Exam Ht

ENG 4U Exam Preparation

Note: The sharing of ideas duri
act as a sounding board only.

1. With your table group, use chart p
these to the class.
   a) Select one form of artist
   standards you would us
   b) When you read your pe
   c) What standards or expe
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2. Read the essay "How Do You Kno
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Unit #4 Trigonometry

~~Radian Measure~~ ~~Another unit of measure~~

$360° = 2\pi$ radians

$180° = \pi$ radians ← we use this to help us convert

converting Radians to degrees

$\times \frac{180}{\pi}$

converting from degrees to radians

$\times \frac{\pi}{180}$

Examples

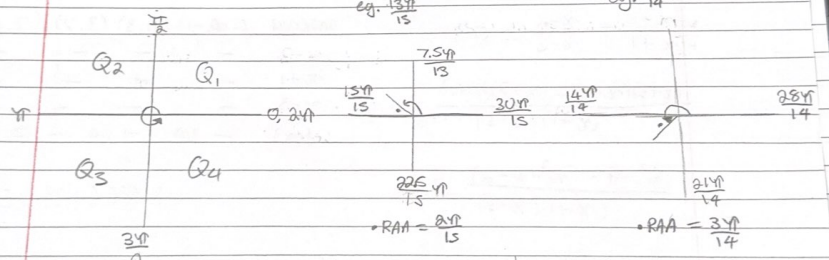
① convert $60°$ to radians

$60 \times \frac{\pi}{180} = \frac{\pi}{3}$

② convert $\frac{\pi}{5}$ to degrees

$\frac{\pi}{5} \times \frac{180}{\pi} = 36°$

Cartesian Plane



Think π

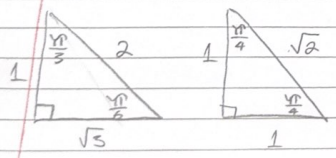
eg. $\frac{13\pi}{15}$

• RAA = $\frac{2\pi}{15}$

eg. $\frac{17}{14}\pi$

• RAA = $\frac{3\pi}{14}$

Special Triangles



	Sine	All
Tangent	Cosine	

Quadrants where the function and its reciprocal are positive

Trigonometric Ratio

Primary

$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$

Examples:

a) Draw in standard

b) $\frac{13\pi}{4}$

$\frac{12\pi}{4}, \frac{4\pi}{4}$

• RAA = $\frac{\pi}{4}$

c) cot $\frac{7\pi}{3}$

② Evaluate - exa

$\frac{3\pi}{3}$ $\frac{6\pi}{3}$

RAA = $\frac{\pi}{3}$

d) sin $\frac{5\pi}{4}$

$\frac{4\pi}{4}$

RAA =

~~AH of reading~~

We use this to help us convert

converting from degrees to radians

$\times \frac{\pi}{180}$

invert $\frac{\pi}{5}$ to degrees

$\frac{\pi}{5} \times \frac{180}{\pi} = 36°$

π

eg. $\frac{17\pi}{14}$

$\frac{5\pi}{3}$

$\frac{30\pi}{15}$ $\frac{14\pi}{14}$ $\frac{28\pi}{14}$

$\frac{31\pi}{14}$

• RAA = $\frac{3\pi}{14}$

All — Quadrants where the function and its reciprocal are positive

Cosine

Trigonometrical Ratios

Primary

$\sin\theta = \frac{opp}{hyp}$ SOH

$\cos\theta = \frac{adj}{hyp}$ CAH

$\tan\theta = \frac{opp}{adj}$ TOA

reciprocal / secondary

$\csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{adj}{opp}$

~~Examples:~~

① Draw in standard position and state the related acute angle

a) $\frac{13\pi}{4}$

$\frac{12\pi}{4}, \frac{4\pi}{4}$ $\frac{8\pi}{4}$

• RAA = $\frac{\pi}{4}$

b) $-\frac{11\pi}{6}$ clockwise

$\frac{6\pi}{6}$ $\frac{12\pi}{6}$

RAA = $\frac{\pi}{6}$

c) $\cot\frac{7\pi}{3}$

② Evaluate - exact values

$= +\cot\frac{\pi}{3}$

$= \frac{1}{\sqrt{3}}$

$\cot = \frac{adj}{opp}$

$\frac{3\pi}{3}$ $\frac{6\pi}{3}$

RAA = $\frac{\pi}{3}$

d) $\sec\left(-\frac{3\pi}{4}\right) = -\sec\frac{\pi}{4}$

$= \frac{-\sqrt{2}}{1}$

$= -\sqrt{2}$ $\sec = \frac{hyp}{adj}$

$\frac{4\pi}{4}$

RAA = $\frac{\pi}{4}$

a) $\sin\frac{5\pi}{4} = -\sin\frac{\pi}{4}$

$= \frac{-1}{\sqrt{2}}$

$\frac{4\pi}{4}$

RAA = $\frac{\pi}{4}$

b) $\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

$\frac{6\pi}{6}$

RAA = $\frac{\pi}{4}$

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Solving $0 \le \theta \le 2\pi$

