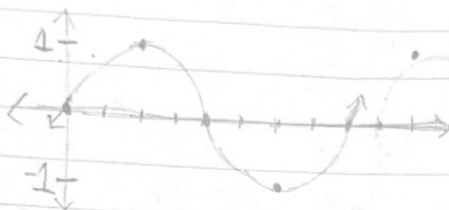


UNIT 5: Trigonometric Functions

Primary & Reciprocal Function graphs

Sine

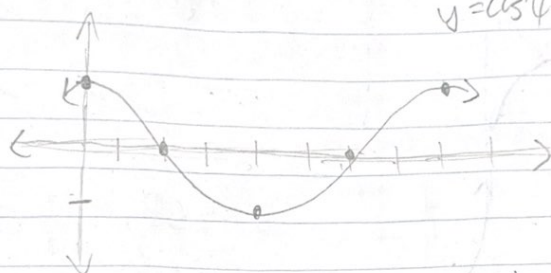
x	$y = \sin x$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



$$D: \{x \in \mathbb{R}\}$$

$$R: \{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$$

x	$y = \cos x$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

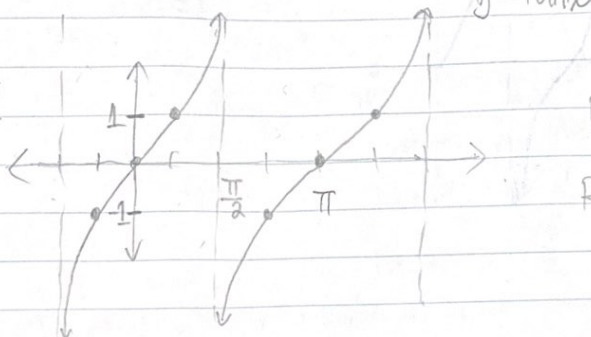


$$y = \cos x$$

$$D: \{x \in \mathbb{R}\}$$

$$R: \{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$$

x	$y = \tan x$
0	0
$\frac{\pi}{2}$	DNE
π	0
$\frac{3\pi}{2}$	DNE
2π	0



$$y = \tan x$$

$$D: \{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + \pi n, n \in \mathbb{Z}\}$$

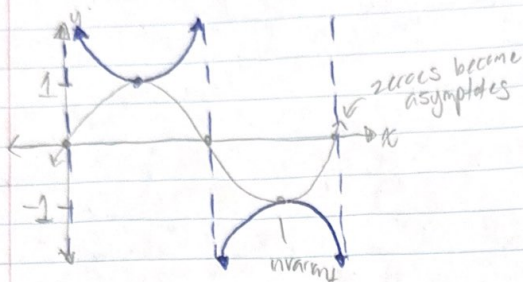
$$R: \{y \in \mathbb{R}\}$$

where n belongs to set of integers

Now Lets Look At Reciprocal Functions

$$y = \sin x$$

$$y = \csc x$$

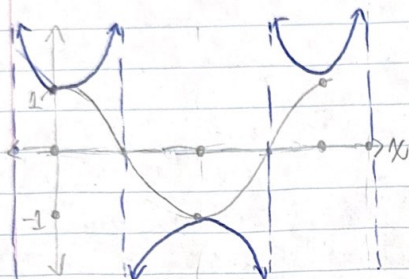


$$D: \{x \in \mathbb{R} \mid x \neq n\pi, n \in \mathbb{Z}\}$$

$$R: \{y \in \mathbb{R} \mid y \geq 1, y \leq -1\}$$

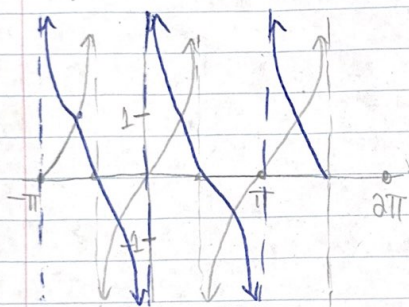
$$y = \cos x$$

$$y = \sec x$$



$$D: \{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}\}$$

$$R: \{y \in \mathbb{R} \mid y \geq 1, y \leq -1\}$$



$$D: \{x \in \mathbb{R} \mid x \neq n\pi, n \in \mathbb{Z}\}$$

$$R: \{y \in \mathbb{R}\}$$

Transfo

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• 7

• 8

• 9

5

$$|x \neq \pi n, n \in \mathbb{Z}$$

$$|z|, y \neq -13$$

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Transformations of Sinusoidal Functions

