Unit #3 Rational Functions

↳ when the denominator has a variable

2 types: Reciprocal functions & Rational functions

↓ denominator only has a variable eg. $f(x) = \frac{1}{x^2-4}$

↓ numerator & denominator both have variables eg. $f(x) = \frac{x+2}{x^2-1}$

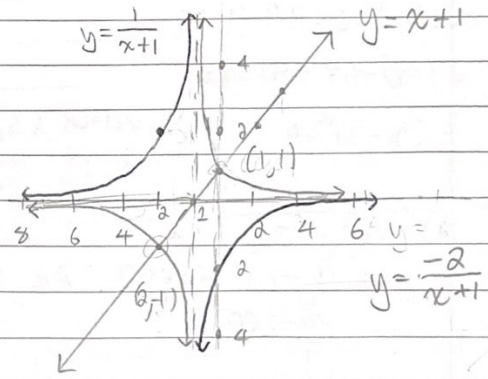
Reciprocal functions

strategy: ① graph the polynomial from the denominator
② x-intercepts become vertical asymptotes (VA)
③ horizontal asymptote (HA) is $y=0$
④ plot the invariand points (where $y = \pm 1$)
⑤ if quadratic, plot the reciprocal of the y-value of the vertex
⑥ use end behaviours near the asymptotes to graph. careful that it passes VLT

Examples:

① Graph $f(x) = \frac{-2}{x+1}$ and

state a) Domain
b) range
c) end behaviours



a) $\{x \in \mathbb{R} \mid x \neq -1\}$

b) $\{y \in \mathbb{R} \mid y \neq 0\}$

c) As $x \to \infty$ $y \to 0^-$ (zero from below)

As $x \to -\infty$ $y \to 0^+$ (zero from above)

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Review the exam instructions an

structions : You are al ulti-text publication days, and lscap provided, write an essay eria for judging artistic work, y

② Graph $g(x) = \frac{1}{x^2-6x+5}$ and state a) Domain
b) Range
c) Behaviours near VA

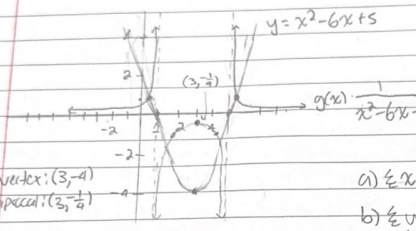
denominator is x^2-6x+5. to graph I will put it into factored form: $(x-5)(x-1)$

x-int: 5, 1

$x_v = \frac{5+1}{2} = 3$

$y_v = 3^2-6(3)+5 = -4$

∴ vertex is (3, -4)



vertex: (3, -4)
inflection: $(3, -\frac{1}{4})$

a) $\{x \in \mathbb{R} \mid x \neq 1, x \neq 5\}$

b) $\{y \in \mathbb{R} \mid y > 0, y \leq -\frac{1}{4}\}$ ← on test

Note: could have put x^2-6x+5 in vertex form by completing the square

$1(x^2-6x)+5 \quad (\frac{-6}{2})^2 = 9$

$= 1(x^2-6x+9-9)+5$

$= 1(x^2-6x+9)-9+5$

$= (x-3)^2-4$ ∴ vertex (3, -4)

x-int: let y=0

$(x-3)^2-4=0$

$(x-3)^2=4$

$x-3 = \pm 2$

$x = 3 \pm 2$

$x_1 = 5 \quad x_2 = 1$

c) VA: $x=1$

As $x \to 1^-$ from left, $y \to \infty$

As $x \to 1^+$ from right, $y \to -\infty$

VA: $x=5$

As $x \to 5^-$, $y \to -\infty$

As $x \to 5^+$, $y \to \infty$

Aside end behaviours

As $x \to \pm\infty$, $y \to 0^+$

Graphing Rational F

strategy:
① Factor fully
② Reduce like factc (open circles on
③ Identify characte

a) Pinhole → fro
b) x-intercept →
c) y-intercept →
d) VA → your
e) HA → com

Degree

f) Crosses HA Set the
g) Interval c

ex: $h(x) = \frac{3x-5}{3x+}$

HA: $y=1$

Cross HA?