$\pi - RAA$	RAA
$\pi + RAA$	$2\pi - RAA$

① find RAA
② use sign of ratio to determine quadrants
③ calculate

Example: find θ; $0 \le \theta \le 2\pi$

① $\cos\theta = \frac{-1}{\sqrt{2}}$

$RAA = \cos^{-1}(\frac{1}{\sqrt{2}})$

$= \frac{\pi}{4}$

S|A T|C

$\therefore \theta_1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\theta_2 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

② $\cot\theta = \frac{\sqrt{3}}{1}$

$\frac{1}{\tan} = \frac{\sqrt{3}}{1}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$RAA = \tan^{-1}(\frac{1}{\sqrt{3}})$

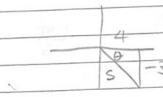
$= \frac{\pi}{6}$

S|A T|C $\therefore \theta_1 = \frac{\pi}{6}$

$\theta_2 = \frac{7\pi}{6}$

Suppose $\sin = \frac{-3}{5}$ and $\cot\theta = \frac{-4}{5}$. What is $\sec\theta$?

S|A T|C Both sine and cotangent are negative in Q4



$\sec\theta = \frac{5}{4}$

ANGULAR VELOCITY

↳ the rate at which the central angle changes with respect to time

eg. A disk rotates 7 times every 3 minutes.
what is the angular velocity in radians per second?

7 rotations = $7 \times 2\pi$ = 14π radians

3 minutes = 3×60 seconds = 180 seconds

∴ angular velocity is $\frac{14\pi}{180} = \frac{7\pi}{90}$ rad/sec

Arc LENGTH

$\theta = \frac{\text{arc length}}{\text{radius}}$

* θ must be in radians

eg. a circle has radius 4.7cm and central angle 1.7 radians
what is the arc length

$\theta = \frac{a}{r}$ $\quad a = (1.7)(4.7)$

$1.7 = \frac{a}{4.7}$ $\quad a = 7.99\,cm$

EQUIVALEN

Based on the CA

$\sin\% = +/- c$
$\cos\% = +/- s$
$\tan\% = +/-$

↑ PICK +/- bo on S|A T|C

Examples
① Given $\sin\frac{2\pi}{5}$ =
find $\cos\frac{\pi}{10}$ $= +s$
$= s$
$= 0.$

$0 \le \theta \le 2\pi$

② $\cot\theta = \sqrt{3}$

$\frac{1}{\tan} = \frac{\sqrt{3}}{1}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$RAA = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$

$\frac{1}{2}A \therefore \theta_1 = \frac{\pi}{6}$

$\theta_2 = \frac{7\pi}{6}$

EQUIVALENT EXPRESIONS

Based on the CAST Rule: S | A / T | C

$\sin x = +/- \cos(\quad)$

$\cos x = +/- \sin(\quad)$

$\tan x = +/- \cot(\quad)$

↑ Pick +/- based on S | A / T | C

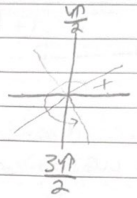
↑ Q2: $\frac{\pi}{2} - x$

Q2: $\frac{\pi}{2} + x$

Q3: $\frac{3\pi}{2} - x$

Q4: $\frac{3\pi}{2} + x$

↑ y-axis



Examples

① Given $\sin\frac{2\pi}{5} = 0.9511$

find $\cos\frac{\pi}{10}$ ← Q_1

$\cos\frac{\pi}{10} = +\sin\left(\frac{\pi}{2} - \frac{\pi}{10}\right)$

$= +\sin\left(\frac{5\pi - \pi}{10}\right)$

$= +\sin\left(\frac{2\pi}{5}\right)$

$= 0.9511$

② Given $\sin\frac{\pi}{5} = 0.5878$

find $\cos\frac{7\pi}{10}$ ← Q_2

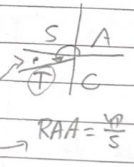
$\cos\frac{7\pi}{10} = +\sin\left(\frac{\pi}{2} + \frac{7\pi}{10}\right)$