Sinusoidal Functions: sine and cosine

$$f(x) = a \sin k(x-d) + c$$

Example: Graph $f(x) = -2 \sin(3x - \pi) + 1$
 $= -2 \sin 3(x - \frac{\pi}{3}) + 1$

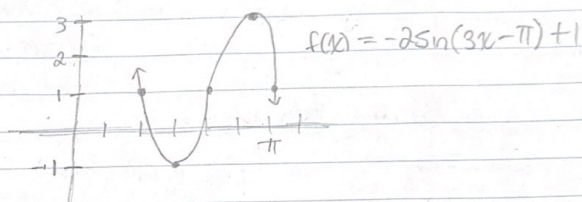
the graphs

- ① write base/parent graph table
- ② $(x, y) \rightarrow (x \div k \pm d, ay \pm c)$
- ③ Plot using smooth curves and arrows

Hint: Use LCD to determine horizontal scale

x	$y = \sin x$	$x: 3 + \frac{\pi}{3}$	$-2y + 1$
0	0	$\frac{\pi}{3} = \frac{2\pi}{6}$	1
$\frac{\pi}{2}$	1	$\frac{\pi}{6} = \frac{3\pi}{6}$	-1
π	0	$\frac{2\pi}{3} = \frac{4\pi}{6}$	1
$\frac{3\pi}{2}$	-1	$\frac{5\pi}{6}$	3
2π	0	$\pi = \frac{6\pi}{6}$	1

LCD is 6 $\rightarrow \pi = \frac{6\pi}{6}$



Key Features

- amplitude = $|a|$
 $= \frac{\text{max} - \text{min}}{2}$
- vertical displacement = c up/down
 $= \frac{\text{max} + \text{min}}{2}$
- phase shift = d left/right
- period = $\frac{2\pi}{k}$

$$f(x) = -2 \sin 3(x - \frac{\pi}{3}) + 1$$

amplitude = 2

vertical displacement = 1 up

phase shift = $\frac{\pi}{3}$ right

period = $\frac{2\pi}{3}$

① A sine function has a maximum value of 11, minimum value of -1, phase shift $\frac{2\pi}{3}$ left and period π

$$\text{amp} = \frac{11 - (-1)}{2} = 6$$

$$\text{vd} = \frac{11 + (-1)}{2} = 5$$

$$\therefore f(x) = 6 \sin 2(x + \frac{2\pi}{3}) + 5$$

$$\frac{2\pi}{k} = \pi$$

$$k = \frac{2\pi}{\pi}$$

$$k = 2$$

Exam Hc

ENG 4U Exam Prepar

Note: The sharing of ideas act as a sounding board on

1. With your table group, use of these to the class.

- Select one form of standards you work
- When you read you
- What standards or
- Where did these st

2. Read the essay "How Do You Once you have read the essay; definitions/ideas on chart paper

- Look up and define connotation; (par. 4 composition; (par. 8 (par. 12) elusive, im
- What is meant
- What is Manner
- What other opi
- To what extent

view the exam instructions

uctions: You are allowed text publication days, and as provided, write an essay for judging artistic work.

Transformations

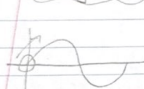
$$f(x) = a \sin b(x - d) + c$$

- a : vertical stretch by factor of " a "
negative: reflection in y-axis
 $0 < a < 1$: vertical compression by " a "
- b : horizontal compression by $\frac{1}{b}$
negative: reflection in y-axis
 $0 < b < 1$: horizontal stretch by $\frac{1}{b}$
- d : horizontal translation " d " units right
- c : vertical translation " c " units down

important $f(x) = -2 \sin 3(x - \frac{\pi}{6}) + 1$

- $y = \sin x$ has been:
 - reflected in the x-axis
 - vertically stretched by a factor of 2
 - horizontally compressed by a factor of $\frac{1}{3}$
 - horizontally translated $\frac{\pi}{6}$ units right
 - vertically translated up 1 unit