Interval
3x-5
3x-1
h(x)

ominator is x^2-6x+5.
graph I will put it into factored
$\therefore (x-5)(x-1)$
$x \neq 5, 1$

$\frac{5+1}{2}$ $y_v = 3^2-6(3)+5$
$= -4$
$\therefore$ vertex is $(3,-4)$

$x \neq 1, x \neq 5$
$y>0, y \neq -\frac{1}{4}, 3$ ← on test

$)^2-4=0$
$(x-3)^2=4$
$x-3=\pm 2$
$x=3\pm 2$
$x_1=5 \quad x_2=1$

end behaviors
$x \pm \infty$
$y \to 0^+$

Graphing Rational Functions

strategy: ① Factor fully
② Reduce like factors ... they create HOLES (open circles on graph)
③ Identify characteristics:

a) Pinhole → from like factors that were reduced
b) x-intercept → your numerator zeroes
c) y-intercept → Let $x=0$ and solve
d) VA → your denominator zeroes
e) HA → compare degrees of your numerator and denominator

Degree numerator	Degree Denominator	
<		HA at $y=0$
=		HA at $y = \frac{LC}{LC}$
>		no HA (oblique asym)

f) crosses HA?
Set the function = HA and solve
g) interval chart

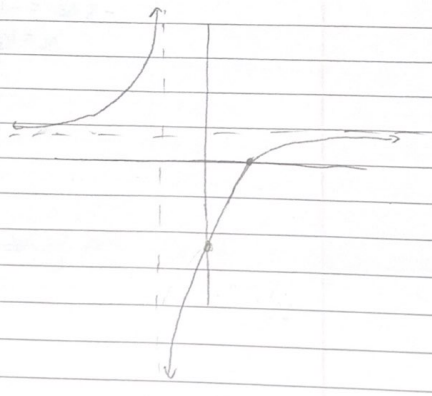
ex: $h(x) = \frac{3x-5}{3x+1}$

HA: $y=1$

cross HA? $\frac{3x-5}{3x+1} = 1$

$3x-5 = 3x+1$
$0x = 6$
$x = \frac{6}{0}$ not possible
$\therefore$ does not cross HA

interval	$(-\infty, -\frac{1}{3})$	$(-\frac{1}{3}, \frac{5}{3})$	$(\frac{5}{3}, \infty)$
$3x-5$	−	−	+
$3x-1$	−	+	+
$h(x)$	+	−	+



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Read the essay "How Do You Kno
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udging artistic work.

Example Graph $f(x) = \frac{x^3-5x^2-x+5}{x^3-2x^2-5x+6}$

$= \frac{(x-5)(x-1)(x+1)}{(x-1)(x-3)(x+2)}$

$= \frac{(x-5)(x+1)}{(x-3)(x+2)}$

x^3-5x^2-x+5
$= x^2(x-5)-1(x-5)$
$= (x-5)(x^2-1)$
$= (x-5)(x-1)(x+1)$

x^3-2x^2-5x+6
$\therefore g(1) = 0$
$\therefore (x-1)$ is a factor

1	1	-2	-5	6
	↓	1	-1	-6
	1	-1	-6	0

$\therefore (x-1)(x^2-x-6)$
$= (x-1)(x-3)(x+2)$

- Pinhole when $x=1$

$f(1) = \frac{(1-5)(1+1)}{(1-5)(1+2)} = \frac{-8}{-6} = \frac{4}{3}$

$\therefore$ pinhole at $(1, \frac{4}{3})$

- x-int: $5, -1$
- y-int: $\frac{5}{6}$
- VA: $x=3, x=-2$
- HA: $y=1$
- cross HA? $\frac{(x-5)(x+1)}{(x-3)(x+2)} = 1$

$x^2-4x-5 = 1(x^2-x-6)$

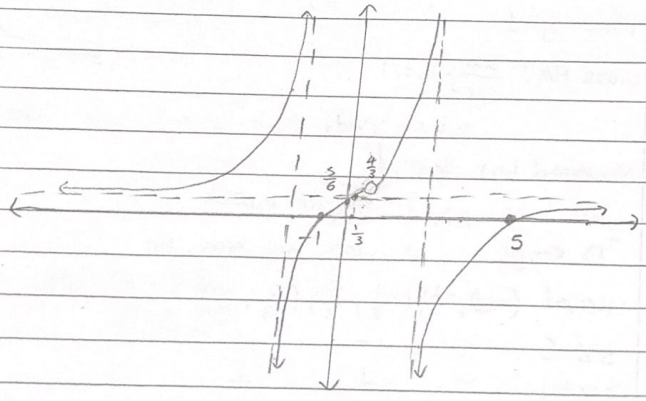
$-3x = -1$

$x = 1/3$

-2 -1 3 5

Interval:	$(-\infty,-2)$	$(-2,-1)$	$(-1,3)$	$(3,5)$	$(5,\infty)$
$x-5$	−	−	−	−	+
$x+1$	−	−	+	+	+
$x-3$	−	−	−	+	+
$x+2$	−	+	+	+	+
$f(x)$	+	−	+	−	+