$= +\sin\left(\frac{12\pi}{10}\right)$

$= +\sin\left(\frac{6\pi}{5}\right)$

$= -\sin\left(\frac{\pi}{5}\right)$

$= -0.5878$



$RAA = \frac{\pi}{5}$

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Review the exam instructions and…

Instructions : You are allowed … multi-text publication days, and a… foolscap provided, write an essa… criteria for judging artistic work, …

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

→ get special angle triangle

Compound Angles

$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$

$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$

$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)}$

Beware: $\sin(x+y) \neq \sin x + \sin y$

Examples:

Expand a) $\cos(4m - 5n)$
$= \cos(4m)\cos(5n) + \sin(4m)\sin(5n)$

b) $\sin(3x - 4y)$
$= \sin(3x)\cos(4y) - \cos(3x)\sin(4y)$

② write as a single trigonometric function

a) $\sin(4t)\cos(2t) - \cos(4t)\sin(2t)$
$= \sin(4t - 2t)$
$= \sin(2t)$

b) $\frac{\tan 5x + \tan 2x}{1 - \tan 5x \tan 2x}$
$= \tan(5x + 2x)$
$= \tan 7x$

③ Evaluate - exact values

a) $\sin\frac{5\pi}{18}\cos\frac{\pi}{36} - \cos\frac{5\pi}{18}\sin\frac{\pi}{18}$
$= \sin\left(\frac{5\pi}{18} - \frac{\pi}{36}\right)$
$= \sin\left(\frac{10\pi}{36} - \frac{\pi}{36}\right)$
$= \sin\left(\frac{1\pi}{4}\right)$
$= \frac{1}{\sqrt{2}}$

b) $\cos\frac{\pi}{8}\cos\frac{3\pi}{8} - \sin\frac{\pi}{8}\sin\frac{3\pi}{8}$
$= \cos\left(\frac{\pi}{8} + \frac{3\pi}{8}\right)$
$= \cos\left(\frac{4\pi}{8}\right)$
$= 0$

c) $\tan\frac{7\pi}{12}$
$= \tan\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$
$= \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$
$= \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$
$= \frac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)}$
$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$

④ If $\tan A = \frac{3}{2}$, $0 \leq A \leq$
Find the exact value

$\sin(A+B) = \sin A \cos B$ …
↑ move

$= \left(\frac{3}{\sqrt{13}}\right)\left(\frac{-5}{6}\right) + \left(\frac{2}{\sqrt{13}}\right)(-$
$= \frac{-9-5}{5\sqrt{13}}$
$= \frac{-17}{5\sqrt{13}}$

Double Angles

$\sin 2\theta = 2\sin\theta\cos$

Evaluate - exact

① $2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$
$= \sin 2\left(\frac{\pi}{8}\right)$
$= \sin\left(\frac{\pi}{4}\right)$
$= \frac{1}{\sqrt{2}}$

∴ S…

I used cast rule its negative cause its going down

④ if $\tan A = \frac{3}{2}$, $0 \le A \le \frac{\pi}{2}$ and $\cos B = \frac{-3}{5}$, $\pi \le B \le \frac{3\pi}{2}$

Find the ~~exact~~ value of $\sin(A+B)$

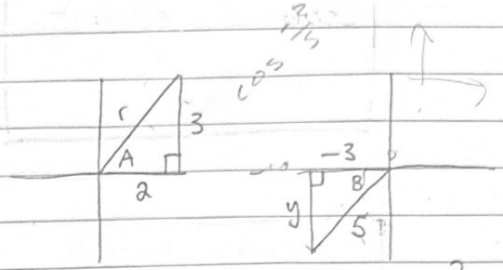
$\sin(A+B) = \sin A \cos B + \cos A \sin B$

↑ have this

$= \left(\frac{3}{\sqrt{13}}\right)\left(\frac{-3}{5}\right) + \left(\frac{2}{\sqrt{13}}\right)\left(\frac{-4}{5}\right)$

$= \frac{-9-8}{5\sqrt{13}}$

$= \frac{-17}{5\sqrt{13}}$



$2^2 + 3^2 = r^2$

$13 = r^2$

$r = \pm\sqrt{13}$

$= \sqrt{13}$

$(-3)^2 + y^2 = 5^2$

$y^2 = 16$

$y = 4$

we use $y = -4$

$y \pm \sqrt{}$

Double Angles

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 1 - 2\sin^2\theta$
$= 2\cos^2\theta - 1$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Evaluate - ~~exact~~ values