Graphing Equations



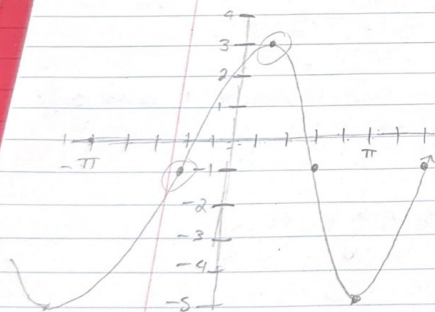
sine graphs start at vertical displacement and goes up



cosine graph starts at maximum value

use these to determine phase shift

EXAMPLES



$$\text{amp} = \frac{\text{max} - \text{min}}{2} = \frac{3 - (-5)}{2} = 4$$

$$\text{v.d} = \frac{\text{max} + \text{min}}{2} = \frac{3 + (-5)}{2} = -1$$

$$\text{Period is } \frac{8\pi}{5}; \frac{8\pi}{5} = \frac{2\pi}{k}$$

$$8\pi k = 10\pi$$

$$k = \frac{10\pi}{8\pi}$$

$$k = \frac{5}{4}$$

$$\therefore f(x) = 4 \sin \frac{5}{4}(x + \frac{\pi}{5}) - 1$$

$$\therefore f(x) = 4 \cos \frac{5}{4}(x - \frac{\pi}{5}) - 1$$

Solving Trigonometric Equations

- Isolate
- Find
- CAS
- Final

Examples: solve

$$1) 2 \cos^2 \theta - 1 =$$

$$2 \cos^2 \theta =$$

$$\cos^2 \theta =$$

$$\cos \theta =$$

$$\cos \theta =$$

where cosine is + and - RAA

$$\frac{\pi}{2}$$

$$\therefore \theta_1 =$$

$$\theta_2 =$$

$$\theta_3 =$$

$$\theta_4 =$$

vertical translation c units up
or translation c units down

vertical translation " d " units

in y -axis
compression by $\frac{1}{k}$
stretch by k

axis
a factor of 2
a factor of $\frac{1}{2}$
units right
1 unit

cosine graphs
starts at maximum
value

$$T = 10\pi$$

$$= \frac{10\pi}{8\pi}$$

$$= \frac{5}{4}$$

Solving Trigonometric Equations

- Strategy: ① Isolate or Factor
② Find RAA (related acute angle)
↳ use the positive value of the ratio
③ CAST Rule
④ Find θ

$$0 \leq \theta \leq 2\pi$$

$$\pi - \text{RAA} \quad \text{RAA}$$

$$\pi + \text{RAA} \quad 2\pi - \text{RAA}$$

Examples: solve $0 \leq \theta \leq 2\pi$

$$① 2\cos^2\theta - 1 = 0$$

$$2\cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \pm\sqrt{\frac{1}{2}}$$

$$\cos\theta = \pm\frac{1}{\sqrt{2}}$$

where cosine
is + and -

$$\text{RAA} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\pi}{4}$$

$$\therefore Q_1 = \frac{\pi}{4}$$

$$Q_2 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$Q_3 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$Q_4 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$② \sec^2\theta + \sec\theta - 2 = 0$$

Quadratic... factor or QF

$$m^2 + m - 2 = 0 \rightarrow (m+2)(m-1) = 0$$

$$(\sec\theta + 2)(\sec\theta - 1) = 0$$

$$\therefore \sec\theta + 2 = 0 \quad \text{or} \quad \sec\theta - 1 = 0$$

$$\sec\theta = -2$$

$$\sec\theta = 1$$

$$\frac{1}{\cos\theta} = -\frac{1}{2}$$

$$\frac{1}{\cos\theta} = \frac{1}{1}$$

$$\cos\theta = -\frac{1}{2}$$

$$\cos\theta = 1$$

$$\text{RAA} = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{2\pi}{3}$$

$$\therefore Q_1 = \pi - \frac{2\pi}{3} = \frac{4\pi}{3}$$

$$Q_2 = \pi + \frac{2\pi}{3} = \frac{5\pi}{3}$$

use graph when
 $= \pm 1, 0$



$$Q_3 = 0$$

$$Q_4 = 2\pi$$