+ means above x-axis
− means below x-axis



Unit 3

RATIONAL FUNCTIONS

Strategy: ① Factor fully
② Reduce like factors... they create HOLES, open circles on graph (else it would be $\frac{0}{0}$, undefined)
③ Identify important characteristics to graph

a) Pinhole → from reduced like factors
b) x-intercept → your numerator zeroes
c) y-int → let $x=0$ and solve
d) VA → your denominator zeroes
e) HA → compare degrees of numerator and denominator (degree denominator is 1 less than numerator)

Degree numerator		Degree denominator	
	<		HA at 0
	=		HA at $y = \frac{\text{LC num}}{\text{LC den}}$
	>		no HA (or oblique)

f) check if function crosses HA, set function = to HA and solve
g) Interval chart, use intervals to graph

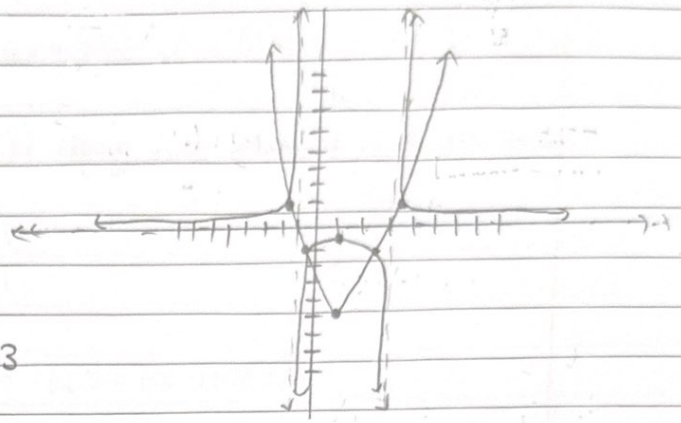
$-2 \quad x^2$
$x^2 \quad - \quad + \quad +$

Steps
- Pinhole
- HA
  - crosses?
- VA
- x-int
- y-int
- interval chart
- end behaviours, just sub in large value and see if it approaches HA from above or below

Putting it together ** need a invariant points

① graph $y = \frac{1}{x^2-2x-3}$

$y = \frac{1}{(x-3)(x+1)}$



VA: 3, -1

HA: $y=0$

$x_v = \frac{3+(-1)}{2}$ $y_v = (1)^2-2(1)-3$

$= 1$ $= -5$

vertex: $(1,-5)$

reciprocal $(1, -\frac{1}{5})$

a) Domain $\{x \in \mathbb{R} \mid x \neq 3, x \neq -1\}$

b) Range $\{y \in \mathbb{R} \mid y \neq 0, y \leq -\frac{1}{5}\}$

c) Behaviours near VA

VA: $x = -1$

As $x \to -1^-$ $y \to \infty$

As $x \to -1^+$ $y \to -\infty$

VA: $x = 3$

As $x \to 3^-$ $y \to -\infty$

As $x \to 3^+$ $y \to \infty$

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$P(x) = \frac{f(x)}{g(x)}$ where $f(x) = -3(x^3 - 2x^2 - 3x)$ and
$g(x) = x^3 - 21x + 20$

List the key features and graph $P(x)$

Solving RATIONAL Expressions

Strategy: ① Factor fully
② Multiply each term by the LCD
③ solve

Examples: solve - exact values

① $\frac{x+4}{x+3} + \frac{x^2-3}{x^2+2x-3} = \frac{x-3}{x-1}$

$\frac{x+4}{x+3} + \frac{x^2-3}{(x-1)(x+3)} = \frac{x-3}{x-1}$ LCD $= (x-1)(x+3)$

$(x-1)(x+3)\left(\frac{x+4}{x+3}\right) + (x-1)(x+3)\left(\frac{x^2-3}{(x-1)(x+3)}\right) = (x-1)(x+3)\left(\frac{x-3}{x-1}\right)$

