1}{k}$

$= \dfrac{(n+k-1)!}{(k)!\,((n+k-1)-(k))!}$

$= \dfrac{((k+1)+(n-1)-1)!}{k!\,(n-1)!}$

$= \dfrac{((k+1)+(n-1)-1)!}{(n-1)!\,((k+1)+(n-1)-1-(n-1))!}$

$= \binom{(k+1)+(n-1)-1}{n-1} = \left(\!\binom{k+1}{n-1}\!\right)$

Problem 4

(a) $4^4 - \binom{4}{1}3^4 + \binom{4}{2}2^4 - \binom{4}{3}1^4 = 24.$
An onto function from a set of 4 elements to a set of 4 elements must have $4^4 - \binom{4}{1}3^4 + \binom{4}{2}2^4 - \binom{4}{3}1^4 = 256 - 4(81) + 6(16) - 4 = 24$ different functions.

(b) $3^4 - \binom{3}{1}2^4 + \binom{3}{2}1^4 = 36$
An onto function from a set of 4 elements to a set of 3 elements must have $3^4 - \binom{3}{1}2^4 + \binom{3}{2}1^4 = 36$ different functions.

(c) $3^5 - \binom{3}{1}2^5 + \binom{3}{2}1^5 = 150$
This is the equivalent of asking how many onto functions are there from a set of 5 elements to a set of 3 elements. An onto function from a set of 5 elements to a set of 3 elements must have $3^5 - \binom{3}{1}2^5 + \binom{3}{2}1^5 = 150$ different functions.

Problem 5

(a) $(2)(2) = 4$.
If we are looking at derangements of $(1, 2, 3, 4, 5, 6)$ which start with some derangement of (1,2,3), then we can have 2 different derangements - (3,1,2,x,x,x) and (2,3,1,x,x,x). where the x's are the look at deragements of (x,x,x,4,5,6) - (5,6,4),(6,4,5). There are (2)(2)=4 different arrangements of these 2 derangements.

(b) $(3!)(3!) = 36$
If we are looking at derangements of (1,2,3,4,5,6) which end with some derangement of (1,2,3), then we can have 3! arrangements. Then we have to look at 3! arragements of (4,5,6). There are (3!)(3!)=36 different derangements.

(c) $6! - \binom{3}{1}5! + \binom{3}{2}4! - \binom{3}{3}3! = 426$
Using inclusion exclusion, we know there are 6! different ways to rearrange (1,2,3,4,5,6). From tose 6! ways we have to subtract the ones where even numbers are in their initial positions. From this, we get $6! - \binom{3}{1}5! + \binom{3}{2}4! - \binom{3}{3}3! = 426$.