① $2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$= \sin 2\left(\frac{\pi}{8}\right)$

$= \sin\left(\frac{\pi}{4}\right)$

$= \frac{1}{\sqrt{2}}$

② $\frac{2\tan\frac{\pi}{6}}{1-\tan^2\left(\frac{\pi}{6}\right)}$

$= \tan 2\left(\frac{\pi}{6}\right)$

$= \tan\frac{\pi}{3}$

$= \sqrt{3}$

③ $\sin\frac{\pi}{8}$

$\frac{\pi}{8}$ is not a special triangle angle from our special Δ's but $2\left(\frac{\pi}{8}\right) = \frac{\pi}{4}$ is!

so we want $\sin\frac{\pi}{8}$ in terms of $\frac{\pi}{4}$

use $\cos 2\theta = 1 - 2\sin^2\theta$ where

$1 - 2\sin^2\left(\frac{\pi}{8}\right) = \cos 2\left(\frac{\pi}{8}\right)$

$1 - 2\sin^2\left(\frac{\pi}{8}\right) = \cos\frac{\pi}{4}$

$1 - 2\sin^2\left(\frac{\pi}{8}\right) = \frac{1}{\sqrt{2}}$

$-2\sin^2\left(\frac{\pi}{8}\right) = \frac{1}{\sqrt{2}} - 1$

$\sin^2\left(\frac{\pi}{8}\right) = \frac{1-\sqrt{2}}{\sqrt{2}} \times \frac{1}{-2}$

$\sin\left(\frac{\pi}{8}\right) = \pm\sqrt{\frac{1-\sqrt{2}}{-2\sqrt{2}}}$

$\because \sin\frac{\pi}{8}$ is in Q_1 and sin is positive in Q_1

$\therefore \sin\frac{\pi}{8} = \sqrt{\frac{1-\sqrt{2}}{-2\sqrt{2}}}$

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Trigonometric Identities

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Compound Angles

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

Double Angles

$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

Form LS = original

=

=

RS = original

=

NOTE
$\cos^2\theta$
$= (\cos\theta)^2$

$\therefore$ LS = RS

$\therefore$ QED

Examples: Prove

① $\csc 2\theta = \frac{1}{2}\sec\theta\csc\theta$

LS $= \csc 2\theta$

$= \frac{1}{\sin 2\theta}$

RS $= \frac{1}{2}\sec\theta\csc\theta$

$= \frac{1}{2}\cdot\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$

$= \frac{1}{2\cos\theta\sin\theta}$

$= \frac{1}{\sin 2\theta}$

$\therefore$ LS = RS

$\therefore$ QED

② $\sec A = \frac{\sin 2A}{\sin A} - \frac{\cos 2A}{\cos A}$

LS $= \sec A$

$= \frac{1}{\cos A}$

RS $= \frac{\sin 2A}{\sin A} - \frac{\cos 2A}{\cos A}$

$= \frac{2\sin A\cos A}{\sin A} - \frac{2\cos^2 A - 1}{\cos A}$

$= \cos A - \frac{2\cos^2 A - 1}{\cos A}$

$= \frac{2\cos^2 A - 2\cos^2 A + 1}{\cos A}$

$= \frac{1}{\cos A}$

$\therefore$ LS = RS

$\therefore$ QED

③ $\frac{1-\sin 2x}{\cos 2x} = \frac{\cos 2x}{1+\sin 2x}$

LS $= \frac{1-\sin 2x}{\cos 2x}$

$= \frac{1-\sin 2x}{\cos 2x} \times \frac{1+\sin 2x}{1+\sin 2x}$

$= \frac{1+\sin 2x - \sin 2x - \sin^2 2x}{\cos 2x(1+\sin 2x)}$

$= \frac{1-\sin^2 2x}{\cos 2x(1+\sin 2x)}$

$= \frac{\cos^2(2x)}{\cos 2x(1+\sin 2x)}$

$= \frac{\cos 2x}{1+\sin 2x}$

RS $= \frac{\cos 2x}{1+\sin 2x}$

$\therefore$ LS = RS

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④ $\cos^4\alpha - \sin^4\alpha = \cos 2\alpha$

LS $= \cos^4\alpha - \sin^4\alpha$ RS $= \cos 2\alpha$

$= (\cos^2\alpha - \sin^2\alpha)(\cos^2\alpha + \sin^2\alpha)$

$= 1(\cos^2\alpha - \sin^2\alpha)$

$= \cos^2\alpha - \sin^2\alpha$

$= \cos 2\alpha$

$\therefore$ LS = RS

$\therefore$ QED