$(x-1)(x+4) + (x^2-3) = (x+3)(x-3)$

$x^2+4x-x-4+x^2-3 = x^2+3x-3x-9$

$x^2+3x+2 = 0$

$(x+2)(x+1) = 0$

$\therefore x = -2, -1$ ← double check restrictions $x \neq -3, 1$

* test if $x = R$ ↓ don't include

② $x - \frac{x}{x+1} = 2$ LCD $= x+1$

$(x+1)(x) - (x+1)\left(\frac{x}{x+1}\right) = (x+1)(2)$

$x^2+x-x = 2x+2$

$x^2-2x-2 = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$

$= \frac{2 \pm \sqrt{12}}{2}$

$= \frac{2 \pm 2\sqrt{3}}{2}$

$= 1 \pm \sqrt{3}$ restriction: $x \neq 1$

$(x+5)$ $-$ $-$ $-$ $+$

$(x+1)$ $-$ $-$ $+$ $+$

$(2x-1)$ $-$ $-$ $-$ $+$

$(x-3)$ $-$ $-$ $-$ $+$

$f(x)$ $+ \; 2 \; - \; 2 \; + \; VA \; - \; VA \; +$

$\therefore x \in (-5,-1) \cup (\frac{1}{2}, 3)$

interval $(-\infty,-1)$ $(-1,3)$ $(3,7)$ $(7,\infty)$

$x-7$ $-$ $-$ $-$ $+$

$x+1$ $-$ $+$ $+$ $+$

$x-3$ $-$ $-$ $+$ $+$

$f(x)$ $- \; VA \; + VA \; - \; 2 \; +$

$\therefore x \in (-1,3) \cup [7,\infty)$

$-\geq 0$

≥ 0

≥ 0

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you try!

$$\frac{1}{x-4}+\frac{x}{x-2}=\frac{2}{x^2-6x+8}$$

$$\frac{1}{x-4}+\frac{x}{x-2}=\frac{2}{(x-2)(x-4)} \quad \text{LCD: } (x-2)(x-4)$$

$$(x-2)(x-4)\left(\frac{1}{x-4}\right)+(x-2)(x-4)\left(\frac{x}{x-2}\right)=(x-2)(x-4)\frac{2}{(x-2)(x-4)}$$

$$x-2+x^2-4x=2$$
$$x^2-3x-2=2$$
$$x^2-3x-4=0$$
$$(x-4)(x+1)=0 \leftarrow \text{restriction: } 4, 2$$

∴ since $x \neq 4$

$x=1$

$x=-1, x=4$ ← ∴ $x \neq 4, x \neq 2$

∴ $x=-1$

Solving Rationa

Strategy: ① set eq
② combin
③ facto
④ interv

Examples: solve

① $\frac{x^2+6x+5}{2x^2-7x+3}$

$\frac{(x+5)(x+1)}{(2x-1)(x-3)}$

$-5 \quad -1 \quad 1/2$

② $\frac{x+3}{x+1} \geq$

$\frac{x+3}{x+1} - \frac{x}{x}$

$\frac{(x+3)(x-3)}{(x}$

x^2-9

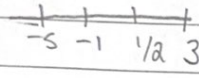
Solving Rational Inequalities

Strategy: ① set inequality to zero
② combine to write as one term
③ factor fully
④ interval chart ★ be careful with ≤ and ≥ and the VA!

Examples: solve algebraically

① $\frac{x^2+6x+5}{2x^2-7x+3} < 0$

$\frac{(x+5)(x+1)}{(2x-1)(x-3)} < 0$



Interval	$(-\infty,-5)$	$(-5,-1)$	$(-1,\frac{1}{2})$	$(\frac{1}{2},3)$	$(3,\infty)$
$(x+5)$	−	+	+	+	+
$(x+1)$	−	−	+	+	+
$(2x-1)$	−	−	−	+	+
$(x-3)$	−	−	−	−	+
$f(x)$	+	0 −	0 +	VA −	VA +

$\therefore x \in (-5,-1) \cup (\frac{1}{2},3)$

② $\frac{x+3}{x+1} \geq \frac{x-2}{x-3}$

$\frac{x+3}{x+1} - \frac{x-2}{x-3} \geq 0$

$\frac{(x+3)(x-3)-(x-2)(x+1)}{(x+1)(x-3)} \geq 0$

$\frac{x^2-9-(x^2-x-2)}{(x+1)(x-3)} \geq 0$

$\frac{x-7}{(x+1)(x-3)} \geq 0$

Interval	$(-\infty,-1)$	$(-1,3)$	$(3,7)$	$(7,\infty)$
$x-7$	−	−	−	+
$x+1$	−	+	+	+
$x-3$	−	−	+	+
$f(x)$	− VA	+ VA	− 0	+

$\therefore x \in (-1,3) \cup [7,\infty)$